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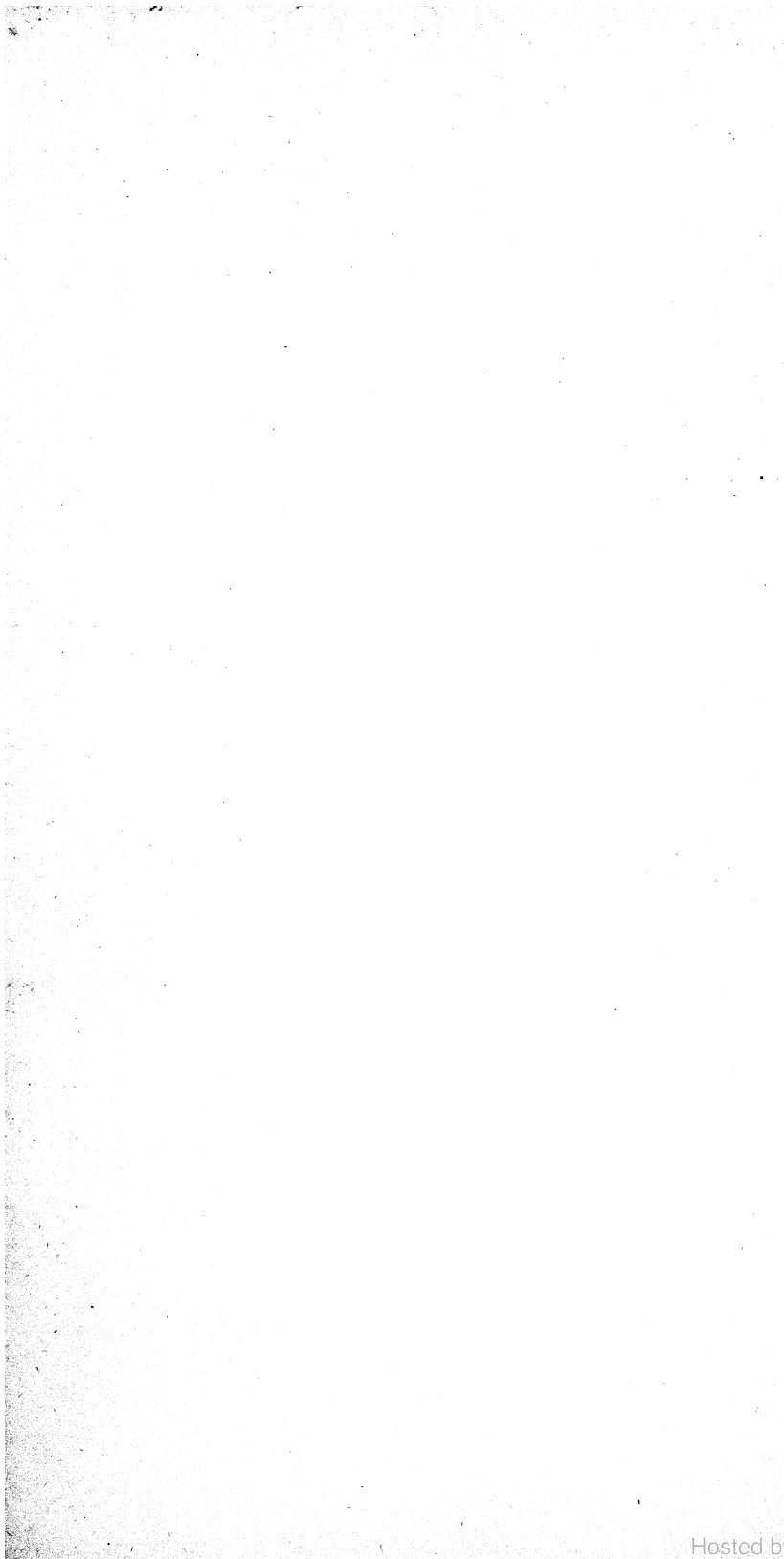
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THE
CIVIL-ENGINEER & SURVEYOR'S
MANUAL:

COMPRISING
Surveying, Engineering, Practical Astronomy,
GEODETICAL JURISPRUDENCE,
ANALYSES OF
MINERALS, SOILS, GRAINS, VEGETABLES,
VALUATION OF
LANDS, BUILDINGS, PERMANENT STRUCTURES, ETC.

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COLLEGE OF PHARMACY, AND THE CHICAGO CHEMICAL ASSOCIATION.

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AUTOBIOGRAPHY.

I have been born on the 10th day of Sep., 1810, in the village of Kilmore, near Castlekelly, in the County of Galway, Ireland. My mother, Ellen Nolan, daughter of Doctor Nolan, was of that place, and my father Michael McDermott was from Flaskagh, near Dunmore, in the same County, where I spent my early years at a village school kept by Mr. James Rogers, for whom I have an undying love through life. Of him I learned arithmetic and some book-keeping. He read arithmetic of Cronan and Roach, in the County of Limerick. They excelled in that branch. John Gregory, Esq., formerly Professor of Engineering and Surveying in Dublin; but now of Milwaukee, read of Cronan, which enabled him to publish his "Philosophy of Arithmetic," a work never equalled by another. By it one can solve quadratic and cubic equations, the diophantine problems, and summation of series.

After having been long enough under my friend Mr. Rogers, I went to the Clarenbridge school, kept by the brothers of St. Patrick, under the patronage of the good lady Reddington. I lived with a family named Neyland, at the Weir, about two miles from the school, where I had a happy home on the sea-side. There I read algebra, grammar, and book-keeping. After being nearly a year in that abode of piety and learning, I went to Mathew Collin's Mathematical school, in Limerick. He was considered then, and at the time of his death, the best mathematician in Europe. His correspondence in the English and Irish diaries on mathematics proves that he stood first. I left him after eight months studying geometry, etc., and went to Castleircan, near Cahirconlish, seven miles from Limerick, where I entered the mathematical school, kept by Mr. Thomas McNamara, familiarly known as Tom Mac, and Father of X, on account of his superior knowledge of algebra, he was generally known by the name of "Father of X." Of him I read algebra and surveying; lived with a gentleman farmer—named William Keys, Esq., at Drimkeen, about one and one-half miles south-east of the school. Mr. Mac had a large school, exclusively mathematical, and was considered the best teacher of surveying. After being with him nearly a year, I left and went to Bansha, four miles east of the town of Tipperary. Here Mr. Simon Cox, an unassuming little man, had the largest mathematical class in Ireland, and probably in the world, having 157 students, gathered from every County in Ireland, and some from England. Like Mr. McNamara, he had special branches in which he excelled; these were the use of the globes, spherical astronomy, analytical geometry, and fluxions. The differential and integral calculus were then slowly getting into the schools. I lived

with Dairyman Peters, near the bridge of Aughahall, about three miles east of Bansha. I remained two years with Mr. Cox, and then bade farewell to hospitable and learned Munster, where, with a few exceptions, all the great mathematical and classical schools were kept, until the famine plague of 1848 broke them up. I next found myself in Athleague County, Roscommon, with Mr. Mathew Cuniff, who was an excellent constructor of equations, and showed the application to the various arts.

I received my diploma as certified Land Surveyor on the sixth of September, 1836, after a *rough* examination by Mr. Fowler, in the theoretical, and William Longfield, Esq., in the practice of surveying. I soon got excellent practice, but wishing for a wider field of operation, for further information, I joined the Ordnance Survey of Ireland. Worked on almost every department of it, such as plotting, calculating, registering, surveying, levelling, examining and translating Irish names into English. Having got a remunerative employment from S. W. Parks, Esq., land surveyor and civil engineer, in Ipswich, County of Suffolk, England. I left my native Isle in April, 1838. Surveyed with Mr. Parks in the counties of Suffolk, Norfolk, and Essex, for two years, then took the field on my own account. I left happy, hospitable, and friendly England in April, 1842, and sailed for Canada. Landed in Quebec, where I soon learned that I could not survey until I would serve an apprenticeship, be examined, and receive a diploma.

I sailed up the St. Lawrence and Ottawa Rivers to Bytown,—then a growing town in the woods,—but now called Ottawa, the seat of the Government of British America. I engaged as teacher in a school in Aylmer, nine miles from Bytown (now Ottawa). At the end of my term of three months, I joined John McNaughton, Esq., land surveyor, and justice of the peace, until I got my diploma as Provincial Land Surveyor for Upper Canada, dated December 16, 1843, and my diploma or commission for Lower Canada, dated September 12, 1844.

I spent my time about equally divided between making surveys for the Home (British) Government four years, and the Provincial Government, and private citizens, until I left Bytown in September, 1849, having thrown up an excellent situation on the Ordnance Department. I never can forget the happy days I have been employed on ordnance surveys in Ireland, under Lieutenants Broughton and Lancy. In Canada, under the supervision of Lieutenants White and King, and Colonel Thompson, of the Royal Engineers. In my surveys for the Provincial Government of Canada, I always found Hon. Andrew Russell and Joseph Bouchette, Surveyor-Generals, and Thomas Devine, Esq., Head of Surveys, my warmest friends. They are now—October 7, 1878—living at the head of their respective old Departments, having lived a long life of usefulness, which I hope will be prolonged. To Sir William Logan, Provincial Geologist, I am indebted for much information. I lived nearly eight years in Ottawa, Canada, where my friends were very numerous. The dearest of all to me was Alphonso Wells, Provincial Land Surveyor, who was the best surveyor I ever met. He had been so badly frost-bitten on a Government survey that it was the remote cause of his death.

On one of my surveys, far North, I and one of my men were badly frost-bitten. He died shortly after getting home. I lost all the toes of my left foot and seven fingers, leaving two thumbs and the small finger on the

right hand. After the amputation, I soon healed, which I attribute to my strictly temperate habits, for I never drank spirituous liquor nor used that narcotic weed—tobacco.

In Sept., 1849, I left the Ordnance Survey, near Kingston. Having surveyed about 120 miles of the Rideau Canal, in detail, with all the Government lands belonging to it. On this service I was four years employed. I came to the City of Milwaukee, September, 1849; could find no surveying to do. I opened a school, October 1. Soon gathered a good class, which rewarded me very well for my time and labor. Here I made the acquaintance of many of the learned and noble-hearted citizens of the Cream City—Milwaukee, amongst whom I have found the popular Doctors Johnson and Hubeschman; I. A. LAPHAM; Professor Buck; Peters, the celebrated clock-maker; Byron Kilbourne, Esq.; Aldermen Edward McGarry, Moses Neyland, James Rogers, Rosebach, Furlong, Dr. Lake; John Furlong, etc., etc. I found extraordinary friendship from all Americans and Germans, as well as Irishmen. I was appointed or elected by the City Council, in the following April, as City Engineer, for 1850 and part of 1851. I was reappointed in April, 1851, and needed but one vote of being again elected in 1852. I made every exertion not to have my name brought up for a third term, because, in Milwaukee the correct rule, "Rotation in office is true democracy," was adhered to. In accordance with a previous engagement, made with Wm. Clogher, Esq., many years City Surveyor of Chicago, I left Milwaukee with regret, and joined Mr. Clogher, as partner, in April, 1852, immediately after the Milwaukee election. Worked together for one year, and then pitched my tent here since, where I have been elected City Surveyor, City Supervisor, and had a hand in almost if not all the disputed surveys that took place here since that time.

I have attended one course of lectures on chemistry, in Ipswich, England, in 1840, and two courses at Rush Medical College, under the late Prof. J. V. Z. Blaney, and two under Dr. Mahla, on chemistry and pharmacy. By these means, I believe that I have given as much on the subject of analysis as will enable the surveyor or engineer, after a few days application, to determine the quality and approximate quantity of metal in any ore. To the late Sir Richard Griffith, I am indebted for his "Manual of Instructions," which he had the kindness to send me, May 23, 1861. He died Sept. 22, 1878, at the advanced age of 94 years; being the last Irishman who held office under the Irish Government, before the Union with England. He was in active service as surveyor, civil engineer, and land valuator almost to the day of his death.

The principles of geometry and trigonometry are well selected for useful applications. The sections on railroads, canals, railway curves, and tables for earthwork are numerous.

The Canada and United States methods of surveying are given in detail, and illustrated with diagrams. Sir Richard Griffith's system of valuation on the British Ordnance Survey, and the various decisions of the Supreme Courts of the United States are very numerous, and have been sometimes used in the Chicago Courts as authority in surveys. Hydraulics, and the sections on building walls, dams, roofs, etc., are extensive, original, and comprehensive. The sections and drawings of many bridges and tunnels are well selected, and their properties examined and defined. The tables of sines and tangents are in a new form, with guide lines at every five min-

minutes. The traverse table is original, and contains 88 pages, giving latitude and departures for every minute of four places, and decimals, and for every number of chains and links. The North and South polar tables are the results of great labor and time. The table of contents is full and explicit. I believe the surveyors, engineers, valuers, architects, lawyers, miners, navigators, and astronomers will find the work instructive.

I commenced my traverse table, the first of my Manual, on the 15th of October, 1833, and completed my work on the 8th of October, 1878.

The oldest traverse table I have seen was published by D'Burgh, Surveyor General, in Ireland, in 1723, but only to quarter degrees and one chain distance. The next is that by Benjamin Noble, of Ballinakill, Ireland, entitled "*Geodesia Hibernica*," printed in 1768, were to $\frac{1}{4}$ degrees and 50 chains. The next, by Harding, were to $\frac{1}{4}$ degrees and 100 chains. In my early days, these were scarce and expensive; that by Harding, sold at two pounds two shillings Sterling, (about \$10.50).

Gibson's tables, so well known, are but to $\frac{1}{4}$ degrees and one chain distance.

Those by the late lamented Gillespie, were but to $\frac{1}{4}$ degrees, three places of decimals, and for 1 to 9 chains. Hence appears the value of my new traverse table, which is to every minute, and can be used for any required distances.

Noble gave the following on his title-page: "Ye shall do no unrighteousness in meteyard, in weight, or in measure." Leviticus, chap. xix, 35; "Cursed be he that removeth his neighbor's landmark." Deuteronomy, chap. xxvii, 17.

I lost thirty-two pages of the present edition of 1000 copies in the great Chicago fire, Oct. 9th and 10th, 1871, with my type and engravings; this caused some expense and delay.

The Manual has 524 pages, strongly bound, leather back and corners.

MICH'D McDERMOTT.

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THE SURVEYOR AND CIVIL ENGINEER'S MANUAL.

STRAIGHT-LINED AND CURVILINEAL FIGURES.

OF THE SQUARE.

1. Let A B C D (Fig. 1) be a square. Let A B = a, and A D = d, or diagonal.
2. Then $a \times a = a^2 =$ the area of the square.
3. And $\sqrt{2} a^2 = a \sqrt{2} = a \times 1,4142136 =$ diagonal.
4. Radius of the inscribed circle = O E = $\frac{a}{2}$
5. Radius of the circumscribing circle = O D = $\frac{a \times 1,4142136}{2} = a \times 0,707168.$
6. Perimeter of the square = A B + B D + D C + C A = 4 a.
7. Side of the inscribed octagon F G = $a \sqrt{2} - a = a \times 1,4142136 - a = a \times 0,414214$, i. e., the side of the inscribed octagon is equal to the difference between the diagonal A D and the side A B of a square.
8. Area of the inscribed circle = $a^2 \times 0,7854.$
9. Area of the circumscribed circle $0,7854 \times 2 a^2.$
10. Area of a square circumscribing a circle is double the square inscribed in that circle.

RHOMBUS.

11. (Fig. 3.) In a rhombus the four sides are equal to one another, but the angles not right-angled.
12. The area = the product of the side $\times$ perpendicular breadth = A B $\times$ C E.
13. Or, area = $a^2 \times$ natural sine of the acute angle C A B; i. e., A B $\times$ A C $\times$ nat. sine of the angle C A B = the area.

OF THE RECTANGLE OR PARALLELOGRAM.

14. (Fig. 2.) Let A. B = a, B D = b, and A D = d.
15. $A D = d = \sqrt{a^2 + b^2}.$
16. $\frac{d}{2} =$ radius of the circumscribing circle.
17. Area = a b or the length $\times$ by the breadth.
18. When a = 2 b, the rectangle is the greatest in a semi-circle.
19. When a = 2 b, the perimeter, A C + C D + D E contains the greatest area.

20. *Hydraulic mean depth* of a rectangular water-course is found by dividing the area by the wetted perimeter; *i. e.*, = area divided by the sum of $2 A C + C B$.

21. When the breadth is to the depth as $1 : \sqrt{2}$, *i. e.*, as $1 : 1.4152$, the rectangular beam will be the strongest in a circular tree.

22. When the breadth is to the depth as $1 : \sqrt{3}$, *i. e.*, as $1 : 1.732$, the beam will be the stiffest that can be cut out of a round tree.

23. *Rhomboid.* (Fig 4.) In a rhomboid the four sides are parallel. Area = longest side $\times$ by the perpendicular height = $A B \times C E = A B \times A C \times \text{nat. sine } \angle C A B$.

24. *Trapezoid.* In a trapezoid only two of its sides are parallel to one another. Let $A D E B$ (fig. 4) be a trapezoid.

Area = $\frac{1}{2} (C D + A B) \times$ by the perpendicular width $C E$.

OF THE TRIANGLE.

25. Let $A B C$ (Fig. 5) be a triangle.

26. If one of its angles, as B , is right-angled, the area = $\frac{A B}{2} \times B C$
 = $\frac{B C}{2} \times A B = \frac{1}{2} (A B \times B C)$

27. Or, area = $\frac{1}{2} A B \times$ tangent of the angle $B A C$.

28. When the triangle is not right-angled, measure any side; $A C$ as a base, and take the perpendicular to the opposite angle, B ; then the area = $\frac{1}{2} (A C \times E B)$.

In measuring the line $A C$, note the distance from A to E and from E to C , E being where the perpendicular was erected.

29. Or, area = $\frac{A C \times A B}{2} \times \text{nat. sine of the angle } C A B$.

When the perpendicular $E B$ would much exceed 100 links, and that the surveyor has not an instrument by which he could take the perpendicular $E B$, or angle $C A B$, his best plan would be to measure the three sides, $A B = a$, $B C = b$, and $A C = c$. Then the area will be found as follows:

30. Add the three sides together, take half their sum; from that half sum take each side separately; multiply the half sum by the three differences. The square root of the last product will be the *area*.

$$31. \text{ Area} = \left\{ \left(\frac{a+b+c}{2} \right) \cdot \left(\frac{a+b+c}{2} - a \right) \cdot \left(\frac{a+b+c}{2} - b \right) \cdot \left(\frac{a+b+c}{2} - c \right) \right\}^{\frac{1}{2}}$$

32. Let s equal half the sum of the three sides then

$$\text{Area} = \sqrt{\left\{ s \cdot (s-a) \cdot (s-b) \cdot (s-c) \right\}}$$

$$33. \text{ Or, area} = \frac{1}{2} \left\{ \log s + \log (s-a) + \log (s-b) + \log (s-c) \right\} \text{ i. e.,}$$

to the logarithm of half the sum add the logs of the three differences, divide the sum by 2, and the quotient will be the log of the required area.

34. Or, to the log of A C add the log of A B and the log sine of the contained angle C A B. The number corresponding to the sum of these three logs will be double the area, *i. e.*,

$$\text{Log } a + \text{log } c + \text{log sine angle C A B} = \text{double the area.}$$

35. Or, by adding the arithmetical compliment of 2, which is $\bar{1},698970$, we have a very concise formula.

$$\text{Area} = \text{log } a + \text{log } c + \text{log sine angle C A B} + \bar{1},698970.$$

Example. Let A B = $a = 18,74$, and A C = $c = 1695$ and the contained angle C A B = $29^\circ 43'$

Log 18,47 chains,	-	-	-	-	-	-	1,2664669
Log 16,95 chains,	-	-	-	-	-	-	1,2291697
Log sine $29^\circ 43'$	-	-	-	-	-	-	9,6952288
Constant log,	-	-	-	-	-	-	<u>1,6989700</u>
							11,8898354
Reject the index 10,	-	-	-	-	-	-	<u>10</u>
							1,8898354

The natural number corresponding to this log will be the required area = 77,5953 square chains, which, divided by 10, will give the area = 7,75953 acres.

35a. In Fig. 5, let the sides A C and B C be inaccessible. Measure A B = a ; take the angles A and B, then the area = $\frac{a^2 \text{ sine A} \times \text{sine B}}{2 \text{ sine C}}$.

which, in words, is as follows:

Multiply together the square of the side, the natural sines of the angles A and B; divide the contained product by twice the sine of the angle C. The quotient will be the required area.

Or thus: Add together twice the log of a , the log sine A, and the log sine B; from the sum subtract $\text{log } 2 + \text{log sine C}$. The difference will be the log of the area.

Example. Let the $\angle A = 50^\circ$, angle B = 60° , and by Euclid I. 32, the $\angle C = 70^\circ$; and let A B = $a = 20$ chains to find the area of the triangle:

Log 20,	-	-	-	-	-	-	1,3010200
							<u>2</u>
							2,6020400
Angle A = 50° , log sine,	-	-	-	-	-	-	9,8842540
Angle B = 60° , log sine,	-	-	-	-	-	-	<u>9,9375206</u>
							(A) = 22,4238146
Constant log of 2 =	-	-	-	-	-	-	0,3010300
Angle C = 70° , log sine,	-	-	-	-	-	-	<u>9,9729858</u>
							(B) = 10,2740158
							2,1498288

From the sum A subtract the sum B, the difference, having rejected 10 from the index will be the log of the natural number corresponding to the area 141,198 square chains, which divided by 10 gives the area = $14\frac{1198}{10000}$ acres.

Or thus: By using the table of natural sines. Having used Hutton's logs, we will also use his nat. sines.

See the formula (34) $a^2 = 20 \times 20$, - - - 400
 Nat. sine $50^\circ = \text{nat. sin.} < A =$ - - - ,7660444

Product, - - - - - 306,4177600

Let us take this = - - - 306,418

Nat. sine $60^\circ = \text{nat. sin.} < B =$ - - - ,86603+

Product, - - - - - 265,367007334

Nat. sine of $70^\circ =$,939693

2

Divisor, = 1,879386) 265,367007334

Quotient, = 141,198 square chains, which, divided by 10, gives $14\frac{1198}{10000}$ acres. Q. E. F.

35b. If on the line AB the triangles ACB , ADB , AEB , etc., be described such that the difference of the sides AC and CB , of AD and DB , and of AE and EB is each equal to a given quantity, the curve passing through the points C , D and E is a hyperbola.

36. If the sum of each of the above sides $AC + CB$, $AD + DB$, $AE + EB$ is equal to a given quantity, the curve is an ellipsis.

37. In the $\triangle ACB$, (Fig. 5,) if the base CE is $\frac{1}{3}$ of the line AC , the $\triangle CEB$ will be $\frac{1}{3}$ of the $\triangle ACB$, and if the base AC be n times the base CE , the $\triangle ACB$ will be n times the area of the $\triangle CEB$.

38. From the point P in the $\triangle ACB$, (Fig. 11,) it is required to draw a line PE , so that the $\triangle APE$ will be $\frac{1}{4}$ the area of the $\triangle ACB$.

Divide the line AB into 4 equal parts, let $AD =$ one of these parts, join D and C and P and C , draw DE parallel to PC , then the $\triangle AEP$ will be $= \frac{1}{4}$ of the $\triangle ACB$; for by Euclid I. 37, we find that the $\triangle EOC = \triangle ODP \therefore$ the $\triangle AEP = \triangle ACD = \frac{1}{4}$ the $\triangle ACB$, Q.E.F.

39. From the $\triangle ACB$, required to cut off a $\triangle ADE =$ to $\frac{1}{5}$ of the $\triangle ACB$ by a line DE parallel to BC .

By Euclid VI. 20, $\triangle ADE : \triangle ACB :: AD^2 : AB^2$; therefore, in this case, divide AB into two such parts, so that $AD^2 = \frac{1}{5}$ the square of AB . Let D be the required point, from which draw the line DE parallel to BC , and the work is done.

40. In the last case we have $\triangle ADE : \triangle ACB :: AD^2 : AB^2$; i. e., $1 : 5 :: AD^2 : AB^2$. Generally, $1 : n :: AD^2 : AB^2$; and by Euclid VI. 16, $n \times AD^2 = AB^2$; therefore, $AD = \frac{AB}{\sqrt{n}}$, which is a general formula.

Example. Let $AB = 60$ and $n = 5$; then $AD = \frac{60}{2,236} = 26,7$.

41. If D be a point in the $\triangle ACB$, (Fig. 13,) through which the line PE is drawn parallel to CB , make $CE = EF$, join FD , and produce it to meet CB in G , then the line FDG will cut off the least possible triangle.

42. By Euclid VI. 2, $FD = DG$, because $FE = EC$.

43. To bisect the $\triangle A C B$ (Fig. 16,) by the shortest line $P D$. Let $A C = b$, $B C = a$, $C P = x$, and $C D = y$, $\triangle C P D = \frac{1}{2} \triangle A C B$, conditions which will be fulfilled when $x = C P = \sqrt{\frac{a b}{2}}$ and $y = C D = \sqrt{\frac{a b}{2}}$. Hence it follows that $C P = C D$. (See Tate's Differential Calculus, p. 65.)

44. The greatest rectangle that can be inscribed in any $\triangle A C B$, is that whose height $n m$, is $= \frac{1}{2}$ the height $n C$ of the given triangle (see Fig 14,) $A B C$. Hence the construction is evident. Bisect $A C$ in K , draw $K L$ parallel to $A B$, let fall the perpendiculars $K D$ and $L I$, and the figure $K L I D$ will be the required rectangle.

45. The centre of the circumscribing circle $A C B$, (Fig. 7,) is found by bisecting the sides $A B$, $A C$, and $C B$, and erecting perpendiculars from the points of bisection; the point of their bisection will be the required centre. (See Euclid IV. 5.)

46. The centre of the inscribed circle (Fig. 6,) is found by bisecting the angles A , B and C , the intersection of these lines will be the required centre, O , from which let fall the perpendicular $O E$ or $O D$, each equal to the perpendicular $O F =$ to the required radius.

47. Let $R =$ radius of circumscribing circle and $r =$ radius of the inscribed circle, and the sides $A B = a$, $B C = b$, and $A C = c$ of the $\triangle A B C$; then $R = \frac{a b c}{2 r (a+b+c)}$

$$\text{and } r = \frac{a b c}{2 R (a+b+c)}$$

48. To find r , the radius of the inscribed circle in (Fig. 6.)

$$\frac{r}{2} (a+b+c) = \text{area of the } \triangle A B C = A,$$

$$\therefore r = \frac{2 A}{a+b+c} = \frac{A}{\frac{1}{2} (a+b+c)} = \text{area divided by half of the sum of the sides of the } \triangle,$$

$$R = \frac{a b c}{2 r \cdot (a+b+c)} = \frac{a b c}{(a+b+c)} \div \frac{4 A}{(a+b+c)} \text{ i. e.,}$$

$$R = \frac{a b c \cdot (a+b+c)}{4 A \cdot (a+b+c)} = \frac{a b c}{4 A} \text{ i. e.,}$$

49. Radius of the circumscribing circle is equal to the product of the three sides divided by 4 times the area of the triangle, and substituting the formula in § 31 for the area of the triangle, we have

$$R = \frac{a b c}{4 A} = \frac{a b c}{2 r (a+b+c)} \text{ i. e.,}$$

$$R = \frac{a b c}{4 \left\{ s \cdot (s-a) \cdot (s-b) \cdot (s-c) \right\}^{\frac{1}{2}}} \text{ where } s \text{ is } \frac{1}{2} \text{ the sum of the sides,}$$

$$\text{but } (a+b+c) \frac{r}{2} = A; \text{ therefore,}$$

$$50. \quad r = \frac{2 A}{a+b+c}$$

51. The area of any $\triangle CKL$ (Fig. 14,) will be subtended by the least line KL , when $CK = CL$. Let $x = CK = CL$, and $A =$ the

$$\text{required area, then } x = \frac{2A}{\text{nat. sine } C}$$

52. Of all the triangles on the same base and in the same segment of a circle, the isosceles $\triangle$ contains the greatest area.

53. The greatest isosceles $\triangle$ in a circle will be also equi-lateral and will have each side $= r\sqrt{3}$ where $r =$ radius of the given circle.

54. In a right-angled $\triangle$, when the hypotenuse is given, the area will be a maximum when the $\triangle$ is isosceles; that is, by putting h for the hypotenuse the base and perpendicular will be each $= \frac{h}{\sqrt{2}} = \frac{h}{1.414313}$

55. The greatest rectangle in an isosceles right-angled $\triangle$ will be a square.

56. In every triangle whose base and perpendicular are equal to one another, the perimeter will be a maximum when the triangle is isosceles.

57. Of all triangles having the same perimeter, the equi-lateral $\triangle$ contains the greatest area.

58. In all retaining walls (*walls built to support any pressure acting laterally*) whose base equals its perpendicular, or whose hypotenuse makes an angle of 45° with the horizon, will be the strongest possible.

OF THE CIRCLE.

Let $\log$ of 3,1416 = 0,4971509, of 0,7854 = $\overline{1}$,8950909, and of 0,07958 = $\overline{2}$,9008039.

59. Let $a =$ area, $d =$ diameter and $c =$ circumference. $n = 3,1416$ and $m = 0,7854$. Const. $\log 3,1416 = 0,4971509$. $d \times 3,1416 =$ circumference, or $\log d + \log 0,4971509 = \log$ circumference.

60. $d^2 \times 0,7854 = \text{area} = \text{twice } \log d + \text{constant } \log \text{ of } 0,7854 =$
 $(\overline{1},8950909)$, and $c^2 \times 0,07959 = \text{area} = \frac{d}{2} \times \frac{c}{2} = \frac{dc}{4}$,

$\log \text{ of area} = 2 \log c + \text{constant } \log \overline{2},9008039$.

61. *Example.* Let $d = 46$, then $46 \times 3,1416 = 144,5136 =$ circumference; or, by logarithms,

$$\begin{array}{r} 46, \log = 1,6627578 \\ 3,1416 \text{ constant } \log 0,4971509 \\ \hline 2,1599087 = 144,5136 \\ 8979 \text{ circumference.} \\ \hline 108 \\ 90 \\ \hline 18 \end{array}$$

62. $d = \frac{c}{3,1416}$ or $c = 144,5136$ $\log = 2,1599087$

$$\begin{array}{r} 3,1416 \quad \log 0,4971509 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Difference, } 1,6627578 \\ d = 46 \end{array}$$

$$63. \text{ Area} = d^2 \times 0,7854 = \frac{cd}{4} = \frac{c}{4} \cdot d = \frac{d}{4} \cdot c = c^2 \cdot 0,07958.$$

Log area = twice log d + log 1,8950909, the nat. number of which will give the required area.

Example. Let $d = 45$, its log = $\begin{matrix} 1,6627578 \\ 1,6627578 \end{matrix}$
 Constant log of 0,7854, $\begin{matrix} 1,8950909 \\ \hline \end{matrix}$
 Area = 1661,909 = 3,2206065

$$64. = c^2 \times 0,07958 = \text{twice log } c + \text{log of } 0,07958 = \text{log area.}$$

Example. Let $c = 154$.

$$\begin{array}{r} \text{Log } c = 2,1875207 \\ 2 \\ \hline \text{Log } c^2 = 4,3750414 \\ \text{Constant log of } 0,07958 = 2,9008039 \\ \hline \text{Log area} = 3,2758453 \\ \text{Area} = 1887,3191 \end{array}$$

$$65. d = \left(\frac{a}{0,7854}\right)^{\frac{1}{2}} \text{ and } c = \left(\frac{a}{0,07958}\right)^{\frac{1}{2}}$$

66. *Area of a Circular Ring* = $(D^2 - d^2) \times 0,7854$. Here D = diameter of greater circumference, and d , that of the lesser circumference.

67. *Area of a Sector of a Circle.* (See Fig. 8.) Arc $E G F$ is the arc of the given sector $O E G F$, area = $\frac{r}{2} \cdot \text{arc } E G F$ or area = $r \cdot \frac{E G F}{2}$; but arc $E G F$ = 8 times the arc $E G$, less the chord $E F$, the difference divided by three = arc $E G F$ (*i. e.*)

$$\text{Arc } E G F = \frac{8 E G - E F}{3}, \therefore \text{area of sector} = \frac{r}{2} \times \frac{8 E G - E F}{3},$$

68. *i. e.*, Area = $\frac{r}{6} (8 E G - E F)$. $E G$, the chord of $\frac{1}{2}$ the arc, may be found by Euclid I. 47. For we have $O E$ = to the hypotenuse, given, also $\frac{1}{2}$ the chord $E F$ = $E H$, $\therefore \sqrt{(O E^2 - E H^2)} = O H$, and $O E - O H = H G$, then $\sqrt{(E H^2 + H G^2)} = E G$.

69. Area = degrees of the $\angle E O F \times \text{diameter} \times \text{by the constant number, or factor } 0,008727$, *i. e.*, area = $d a \times 0,008727$ where $a \angle = E O F$ in degrees and decimals of a degree.

70. *Segment of a Ring.* $N K M F G E$, the area of this segment may be found by adding the arcs $N K M$ and $E G F$ of the sector $O N K M$ and multiplying $\frac{1}{2}$ their sum by $E N$, the height of the segment of the ring, *i. e.*, area = $\frac{\text{arc } N K M + \text{arc } E G F}{2} \times G K$.

71. *Segment of a Circle.* Let $E G F$ be the given segment whose area is required. By § 67 find the area of the sector $O E F$, from which take the area of the $\triangle O E F$, the difference will be the required area.

72. Or, area = $\frac{2 \text{ E F} \times \text{G H}}{3} + \frac{\text{G H}^3}{2 \text{ E F}}$; i. e., to $\frac{2}{3}$ of the product of the chord by the height, add the cube of the height divided by twice the chord of the segment, the sum will be the required area.

73. Or, divide the height G H by the diameter G L of the circle to three places of decimals. Find the quotient in the column Tabular Heights of Table VII., take out the corresponding area segment; which, when multiplied by the square of the diameter, will give the required area.

74. When G H, divided by the diameter G L, is greater than .5, take the quotient from 0,7854, and multiply the difference by the square of the diameter as above, when G H divided by G L does not terminate in three places of decimals, take out the quotient to five places of decimals, take out the areas less and greater than the required, multiply their difference by the last two decimals of the quotient, reject two places of decimals, add the remainder of the product to the lesser area, the sum will be the required tabular area.

Example. Let G H = 4, and $\frac{1}{2}$ the chord = E H = 9 = $\frac{1}{2}$ E F. By Euclid III. 35, H G $\times$ H L = E H $\cdot$ H F = E H² = 81; $\therefore \frac{81}{4} = 20,25 = \text{H L}$; consequently, by addition, $20,25 + 4 = 24,25 = \text{G L} = \text{diameter}$. And 4 divided by 24,25 = 0,16494 = tabular number.

Area corresponding to 0,164 = ,084059

“ “ “ 0,165 = ,084801

,000742

94

,000697,48

Lesser area for ,164 ,084059

Correction to be added for 00094 = 697

Corrected tabular area, ,084756; which, multiplied by the square of the diameters will give the required area.

OF A CIRCULAR ZONE.

75. Let E F V S (Fig. 8.) be a circular zone, in which E F is parallel to S V, and the perpendicular distance E t is given; consequently E S = t v may be found by Euclid I. 47, $s t = \frac{1}{2} (S v - E F) = d$, and $S v - d = t v$, and by Euclid III. 35, $\frac{s t \times t v}{E t} = t W$, $\therefore E t + t U = E U$ is given.

And by Euclid I. 47, the diameter U F is = $\sqrt{(E U^2 + E F^2)}$

And by Euclid III. 3, by bisecting the line, O Z is at right angles to F V; and by Euclid III. 31, the $\angle U V F$ is a right angle; and by Euclid VI. 2 and 4, $U V = 2 o x$.

And E t : E S :: v t : V U, by substitution we have

E t : E S :: v t : 2 o x.

By Euclid VI. 16, $o x = \frac{1}{2} (E S \times v t) \div E t = \frac{E S \times v t}{2 E t}$

Now having ox and oy = radius, we can find the height of the segment xy ; $\therefore$ having the height of the segment xy , and diameter WF of the segment FYV , we can find its area as follows:

The area of the trapezium $EFVS = \frac{1}{2} (EF + VS) \times Et$, to which add twice the segment FYV , the sum will be the required area of the zone $EFVS$.

In fig. 8, let $EF = a$, $SV = b$, $Et = p$, $St = d = \frac{1}{2} (Sv - EF)$,

and $Tv = e$, $EW = p + \frac{ed}{p} = \frac{p^2 + ed}{p}$, and by Euclid I. 47.

$$WF = \left\{ \left(\frac{p^2 + ed}{p} \right)^2 + a^2 \right\}^{\frac{1}{2}} \text{ i. e.,}$$

$$WF = \sqrt{\frac{(p^4 + 2p^2ed + e^2d^2 + p^2a^2)}{p}}$$

$$ES = (p^2 + d^2)^{\frac{1}{2}}$$

$$\text{Because } Et : ES :: Vt : VW$$

$$Et : ES :: Vt : 2ox$$

$$\therefore ox = \frac{ES \cdot Vt}{2Et}$$

And by substituting the values of ES , Vt and $2Et$, we have

$$ox = \frac{e(p^2 + d^2)^{\frac{1}{2}}}{2p}$$

$$xy = \frac{WF}{2} - ox$$

$$WF = 2xy + 2ox$$

Example. Let $EF = a = 20$, and $sv = b = 30$, $Et = p = 25$, $St = d$, and $tv = e$, to find the diameter WF and height xy . Here $d = 5$ and $tv = e = 25$.

$$ES = \sqrt{650} = 25,494.$$

$$ox = \frac{25 \sqrt{625 + 25}}{50} = \frac{25 \sqrt{650}}{50} = \frac{25 \times 25,495}{50}, \text{ i. e.,}$$

$$ox = 12,747,$$

$$WF = \frac{1}{p} \sqrt{390625 + 156250 + 15625 + 390625} = \frac{908,05}{25}$$

therefore $WF = 36,12 =$ required diameter.

$WF = 36,07 =$ diameter; and having the diameter WF and height xy , the area of the segment, subtended by the chords Fv and ES , can be found by Table VII., and the trapezium $EFvt$ by section 24.

OF A CIRCULAR LUNE.

76. Let $ACBD$, fig. 10, represent a lune. Find the difference between the segment ACB and ADB , which will be the required area.

b

77. Hydraulic mean depth of a segment of a circle is found by dividing the area of the segment by the length of the arc of that segment. Of all segments of a circle, the semi-circular sewer or drain, when filled, has the greatest hydraulic mean depth.

78. The greatest isosceles $\triangle$ that can circumscribe a circle will be that whose height or perpendicular C F is equal to 3 times the radius O E.

79. Areas of circles are to one another as the squares of their diameters; *i. e.*, in fig. 8, circle A K B I is to the area of the circle C G V L as the square of A B is to the square of C D.

80. In any circle (fig. 9), if two lines intersect one another, the rectangle contained by the segments of one is = to the rectangle contained by the segments of the other; *i. e.*, $OM \times MC = FM \times MH$, or $OA \times AC = FA \times AH$.

81. In fig. 8, $a \times b \times T = IT \times KT = \text{square of the tangent } TM$.

82. In a circle (fig. 9), the angle at the centre is double the angle at the circumference; *i. e.*, $\angle CAB = 2 \angle COB$. Euclid III. 20.

83. By Euclid III. 21, equal angles stand upon equal circumferences; *i. e.*, $\angle COB = \angle CLB$.

84. By Euclid III. 26, the $\angle BCL = \angle BLC = \angle COB$.

85. By Euclid III. 32, the angle contained by a tangent to a circle, and a chord drawn from the point of contact, is equal to the angle in the alternate segment of the circle; *i. e.*, in fig. 9, the $\angle TBC = \angle BOC = \frac{1}{2} \angle CAB$. This theorem is much used in railway engineering.

86. The angle T B C is termed by railroad engineers the tangential angle, or angle of half deflection.

87. To draw a tangent to a circle from the point T without the circle. (See fig. 9.) Join the centre A and the point T, on the line A T describe a semi-circle, where A cuts the circle, in B. Join T and B, the line T B will be the required tangent or the square root of any line $QTH = TB$; *i. e.*, $\sqrt{QTH} = TM$.

Then from the point T with the distance T B, describe a circle, cutting the circle in the point B, the line T B is the required tangent.

In Section 81, we have $Ta \cdot TB = TM^2$, $\therefore \sqrt{Ta \cdot TB} = TM$, and a circle describe with T as centre and T M as radius will determine the point M.

OF THE ELLIPSE.

88. An ellipse is the section of a cone, made by a plane cutting the cone obliquely from one side to the other.

Let fig. 39 represent an ellipse, where A B = the transverse axis, and D E = the conjugate axis. F and G the foci, and C the centre.

Construction.—An ellipse may be described as follows: Bisect the transverse axis in C, erect the perpendicular C D equal to the semi-conjugate, from the point D, as centre with A C as distance describe arcs cutting the transverse axis in the foci F and G. Take a fine cord, so that when knotted and doubled, will be equal to the distance A G or F B. At

the points or foci F and G put small nails or pins, over which put the line, and with a fine-pointed pencil describe the curve by keeping the line tight on the nails and pencil at every point in the curve.

89. *Ordinates* are lines at right angles to the axis, as O I is an ordinate to the transverse axis A B.

90. *Double ordinates* are those which meet the curve on both sides of the axis, as H V is a double ordinate to the transverse axis.

91. *Abscissa* is that part of the axis between the ordinate and vertex, as A O and O B are the abscissas to the ordinate O I; and A G and G B are abscissas to the ordinate G H.

92. *Parameter or Latus rectum* is that ordinate passing through the focus, and meeting the curve at both sides, as H. V.

93. *Diameter* is any line passing through the centre and terminated by the curve, as Q X or R I.

94. *Ordinate to a diameter* is a line parallel to the tangent at the vertex of that diameter, as Z T is the ordinate being parallel to the tangent X Y drawn to the vertex X of the diameter X Q.

95. *Conjugate to a diameter* is that line drawn through the centre, terminated by the curve, and parallel to the tangent at the vertex of that diameter, as C b is the semi-conjugate to the diameter Q X.

96. *Tangent to any point H' in the curve*, join H F and G H, bisect the angle L H G by the line H K, then H K will be the required tangent.

97. *Tangent from a point without*, let P be the given point, (see fig. 40) join P F; on P F and A B describe circles cutting one another in X, join P X and produce it to meet the ellipse in T, then P T will be the required tangent, and $H K' = \text{tangent to the point h}$.

98. *Focal tangents*, are the tangents drawn through the points where the *latus rectum* meets the curve, K H is the focal tangent to the point H.

99. *Normal* is that line drawn from the point of contact of the tangent with the curve, and at right angles to the tangent, H N is normal to K H.

100. *Subnormal* is the intercepted distance between the point where the normal meets the axis, and that point where an ordinate from the point of tangents contact with the curve meets the axis, as N O' is the subnormal to the point H.

101. *Eccentricity* is the distance from the focus to the centre, as C G.

102. All diameters bisect one another in the centre C; that is, $C X = C Q$ and $C I = C R$.

103. *To find the centre of an ellipse*. Draw any two cords parallel to one another, bisect them, join the points of bisection and produce the line both ways to the curve, bisect this last line drawn, and the point of bisection will be the centre of the ellipse.

104. $A B = F D + G B = F I + G I = F H + G H$, etc.; that is, the sum of any two lines drawn from the foci to any point in the curve, is equal to the transverse axis.

105. The square of half the transverse, is to the square of half the conjugate, as the rectangle of any two abscissas is to the square of the ordinate to these abscissas; *i. e.*,

$$A C^2 : C D^2 :: A O \cdot O B : O I^2; \text{ therefore,}$$

$$O I = \sqrt{\left(\frac{A O \cdot O B \cdot C D^2}{A C^2}\right)} = \frac{C D}{A C} \cdot \sqrt{(A O \cdot O B)}$$

Let us assume $\frac{C D}{A C}$ equal to n , then

$$G H' = \sqrt{(A G \cdot G B)} \cdot n.$$

106. Rectangles of the abscissas are to one another as the squares of their ordinates; *i. e.*,

$$A O \cdot O B : A G \cdot G B :: O I^2 : G H'^2$$

107. The square of any diameter is to the square of its conjugate, as the rectangle of the abscissas to that diameter, is to the square of the ordinate to these abscissas; *i. e.*,

$$Q X^2 : H' b^2 :: Q T \cdot T X : T Z^2; \text{ i. e.,}$$

$$C X^2 : C b^2 :: Q T \cdot T X : T Z^2.$$

108. To find where the tangent to the point H will meet the transverse axis produced:

$C O' : A C :: A C : C K'$. Substituting x for $C O'$, and a for $A C$

$$x : a :: a : C K'; \therefore C K' = \frac{a^2}{x}; \text{ therefore,}$$

$$O K' = \frac{(a+x) \cdot (a-x)}{x} = \frac{a^2 - x^2}{x}. \text{ Now, by having the ordinate } O I = y, \text{ we have}$$

$$H K' = \left\{ y^2 + \left(\frac{a^2 - x^2}{x} \right)^2 \right\}^{\frac{1}{2}} = \left\{ y^2 + \frac{a^4 - 2 a^2 x^2 + x^4}{x^2} \right\}^{\frac{1}{2}}$$

$$109. \text{ Tangent } H K' = \sqrt{\frac{(x^2 y^2 + a^4 - 2 a^2 x^2 + x^4)}{x}}, \text{ here } x = C O.$$

$$110. \text{ Equation to the ellipse } \frac{y^2}{b^2} + \frac{x^2}{a^2} = 1;$$

$$\text{or, } y = \left\{ \frac{b^2}{a^2} \cdot (a^2 - x^2) \right\}^{\frac{1}{2}} \text{ here } y = \text{any ordinate } O H.$$

Having the semi-transverse axis = a , the semi-conjugate = b
 $O = O H = \text{any ordinate, } x = C O = \text{co-ordinate of } y.$ Let

$A O = S = \text{greater abscissa, and } O B = s = \text{lesser abscissa. We will from the above deduce formulas for finding either } a, b, S, s, O \text{ or } x.$

$$111. O H = o = \left(\frac{S \cdot s \cdot b^2}{a^2} \right)^{\frac{1}{2}} = \text{ordinate} = \frac{b}{a} \sqrt{S \cdot s}.$$

$$112. A C = a = \frac{b \cdot S}{o^2} \left\{ b + \sqrt{(b^2 - o^2)} \right\} = \text{semi-transverse.}$$

$$113. \quad CD = b = \sqrt{\left(\frac{a^2 - o^2}{S \cdot s}\right)} = a \cdot o \sqrt{\frac{1}{S \cdot s}} = \text{semi-conjugate.}$$

$$114. \quad AO = S = a + \frac{a}{b}(b^2 - o^2)^{\frac{1}{2}} = \text{greater abscissa.}$$

$$115. \quad \text{Area of an ellipse} = AB \times DE \times ,7854 = 4ab \cdot 7854 = 3,1416 \times ab.$$

116. *Area of an elliptical segment.*—Let h = height of the segment. Divide the height h , by the diameter of which it is a part; find the tabular area corresponding to the quotient taken from tab. VII; this area multiplied by the two axes will give the required area, *i. e.*,

$$\text{Tab. area } \frac{h}{2a} \cdot 4ab, \text{ when the base is parallel to the conjugate axis;}$$

$$\text{or, tab. area} = \frac{h}{2b} \cdot 4ab, \text{ when the base is parallel to the transverse axis.}$$

$$117. \quad \text{Circumference of an ellipse} = \sqrt{\left(\frac{4a^2 + 4b^2}{2}\right)} \cdot 3,1416; \text{ i. e.,}$$

$$\text{Circumference} = \sqrt{(2a^2 + 2b^2)} \cdot 3,1416.$$

118. *Application.*—Let the transverse = 35, and conjugate = 25.
Area = $35 \times 25 \times ,7854 = 875 \times ,7854 = 687,225.$

$$\text{Circumference} = \sqrt{\left(\frac{35^2 + 25^2}{2}\right)} \cdot 3,1416 = 22,09 \times 3,1416 = 69,3979.$$

Let $AO = 28$ = greater abscissa, then 7 = the lesser abscissa, to find the ordinate OH .

$$OH = \left(\frac{28 \times 7 \times 25^2}{35^2}\right)^{\frac{1}{2}} = \sqrt{100} = 10.$$

$$\text{or, } OH = \frac{25}{35} \sqrt{28 \times 7} = 10. \quad (\text{See section 111.})$$

$$\text{Abscissa } AO = 17,5 + \frac{17,5}{12,5} \sqrt{625 - 100} = 17,5 + 1,4 \times 7,5 = 28.$$

OF THE PARABOLA.

122. A parabola is the section of a cone made by a plane cutting it parallel to one of its sides (see fig. 41).

123. *To describe a parabola.*—Let DC = directrix and F = focus; bisect AF in V ; then V = vertex; apply one side of a square to the directrix CD ; attach a fine line or cord to the side HI ; make it fast to the end I and focus F ; slide one side of the square along the edge of a ruler laid on the directrix; keep the line by a fine pencil or blunt needle close to the side of the square, and trace the curve on one side of the axis.

Otherwise, Assume in the axis the points $F B B' B'' B''' B''''$ etc., at equal distances from F ; from these points erect perpendicular ordinates to the axis, as $F Q, B P, B' O, B'' N, B''' M$; from the focus F , with the distances $A F, A B, A B', A B'', A B'''$, describe arcs cutting the above ordinates in the points Q, P, O, N, M , etc., which points will be in the curve of the required parabola; by marking the distances $F B = B B' = B' B''$, etc., each distance equal about two inches, the curve can be drawn near enough; but where strict accuracy is required, that method given in sec. 122 is the best.

124. *Definitions*.— $C D$ is the directrix, F = focus, V = vertex, $A B$ = axis. The lines at right angles to the axis are called ordinates. The double ordinate $Q R$ through the focus is equal to four times $F V$, and is called *parameter*, or *latus rectum*.

Diameter to a parabola is a line drawn from any point in the curve parallel to the axis, as $S Y$.

Ordinate to a diameter is the line terminated by the curve and bisected by the diameter.

Abscissa is the distance from the vertex of any diameter to the intersection of an ordinate to that diameter, as $V B$ is the abscissa to the ordinate $P B$.

124a. Every ordinate to the axis is mean proportional between its abscissa and the latus rectum; that is $4 V F \times B'' V = B'' N^2$, consequently having the abscissa and ordinate given, we find the latus rectum

$$= 4 V F = \frac{B'' N^2}{B'' V}; \text{ also the distance of the focus } F \text{ from the vertex}$$

$$= F V = \frac{B'' N^2}{4 B'' V}.$$

125. Squares of the ordinates are to one another as their abscissas; i. e., $B P^2 : B' O^2 :: V B : V B'$.

$$126. F Q = 2 F V \therefore Q R = 4 F V.$$

127. The ordinate $B S^2 = V B \cdot 4 V F$; hence, the equation to the curve is $y^2 = p x$, where y = ordinate = $B S$, and x = abscissa $V B$, and p = parameter or latus rectum.

128. To draw a *tangent* to any point S in the curve, join $S F$; draw $Y S L$ parallel to the axis $A B$; bisect the angle $F S L$ by the line $X S$, which will be the required tangent.

Otherwise, Draw the line from the focus to the directrix, as $F L$; bisect $F L$ in m ; draw $m X$ at right angles to $F L$; then $m X S$ will be the tangent required, because $S L = S F$.

Otherwise, Let S be the point from which it is required to draw a tangent to the curve; draw the ordinate $S B$, produce $B'' V$ to G , making $V G = V B$; then the line $G S$ will be the required tangent.

129. *Area* of a parabola is found by multiplying the height by the base, and taking two-thirds of the product for the area; i. e., the area of the parabola $N V U = \frac{2}{3} (B'' V \cdot N W)$.

130. To find the length of the curve N V B of a parabola:

Rule.—To the square of the ordinate N B'' add four thirds of the square of the abscissa V B''; the square root of the product multiplied by 2 will be the required length. Or, by putting $a = \text{abscissa} = V B''$, and $d =$

ordinate N B''; length of the curve N V U $= \sqrt{\left(\frac{a d^2 + 4 a^2}{3}\right)} \cdot 2$, i. e.,

Length of the curve N V U $= \sqrt{(3 d^2 + 4 a^2)} \times 1,155$.

Rule II.—The following is more accurate than the above rule, but is more difficult.

Let $q = \frac{2 d}{4 V F}$ = to the quotient obtained by dividing the double ordinate by the parameter.

Length of the curve $= 2 d \cdot \left(1 + \frac{q^2}{2 \cdot 3} - \frac{q^4}{2 \cdot 4 \cdot 5} + \frac{3 q^6}{2 \cdot 4 \cdot 6 \cdot 7}\right)$ etc.

131. By sec. 57, of all triangles the equilateral contains the greatest area enclosed by the same perimeter; therefore, in sewerage, the sewer having its double ordinate, at the spring of the arch, equal to d ; then its depth or abscissa will be ,866 d ; i. e., multiply the width of the sewer at the spring of the arch by the decimal ,866. The product will be the depth of that sewer, approximately for parabolic sewer.

132. The great object in sewerage is to obtain the form of a sewer, such that it will have the greatest hydraulic mean depth with the least possible surface in contact.

OF THE PARABOLIC SEWER.

133. Given the area of the parabolic sewer, N V U = a to find its abscissa V B'' and ordinate B'' N such that the hydraulic mean depth of the sewer will be the greatest possible.

Let $x = \text{abscissa} = V B''$

and $y = \text{ordinate N B''}$; then N U = $2 y$.

By section 129, $\frac{4 y x}{3} = a$; i. e., $4 y x = 3 a$

$$y = \frac{3 a}{4 x} = ,75 \frac{a}{x} = \frac{,75 a}{x}$$

To find the length of the curve N V U.

$$2 \sqrt{\frac{,5625 a^2}{x^2} + \frac{4 x^2}{3}} = \text{perimeter.}$$

$$2 \sqrt{\left(\frac{1,6875 a^2 + 4 x^4}{3 x^2}\right)} = \frac{2 \sqrt{1,6875 a^2 + 4 x^4}}{1,732 x} = \text{perimeter.}$$

i. e., $\frac{1,155\sqrt{1,6875 a^2 + 4 x^4}}{1,732 x} = \text{perimeter, which, divided into the given area, (a) will give}$

$$\frac{a x}{1,155\sqrt{1,6875 a^2 + 4 x^4}} = \text{hydraulic mean depth.}$$

$$\text{i. e., } \frac{a x}{1,155\sqrt{1,6875 a^2 + 4 x^4}} = \text{maximum.}$$

And by differentiating this expression, we have

$$\text{Differential } u = a dx \cdot \frac{1,155\sqrt{1,6875 a^2 + 4 x^4} - a x \left(\frac{1,155 \cdot 8 x^3 dx}{\sqrt{1,6875 a^2 + 4 x^4}} \right)}{1,155\sqrt{1,6875 a^2 + 4 x^4}}$$

rejecting the denominator and bringing to the same common denominator.

$$\frac{du}{dx} = a \cdot 1,155 (1,6875 a^2 + 4 x^4) - a x (9,24 x^3) = 0.$$

$$\text{i. e., } 1,949 a^2 + 4,62 a x^4 - 9,24 a x^4 = 0.$$

$$1,949 a^2 = 4,62 a x^4$$

$$x^4 = ,4218 a^2$$

$$x^2 = ,6494 a$$

$$x = ,806\sqrt{a} = \sqrt{,649 a} = \text{required abscissa.}$$

$$y = \frac{3 a}{4 x} = \frac{0,75 a}{x} = \text{required ordinate.}$$

Example.—Let the area = 4 feet = a;

$$\text{then } ,806\sqrt{a} = ,806 \cdot 2 = 1,612 = \text{abscissa} = x;$$

$$\text{and } y = \text{ordinate} = \frac{3 a}{4 x} = \frac{12}{6,448} = 1,863.$$

Now we have the abscissa $x = 1,612$, and ordinate = 1,863.

By Sec. 130, we find the length of the curve $N V U = 5,26$; and by dividing the perimeter, 5,26, into the area of the sewer, we will have the

$$\text{hydraulic mean depth} = \frac{4}{5,16} = 0,76 \text{ feet.}$$

134. The circular sewer, when running half full, has a greater hydraulic mean depth than any other segment; but as the water falls in the sewer, the difference between the circular and parabolic hydraulic mean depths, decreases until in the lower segments, where the *debris* is more concentrated in the parabolic, than in the circular, the parabolic sewer with the same sectional area will give the greatest hydraulic mean depth. This will appear from the following calculations: Where the segment of a circle is assumed equal to a segment of a parabola, which parabola is equal to one-half of the given circle. The method of finding the length of the curve, area and hydraulic mean depth, will also appear.

That the parabolic sewer, whose abscissa $= 0,806\sqrt{a}$ and ordinate $= \frac{\sqrt{a}}{1,075}$ (where a = given area), is better than either the circular or egg-shaped sewer, will appear from the following table and calculations.

135. TABLE, SHOWING THE HYDRAULIC MEAN DEPTH IN SEGMENTS OF PARABOLIC AND CIRCULAR SEWERS, EACH HAVING THE SAME SECTIONAL AREA. THE DIMENSIONS OF THE PRIMITIVE PARABOLA AND CIRCULAR ARE AT THE TOP.

PARABOLA, *Latus Rectum* 2,7. SEMICIRCLE, *Diameter* = 4 feet.

Abscissa or depth filled.	Ordinate or half chord filled.	Area of segment filled.	Perimeter of segment filled.	Hydraulic mean depth of segment filled.	Abscissa or depth filled.	Ordinate or half chord filled.	Area of segment filled.	Perimeter of segment filled.	Hydraulic mean depth of segment filled.
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
2,019	2,335	6,286	6,737	0,933	2,00	2,00	6,286	6,283	1,0
2,0	2,324	6,197	6,553	0,946	1,98	1,999	6,197	6,241	0,993
1,9	2,265	5,738	6,307	0,909	1,86	1,995	5,738	6,002	0,956
1,8	2,205	5,292	6,060	0,873	1,75	1,984	5,292	5,781	0,912
1,7	2,142	4,855	5,811	0,835	1,64	1,967	4,855	5,560	0,873
1,6	2,079	4,435	5,562	0,797	1,53	1,944	4,435	5,334	0,831
1,5	2,013	4,026	5,311	0,758	1,43	1,917	4,026	5,121	0,786
1,4	1,944	3,629	5,056	0,719	1,32	1,881	3,629	4,900	0,741
1,3	1,874	3,248	4,802	0,676	1,22	1,842	3,248	4,680	0,694
1,2	1,800	2,880	4,543	0,634	1,12	1,796	2,880	4,462	0,645
1,1	1,723	2,527	4,281	0,596	1,02	1,744	2,527	4,224	0,598
1,0	1,643	2,191	4,016	0,545	0,92	1,683	2,191	4,001	0,547
0,9	1,559	1,871	3,747	0,499	0,83	1,622	1,871	3,784	0,494
0,8	1,470	1,568	3,472	0,451	0,73	1,544	1,568	3,530	0,444
0,7	1,375	1,283	3,190	0,402	0,64	1,466	1,283	3,291	0,389
0,6	1,273	1,018	2,898	0,351	0,54	1,367	1,018	3,010	0,338
0,5	1,162	0,775	2,595	0,299	0,45	1,264	0,775	2,737	0,283
0,4	1,039	0,559	2,274	0,246					

Because the hydrostatic or scouring force in a sewer is found by multiplying the sectional area by the depth and $62\frac{1}{2}$ pounds, and that the depths of the segments of a parabola are greater than in the segments of the semicircle, each being equal to the same given area; therefore, from inspecting the above table, it will appear that the parabolic sewers have greater hydrostatic depths and pressure than the circular segments. It also appears that in the lower half depth of the semicircle, and in all other depths lower than half the radius, the hydraulic mean depth is greater than in circular segments of the same areas.

Calculation of the foregoing Table.

Example. Required, the ordinate at abscissa 1,2 of the given parabola, whose abscissa $= 2,019$, and ordinate 2,335, and *latus rectum* 2,7.

Rule. Multiply the *latus rectum* by the abscissa of the parabolic segment. The square root of product will be the required ordinate.

Or by logarithms, let log of 2,7 $= 0,431364$
 log of the given abscissa $= 0,041393$
 log of the product of abscissa and *latus rectum* $= 0,472757$
 which divide by 2 will give the log of the square root of the product $= 0,236378$
 the natural number corresponding to which gives the ordinate $= 1,800$

c

To Find the Area.

The given ordinate = 1,800.

The chord or double ordinate = 3,600.

$$\begin{array}{r} \text{abscissa} \quad 1,2 \\ \hline 4,32 \end{array}$$

This product multiplied by 2 and divided by 3, gives the area = 2,88.

That is, two-thirds of the product of the abscissa and double the ordinate is equal to the required area.

To Find the Perimeter of the given Segment.

136. *Rule.* To one and one-third times the square of the abscissa, add the square of the given ordinate. The square root of the sum, if multiplied by 2, will give the perimeter.

In the example, abscissa = 1,2, and ordinate = 1,80.

$$\text{Abscissa squared} = (1,2)^2 = 1,44$$

$$\text{one-third of } (1,2)^2 = 48$$

$$\text{square of the ordinate} = (1,8)^2 = 3,24$$

$$\hline 5,16$$

$$\text{the square root of } 5,16 = 2,2715$$

$$\hline 2$$

$$\text{Required perimeter} = 4,5430$$

To Find the Hydraulic Mean Depth.

Rule. Divide the area of the segment by the wetted perimeter. The quotient will be the hydraulic mean depth.

$$\text{That is, } \frac{2,880}{4,548} = \text{hydraulic mean depth} = 0,634.$$

To Find the Height and Chord of a Circular Segment.

137. To find the chord corresponding to a circular segment whose area = that of the parabolic segment (see segment No. 10 in table), where area = $a = 1,880$, $\frac{a}{d^2}$ = tabular segment area, opposite tab. ver. sine. This multiplied by the diameter will be the height of the segment.

Here we have $a = 2,880$.

$$d^2 = 4 \times 4 = 16, \text{ and the quotient } \frac{a}{d^2} = 0,18000.$$

Tab. area segment = ,18000.

Corresponding ver. sine = ,280 (by TAB. VII).

$$\begin{array}{r} \hline 4 \\ \text{therefore,} \quad 1,120 = \text{depth or abscissa.} \end{array}$$

To Find the Chord or Ordinate to this depth.

138. Diameter of the circle, 4 feet,
given height or depth of wet segment = $a = 1,12$
remaining or dry segment = $b = 2,88$

$$\hline 1,12$$

$$\text{product} = a b \quad 3,2256$$

the square root of this product will (Euclid III, prop. 35) give the ordinate or half chord = 1,796, and the chord of the segment = $c = 3,592$.

To Find the Perimeter.

139. We have the height of the segment = $a = 1,12$,
 the chord or double ordinate, $c = 3,592$.
 Then by TAB. VI, find the tabular length corresponding to the quotient
 in column tabular length. The tabular number thus found, multiplied by
 the chord, will be the required length.

$$\begin{array}{r} 3,592 \overline{) 1,12} \end{array}$$

quotient, $,3118$,

whose tabular length = $1,2419$,

which multiplied by the chord $c = 3,592$,

will give the product = the required perimeter = $4,461$, and the perimeter
 divided into the given area will give the hydraulic mean depth, $0,645$.

EGG-SHAPED SEWER.

140. The egg-shaped sewer, in appearance, resembles a parabola, and
 is that now generally adopted in the new sewerage of London and Paris
 since 1857.

Let AB (fig 41) = width of sewer at the top. Bisect AB in O , erect
 the perpendicular $OC = AB$.

On AB describe the circle $EADB$, and on DC describe the circle
 $DICK$. Produce AB both ways. Making $AG = BH$ = the total height
 CE , join GF and HF . Produce them to the points I and K . From G
 as centre describe the arc ALI , and from H as centre describe the arc BK .

Let $AB = 4$ feet, then $DC = 2$, and $CE = 6$, and $OC = 4$, and OF
 $= 3$. Also $HB = AG = GI = HK = 6$, and $HA = BG = 2$ ∴
 $HG = 8$. Because $GQ = AG$ ∴ $GQ^2 - GO^2 = OQ^2$.

In this example, $OQ^2 = 6^2 = 36$,

$$OG^2 = 4^2 = 16.$$

The square root of $20 = 4,472 = OQ$.

To Find the $\angle O G Q$ by Trigonometry.

$4,472$ divided by radius $6 = 0,745333$, which is the natural cosine of
 $41^\circ 49' 2''$, and OF divided by $GO = 0,75 =$ nat. tangent of $\angle AGF$
 $= 36^\circ 52'$. (By sec. 69) $d^2 \times n \times ,00218175 = 12^2 \times 36^\circ, 86667 \times$
 $,00218175 =$ area $GAI = 11,5825$. Here $d^2 =$ diameter $= 12$, and n
 $= 36^\circ 52' = 36,86967$.

$$\text{Area of the } \triangle GOF = \frac{GO \times OF}{2} = 2 \times 3 = 6$$

$$\text{Sector } GAI - \triangle GOF = 5,5825.$$

To Find the Sector IFC .

Because the angle $GOF = 90^\circ$, and the angle $OGF 36^\circ 52'$, their
 sum $126^\circ 52'$ taken from 180° will give $\angle FGO = 53^\circ 8'$; but Euclid I,
 prop. 15, the angle $GFO = \angle IFC = 53^\circ 8'$, and $FC =$ radius $= 1$,
 consequently $d^2 = 4$;

And by section 69, $d^2 \times n \times ,00218175 = 0,4636$, etc.;

Or by TAB. V, length of the arc corresponding to the angle $IFC 53^\circ 8'$
 $= 53^\circ, 13333 = 0,927351$. This multiplied by $\frac{1}{2} = \frac{1}{2}$ the radius, will
 give the area $ICF = 0,4636$, etc.

And from above we have the area A I G = 11,5825.

The sum of these two areas = area of the figure G O A I C F G = 12,0461

From this area deduct the $\triangle$ G O F found above, = $\frac{6}{\text{-----}}$

There remains the area of half the sewer below the spring of the arch, 6,0461

This multiplied by 2 gives the area of sewer to the spring of the arch ;
that is, area of A O B K C I = 12,0922

Length of the curve A I may be found by TAB. V.

$\angle$ O G F = $36^\circ 52' = 36^\circ, 86$, length of arc to radius 1 = ,653444

radius G Q = $\frac{6}{\text{-----}}$

arc A I = 3,920664

arc I C from above = 0,927351

length of arc A I C = $\frac{4,848}{\text{-----}}$

2

do. A I C K B = perimeter = $\frac{9,696}{\text{-----}}$

This perimeter, 9,696, if divided into the area, 12,0922, will give the
hydraulic mean depth of the sewer below the spring of the arch = 1,247
feet.

141. *To Find the Diameter of a Circle whose Semicircular Area = 12,0922.*

12,0922

$\frac{2}{\text{-----}}$

Area of required circle = 24,1844

This divided by 0,7854, will give the square of the required diameter =
30,792462, square root = diameter = 5,550. Half of the diameter multi-
plied by 3,1416 = perimeter of semicircle = 8,718. This perimeter
divided into the area 12,0922 = hydraulic mean depth 1,387.

Let us Find a Parabolic Sewer equal in area to 12,0922.

142. Abscissa = 0,806 $\sqrt{a} = 0,806 \sqrt{12,092} = 2,803$. By sec. 133.

Ordinate = $\frac{\sqrt{a}}{1,075} = \frac{3,4774}{1,075} = 3,2344$.

Double ordinate, 6,4688.

Area corresponding to double ordinate 6,4688, and abscissa 2,803 =
12,088.

To Find Perimeter of this Parabolic Sewer.

143. Abscissa squared = $(2,803)^2 =$ 7,856809

one-third of do. = 2,618936

Ordinate squared = $(3,2344)^2 =$ 10,461343

$\frac{20,937098}{\text{-----}}$

The square root of the sum = 4,575

$\frac{2}{\text{-----}}$

Perimeter of wetted parabola = 9,15

This perimeter divided into 12,088, gives H. M. D. = 1,321.

Now we have the following summary :

	Circular Sewer.	Parabolic Sewer.	Egg-shaped Sewer.
Area filled in sewer,	12,0922	12,088	12,0922
Depth of water in sewer,	2,775	2,803	4,000
Hydraulic mean depth of part filled,	1,387	1,321	1,247
Hydrostatic pressure on bottom of sewer = depth of water $\times$ by $62\frac{1}{2}$ lbs. $\times$ sectional area,	2097 lbs.	2271 lbs.	3241 lbs.

Hence it appears that the scouring force, or hydrostatic pressure, is greater in a parabola than in the semicircle, and greater in the egg-shaped sewer than in the parabolic sewer.

And that the hydraulic mean depth, and consequently the discharge, is greater in the parabolic than in the egg-shaped, and greater in the circular than in the parabolic.

The great depths required by the egg-shaped, renders them impracticable excepting where sufficient inclinations can be obtained.

The parabolic segments will give greater hydraulic mean depths than circular or egg-shaped segments, and are as easily constructed as the egg-shaped sewers; therefore, ought to be preferred.

Having so far discussed curvilinear water courses or sewers, we will now proceed to the discussion of

RECTILINEAL WATER COURSES.

144. Let the nature of the soil require that the best slope to be given to the sides be that which makes the $\angle DCA = Q$. Let the required area of the section $ABDC$ be a , and h the given depth, to find the width $AB = x$.

Let $x = AB = EF$, and having the $\angle DCA$, we have its complement $\angle CAE$. By Trigonometry, $h \times \cotangent Q = CE = FD$, and $h \times \cot. Q \times h = h^2 \times \cot. Q = \text{area of the triangles } CEA + \triangle BFD$, and $h \times x = \text{area of the figure } AEFB$; therefore,
 $hx + h^2 \cot. Q = a$,

$$x + h \cot. Q = \frac{a}{h},$$

$$x = \frac{a}{h} - h \cot. Q. \quad \text{A general formula.} \quad (1.)$$

$$\text{Or, } x = \frac{a}{h} - h \tan. \text{comp. } Q. \quad (2.)$$

When the $\angle CAE = 0$ then AC , coincides with AE , and $-h \cot. Q$ vanishes; then

$x = \frac{a}{h}$ = value for rectangular figures, where h the depth is limited, as in the case of canals; but if it were required to enclose the area a in a rectangular figure, open at top, so that the surface will be a minimum.

Here we have $A B = x$, and $A C = B D = \frac{a}{x}$ $\therefore$ perimeter $C A B D =$
 $x + \frac{2a}{x} = \frac{x^2 + 2a}{x}$;

that is, $y = \frac{x^2 + 2a}{x}$, and by differentiating this expression,

$$d y = \frac{2 x^2 d x - x^2 d x - 2 a d x}{x^2} = \frac{x^2 d x - 2 a d x}{x^2}$$

$$\frac{d y}{d x} = \frac{x^2 - 2 a}{x^2} = 0,$$

$$x^2 - 2 a = 0,$$

$$x = \sqrt{2 a} = A B,$$

$$\text{and } \frac{a}{\sqrt{2 a}} = A C = \frac{\sqrt{a} \cdot \sqrt{a}}{\sqrt{a} \cdot \sqrt{2}} = \frac{\sqrt{a}}{\sqrt{2}}. \quad \text{Multiply this by } \sqrt{2};$$

$$\text{then } = \frac{\sqrt{2} \cdot \sqrt{a}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2 a}}{\sqrt{2}} = \frac{1}{2} \sqrt{2 a} = A C.$$

$$\text{But } \sqrt{2 a} = A B.$$

Consequently, $A B =$ twice $A C$, as stated in sec. 19.

Having determined the natural slope from observing that of the adjacent hills—and if no such hills are near, it is to be determined from the nature of the soil,—

Let $A C =$ required slope, making angle n degrees with the perpendicular $A E$; then $C E =$ tangent of angle n to radius $A E$.

Let $s =$ secant of the angle $C A E$; then $A C =$ secant to radius $A E$ and angle n degrees. See fig. 42.

Let $x =$ height of the required section, and $a =$ area of the required section $C A B D$, to find the height x and base $A B$, $n x^2 =$ area of the two triangles $A C E + B F D$, because $C E = n x$, and $A E = x$, $\therefore n x^2 =$ double area of triangle $A C E$.

Now, we have $a - n x^2 =$ area of the rectangle $A B E F$ $\therefore$

$$\frac{a - n x^2}{x} = A B. \quad \text{But } s x = A C, \text{ and } 2 s x = C A + B D;$$

$$\text{therefore, } \frac{a - n x^2}{x} + 2 s x = \text{perimeter } C A B D = \text{a minimum};$$

$$\text{i. e., } \frac{a - n x^2 + 2 s x^2}{x} = \frac{2 s x^2 - n x^2 + a}{x} = \frac{x^2 \cdot (2 s - n) + a}{x}$$

and by differentiating the last expression,

$$\text{we have } d y = \frac{4 s x^2 d x - 2 n x^2 d x + n x^2 d x - a d x}{x^2},$$

$$\text{and } \frac{d y}{d x} = 2 s x^2 - n x^2 - a = 0,$$

$$\text{and } x^2 = \frac{a}{2 s - n},$$

$$\text{and } x = \left(\frac{a}{2 s - n} \right)^{\frac{1}{2}} = A E = \text{height, or required depth.} \quad (3.)$$

When there is no slope, $A C$ coincides with $A E$, and $S = 1$, and $n = 0$;

$$\text{then for rectangular conduits } x = \left(\frac{a}{2} \right)^{\frac{1}{2}} \quad (4.)$$

Example. What dimensions must be given to the transverse profile (or section) of a canal, whose banks are to have 40° slope, and which is to conduct a quantity of water Q , of 75 cubic feet, with a mean velocity of 3 feet per minute?—*Weisbach's Mechanics*, vol. 1, p. 444.

Here we have the $\angle DCA = 40^\circ$, consequently $\angle CAE = 50^\circ$, and the sectional area of figure $CABD = a = 25$ feet.

By formula 3, $x = \left(\frac{a}{2s - n}\right)^{\frac{1}{2}}$ where $s = \secant\ of\ 50^\circ = 1,555724$,
and $n = \text{tangent of } 50^\circ,$ 1,191754.

$$2s = 3.111448$$

$$n = 1,191754$$

1.919694 divided into 25, gives 13,022868,

the square root of which $= x = \text{depth } AE = 3,6087 = 3,609$ nearly,
and tangent $= 1,191754$ if multiplied by $3,609 \times 3,609 = \text{area of the}$
triangles $ACE + BFD = 15,522309$, which taken from 25, will leave
the rectangle $AEFB =$ 9,477691

This divided by the height, 3,609, gives $AB =$ 2,626

But $3,609 \times 1,191754 = CE =$ 4,301

and $FD,$ 4,301

Upper breadth $CD = 11,228$

Bottom AB 2,6260

$1,555724 \times 3,609 = AC =$ 5,6146

and $BD =$ 5,6146

$p = \text{perimeter} = AC + AB + BD =$ 13,8553

which is the least surface with the given slopes, and containing the given
area $= 25$ feet.

The results here found are the same as those found by Weisbach's formula, which appears to me to be too abstruse.

145. From the above, the following equations are deduced:

$$AE = BF = x = \left(\frac{a}{2s - n}\right)^{\frac{1}{2}}$$

$$AC = BF = \left(\frac{a}{2s - n}\right)^{\frac{1}{2}} \cdot s = \left(\frac{as^2}{2s - n}\right)^{\frac{1}{2}}$$

$$AB = \frac{a - nx^2}{1} \times \frac{1/\sqrt{a}}{\sqrt{2s - n}} = (a - nx^2) \frac{\sqrt{(2s - n)}}{\sqrt{a}}$$

146. Hence it appears that the best form of *Conduits* are as follows:

Circular, when it is always filled.

Rectangular, that whose depth is half its breadth.

Triangular, when the triangle is equilateral.

Parabolic, when the depth of water is variable and conduit covered, and
in accordance with section 133.

Rectilinear, when opened, and in accordance with section 144.

For the velocity and discharge through conduits, also for the laying out of canals, and calculating the necessary excavation and embankment, see *Sequel*.

147. TABLE, SHOWING THE VALUE OF THE HEIGHT $AE = x$, in the equation $x = \left(\frac{a}{2s-n}\right)^{\frac{1}{2}}$, where a = area of the given section, having given slopes, and such that the area a is inclosed by the least surface or perimeter in contact. s = secant and n = tangent of the angle DBF , or complement of the angle of repose (see fig. 42).

Ratio of base BG to perpendicular BF.	Angle of repose or angle DBG.	Angle Q or $\angle DBF$.	Value of $x = \left(\frac{a}{2s-n}\right)^{\frac{1}{2}}$ or AE.
Perpendicular 0 to 1	90° 00'	00° 00'	$x = \sqrt{\left(\frac{a}{2}\right)}$
1 to 1	45° 00'	45° 00'	$x = \sqrt{\left(\frac{a}{1,828427}\right)}$
1,5 to 1	23° 41'	66° 49'	$x = \sqrt{\left(\frac{a}{2,745287}\right)}$
2 to 1	26° 34'	63° 36'	$x = \sqrt{\left(\frac{a}{2,472025}\right)}$
2,5 to 1	21° 48'	68° 12'	$x = \sqrt{\left(\frac{a}{2,885318}\right)}$
3 to 1	18° 26'	81° 34'	$x = \sqrt{\left(\frac{a}{6,892288}\right)}$
3,5 to 1	15° 56'	74° 04'	$x = \sqrt{\left(\frac{a}{3,782686}\right)}$
4 to 1	14° 02'	75° 58'	$x = \sqrt{\left(\frac{a}{4,247024}\right)}$
5 to 1	11° 19'	48° 41'	$x = \sqrt{\left(\frac{a}{1,891684}\right)}$
Perfectly dry soil,	38° 49'	51° 11'	$x = \sqrt{\left(\frac{a}{1,947647}\right)}$
Moist soil,	42° 43'	47° 17'	$x = \sqrt{\left(\frac{a}{1,865171}\right)}$
Very dry sand,	30° 58'	59° 02'	$x = \sqrt{\left(\frac{a}{2,220497}\right)}$
Rye seed,	30° 00'	60° 00'	$x = \sqrt{\left(\frac{a}{2,267949}\right)}$
Fine shot,	25° 00'	65° 00'	$x = \sqrt{\left(\frac{a}{2,587897}\right)}$
Finest shot,	22° 30'	67° 30'	$x = \sqrt{\left(\frac{a}{2,812038}\right)}$

Slopes for the sides of canals, in very compact soils, have $1\frac{1}{2}$ base to 1 perpendicular; but generally they are 2 base to 1 perpendicular, as in the Illinois and Michigan Canal.

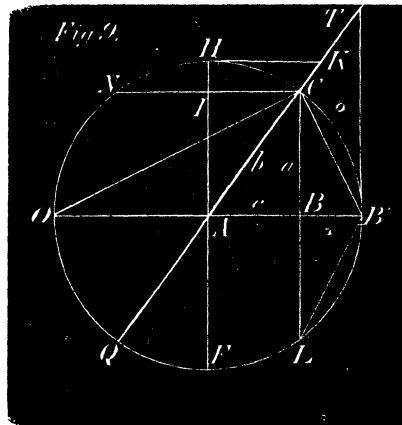
Sea banks, along sea shores, have slopes whose base is 5 to 1 perpendicular for the height of ordinary tides; base 4 to 1 perpendicular for that part between ordinary and spring tides; and slopes 3 to 1 for the upper part. By this means the surface next the sea is made hollow, so as to offer the least resistance to the waves of the sea. The lower part is faced with gravel. The centre, or that part between ordinary and spring tides, is faced with stone. The upper part, called the swash bank, is faced with clay, having to sustain but that part of the waves which dashes over the spring tide line. (See *Embankments*.)

PLANE TRIGONOMETRY.

RIGHT ANGLED TRIANGLES.

148. Let the given angle be $\angle CAB'$ (fig. 9). Let $AB = c$, $CB = a$, and $AC = b$, be the given parts in the right angled triangle ACB .
149. Radius $= AB' = AC$.
150. Sine $\angle CAB' = CB = \text{cosine of the complement} = \cos. \angle ACD$.
151. Cos. $\angle CAB = AB = \text{sine of the comp. of } \angle CAB = \text{sine } \angle ACB$.
152. Tangent $\angle CAB' = BT = \text{cot. of its complement} = \text{cot. } \angle HAC$.
153. Cotangent $\angle CAB' = HK = \text{tan. of its complement} = \text{tan. } \angle HAC$.
154. Secant $\angle CAB' = AT = \text{cosec. of its complement} = \text{cosec. } \angle HAC$.
155. Cosecant $\angle CAB' = AK = \text{sec. of its comp.} = \text{sec } \angle CAH$.
156. Versed sine $\angle CAB' = BB'$.
157. Covered sine $\angle CAB' = HI = \text{versed sine of its complement}$.
158. Chord $\angle CAB' = CB' = \text{twice the sine of } \frac{1}{2} \text{ the } \angle CAB'$.
- 158a. Complement of an angle is what it wants of being 90° .
- 158b. Supplement of an angle is what it wants of being 180° .
- 158c. Arithmetical complement is the log. sine of an angle taken from 10, or begin at left hand and subtract from 9 each figure but the last, which take from 10.
159. Let ACB (fig. 9) represent a right angled triangle, in which $AB = c$, $BC = a$, and $AC = b$, and A, B, C , the given angles.

160. Sine $\angle A = \frac{a}{b}$
161. Cos. $\angle A = \frac{c}{b}$
162. Tan. $\angle A = \frac{a}{c}$
163. Sine $C = \frac{c}{b}$
164. Cos. $C = \frac{a}{b}$
165. Tan. $C = \frac{c}{a}$
166. Sec. $A = \frac{b}{c}$



And the sides can be found as follows:

167. $a = c \tan. A$.
168. $a = b \sin. A$.

d

$$169. a = b \cos. C.$$

$$170. b = c \sec. A = a \sec. < C = \frac{c}{\cos. A} = \frac{a}{\cos. C} = \frac{a}{\sin. A}$$

$$171. c = b \cos. A = b \sin. C = a \tan. C = \frac{b}{\sec. A}$$

Examples. Let A C = the hypotenuse = 480, and the angle at A $53^{\circ} 8'$, to find the base A B and perpendicular A C.

By sec. 168, natural sine of $< A$, 8000 = departure of $53^{\circ} 8'$

$$A C = 480$$

$$B C = a = 384 = \text{product.}$$

Or by logarithms:

$$\text{Log. sine of } < A (53^{\circ} 8') = 8,9031084$$

$$\text{Log. of } b = \text{log. of } 480 \quad 2,6812412$$

$$B C = 384 = 2,5843496$$

And by having the $< A = 53^{\circ} 8' \therefore$ the $< C = 36^{\circ} 52'$.

Nat. sine of $36^{\circ} 52' = ,6000$

$$A C = 480$$

$$A B = 280 = \text{product.}$$

Otherwise,

$$36^{\circ} 52' \text{ Log. sine} = 9,7781186$$

$$\text{Log. of } 480 = 2,6812412$$

$$288 \text{ nearly} = 2,4593598$$

$$\text{or } 287,978 = A B.$$

171a. Let the side B C = a = 384, and the angle C = $36^{\circ} 52'$ be given to find c, b, and the angle A.

$$90^{\circ} - 36^{\circ} 52' = < A = 53^{\circ} 8',$$

$$\text{and } a \tan. C = c, \text{ that is } 384 \times 0,7499 = A B = 288 \text{ nearly.}$$

171b. Let the sides be given to find the angles A and C.

$$\text{Sine } A = \frac{a}{b} (\text{per sec. 160}) = \frac{384}{480} = 0,8000 = 53^{\circ} 8' \text{ nearly.}$$

$$\text{Sec. } A = \frac{b}{c} (\text{per sec. 166}) = \frac{480}{288} = 1,6666 = 53^{\circ} 8' \text{ nearly.}$$

$$\text{Cos. } A = \frac{c}{b} (\text{per sec. 161}) = \frac{288}{480} = 0,6000 = 53^{\circ} 8' \text{ nearly.}$$

$$\text{Tan. } A = \frac{a}{c} (\text{per sec. 162}) = \frac{384}{288} = 1,3333 = 53^{\circ} 8' \text{ nearly.}$$

In like manner the angle C may be found.

These examples are sufficient to enable the surveyor to find the sides and angles.

The calculations may be performed by logarithms as follows:

$$\text{Log. } a = +, \text{ etc.}$$

$$\text{Log. } b = -, \text{ etc.}$$

Sine of angle A

$$\text{Log. sine of } < A.$$

OBLIQUE ANGLED TRIANGLES.

171c. The following are the algebraic values for the four quadrants:

	From 0 to 90.	From 90 to 180.	From 180 to 270.	From 270 to 360
Sine,	+	+	—	—
Cosine,	+	—	—	+
Tangent,	+	—	+	—
Cotangent,	+	—	+	—
Secant,	+	—	—	+
Cosecant,	+	+	—	—
Versed sine,	+	+	+	+

	0°	90°	180°	270°	
Sine,	0	1	0	—1	<i>Note.</i> Here the symbol <i>inf</i> signifies a quantity which is infinitely great.
Cosine,	1	0	—1	0	
Tangent,	0	<i>inf</i>	0	<i>inf</i>	
Cotangent,	<i>inf</i>	0	<i>inf</i>	0	
Secant,	1	<i>inf</i>	—1	<i>inf</i>	
Cosecant,	<i>inf</i>	1	<i>inf</i>	—1	
Versed sine,	0	1	2	1	

172. $a^2 = b^2 + c^2 - 2bc \cdot \cos. A.$

173. $b^2 = a^2 + c^2 - 2ac \cdot \cos. B.$

174. $c^2 = a^2 + b^2 - 2ab \cdot \cos. C.$

Now, from 172, 173, and 174, we find the cosines of the angles A, B, and C.

175. $\cos. A = \frac{b^2 + c^2 - a^2}{2bc}$

176. $\cos. B = \frac{a^2 + c^2 - b^2}{2ac}$

177. $\cos. C = \frac{b^2 + a^2 - c^2}{2ba}$ And by substituting $s = \frac{1}{2}$ the sum of

the three sides $= \frac{1}{2}(a + b + c)$, we find—

178. $\sin. A = \frac{2}{bc} \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$

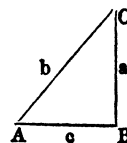
179. $\sin. B = \frac{2}{ac} \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$

180. $\sin. C = \frac{2}{ab} \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$

181. $\cos. \frac{A}{2} = \sqrt{\frac{s \cdot (s - a)}{bc}}$

182. $\cos. \frac{B}{2} = \sqrt{\frac{s \cdot (s - b)}{ac}}$

183. $\cos. \frac{C}{2} = \sqrt{\frac{s \cdot (s - c)}{ab}}$ Also, we find in terms of the tangent—



$$184. \quad \tan. \frac{A}{2} = \sqrt{\frac{(s-b) \cdot (s-c)}{s \cdot (s-a)}}$$

$$185. \quad \tan. \frac{B}{2} = \sqrt{\frac{(s-a) \cdot (s-c)}{s \cdot (s-b)}}$$

$$186. \quad \tan. \frac{C}{2} = \sqrt{\frac{(s-a) \cdot (s-b)}{s \cdot (s-c)}} \quad \text{We can find in terms of sine—}$$

$$187. \quad \sin. \frac{A}{2} = \sqrt{\frac{(s-b) \cdot (s-c)}{bc}}$$

$$188. \quad \sin. \frac{B}{2} = \sqrt{\frac{(s-a) \cdot (s-c)}{ac}}$$

$$189. \quad \sin. \frac{C}{2} = \sqrt{\frac{(s-a) \cdot (s-b)}{ab}}$$

$$190. \quad \text{Radius of the inscribed circle in a triangle} = r = \frac{\sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}}{s} \quad \text{which is the same as given in sec. 48.}$$

$$191. \quad \text{Radius of the circumscribing circle} = R = 4 \left\{ s \cdot (s-a) \cdot (s-b) \cdot (s-c) \right\}^{\frac{1}{2}}.$$

$$192. \quad \text{By assuming } D = \text{the distance between the centres of the inscribed and circumscribed circles, we have } D^2 = R^2 - 2Rr, \text{ and } D = (R^2 - 2Rr)^{\frac{1}{2}}$$

$$193. \quad \text{Area of a quadrilateral figure inscribed in a circle is equal to } \left\{ (s-a) \cdot (s-b) \cdot (s-c) \cdot (s-d) \right\}^{\frac{1}{2}}, \text{ where } s \text{ is equal to the sum of the sides.}$$

Sides are to one another as the Sines of their Opposite Angles.

$$194. \quad a : c :: \sin. A : \sin. C.$$

$$195. \quad a : b :: \sin. A : \sin. B.$$

$$196. \quad b : c :: \sin. B : \sin. C. \quad \text{And by alternando—}$$

$$197. \quad a : \sin. A :: c : \sin. C.$$

$$198. \quad a : \sin. A :: b : \sin. B.$$

$$199. \quad b : \sin. B :: c : \sin. C. \quad \text{And by invertendo—}$$

$$200. \quad \sin. A : a :: \sin. C : c.$$

$$201. \quad \sin. A : a :: \sin. B : b.$$

$$202. \quad \sin. B : b :: \sin. C : c.$$

Having two Sides and their contained Angle given to Find the other Side and Angles.

203. *Rule.* The sum of the two sides is to their difference, as the tangent of half the sum of the opposite angles is to the tangent of half their difference; i. e., $a + b : a - b :: \tan. \frac{1}{2}(A + B) : \tan. \frac{1}{2}(A - B).$

Here a is assumed greater than b . $\therefore$ the $\angle A$ is greater than B .—E. I., 19.
(See fig. 12.)

Now, having half the difference and half the sum, we can find the greater and lesser angles of those required for half the sum, added to half the difference = greater $\angle$, and half the difference taken from the half sum = lesser $\angle$.

When the Three Sides of the Triangle are given to Find the Angles.

205. *Rule.* As twice the base or longest side $AC = b$ is to the other two sides, so is the difference of these two sides to the distance of a perpendicular from the middle of the base; that is, $2b : a + c :: a - c : DE$.

Here BD is the perpendicular, and BE the line bisecting the base; because $BC = a$ is greater than $AB = c$, CD is greater than AD ; because $\angle A$ is greater than $\angle C$, the $\angle ABD$ is less than $\angle CBD$; therefore, the area of the $\triangle CDB$ is greater than $\triangle ADB$; consequently, the base CD is greater than AD .

Let $DE = d$; now the $\triangle ABC$ is divided into two right angled triangles ABD and CBD , having two sides and an angle in each given to find the other angles.

$$\text{In the } \triangle ABD \text{ is given } AD = \frac{b}{2} - d = \frac{b - 2d}{2}$$

$$\text{And } AB = c, \text{ and } BC = a, \text{ and } CD = \frac{b}{2} + d = \frac{b + 2d}{2}$$

$$\text{By sec. 161, } \cos. A = \frac{b - 2d}{2c} \quad \text{And in like manner,}$$

$$\cos. C = \frac{b + 2d}{2a} \quad \text{And by Euclid I. 32, angle } B \text{ is found.}$$

Cosine A may be found by sec. 175, and cosine C by sec. 177.

206. *Example.* Let the $\angle A = 40^\circ$ (fig. 5), $\angle B = 50^\circ$, and the side BC equal to 64 chains, to find the side AC .

By sec. 194, $\sin 40^\circ : 64 \text{ chains} :: \sin 50^\circ : AC$
 Nat. sine $50^\circ = 0.76604$
 Nat. number $= 64$
 Product $= 49.02656$
 Nat. sine $40^\circ = 0.64279$
 Quotient, $76.272 = AC$.

Or thus:
 Log. sine $50^\circ = 9.884254$
 Log. 64 $= 1.806180$
 Sum 11.690434
 Log. sine $40^\circ = 9.882366$
 Dif. 1.882366
 Nat. No. $= 76.272 \text{ chains} = AC$.

In like manner, by the same section, AB may be found, because angles A and B together $= 90^\circ \therefore \angle C = 90^\circ$.

207. In the $\triangle ABC$ (fig. 12), let the angle $A = 40^\circ$, angle $B = 60^\circ$, consequently, $\angle C = 80^\circ$. Let $BC = 64$, to find the side AC .

Nat. sine $60^\circ = 0.86602$	Or thus:	Or thus:
Nat. number $= 64$	Log. sine $= 9.937531$	Log. sine $= 9.937531$
Product, $= 55.42528$	Log. $= 1.806180$	Log. $= 1.806180$
Nat. sine $40^\circ = 0.64279$	Sum $= 11.743711$	Ar. comp. $= 0.191982$
Quotient $86.277 = \text{side } AC$	Log. sine $= 9.808068$	Sum $= 1.983643$
	Dif. $= 1.935643$	$= 86.227$
	Nat. No. $= 86.227 = AC$	$= AC$

AB may be found by sec. 200.

Note. Here ar. comp. signifies arithmetical complement. It is log. sine 40° taken from 10 (see sec. 158 c), or it is the cosecant of 40° .

Given Two Sides and the Contained Angle to Find the Other Parts.

208. *Example.* Let $AC = 120$, $BC = 80$, and $\angle ACB = 40^\circ$, to find the other side, AB , and angles A and B .

By sec. 203, $120 + 80 : 120 - 80 :: \tan. 70^\circ : \text{tangent of the half difference between the angles } B \text{ and } A$.

$$i. e., 200 : 40 :: \tan. 70^\circ : \tan. \frac{1}{2} \text{ dif. } B - A.$$

$$i. e., 5 : 1 :: 2,747477 : 0,549495 = 28^\circ 47'.$$

$$\therefore 70^\circ + 28^\circ 47' = 98^\circ 47' = \angle B.$$

$$\text{And } 70^\circ - 28^\circ 47' = 41^\circ 13' = \angle A.$$

By sec. 194, $\text{sine } 41^\circ 13' : 80 :: \text{sine } 40^\circ : AB$.

Nat. sine 40°	0,64279	Or thus:		Or thus:	
Nat. number 80 =	80	Log. sine	9,808068	Log. sine 40°	9,808068
Product	51,42320	Log.	1,903090	Log. 80	1,903090
Nat. sine $41^\circ 13'$	0,65891	Sum	11,711158	Ar. comp. $40^\circ 13'$	0,181175
Quotient, 78,043 = AB .		Log. sine	9,818825		1,892333
		Dif.	1,892333		= 78,043 = AB .
			= 78,043 = AB .		

Given the Three Sides to Find the Angles.

209. *Example.* $AB = b = 142,02$, $AC = c = 70$, and $BC = a = 104$, to find the angles at A , B and C . (See fig. 5.)

$$\text{By sec. 205, } 284,04 : 174 :: 34 : DE = 20,828$$

$$\text{But } AD = DB = 71,010$$

$$\text{Therefore, } AE = 91,838 = \cos. \angle A \times AC.$$

$$\text{And } BE = 50,182 = \cos. \angle B \times BC.$$

$$\text{Consequently } 50,182 \div 70 = 0,716885 = \cos. \angle A = 44^\circ 12'$$

$$\text{and } 91,838 \div 104 = 0,88305 = \cos. \angle C = 27^\circ 59'$$

*

Having the angles A and C , the third angle at B is given.

Or thus by sec. 175:

$$b^2 = (142,02)^2 = 20169,6804$$

$$a^2 = (104)^2 = 10816,$$

$$\text{sum, } 30985,6804$$

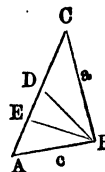
$$c^2 = (70)^2 = 4900,$$

$$2ba = 29540) 26085,6804 \text{ quotient} = 0,88306$$

(Divisor.) (Dividend.)

Which is the cosine of the $\angle C = 27^\circ 59'$

210. Or thus by sec. 183:



$b = 142,02$, $b = 104$, and $a = 70$.

$a = 104$,

$c = 70$,

$2)816,02 = \text{sum.}$

$s = 158,01 = \text{half sum,} \quad \log. = 2,1986846$

$s - c = 88,01, \quad \log. = 1,9445320$

$a = 104, \log. = 2,0170333, \text{ ar. comp. } 7,9829667$

$b = 142,02, \log. 2,1523495, \text{ ar. comp. } 7,8476505$

$2)19,9738338$

$\text{Cos. } \frac{1}{2} < C = 13^\circ 59' 36'' = \log. \text{ sine } 9,986169$

$\therefore \text{ the angle } A = 27^\circ 59' 12''.$

In like manner, $\cos. \frac{1}{2} < B$ may be found by sec. 176.

The same results could be obtained by using the formulas in sections 184 and 188.

HEIGHTS AND DISTANCES.

211. In chaining, the surveyor is supposed to have his chain daily corrected, or compared with his standard. He uses ten pointed arrows or pins of iron or steel, one of which has a ring two inches in diameter, on which the other nine are carried; the other nine have rings one inch in diameter. The rings ought to be soldered, and have red cloth sewed on them. He carries a small axe, and plumb bob and line, the bob having a long steel point, to be either stationary in the bob or screwed into it, thus enabling the surveyor to carry the point without danger of cutting his pocket. A plumb bob and line is indispensable in erecting poles and pickets; and in chaining over irregular surfaces, etc., he is to have steel shod poles, painted white and red, marked in feet from the top; flags in the shape of a right angled triangle, the longest side under; some flags red, and some white. For long distances, one of each to be put on the pole. For ranging lines, fine pickets or white washed laths are to be used, set up so that the tops of them will be in a line. Where a pole has to be used as an observing station, and to which other lines are to be referred, it would be advisable to have it white-washed, and a white board nailed near the top of it.

His field books will be numbered and paged, and have a copious index in each. In his office he will keep a general index to his surveys, and also an index to the various maps recorded in the records of the county in which he from time to time may practice. In his field book he keeps a movable blotting sheet, made by doubling a thin sheet of drawing paper, on which he pastes a sheet of blotting paper, by having a piece of tape, a little more than twice the length of the field book. The sheet may be moved from folio to folio. One end of the tape is made fast at the top edge and back, brought round on the outside, to be thence placed over the blotting sheet to where it is brought twice over the tape on the outside, leaving about one inch projecting over the book. He has offset poles,—one of ten links, decimally divided, and another of ten or six feet, similarly divided, mounted with copper or brass on the ends. One handle of the

chain to have a large iron link, with a nut and screw, so as to adjust the chain when the correction is less than a ring. By this contrivance the chain can be kept of the exact length. Some surveyors keep their chains to the exact standard, but most of them allow the thickness of an arrow, to counteract any deflections—that is allowing one-tenth of an inch to every chain.

In surveying in towns and cities, where the greatest accuracy is required, the best plan is to have the chain of the exact length, and the fore chain bearer to draw a line at the end of the chain, and mark the place of the point at the middle of the handle. Turn the arrow so as to make a small hole, if in a plank or stone; if in the earth, hold the handle vertically, so as to make the mark on the handle come to the side of the arrow next the hind chainman. Where permanent buildings are to be located, surveyors use a fifteen feet pole, made of Norway pine, and decimally marked. This, with the plumb line, will insure the greatest accuracy.

In locating buildings, the surveyor gives lines five feet from the water table, so as to enable cellars or foundations to be dug. When the water table is laid, the surveyor *ought to go on the ground* and measure the distance from the water table and face of the walls from the true side or sides of the street or streets and sides of the lot.

In making out his plan and report of the survey, he ought to state the date, chainmen, the builder and owner of the lot and building, at what point he began to measure, and his data for making the survey. A copy of this he files in his office, in a folio volume of records, and another is given to him for whom the survey has been made, on the receipt of his fees. If any of his base lines used in measuring said land pass near any permanent object, he makes a note of it in his report.

In chaining in an open country, he leaves a mark, dug at every ten chains, made in the form of an isosceles triangle, the vertex indicating the end of the ten chains, or 1000 feet or links. Out of the base cut a small piece about two by four inches, to show that it is a ten chain mark, and to distinguish it from other marks made near crossings of ditches, drains, fences, or stone walls. Some of the best surveyors I have met in the counties of Norfolk, Suffolk and Essex, in England, amongst whom may rank Messrs. Parks, Molton and Encies, had small pieces of wood about six inches long, split on the top, into which a folded piece of paper, containing the line and distance, was inserted. This was put at the pickets or triangular marks made in the ground, and served to show the surveyor where other lines closed.

In woodland, drive a numbered stake at every ten chains. In open country, note buildings, springs, water courses, and every remarkable object, and take minute measurements to such as may come within one hundred feet of any boundary lines, *for future reference*.

In laying out towns and villages, stones 4 feet long and 6 inches square, at least, ought to be put at every two blocks, either in the centre of the streets, or at convenient distances from the corners, such as five feet; the latter would be best, as paving, sewerage, gasworks or public travel would not interfere with the surveyor's future operations. All the angles from stone to stone ought to be given, and these angles referred, if possible, to some permanent object, such as the corner of a church tower, steeple, or brick building; or, as in Canada, refer them to the true meridian.

This latter, although troublesome, is the most infallible method of perpetuating these angles. When the hole is dug for the stone, the position of its centre is determined by means of a plumb line; a small hole is then made, into which broken delf or slags of iron or charcoal is put, and the same noted in the surveyor's report or *proces verbal*. These precautions will forever prevent 99-100th parts of the litigations that now take place in our courts of justice. The office of a surveyor being as responsible as it is honorable, he ought to spare no pains or expense in acquiring a theoretical and practical knowledge of his profession, and to be supplied with good instruments. Where a difference exists between them, it ought to be their duty to make a joint survey, and thus prevent a lawsuit. This appears indispensable when we consider the difficulty of finding a jury who is capable of forming a correct judgment in disputed surveys.

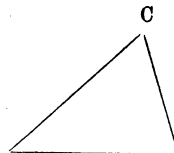
When in woodland, we mark trees near the line, blazing front, rear, and the side next the line, and cutting in the side next the line, a notch for every foot that the line is distant from the tree, which notches ought to be lower than where the trees will be cut, so as to leave the mark for a longer time, to be found in the stumps. State the kind of tree marked, its diameter, and distance on the line. Where a post is set in woodland, take three or four bearing trees, which mark with a large blaze, facing the post. Describe the kind of each tree, its diameter, bearing, and distance from the post. For further, see United States surveying.

In order to make an accurate survey, the surveyor ought to have a good transit instrument or theodolite, as *the compass cannot be relied on, owing to the constant changing of the position of the needle*. By a good theodolite, the surveyor is enabled to find the true time, latitude, longitude, and variation of any line from the true meridian. If packed in a box, covered with leather or oiled canvas, it can be carried with as little inconvenience as a soldier carries his knapsack,—only taking care to have the box so marked as to know which side to be uppermost. The box ought to have a space large enough to hold two small bull's eye lamps and a square tin oil can; this space is about 9 inches by 3. Also, a place for an oil cap covering for the instrument in time of rain or dust; two tin tubes, half an inch in diameter and five inches long; with some white lead to clean the tubes occasionally. These tubes are used when taking the bearing of a line at night, from the true meridian. One of the tubes is put on the top of a small picket, or part of a small tree: this we call the tell-tale. The other is made fast to the end of a pole or picket, and set in direction of the required line, or line in direction of the pole star when on the meridian, or at its greatest eastern or western elongation. Some spider's web on a thick wire, bent in the shape of a horse shoe, about six inches long and two and a half inches wide, having the tops bent about a third of an inch, and a lump of lead or coil of wire on the middle of the circular part. This put in a small box, with a slide a fourth of an inch over the wire, so as to keep the web clean. Have a small phial full of shellac varnish, to put in cross hairs when required. In order to have the instrument in good adjustment, have about two pounds of quicksilver, which put in a trough or on a plate, if you have no artificial horizon. In order to have the telescope move in a vertical position, place the instrument, leveled, so that you can see some remarkable point above the horizon, and reflected in

the mirror or quicksilver. Adjust the telescope so as to move vertically through these points. Mark on the lid of the box the index error, if any, with the sign +, if the error is to be added, and —, if it is to be subtracted.

On the last page of each field book pencil the following questions, which read before leaving home: Have I the true time,—necessary extracts from the Nautical Almanac,—latitude and longitude of where the survey is to be made,—expenses, axes, flags, poles, instrument, tripod, keys, necessary clothing, etc.,—field notes, sketches, and whatsoever I generally carry with me, according to the nature of the survey. It ought to be the duty of one of the chainmen every morning, on sitting to breakfast, to say, "*Wind your chronometer, sir.*" These precautions will prevent many mistakes. The surveyor ought to carry a pocket case filled with the necessary medicines for diarrhoea, dysentery, ague and bilious fever, and some salves and lint for cuts or wounds on the feet; some needles and strong thread, and all things necessary for the toilet; a copy of Simms or Heather on Mathematical Instruments, and McDermott's Manual, and the surveyor is prepared to set out on his expedition. If it so happens that he is to be a few days from home, he ought to have drawing instruments and cart-ridge paper, on which to make rough outlined maps every night, after which he inks his field notes. He makes no erasures in his report or field notes. When he commits an error, he draws the pen twice over it, and writes the initials of his name under it. This will cause his field book to be deserving of more credit than if it had erasures. The surveyor ought to leave no cause for suspecting him to have acted partially.

212. Let it be required at station A (fig. 12) to find the $\angle BAC$, where the points B and C are at long distances from A. Let the telescope be directed to C, and the limb read 0. Move the telescope to B; let the limb now be supposed to read $20^\circ +$. Direct the whole body with the index at $20^\circ +$ on C, clamp the under plate and loosen the upper. Bring the A Fig. 12. B telescope again on B, reading $40^\circ +$. Repeat the same operation, bringing the telescope a third time on B, and reading $60^\circ 23'$, which being three times the required angle, $\therefore$ the $\angle BAC = 20^\circ 7' 20''$.



By this means, with a five inch theodolite, angles can be taken to within twenty or thirty seconds, which is equal to six inches in a mile, if read to twenty seconds. In setting out a range of pickets, one of the cross hairs ought to be made vertical, by bringing it to bear on the corner of a building, on a plumb line suspended from a tree or window. The plumb-bob ought to be in water to prevent vibration. Two corresponding marks may be cut,—one on the Ys and the other on the telescope. These two marks, when together, indicate that the vertical hair is adjusted. Where the surveyor has an artificial horizon or quicksilver, he can, by the reflection of the point of a rod or stake, or any other well defined point, adjust the vertical hair, and then mark the Y and telescope for future operations.

213. All the interior angles of any polygon, together with four right angles, are equal to twice as many right angles as the figure has sides.

<i>Example.</i> Interior angles A, B, C, D, E, F = n°			
4 right angles,			360
Sum	= n° +	360°	
Number of sides = 6 $\therefore$ 6 $\times$ 2 right angles =		1080°	
By subtraction	n°	=	720°

Having the Interior Angles, to Reduce them to Circumferentor Bearings, and thence to Quarter Compass Bearings.

214. Assume any line whose circumferentor bearing is given. *Always keep the land on the right as you proceed to determine the bearings.*

Rule 1. If the angle of the field is greater than 180 degrees, take 180 from it, and add the remainder to the bearing at the foregoing station. The sum, if less than 360 degrees, will be the circumferentor bearing at the present station—that is, the bearing of the next line (forward). But if the sum be more than 360°, take 360 from it, and the remainder will be the present bearing.

Rule 2. If the angle of the field be less than 180, take it from 180, and from the bearing at the foregoing station take the remainder, and you will have the bearing at the present station. But if the bearing at the foregoing station be less than the first remainder to this foregoing bearing, add 360, and from the sum subtract the first remainder, and this last remainder will be the present bearing.

To Reduce Circumferentor Bearings to Quarter Compass Bearings.

Rule 3. If the circumferentor bearings are less than 90, they are that number in the N. W. Quadrant.

Rule 4. If the circumferentor bearings are between 90 and 180, take them from 180. The remainder is the degrees in the S. W. Quadrant.

Rule 5. If the degrees are between 180 and 270, take 180 therefrom, and the remainder is the degrees in the S. E. Quadrant.

Rule 6. If the circumferentor bearing is between 270 and 360, take them from 360, and the remainder is the degrees in the N. E. Quadrant.

Rule 7. 360, or 0, is N., 180 is S., 90 is W., and 270 is E.

These rules are from Gibson's Surveying, one of the earliest and best works on practical surveying. Why so many editions of his Surveying have been published omitting these rules, plainly shows, that too many of our works on Surveying have been published by persons having but little knowledge of what the practical surveyor actually requires.

We will give the same example as that given by Mr. Gibson in the unabridged Dublin edition, page 269 :

The following example shows the angles of the field, and method of reduction. The bearing of the first line is given = 262 degrees.

Stat'n.	Angle Field.		Cir. B.	Q. C. B.
1 A	159			
2 B	200	$200 - 180 = 20, 262 + 20$	$= 282 =$	N. E. 78
3 C	270	$270 - 180 = 90, 282 + 90 = 372, 372 - 360 = 12$	$= 12 =$	N. W. 12
4 D	80	$180 - 80 = 100, 12 + 360 = 372, 372 - 100$	$= 272 =$	N. E. 88
5 E	98	$180 - 98 = 82, 272 - 82$	$= 190 =$	S. E. 10
6 F	100	$180 - 100 = 80, 190 - 80$	$= 110 =$	S. W. 70
7 G	230	$230 - 180 = 50, 110 + 50$	$= 160 =$	S. W. 20
8 H	90	$180 - 90 = 90, 160 - 90$	$= 70 =$	N. W. 70
9 I	82	$180 - 82 = 98, (70 + 360 - 98) = 430 - 98$	$= 332 =$	N. E. 28
10 K	191	$191 - 180 = 11, 332 + 11$	$= 343 =$	N. E. 17
11 L	120	$180 - 120 = 60, 343 - 60$	$= 283 =$	N. E. 77
Sum,	1620			
Add,	360	$180 - 159 - 21, 283 - 21$	$= 262 =$	S. E. 82

$90 \times 11 \times 2 = 1980$, which proves that the angles of the field have been correctly taken. Also finding 262 to be the same as the bearing first taken by the needle, is another proof of the correctness of the work.

215. Having selected one of the sides as meridian, for example, a line that is the most easterly. This may be called a north and south line; the north, or 360, or zero, being the back station, and 180 the forward station. Let the angles, as you proceed round the land, *keeping it on the right*, be A, B, C, D, E, and let the line A B be assumed N and S. A = north and B = south. Then the circumferentor bearing of the line A B from station A, is $= 180^\circ$. If the surveyor begins on the east side of the land, and sets his telescope at zero on the forward station, and then clamps the body, he then turns it on the back station. The reading on the limb will be the interior angle. But if the telescope be first directed to the back station, and then to the forward station, the difference of the readings will be the exterior angle of the field, which taken from 360 will be the interior angle.

The circumferentor is numbered like the theodolite, from north to east, thence south-west, etc., to the place of beginning. But the bearings found by the circumferentor are not the same as those found by the ordnance survey method, where any line is assumed as meridian, as A B.

ORDNANCE METHOD.

216. The following method is that which has been used on the ordnance survey of Ireland:

Assume any line as meridian or base, so as to keep the land to be surveyed *on the left* as you proceed around the tract to be surveyed. Let the above be the required tract, whose angles are at A, B, C, D, E, F, G, H, I, K and L. In taking the interior angles for to determine the circumferentor bearings, the land is *kept on the right*; but by this method the land is *kept on the left*. To determine by this method all the interior angles, we proceed from A to L, L to K, K to I, I to H, H to G, G to F, F to E, E to D, D to C, C to B, and B to A.

Let B to A be the first line, and B the first station. Let the magnetic or true bearing of A to B = S. 82° E.

		Angle.
Let the theodolite at A read on B	= 0	0
on L read	= 159	A = 159°
Theodolite at L read on A	= 159	
on forward K, read	= 279	L = 120
Theodolite at K, read on L back	= 279	
read forward on I	= 110	K = 191
Theodolite at I, read back on K	= 110	
read forward on H	= 192	I = 82
Theodolite at H, read back on I	= 192	
read forward on G	= 282	H = 90
Theodolite at G, read back on H	= 282	
read forward on F	= 152	G = 230
Theodolite at F, read back on G	= 152	
read forward on E	= 252	F = 100
Theodolite at E, read back on F	= 252	
read forward on D	= 350	E = 98
Theodolite at D, read back on E	= 350	
read forward on C	= 70	D = 80
Theodolite at C, read back on D	= 70	
read forward on B	= 340	C = 270
Theodolite at B, read back on C	= 340	
read forward on A	= 180	B = 200

• When at B, 360 was on station A, and 180 on station B. Now when at A, 180 is on B,—a proof that the traverse has been correctly taken.

217. In traversing by the ordnance method where the survey is extensive, it is necessary to run a check-line, or lines running through the survey, beginning at one station and closing on some opposite one. This will serve in measuring detail, such as fields, houses, etc., and will divide the field into two or more polygons, and enable the surveyor to detect in which part of the survey any error has been committed, and whether in chaining or taking the angles. I consider it unsafe for a surveyor to equate his northings and southings, eastings and westings, where the difference would be one acre in a thousand. When the error is but small, *equate or balance* in those latitudes and departures which increase the least in one degree.

DeBurgh's method—known in America as the Pennsylvania method—is as follows:

As the sum of the sides of the polygon is to one of its sides, so is the difference between the northing and southing to the correction to be made in that line.

Half the difference to be applied to each side; as, for example,

Let sum of the sides = 24000 feet, and one of them = 600 feet, whose bearing is N. 40° E.

And that the northings	= 56,20	equated	56,30
And sum of the southings	= 26,40	equated	56,30
		diff. 20 and half dif.	= 10

As 24000 : 600 :: 0,10 : cor. = 0,0025, correction to be added, because the northings is less than the southings.

218. TABLE. *To Change Degrees taken by the Circumferentor to those of the Quartered Compass, and the Contrary.*

Degrees.		Degrees.		Degrees.		Degrees.		Degrees.		Degrees.	
Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.
360	North.	60	N. W. 60	120	S. W. 60	180	South.	240	S. E. 60	300	N. E. 60
1	N. W. 1	61	61 121	59	181	S. E. 1	241	61	301	59	
2	2	62	62 122	58	182	2	242	62	302	58	
3	3	63	63 123	57	183	3	243	63	303	57	
4	4	64	64 124	56	184	4	244	64	304	56	
5	5	65	65 125	55	185	5	245	65	305	55	
6	6	66	66 126	54	186	6	246	66	306	54	
7	7	67	67 127	53	187	7	247	67	307	53	
8	8	68	68 128	52	188	8	248	68	308	52	
9	9	69	69 129	51	189	9	249	69	309	51	
10	10	70	70 130	50	190	10	250	70	310	50	
11	11	71	71 131	49	191	11	251	71	311	49	
12	12	72	72 132	48	192	12	252	72	312	48	
13	13	73	73 133	47	193	13	253	73	313	47	
14	14	74	74 134	46	194	14	254	74	314	46	
15	15	75	75 135	45	195	15	255	75	315	45	
16	16	76	76 136	44	196	16	256	76	316	44	
17	17	77	77 137	43	197	17	257	77	317	43	
18	18	78	78 138	42	198	18	258	78	318	42	
19	19	79	79 139	41	199	19	259	79	319	41	
20	20	80	80 140	40	200	20	260	80	320	40	
21	21	81	81 141	39	201	21	261	81	321	39	
22	22	82	82 142	38	202	22	262	82	322	38	
23	23	83	83 143	37	203	23	263	83	323	37	
24	24	84	84 144	36	204	24	264	84	324	36	
25	25	85	85 145	35	205	25	265	85	325	35	
26	26	86	86 146	34	206	26	266	86	326	34	
27	27	87	87 147	33	207	27	267	87	327	33	
28	28	88	88 148	32	208	28	268	88	328	32	
29	29	89	89 149	31	209	29	269	89	329	31	
30	N. W. 30	90	West. 150	S. W. 30	210	S. E. 30	270	East. 330	N. E. 30		
31	31	91	S. W. 89	151	29 211	31	271	N. E. 89	331	29	
32	32	92	88 152	28 212	32 272	88	332	28			
33	33	93	87 153	27 213	33 273	87	333	27			
34	34	94	86 154	26 214	34 274	86	334	26			
35	35	95	85 155	25 215	35 275	85	335	25			
36	36	96	84 156	24 216	36 276	84	336	24			
37	37	97	83 157	23 217	37 277	83	337	23			
38	38	98	82 158	22 218	38 278	82	338	22			
39	39	99	81 159	21 219	39 279	81	339	21			
40	40	100	80 160	20 220	40 280	80	340	20			
41	41	101	79 161	19 221	41 281	79	341	19			
42	42	102	78 162	18 222	42 282	78	342	18			
43	43	103	77 163	17 223	43 283	77	343	17			
44	44	104	76 164	16 224	44 284	76	344	16			
45	45	105	75 165	15 225	45 285	75	345	15			
46	46	106	74 166	14 226	46 286	74	346	14			
47	47	107	73 167	13 227	47 287	73	347	13			
48	48	108	72 168	12 228	48 288	72	348	12			
49	49	109	71 169	11 229	49 289	71	349	11			
50	50	110	70 170	10 230	50 290	70	350	10			
51	51	111	69 171	9 231	51 291	69	351	9			
52	52	112	68 172	8 232	52 292	68	352	8			
53	53	113	67 173	7 233	53 293	67	353	7			
54	54	114	66 174	6 234	54 294	66	354	6			
55	55	115	65 175	5 235	55 295	65	355	5			
56	56	116	64 176	4 236	56 296	64	356	4			
57	57	117	63 177	3 237	57 297	63	357	3			
58	58	118	62 178	2 238	58 298	62	358	2			
59	59	119	61 179	1 239	59 299	61	359	1			
60	N. W. 60	120	S. W. 60	180	South.	240	S. E. 60	300	N. E. 60	360	North.

218a. *Traverse surveying is to be preferred to triangulation.* In triangulation, the various lines necessary will have to pass over many obstacles, such as trees, buildings, gardens, ponds, and other obstructions; whereas in a traverse survey, we can make choice of good lines, free from obstructions, and which can be accurately measured, and the angles correctly taken, without doing much damage to any property on the land.

In every Survey which is truly taken, the sum of the Northings or North Latitudes is equal to the sum of the Southings or South Latitudes, and the sum of the Eastings or East Departure is equal to the sum of the Westings or West Departure.

219. Let A, B, C, D, E, F, G, H, I, K, be the respective stations of the survey, (see fig. 17b), and NS the meridian, N = north and S = south. Consequently, all lines passing through the stations parallel to this meridian will be meridians; and all lines at right angles to these meridians, and passing through the stations, will be east and west lines, or departures.

Let fig. 17b represent a survey, where the first meridian is assumed on the west side of the polygon.

Here we have the northings = $AB + Bc + Cd + do + RA = RQ$, and the southings = $nF + FG + mI + iL = PL$.

But $RQ = PL$ ∴ the sum of the northings = sum of the southings, and the eastings $Cc + oE + En + Gm$.

But $Cc = Dd + Dh$. Therefore the

eastings = $Dd + Dh + Qn + Gm = QP + Dh$,

and westings = $Dh + LR$; but $LR = QP$, and $Dh = Dh$. Consequently the sum of the eastings is equal to the sum of the westings.

Example 2. Let fig. 17c, being that given by Gibson at page 228, and on plate IX, fig. 1, represent the polygon $abcdefg$. Let a be the first station, b the second, c the third, etc. Let NS be a meridian line; then will all lines parallel thereto which pass through the several stations be also meridians, as ao, bs, cd , etc., and the lines bo, cs, dc , etc., perpendicular to those, will be east or west lines or departures.

The northings are $ei + go + hq = ao + bs + cd + fr$, the southings.

Let the figure be completed,—then it is plain that $go + hq + rk = ao + bs + cd$, and $ei - rk = fr$. If we add $ei - rk$ to the first, and fr to the latter, we have $go + hq + rk + ei - rk = ao + bs + cd + fr$.

i. e., $go + hq + ei = ao + bs + cd + fr$. Hence the sum of the northings = sum of the southings.

The eastings $cs + qa = ob + de + if + rg + oh$, the westings.

For $aq + yo = aq + az = de + if + rg + oh$, and $bo = cs - yo$; therefore $aq + yo + cs - yo = de + if + rg + oh + bo$.

i. e., $aq + cs = bo + de + if + rg + oh$; that is, the sum of the eastings = the sum of the westings.

220. *Method of Finding the Northings and Southings, and Eastings and Westings. (Fig. 17b.)*

	Bearing.	Distance.	Northing.	Southing.	Easting.	Westing.
A B	North	29,18	29,1800			
B C	N. 40° E.	8,00	6,1283		5,1428	
C D	N. 10° W.	9,00	8,8633			1,5629
D E	N. 50° E.	12,00	7,7135	8,6603	9,1925	
E F	S. 30° E.	10,00		17,0000	5,0000	
F G	South	17,00				
G H	East	11,00			11,0000	
H I	S. 20° E.	20,00		18,7938	6,8404	
I K	S. 60° W.	21,00		10,5000		18,1866
K A	N. 80° W.	17,69	3,0726			17,4257
			54,9577	54,9541	37,1752	37,1552

If the above balance or trial sheet showed a difference in closing, we proceed to a resurvey, if the error would cause a difference of area equal to one acre in a thousand. But if the error is less than that, we equate the lines, as shown in sec. 217.

By Assuming any Station as the Point of Beginning, and Keeping the Polygon on the Right, to Find the most Easterly or Westerly Station.

221. Let us take the example in section 220, and assume the station F as the place of beginning (see fig. 17b).

		Easting.	Total Easting.	Westing.	Total Westing.	
F G	South	11,00				
G H			11,00			
H I		6,84	17,84			I = most easterly station.
I K				18,19	18,19	
K A				17,43	35,62	
A B	North					A and B the most westerly stations.
B C		5,14				
C D				1,56		
D E		9,19				
E F		8,66				

Here we see that the point I has a departure east = 17,84
 after which follow west departure to A = 35,62
 Therefore the point A is west of F = 17,78

Then follows E. dep. 5,14, and W. dep. = 1,56, which leaves points A and B west of C, D, E and F. Consequently point I is the most easterly, and points A and B, or line A B, the most westerly.

In calculating by the traverse method, the first meridian ought to pass through the most easterly or westerly station. This will leave no chance of error, and will be less difficult than in allowing it to pass through the polygon or survey. However, each method will be given; but we ought to adopt the simplest method, although it may involve a few more figures in calculating the content. For the first method, see next page.

INACCESSIBLE DISTANCES.

Let A B (Fig. 17a) be a Chain Line, C D, a part of which passes through a house, to find C D.

221a. Find where the line meets the house at C; cause a pole to be held perpendicularly at D, on the line A B; make $D e = C f$; then Euclid I, 34, $f e = C D$.

221b. When the pole cannot be seen over the house, measure any line, A R, and mark the sides of the building; if produced, meet the line A K, in the points i and K. Then by E. VI, 4, $A i : C i :: A K : K D$. K D is now determined. Let C i be produced until $C m = D K$. Measure m K, which will be the length required. Distance C D.

221c. Or, at any points, A and G on the line A B, erect the perpendiculars A O and G' H equal to one another, and produce the line O H far enough to allow perpendiculars to be erected at the points L' and M, making $L B = M N = A O = H G$. $\therefore$ the line B N will be in the continuation of the line A B; and by measuring D N and A C, and taking their sum from O W, the difference will be equal to C D.

222. When the obstruction is a river. In fig. 18, take the interior angles at C and D; measure C D; then $\text{sine } \angle E : C D :: \text{sine } \angle D : C E$. When the line is clear of obstructions to the view, make the $\angle D$ equal to half the complement of the $\angle C$. Then the line $C E = C D$.

As, for example, when the $\angle$ at C is 40° , the half of the complement is $70^\circ = \text{angle at D} = \angle C E D$; consequently (E. I, 5), $C E = C D$. In this case the flagman is supposed to move slowly along the line A B, until the surveyor gives him the signal to halt in direction of the line D E, the surveyor having the telescope making $\angle C D E = 70^\circ$.

223. Or, take (fig. 19) C D perpendicular to A B. If possible, let C D be greater than C E. Take the $\angle$ at D; then, by sec. 167, $C D \times \tan. \angle D = C E$. Or by the chain only (fig. 20), erect C D and K L perpendicularly to A B; make $C F = F D$ and $K L = C D$; produce E F to meet D L in G; then $G D = C E$, the required distance. See Euclid I, prop. 15 and 26.

224. Let A C (fig. 20a) be the required distance. Measure A B any convenient distance, and produce A B, making $B E = A B$; make E G parallel to A C; produce C B to intersect the line E G in F. Then it is evident, by Euclid VI, 4, that $E F = A C$ and $B F = B C$.

225. Let fig. 21 represent the obstruction (being a river). Measure any line A B = c, and take the angles H A C, C A B, and A B C, C being a station on the opposite shore. Again, at C take the $\angle A C G$ and A C B, E being the object. Now, by having the length to be measured from C towards G = C E, E will be a point on the line A F.

By sec. 194 we find A C, and having the angles E A C and A C E, we find (E. I, 32) the $\angle A E C = \angle$ at E. Then $\text{sine } \angle E : A C :: \text{sine } \angle A C E : A E$, and $\text{sine } \angle E : A C :: \text{sine } \angle C A E : C E$; but in the $\triangle C D E$ we have the $\angle$ at D, a right angle, and the $\angle E$ given, $\therefore$ the $\angle E C D$ may be found. Now, C D being given = to the cosine of the $\angle E C D = \text{sine of } \angle E = C D$, we have found A E, C E, and the perpendicular C D; consequently, the line A D E may be found, and continued towards H, and the distances a H, H b, and b D, may be found. $D E = \cos. E \cdot C E$.

f

226. Let the line A F (fig. 22) be obstructed from a to b. Assume any point D, visible from A and C; measure the lines A D and D C; take the angles A C D, C A D, A D C, and C D Y, Y being a station beyond the required line, if possible. In the triangle B C D we have one side C D, and two angles, C B D and C D B, to find the sides C B and D B, which may be found by sec. 194.

227. Or, measure any line A D (fig. 22); take the angle C A D, and make the angle A D G = $180^\circ - \angle C A D$; *i.e.*, make the line D H parallel to A C; take two points in the line A H, such as E and G, so that the lines E B and G F shall be parallel and equal to A D, and such that the line E B will not cut the obstruction a b, and that the lines G F parallel to E B will be far enough asunder from it to allow the line B F to be accurately produced.

As a check on the line thus produced, take the angle F B E, which should be equal to the angle B E D = $\angle C A D$.

228. Let the obstruction on the line A W (fig. 23) be from a to b, and the line running on a pier or any strip of land. At the point C measure the line C D = 800, or any convenient distance, as long as possible; make the $\angle A C D =$ any $\angle$, as 140° , and the interior $\angle C D E =$ any angle, as 130° ; measure D E = 400; make the $\angle D E Y = 70^\circ$, Y being some object in view beyond the line, if possible.

To find the line E B, and the perpendicular E H. In the figure C B E D, we have the interior angles B C D = 40°

$$C D E = 130$$

$$D E Y = D E B = 70$$

$$\underline{240^\circ}$$

$$\text{Let the interior angle } C B E = x^\circ$$

$$\text{Sum, } 240^\circ + x^\circ$$

$$\text{To which add four right angles, } 360$$

$$\underline{600^\circ + x^\circ}$$

$$\text{Should be, by E. I, 32, } 720$$

That is, $600^\circ + x^\circ = 720^\circ \therefore x^\circ = 120^\circ = \angle A B E$; therefore, the angle H B E = 60° .

By E. I, 16, the $A B E = \angle H B E + H E B$, but the angle H B E = $60^\circ \therefore \angle H E B = 30^\circ$; consequently, the interior $\angle D E H = 100^\circ = 70^\circ + 30^\circ$.

Now, we have the interior angles H C D = 40° , bearing N. 40° E.

$$C D E = 130$$

$$D E B = 70$$

$$A B E = 120$$

$$D E H = 100$$

$$C H E = 90$$

The bearings of these lines are found by sec. 218. We assume the meridian A H, making A the south, or 180° , and H the north, or 0° , and keeping the land invariably on the right hand, as we proceed, to find the bearings.

$$\begin{array}{r} 180 \\ 120 \\ \hline 60 \end{array}$$

$$\begin{array}{r} 360 \\ 60 \\ \hline 300 \end{array}$$

$$300 = \text{N. } 60^\circ \text{ E.} = \text{bearing of B E, per quarter compass table.}$$

(See this table, sec. 218.)

180	360
70	110
110	190 = S. 10° E. = bearing of E D.
180	190
130	50
50	140 = S. 40° W. = bearing of D C.
180	140
40	140
140	000 = north = bearing of C B or C H.

Now we have, by reversing these bearings, and finding the northings and southings by traverse table—

Sine.	Chains	Bearing.	Northing.	South.	East.	West.
C D	8,00	N. 40° E.	6,1283 = C d		5,1423	
D E	4,00	N. 10° W.	3,9392 = d H			0,6946
E B		S. 60° W.		x = B H		y = B H
B C		South.		10,0675 — x		
			10,0675	10,0675 — x	5,1423	0,6946 + y

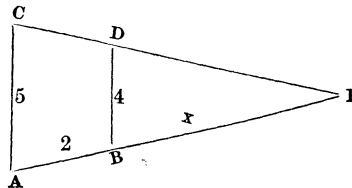
But as the eastings, per sec. 218*a*, is equal to the westings, $y = 5,1423 - 0,6946 = 4,4477 = E H$. Also, from the above, the $\angle H E B = 30$, and the $\angle B H E = 90^\circ$. $\therefore$ we have, in the triangle B H E, given the angles, and side E H, to find E B and B H. For the angle B E H, its latitude or cosine = 0,866, and its sine or departure = 0,500; therefore $E H = 4,4477$, divided by 0,866, gives $5,136 = E B$, and $5,136 \times 0,500 = 2,5680 = B H$; and by taking B H from C H, *i.e.*, $10,0675 - 2,5680 = C B = 7,4995$; and by calling the distances links, we have $C B = 749,95$ links, and $E B = 513,6$.

Note. If, instead of having to traverse but three lines, we had to traverse any number of lines, the line E H, perpendicular to the base A W, will always be the difference of departure, or of the eastings and westings, and B H = difference of latitudes, or of the northings and southings.

229. Chain A C (fig. 25), and at the distance A B, chain B D parallel to A C, meeting the line C E in D; then, by E. VI, 4, and V, prop. D, convertendo, $A E : B E :: A C - B D : B D$. $\therefore$ (E. VI, 16)

$$B E = \frac{A B \times B D}{A C - B D}, \text{ which is a convenient method.}$$

Example. Let B E be required. Let A C = 5, B D = 4, and A B = 2, to find B E. By the last formula, $B E = \frac{2 \times 4}{5 - 4} = 8$ chains.



230. In fig. 26, the line O L is supposed to pass over islands surrounded by rapids, indicated by an arrow. The lines O A, O B, and E F, are measured. From the point B erect the perpendicular B G, and take a point H, from which flag-poles can be seen at O, A, B, C, D, E, and F. Take the angles O H A, A H B, B H C, D H B, E H B, F H B.

The tangents of these angles multiplied by B H, will give the lines B A, O B, B C, B D, B E, B F, and B L.

H B is made perpendicular to O L, and the $\angle O H B$ is given $\therefore$ the angle B O H is given, whose tangent, multiplied by O B, will give the distance B H; consequently, B H multiplied by the tangents of the angles B H C, B H D, B H E, etc., will give the sides B C, B D, B E, etc.

231. If one of the stations, as L, be invisible at H, from L run any straight line, intersecting the line B G in K; take the angle B K L and measure H K; then we have the side B K, and the angle B K L, to find B L in the right angled triangle B K L.

$$\therefore B L = B K \times \tan. \angle B K L.$$

232. But if the line B G cannot be made perpendicular, make the $\angle O B G$ any angle; then having the $\angle O B G$, we have the $\angle L B K$, and having observed the $\angle B K L$, and measured the base B K, we find the distance B L by sec. 131.

In this case we have assumed that B K could be measured; but if it cannot be measured, take the $\angle B O H$ and $\angle O H B$; measure O B; then we have all the angles, and the side O B given in the $\triangle O H B$ to find B H, which can be found by sec. 131. Having B H, measure the remaining part H K.

233. Let the inaccessible distance A B (fig. 27) be on the opposite side of a river. Measure the base C D, and take angles to A and B from the stations C and D, also to D from C, and to C from D. Let $s = C D$, $a = \angle A C B$, $b = \angle B C D$, $c = \angle A D C$, $d = \angle A D B$, $e = \angle C A D$, and $f = \angle C B D$.

$$\text{Sine } e : s :: \text{sine } c : A C.$$

$$\text{Sine } f : s :: \text{sine } b : B D.$$

$$\text{Sine } f : s :: \text{sine } (c + d) : B C.$$

Now having A C and B C, and the included angle, we find (sec. 140) the required line A B.

234. If it be impracticable to measure a line from B (fig. 26), making any angle with the base O L, in order to find the inaccessible distance B C, assume any point H, from which the stations A, B and C are visible.

$$\text{Let } A B = g, B C = x.$$

$$\angle C A H = a = \angle B A H. \quad \angle A H B = c.$$

$$\angle A C H = b. \quad \angle C H B = d.$$

$$\text{Therefore, } \angle A B H = 180 - a - c.$$

$$\text{By sec. 131, sine } c : g :: \text{sine } a : H B = \frac{g \cdot \text{sine } a}{\text{sine } c}$$

$$\text{sine } b : H B :: \text{sine } d : x = \frac{H B \cdot \text{sine } d}{\text{sine } b}$$

Substituting the value of H B in the last equation, we have

$$x = \frac{g \cdot \text{sine } a \cdot \text{sine } d}{\text{sine } c \cdot \text{sine } b} = B C.$$

This formula can be used, by either using the natural or logarithmic sines.

Example. Let $A B = 400$ links = g ,

$$\text{the angle } A H B = c = 60^\circ$$

$$B A H = a = 80^\circ$$

$$\therefore \text{E. I, 32, } A B H = 40^\circ$$

$$C H B = d = 10^\circ \therefore \angle A H C = 70^\circ.$$

$$180 - (B A H + C H B + B H A) = 180 - (80 + 10 + 60) = 30^\circ = A C H = b.$$

$$\text{Log. } g = \text{log. } 400 = 2,6020600$$

$$\text{Log. sine } a = \text{log. sine } 80^\circ = 9,9933515$$

$$\text{Log. sine } d = \text{log. sine } 10^\circ = 9,2396702$$

$$\text{Sum, } 21,8350817$$

$$\text{Log. sine } c = \text{log. sine } 60^\circ = 9,9375306$$

$$\text{Log. sine } b = \text{log. sine } 30^\circ = 9,6989700$$

$$19,6365000$$

$$2,1985811 = 157,98 = B = x.$$

And, as in sec. 163, we have $A B = 400$, and $B C = x = 157,98$, and the included angle $A H C$, the lines $A H$ and $B H$ may be found.

235. Let the land between $C D$ and the river be wood land (see fig. 28). Assume any two random lines, traced from the stations A and B through the wood; let these lines meet at the point C ; trace the lines $C E$ and $E D$ in any convenient direction, so that the point A be visible from E , and the point B visible from the point D ; take the angles $A E C$, $A C E$, $A C B$, $B C D$, and $C D B$, $\therefore$ by E. I, 32, the angles $E A C$ and $C B D$ can be found; and by sec. 131, the sides $A C$ and $C B$ are found; and having the contained angle $A C B$, we find, by sec. 140, the side $A B$.

Note. This case is applicable to hilly countries.

236. The line $A B$ may be found as follows: In direction of the point B (fig. 29) run the random line $P B$, and from A run the lines $A D$ and $A C$ to meet the line $P B$; measure the distance $D C$, and take the angles $A D C = a$, $A C B = c$, $A C D = b$; let the $\angle C A D = d$, and $\angle C A B = e$, and the $\angle A B D = f$. Now, as the angles d , e and f have not been taken, we find them as follows: The angles a and c are given $\therefore$ by E. I, 16, $\angle c = \angle a + \angle d$ $\therefore \angle d = \angle c - \angle a$, and by E. I, 16, we have $\angle b = \angle e + \angle f$, and 180° — the sum of the angles a , d , $e = \angle f$. Now, by sec. 131, $\text{sine } \angle d : D C = s :: \text{sine } \angle a : A C$.

$$\text{i. e., } \text{sine } \angle d : s :: \text{sine } \angle a : \frac{s \cdot \text{sine } \angle a}{\text{sine } \angle d} = A C.$$

$$\text{Also } \text{sine } \angle f : \frac{s \cdot \text{sine } \angle a}{\text{sine } \angle d} :: \text{sine } \angle c : \frac{s \cdot \text{sine } \angle a \cdot \text{sine } \angle c}{\text{sine } \angle d \cdot \text{sine } \angle f} = A B.$$

237. *By the Chain only.* Let it be required to measure the distance $A B$, on the line $O R$ (fig. 30). Measure $A G = G E$ any convenient distances, 50 or 100 links; describe the equilateral triangles $G E D$ and $A G C$ equal to one another; produce $G D$ and $B C$ to meet one another at F ; measure $D F$. Now, because $G F$ and $A C$ are parallel to one another, the $\triangle F D C$ is similar or equiangular to the $\triangle B A C$ (E. VI, 4). $F D : D C :: A C : A B$, but $A C = C D$, because $D C = A C$. $\therefore F D : D C :: D C : A B$, and by E. VI, 16.

$$F D \times A B = D C^2.$$

$$\therefore A B = \frac{D C^2}{F D} = \frac{A G^2}{F D} \text{ which is a convenient formula.}$$

Example. Let $AC = 100$, and $DF = 120$;

$$\text{then } AB = \frac{1000}{120} = 83\frac{1}{3} \text{ links.}$$

This is a practical method, and is the same as that given by Baker in his *Surveying*, London, 1850.

238. The following problem, given by Galbraith in his *Mathematical and Astronomical Tables*, pp. 47 and 48, will be often found of great use in trigonometrical surveying (see fig. 31):

From a convenient station P there could be seen three objects, A , B and C , whose distances from each other were $AB = 8$ miles, $AC = 6$ miles, $BC = 4$ miles. I took the horizontal angles $APC = 33^\circ 45'$, $BPC = 22^\circ 30'$. It is hence required to determine the respective distances of my station P from each object.

Because equal angles stand upon equal or on the same circumferences, the $\angle BPC = \angle DAB$, and $\angle APC = \angle ABD$. In this case the point D is supposed to fall in the original $\triangle ABC$. From this the construction is manifest.

Make the $\angle BAD = \angle ABD$ as above; join C and D , and produce it indefinitely, say to Q ; about the $\triangle ADB$ describe a circle, cutting the line CQ in P ; join A and P , and B and P ; then, by E. III, 21, the $\angle CPB = \angle DAB$, and $\angle APD = \angle ABD$. In this case, the $\angle CPB$ is assumed less than the $\angle CAB$, and the $\angle APB$ less than $\angle ABC$. Now having the three sides of the $\triangle ABC$ by sec. 142, we find the angles A , C and B of the $\triangle ABC$; consequently the $\angle CAD$ is found; also the $\angle CBD$, because, by observation, the $\angle BPC = \angle BAD$, and $\angle ATC = \angle ABC$. In the $\triangle ADB$ are given the side AB and the angles DAB and DBA , to find the sides AD and BD and $\angle ADB$, all of which can be found by sec. 131. Now having the sides AD and AC , and the contained angle BAD , we find (sec. 140) the $\angle ACB$ and the side DC ; and having the angles ACP and ATC given, we find the $\angle CAP$; but above we have found the $\angle CAB$ $\therefore$ the $\angle CAP = \angle CAB = \angle BAP$. In like manner we find the $\angle ABP$; and by sec. 130, and E. I, 32, we find the distances AP and BP . In like manner we proceed to find CP .

COMPUTATION.

$$AC = 6 \text{ miles} = b, \text{ and } APC = 33^\circ 45'.$$

$$CB = 4 = a, \text{ and } CPB = 22^\circ 30'.$$

$$BA = 8 \text{ miles} = c.$$

$$\text{By sec. 125, } \sin \frac{1}{2} A = \left(\frac{(s-b) \cdot (s-c)}{bc} \right)^{\frac{1}{2}}$$

$$\text{Here } s = 9 \text{ miles.}$$

$$b = 6.$$

$$s - b = 3.$$

$$s - c = 9 - 8 = 1.$$

$$(s-b) \cdot (s-c) = 3 \times 1 = 3.$$

$$\text{And } bc = 6 \times 8 = 48; \text{ consequently the value of half the angle } A =$$

$$\left(\frac{3}{48} \right)^{\frac{1}{2}} = \sqrt{\frac{1}{16}} = \frac{1}{4}, \text{ but } \frac{1}{4} = .25 = \sin 14^\circ 28' 39''; \text{ therefore}$$

$$\angle BAC = 28^\circ 57' 18''.$$

By sec. 126, we find $\angle A B C = 46^{\circ} 34' 03''$
 and by sec. 127, $\angle A C B = 104^{\circ} 28' 39''$
 Now we have the $\angle C A B = 28^{\circ} 57' 18''$
 and by observation, the $\angle D A B = 22^{\circ} 30' 00'' = \angle C P B$.
 $\therefore$ the $\angle C A D = 6^{\circ} 27' 18''$
 By observation, we have the $\angle D A B = 22^{\circ} 30' 00''$
 The $\angle D B A = 33^{\circ} 45' 00''$
 Their sum $= 56^{\circ} 15' 00''$
 $\therefore 180^{\circ} - 56^{\circ} 15' = \angle A D B = 123^{\circ} 45' 00''$
 And as the $\angle C A D = 6^{\circ} 27' 18''$, this taken from 180, leaves the $\angle$
 $A D C + \angle A C D = 273^{\circ} 32' 42''$
 and half the sum of these $= 86^{\circ} 46' 21''$
 By sec. 131. As sine $A D B 123^{\circ} 45'$ (arith. complement) $= 0,0801536$
 is, to the side $A B 8$ miles, $\log. 0,9030900$
 so is the sine of the $\angle A B D = 33^{\circ} 45'$ $\log. \text{ sine } 9,7447390$
 to $A D = 5,34543$. $\text{Sum } 0,7279826$
 $A C = 6$, by hypothesis.
 As the sum $= 11,34543$ $\log. 1,0548110$
 is to the difference $0,65457$, $1,8159561$
 so is $\tan. \frac{1}{2} (\angle A D C + \angle A C D) =$
 $86^{\circ} 46' 21''$ $\tan. 11,2487967$
 to the $\tan.$ of half the difference of the
 angles $A D C$ and $A C D$. $16,0099318 = 45^{\circ} 39' 18''$
 $\therefore$ by sec. 140, the $\angle A C P = 41^{\circ} 07' 03''$
 and the $\angle A D C = 132^{\circ} 25' 39''$
 As sine $\angle A P C 33^{\circ} 45'$ $\text{arith. comp. } 0,2552610$
 is to $A C = 6$ miles, $\log. 0,9781513$
 so is $\angle A C P = 41^{\circ} 7'$ $\text{sine } 9,8179654$
 to the distance $A P 7,10195$. $\log. 0,8513777$
 Now we have the $\angle A C B = 41^{\circ} 07' 03''$
 The $\angle A P C = 33^{\circ} 45' 00''$
 Their sum $= 74^{\circ} 52' 03''$
 $180^{\circ} - 74^{\circ} 52' 3'' = \angle P A C = 105^{\circ} 07' 57''$
 By sec. 131, sine $\angle A C P = 41^{\circ} 7' 3''$ $\text{arith. comp. } 0,1820346$
 is to $P A = 7,10195$, $\log. 0,8513777$
 so is sine $\angle P A C = 105^{\circ} 7' 56''$ $\text{sine } 9,9846734$
 to the side $P C = 10,42523$ $\log. 1,0180857$
 We have found the $\angle A B C = 46^{\circ} 34' 03''$
 $\angle B A C = 28^{\circ} 57' 18''$
 Their sum $= 75^{\circ} 31' 21''$, which taken from 180, gives
 the $\angle A C B = 104^{\circ} 28' 39''$.
 But the $\angle A C B$ has been found $= 41^{\circ} 07' 03''$
 $\therefore$ the $\angle B C P = 63^{\circ} 21' 36''$
 and by hypothesis $\angle C P B = 22^{\circ} 30' 00''$
 the sum of the two last angles $= 94^{\circ} 09' 24''$
 $\therefore$ the sine of $\angle C P B = (22^{\circ} 30' 00'')$ $\text{arith. comp. } = 0,4171603$
 is to $B C, 4$ miles, $\log. = 0,6020600$
 so is sine $\angle B C P (63^{\circ} 21' 36'')$ $\text{sine } 9,9512605$
 to $P B, 9,342879$ miles. $\log. 0,9704508$
 Galbraith finds 9,342850 miles by a different method of calculation.

239. *Second Case.* Let us assume the three stations, A, B, W, to be on the same straight, and the angles A P W and W P B to be given (see fig. 31), as in the last example. We find the sides A D and D B. And having the sides A D and A W, and the contained angle, we find the $\angle A D P = \angle A D W$, and the $\angle A P D$ is given by hypothesis $\therefore$ by E. I, 32, we find the $\angle D A P$, and all the angles, and the side A D being given, in the $\triangle A D P$ we can find, by sec. 131, the sides A P and P W. In like manner we find the side P B.

240. *Third Case.* Let us assume the station P to be within the $\triangle A B C$, fig. 32. The $\angle A B D$ is made equal to the supplement of the $\angle A P C$, and the $\angle B A D =$ the supplement of the $\angle B P C$ $\therefore$ as above, we find the sides A D and B D, and having the sides A B, B C, and A C, we find the angles B A C and A B C; consequently, we have the $\angle D A C$. And by sec. 140, we find the angles A D C and A C D, and the $\angle A P C$ being given by hypothesis, $\therefore$ the $\angle C A P$ is found; and by sec. 130, we find the sides P A and P C. In like manner we find the side P B.

Note. When the sum of the two angles at P is 180° , the point P is on the same straight line connecting the stations A, B and C. And when the sum is less than 180° , the point P is without the $\triangle A B C$. When the sum is greater than 180° , the point P is within the $\triangle A B C$.

* 241. In fig. 33, the sum of the angle B P C is supposed $=$ to the sum of the angles C A B + C B A, making the $\angle C A B = \angle C P B$, and the $\angle C B A = \angle A P C$; consequently, the point P is in the circumference of the circumscribing circle about $\triangle A B C$ $\therefore$ the point P can be assumed at any point of the circumference of the segment A P B, and consequently, the problem is indeterminate.

242. The following equation, given by Lacroix in his Trigonometry, and generally quoted by subsequent writers on trigonometry, enables us to find the angles P A C and P B C, and, consequently, the sides A P, C P, and B P. Let $P = \angle A P C$.

Let $a = A C$. $P' = \angle B P C$.

$b = B C$. $R = 360^\circ - P - P' - c$.

$x = \angle P A C$.

$y = \angle P B C$.

$c = \angle A C B$.

$$x = \cot. R \left(\frac{a \cdot \sin P'}{b \cdot \sin P \cdot \cos. R} + 1 \right)$$

$$243. \quad x = \frac{a}{b} (\sin P' \cdot \operatorname{cosec} P \cdot \sec. R \cdot \cot. R + \cot. R)$$

In the problem now discussed, we have

$a = 6$, and $P = 33^\circ 45' 00''$

$b = 4$, and $P' = 22^\circ 30' 00''$

by sec. 238, $104^\circ 28' 39'' = \angle A C B$.

Sum, $160^\circ 43' 39''$

360°

$R = 199^\circ 16' 21''$

$$\text{By sec. 242, } \frac{a}{b} = \frac{6}{4} = \frac{3}{2}$$

From the equation $\cot. x = \cot. R \left(\frac{a \cdot \sin P'}{b \cdot \sin P \cdot \cos. R} + 1 \right)$ (see sec. 242), we have—

$$\begin{aligned} 3 \log. &= 0,4771212 \\ 2 \text{ ar. comp.} &= 9,6989700 \\ P' = 22^\circ 30' \sin &= 9,5828397 \\ P = 33^\circ 45' \text{ ar. comp. sine} &= 0,2552610 \\ R = 199^\circ 16' 21'' \text{ neg. ar. comp. cos.} &= 0,0250452 \\ - 1,09458 \log. &= 0,0392371 \\ + 1, & \\ \hline 0,09458 \log. &= 2,9757993 \end{aligned}$$

$$\cot. R = + 199^\circ 16' 21'' = 10,4563594$$

$$\cot. x, (- 105^\circ 8' 10'') = 9,4321587$$

$$\text{By sec. 131, as sine } 33^\circ 45' \text{ ar. comp.} = 0,2552610$$

$$\text{is to sine } < P A C, (105^\circ 8' 10'') \log. \sin = 9,9846660$$

$$\text{so is } 6 \log. = 0,7781513$$

$$\text{to } P C = 10,4251 \log. = 1,0180783$$

$$\text{By sec. 241, } R - x = y = 199^\circ 16' 21'' - 105^\circ 8' 10'' = 94^\circ 8' 11''.$$

By sec. 131, we can find the lines $A P$ and $P C$.

Note. — $0,09458 \times$ by $+ 199^\circ 16' 21'$, gives a negative product; $\therefore$ the cot. is negative, and the arc is to be taken from 180 , by sec. 108a.

REDUCTION TO THE CENTRE.

244. It frequently happens in extensive surveys that we take angles to spires of churches, corners of permanent buildings, etc. From such points, angles cannot be taken to those stations from which angles were observed. Let C (fig. 34) be the spire of a church. Take any station D , as near as possible to observed station C , from which take the $\angle C D B = D$. Let $\log. \sin 1'' = 4,6855749$; let $\angle C D A = a$, $A D B = b$, and the distance $C D = g$, and $\angle A C B = x$;
then $x = b + \frac{g \sin (b + a)}{B C \cdot \sin 1''} - \frac{g \cdot \sin a}{A C \cdot \sin 1''}$

Great care is required in taking out the sine of the sine of the angles $(a + b)$, and sine of a . The first term, $\frac{g \cdot \sin (b + a)}{B C \cdot \sin 1''}$, will be positive when $(a + b)$ is less than 180° , and the sine of a will be negative.

245. Let A be a station in a ravine, from which it is required to determine the horizontal; distance $A H$ the height of the points D and C above the horizontal line $A H$ (fig. 35).

Trace a line up the hill in the plane of $A D H$, making $A B = g$ feet $= 600$; take the angles $\angle C A H = 3^\circ 10'$, $\angle D A H = 5^\circ 20'$.

$$\text{Therefore } \angle C A D = 2^\circ 10'$$

$$\angle G A B = \angle E B A = 2^\circ 7'$$

$$\text{and } \angle C B E = 1^\circ 7'$$

$$\angle A H C = 90^\circ 0'$$

$$\angle A C H = 86^\circ 50'$$

$$\angle A D C = 84^\circ 40'$$

$$\text{In the triangle } A B C \text{ are given } A B = 600.$$

$$\text{The } \angle A B C = \angle E B A + \angle C B E = 3^\circ 14'$$

$$\text{The } \angle B A C = 180^\circ - \angle C A H - \angle B A G = 174^\circ 43'$$

$$\text{Consequently, } \angle A C B = 2^\circ 3'$$

g

By sec. 131, the sides AC and BC may be found.

And $AC \cdot \cos. CAH = AH$.

And $AC \cdot \sin. CAH = HC$.

And $HA \cdot \tan. CAH = HD$. And by taking the $\angle CBD$, and multiplying its tangent by the line BC , we find the line DC , which added to HC , will give the line HD .

Otherwise,

We have the angles DAC , CAH , and angle at H a right angle.

$180 - 90 - \angle CAH = \angle ACH = 86^\circ 50' = \angle ADC + \angle CAD$.
But $\angle CAD$ being $2^\circ 10'$, $\therefore \angle ADC = 84^\circ 40'$, and $\angle CAD = 2^\circ 10'$, and the side AC may be found; and by sec. 131, CD can be found.

As sine $2^\circ 3'$ ($\angle BCA$)	arith. comp. = 1,4464614
is to AB (600),	log. = 2,7781513
so is sine $3^\circ 14'$ ($\angle ABC$)	log. sine = 8,7512973
to $AC = 946,04$,	log. = 2,9759100
Sine $3^\circ 10'$ ($\angle CAH$)	= 8,7422686
CH 52,26	log. = 1,7181686
Also log. AC	= 2,9759100
Cosine ($\angle CAH = 3^\circ 10'$)	= 9,9993364
$AH = 944,597$	log. = 2,9952464
Tangent ($\angle HAD = 5^\circ 20'$)	= 8,9701350
$HD = 88,182$	log. = 1,9453814
$CH = 52,26$.	
$\therefore CD = 35,922$.	

Or, CD may be found as follows:

As sine ($ADC = 84^\circ 40'$)	arith. comp. 0,0018842
is to the log. AC from above,	2,9759100
so is sine ($\angle DAC = 2^\circ 10'$)	sine 8,5775660
to $CD = 35,922$	log. 1,5553602

INACCESSIBLE HEIGHTS.

246. When the line AB is in the same horizontal plane (fig. 37), required the height BC .

$AB \cdot \tan. \angle CAB = BC$.

247. Let the point B be inaccessible (see fig. 37a). Measure $AD = m$ in the direction of B ; take the $\angle CAB = f$, and $CD B = g$; then, by E. I, 16, $ACD = g - f = h$; and, by E. I, 32, $\angle BCD = 90^\circ - g = k$.

By sec. 131, $CD = \frac{m \cdot \sin f}{\sin h}$

$BC = \frac{m \cdot \sin f \cdot \sin g}{\sin h}$

$DB = \frac{m \cdot \sin f \cdot \cos. g}{\sin h}$

248. Let the inaccessible object C E be on the top of a hill, whose height above the horizontal plane is required (fig. 38).

$$\begin{aligned} \text{As in sec. 246, let } \angle C A B &= f &= 44^\circ 00' \\ &\angle C D B = g &= 67^\circ 50' \\ \text{and E. I, 16, } \angle A C D &= g - f = h &= 23^\circ 50' \\ &\angle E D B = k &= 51^\circ 00' \\ &\angle B C D = p &= 22^\circ 10' \end{aligned}$$

And the horizontal distance A D = m = 134 yards.

$$\text{By sec. 246, } C D = \frac{m \cdot \sin f}{\sin h}$$

$$B C = \frac{m \cdot \sin f \cdot \sin g}{\sin h}$$

$$B D = \frac{m \cdot \sin f \cdot \cos g}{\sin h} = B C \cdot \tan. \angle B C D.$$

And by substituting the value of B C, we have—

$$B D = \frac{m \cdot \sin f \cdot \sin g \cdot \tan. p}{\sin h}$$

$$B E = \frac{m \cdot \sin f \cdot \cos g \cdot \tan. k}{\sin h}$$

$$* \text{ or, } B E = \frac{m \cdot \sin f \cdot \sin g \cdot \tan. p \cdot \tan. k}{\sin h}.$$

Now having B C and B E

given, their difference, C E, may be found.

m = 134 yards,	log.	2,1271048
f = 44° 00'	log. sine	9,8417713
g = 67° 50'	log. sine	9,9666533
h = 23° 50'	cosec. (ar. comp.)	0,3935353
B C = 213,36 yards	log.	2,3290649
< B C D = p = 22° 10'	tan.	9,6100359
< B D E = k = 51° 00'	tan.	10,0916308
B E = 107,33 yards	log.	2,0307314
B C = 213,36.		

∴ C E = 106,03 = height required over the top of the hill.

Note. I have used the formula or value of B E, marked *, which is very convenient. The data of this problem is from Keith's Trigonometry, chap. iii, example 37.

249. Let B C be the height required, situated on sloping ground A B (see fig. 39). At A and D take the vertical angles C A F = a, equal the angle above the horizontal line A F.

$$\begin{aligned} \angle C A B &= f. \\ \angle C D B &= k. \\ \angle A C D &= h = \angle B D C - \angle C A B. \\ \angle A C B &= i = 90^\circ - \angle C A F. \\ \angle F A B &= b. \\ \angle A D &= m, \text{ and } D B = n, \therefore A B = m + n. \\ B F &= (m + n) \cdot \sin b. \\ A F &= (m + n) \cdot \cos b. \\ C F &= (m + n) \cdot \cos b \cdot \tan. a. \end{aligned}$$

Second Method.

250. Measure on the slope A B the distance A D = m; take the $\angle$ C A B = f, and the vertical angles E D B = p and $\angle$ C D E = q.

$$C D = \frac{m \cdot \sin f}{\sin h}$$

$$C E = \frac{m \cdot \sin f \cdot \cos. q}{\sin h}$$

$$D E = \frac{m \cdot \sin f \cdot \cos. q}{\sin h}$$

$$B E = \frac{m \cdot \sin f \cdot \cos. q \cdot \tan. p}{\sin h}$$

Consequently C E — B E = C B.

In this case the distance B D is assumed inaccessible.

Third Method.

251. Having found C D = $\frac{m \cdot \sin f}{\sin h}$, we measure on the continuation of the slope D B = n, making the $\angle$ E D B = as above = p, and the $\angle$ E D C = q. We find B E = n · sine b.

$$C E = \frac{m \cdot \sin f \cdot \sin q}{\sin h}$$

$$\therefore B C = \frac{m \cdot \sin f \cdot \sin q}{\sin h} - n \cdot \sin b.$$

252. Let the land, from A towards B, be too uneven and impracticable to produce the line B A (see fig. 39).

Measure any line, as A G = m; take the horizontal $\angle$ C G A = a.

$$\angle C A G = b.$$

$$\text{Then } 180^\circ - a - b = x$$

$$= \angle A C G = c.$$

Let the vertical angle C A F = o.

$$\angle C A B = f.$$

$$\angle B A F = l.$$

$$\text{By sec. 131, } A C = \frac{m \cdot \sin a}{\sin c}$$

$$C F = \frac{m \cdot \sin a \cdot \sin o}{\sin c}$$

$$B F = \frac{m \cdot \sin a \cdot \cos. o \cdot \tan. b}{\sin c}$$

Consequently, C F — B F = B C = the required height.

Example. Let $\angle a = 64^\circ 30'$ $\angle o = 58^\circ$
 $\angle b = 72^\circ 10'$ $\angle l = 33^\circ$
 $\angle c = 43^\circ 20'$ m = 52 yards, to find C B.

To find C F. We have from this article C F = $\frac{m \cdot \sin a \cdot \sin o}{\sin c}$

m = 52 yards,	log.	1,71600
a = 64° 30'	log. sine	9,95549
o = 58° 00'	sine	9,92842
c = 43° 20'	ar. comp.	0,16352

$$C F = 58,1 \quad \log. \quad 1,76343$$

To find the height B F. We find the value of B F by the last equation of this article.

m =	1,71600
< a	sine 9,95549
< o	cosine 9,72421
< c	ar. comp. 0,16352
< l	tan. = 9,81252
B F = 23,536,	log. = 1,36174
∴ 58 - 23,536 = 34,464 yards = B C.	

253. At sea, at the distance of 20 miles from a lighthouse, the top of which appeared above the horizon; height of the observer's eye above the sea, 16 feet. Required, the height of the lighthouse above the level of the sea. Here 16 feet = 0,003 miles.

Assuming the circumference of the earth 25020 miles, and its semi-diameter 2982 miles.

As 417 : 120 :: 20 miles : $0^{\circ} 17' 16''$ nearly = $\angle B C D$.

And because the angle at D is right angled,

$90 - 0^{\circ} 17' 16'' = 89^{\circ} 42' 44'' = \angle C B D$.

∴ by sec. 131, as sine $\angle B : C D :: \text{rad.} : B C$.

= 3982,003 = C D,	log. = 3,6001013
	rad. = 10
	13,6001013
89° 42' 44''	log. sine = 9,9999945
	3,6001068
	B C = 3982,05
	A C = 3982
	A B = ,05 miles.
	5280
A B = 264 feet,	26400

By sec. 107, $\angle C D \cdot \sec. \angle B C D = B C$. But as the secant in small angles change with little differences, it would be unsafe to use it. In this example, $\angle B C D = 0^{\circ} 17' 16''$, the secants 17' and 18' show no difference for 1'.

254. When the altitude is 45° , the error will be the least possible; in, which case 1' would make an error of $\frac{1}{1718}$ part of the altitude; and generally the error in altitude is to the error committed in taking the altitude, as double the height is to double the observed angle.—*Keith's Trigonometry, chap. iii., example xxix.*

TRAVERSE SURVEYING.

255. Let the figure A, B, C, D, E, F and G (see fig. 17d) be the polygon. This is the same figure given by Gibson on plate 9, fig. 3. Let S N be a meridian assumed west of the polygon; let A W = meridian distance of the point A from the assumed meridian; then M B = mer. dist. of the point B, N C = mer. dist. of point C, D Z = mer. dist. of point D, T E = mer. dist. of E, Q F = mer. dist. of the point F, and G S¹ = mer. dist. of G. Let Y I = mer. dist. to middle of A B, O K = mer. dist. to the middle of B C, L L¹ = mer. dist. to middle of C D, X M = mer. dist. to middle of D E, R R² = mer. dist. to middle of E F, P a = mer. dist. to middle of F G.

It also appears that $W M$ = northing of $A B$, $M N$ = the northing of $B C$, $N Z$ = southing of $C D$, $Z T$ = southing of $D E$, $Q F$ = southing of $E F$, and $Q S^1$ = the northing of $F G$.

By the method of finding the areas of the trapeziums (sec. 24), we have as follows:

	North Area.	South Area.
$W M \cdot Y I$ = area of $A B M W$ =	$W M \cdot Y I$	
$M N \cdot O K$ = area of $B C N M$ =	$M N \cdot O K$	
$N Z \cdot L L^1$ = area of $C D Z N$ =		$N Z \cdot L L^1$
$Z T \cdot M X$ = area of $D E T Z$ =		$Z T \cdot M X$
$T Q \cdot R R^1$ = area of $E F Q T$ =		$T Q \cdot R R^1$
$Q S^1 \cdot P a$ = area of $F G S Q$ =	$Q S^1 \cdot P a$	

Hence appears the following rule, which is substantially the same as Gibson's Theorem III, section v:

256. *Rule.* Multiply the meridian distance taken in the middle of every stationary or chain line by the particular northing or southing of that line.

Put the product of southings in the column of south areas, and the product of northings in the column of north areas. The difference of the area columns will be the required area of the polygon; to which add the offsets, and from the sum take the inlets. The remainder will be the area of the tract which has been surveyed.

To Find the Numbers for Column B, entitled Meridian Distance.

257. Let $A W$ (fig. 17*d*) represent the first number—viz., 61,54 chains, and $N Q$ the first meridian line; and since the map is on the east side of this meridian, all those lines that have east departure will lie farther from the first meridian than those that have west departure; therefore, knowing the length of the line $A W$, the length of the other lines, $I Y$, $B M$, etc., may be found by adding the eastings and subtracting the westings.

The first meridian is supposed to be the length of the whole departure, or the entire easting or westing from the first station; for should the first station be at the easternmost point of the land, the first meridian will then pass through the most westerly point, and the map will entirely be on the east of the first meridian.

But if the meridian distance be assumed less than the whole easting or westing from the most easterly point of the land, then it is plain that the first meridian will pass through the polygon or map, and that part of the land will be east and part west of that meridian. In this case, in that part which would be east of the meridian, we would add the eastings and subtract the westings; but in that part west of the meridian, we would add the westings and subtract the eastings.

In method 1, the sum of all the east departures is assumed as the first meridian distance.

In method 2, the first meridian is made to pass through the most westerly station.

In method 3, the first meridian is made to pass through the most northerly station of the polygon, as station E (see fig. 17*b*).

258. METHOD I.—Commencing Column B with the Sum of all the East Departures (see fig. 17b).

Bearing.	Dist.	N. lat.	S. lat.	E. dep.	W. dep.	A or lat., and $\frac{1}{2}$ dep.	B or mer. dis.	N. Area.	S. Area.
	Ch'ns.						37,1752		
North.	29,18	29,178		0,0000		N. 29,178	37,1752 E.	1084,6979	
N. 40° E.	8,00	6,128		5,1423		0,000	37,1752 E.		
N. 10° W.	9,00	8,863			1,5629	N. 6,128	39,74635 E.	243,5659	
N. 50° E.	12,00	7,714		9,1925		E. 2,57115	42,3175 E.		
S. 30° E.	10,00		8,661	5,0000		N. 8,863	41,53605 E.	368,1336	
South.	17,00		17,001			W. 0,78145	40,7546 E.		
East.	11,00		East. 11,0000			N. 7,714	45,35085 E.	349,8376	
S. 20° E.	20,00		18,794	6,8404		E. 4,59625	49,9471 E.		
S. 60° W.	21,00		10,500		18,1866	S. 8,661	52,4471 E.		454,2445
N. 80° W.	17,694	3,073			17,4257	E. 2,5000	54,9471 E.		
						S. 17,001	54,9471 E.		984,1556
						9,0000	54,9471 E.		
						0,0000	60,4471 E.		
						E. 5,5000	65,9471 E.		
						S. 18,794	69,3673 E.		1303,6890
						E. 3,4202	72,7875 E.		
						S. 10,500	63,6942 E.		668,7891
						W. 9,0933	54,6009 E.		
						N. 3,073	45,888 E.	141,0138	
						W. 8,71285	37,1752 E.		
								2187,2488	3360,8780
									2187,2488
									1173,6232

In column A, the top line of each pair is the north or south latitude, and the under number is half the corresponding departure.

In column B, the sum of all the east departures is assumed as the first meridian distance, thus making the first meridian to be west of the most westerly station.

The meridian distance is found by adding half the eastings twice, and subtracting half the westings twice. These give the meridian distances at half the lines.

Example. The first line is N. lat. 29,178, and departure = 0, $\therefore$ 0

added to 37,152 gives the meridian distance = 37,152, and $37,152 + 0 = 37,152 =$ lower number of the first pair in column B. The next half departure is = 5,57115 east, $\therefore 2,57115 + 37,152 =$ meridian distance = 39,7463; add 2,57118 to 39,7463; it will give the under line of second pair = 42,3175. From 42,3175 take half the next departure, 0,78145, and it gives meridian distance = 41,53605, etc., always adding the eastings and subtracting the westings.

The product of the upper numbers in columns A and B will give the areas. If the upper number in column A is north latitude, the product is put under the heading, north area; but if the upper number in column A be south latitude, then the product is put under the heading, south area.

Having found the last number in column B to agree with the first meridian distance at top, is a proof that the calculation is correct.

The difference between the north area and south area columns determine the area of the given polygon in square chains.

The area could be found in like manner by assuming the principal meridian east of the polygon, and adding the *westings*, or *west departures*, and subtracting the *eastings*, or *east departures*.

Area = $117 \frac{6629}{10000}$ acres.

259. METHOD II.—*The First Meridian passes through the Most Westerly Station (see fig. 17b).*

Bearing.	Dist.	N. lat.	S. lat.	E. dep.	W. dep.	A or lat., and $\frac{1}{2}$ dep.	B or mer. dist.	N. Area.	S. Area.
							0,0000		
North.	29,178	29,178		0,0000		N. 29,178	0,0000		
N. 40° E.	8,00	6,128		5,1423		0,000	0,0000		
N. 10° W.	9,00	8,863			1,5629	N. 6,128	2,57115 E.	15,7563	
N. 50° E.	12,00	7,714		9,1925		E. 2,57115	5,14230 E.		
S. 30° E.	10,00		8,661	5,0000		N. 8,863	4,36085 E.	38,6507	
South.	17,00		17,001	0,0000		W. 0,78145	3,57940 E.		
East.	11,00		East.	11,0000		N. 7,714	8,17565 E.	63,0673	
S. 20° E.	20,00		18,794	6,8404		E. 4,59625	12,77190 E.		
S. 60° W.	21,00		10,500		18,1866	S. 8,661	15,27190 E.		132,2699
N. 80° W.	17,694	3,0730			17,4257	E. 2,5000	17,77190 E.		
						S. 17,001	17,77190 E.		302,1401
						0,0000	17,77190 E.		
						0,0000	22,27190 E.		
						E. 5,5000	28,77190 E.		
						S. 18,794	32,19210 E.		605,0183
						E. 3,4202	35,61230 E.		
						S. 10,500	26,51900 E.		278,4495
						W. 9,0933	17,42570 E.		
						N. 3,0730	18,71285 E.	26,7747	
						W. 8,71285	10,0000		
								144,2490	1317,8768

North area = 144,2490

Area of the polygon = 117,36288 acres.

By first method = 117,36292 acres.

By second method = 117,36288 acres.

This is satisfactory proof.

Note. The surveyor ought to adopt some uniform system, as by this means he will be in less danger of committing errors. I have invariably made the principal meridian pass through the most westerly station of the polygon according to this method, and checked it by the third method, thereby making one method check the other. Making the first meridian pass through the polygon requires less figures, but more care in passing from east to west, and *vice versa*; also in entering the areas in their proper columns, as sometimes the north area is to be put in the south area columns, and the contrary. But in the first and second methods, the north area is always put in north area column, and the south area in south area column.

260. METHOD III.—*The First Meridian passes through the Most Northern Station of the Polygon, as through Station E (see fig. 17b).*

Bearing.	Dist.	N. lat.	S. lat.	E. dep.	W. dep.	A or lat., and $\frac{1}{2}$ dep.	B or mer. dist.	N. Area.	S. Area.
							0,000		
S. 30° E.	10,00		8,661	5,0000		S. 8,661	2,5000 E.		21,525
South.	17,00		17,001	0,0000		E. 2,5000	5,0000 E.		
East.	11,00		0,000	11,0000		S. 17,001	5,0000 E.		85,0050
S. 20° E.	20,00		18,794	6,8404		0,0000	5,0000 E.		
S. 60° W.	21,00		10,500		18,1866	0,0000	10,5000 E.		
N. 80° W.	17,694	3,0730			17,4257	E. 5,5000	16,0000 E.		
North.	29,178	29,178				S. 18,794	19,4202 E.		364,9832
N. 40° E.	8,00	6,128		5,1423		E. 3,4202	22,8404 E.		
N. 10° W.	9,00	8,863			1,5629	S. 10,500	13,7471 E.		144,3446
N. 50° E.	12,00	7,714		9,1925		W. 9,0933	4,6538 E.		

In this method, everything is the same as in methods 1 and 2, except finding the areas.

Rule. The north or south multiplied by their respective east meridian distances, are put in their respective columns of areas, as in methods 1 and 2; but north and south latitudes multiplied by their respective west meridian distances, are put in contrary area columns. That is, S. lat. $\times$ E. mer. dist. is put in south area column; N. lat. $\times$ E. mer. dist. is put in north area column; S. lat. $\times$ W. mer. dist. is put in north area column; N. lat. $\times$ W. mer. dist. is put in south area column.

Area in acres = 117,3687
Second method = 117,3629
First method = 117,3629

The proof of the above rule will appear from the following (see fig. 17b). Draw the meridian E W through the point or station E; let p F, g H, r D, s K, R s, C w, and D x, be the departures respectively.

$$n F \times \frac{1}{2} F p = \text{south} \times \text{by east} = a$$

$$F G \times \frac{1}{2} (F p + G q) = \text{south} \times \text{by east} = a'$$

$$m I \times \frac{1}{2} (H q + I r) = \text{south} \times \text{by east} = a''$$

$$I L \times \frac{1}{2} (I r + K L) \text{ south} \times \text{by east} = a'''$$

This includes figure I r v K + $\triangle$ V K S, S K being the east meridian distance of K; then S K — $\frac{1}{2}$ (K A) = mer. dist. of the middle of the line A K, which is — or east, if S K is more than $\frac{1}{2}$ A K; but if S K is more than $\frac{1}{2}$ A K, then the meridian distance will be + or east, and if the mer. dist. S K is equal to $\frac{1}{2}$ A K, then the mer. dist. of line K A = o.

$\frac{1}{2} h$

North Area Column.	South Area Column.
--------------------------	--------------------------

a

a'

a''

a'''

b

b'

b''

b'''

We now suppose that SK is less than KA ; therefore mer. distance to middle of $KA = SK - \frac{1}{2} Ag =$ west or negative, and $(SK - \frac{1}{2} Ag) \cdot gK =$ figure $gKsy - \triangle AgK =$ figure $gKvy + Kvs - \triangle AgK$; but the meridian distance being negative, $\therefore$ the product must be negative; that is, the above product $\triangle AgK - gKvy + Kvs$, which is equal to the $\triangle Ayv$, because we have to deduct $gKvy + Kvs$, which have been including the figure $Kirs$; consequently north by west is to be added or put in south area column. Let this area be equal to b , and entered in the south area column. The mer. dist. of A is the same as that of B , and is found by adding $\frac{1}{2} Ag$ to the last mer. dist. to the middle of AK . That mer. dist. $\times$ by AB , gives an area to be added $=$ figure $gABb = b'$, which is put in south area column. Also the mer. dist. in middle of BC is west, which multiplied by BC , will give the area $BCwb = b''$, which put in south area column. In like manner we find the area $C Dxw = b'''$, which put in south area column; and the area of DEx is west of the meridian b'''' , and is to be put in south area column.

Hence it appears that those areas derived from east meridian distances are put under their respective heads, S. and N.; but those having west meridian distances, are put in their contrary columns.

261. *Calculating the Offsets and Inlets. (See fig. 17e.)*

Line 1.	Base.	Sum of offsets	Double area. add.	Double area. Sub'tct
On a to b	40	14	1960	—
	107	78	8346	—
	103	84	8652	—
	116	14	1604	—
On b to F	98	16	—	1568
	190	46	—	8740
	102	50	—	5100
	94	30	—	2820
Sum of addition,			20562	18228
Sum of subtraction,			18228	
Difference,			2334,	to be added to the area of the polygon.

The letters a, b , etc., show between what points on the line the areas are calculated.

When the area, and not the double area, of the polygon is given, then we take half the double area of the difference of the offset and inlet columns, and add or subtract to or from the area of the polygon, as may be the case.

In making out the bases, we subtract 150 from 190; put the difference, 40, in base column, and opposite which, in offset column, put 14; then 40×14 will give double the area of the $\triangle$ between 150 and 190.

Again, take 190 from 297; the difference, 107, is put in base column, opposite to which, in offset column, is put $78 = 14 + 64$; then $107 \times 78 =$ double the area of the trapezium between 190 and 297.

This method of keeping field notes facilitates the computation of offsets and plotting detail.

We begin at the bottom of the page or line, and enter the field notes as we proceed toward the top or end of the line. The chain line may be a space between two parallel lines, or a single line, as in fig. 17e. If the field book is narrow, only one line ought to be on the width of every page, and that up the middle (see sec. 211).

ORDNANCE METHOD.

262. *Field Book, No. 16, Page 64.*

On the first day of May, 1838, I commenced the survey of part of Flaskagh, in the parish of Dunmore, and county of Galway, Ireland, surveyed for John Connolly, Esq.

MICH'L McDERMOTT, *C. L. S.*

THOMAS LYNSEY, } *Chain bearers.*
THOMAS KING, }

The angles have been taken by a theodolite, the bearing of one line determined, from which the following bearings have been deduced (see fig. 17e). *Land kept on the right.*

We begin at the most northerly station, as by this means we will always add the south latitudes and subtract the north latitudes.

Explanation. On line 1, at distance 210, took an offset to the left, to where a boundary fence or ditch, etc., jutted. The dotted line along said fence shows that the face next the dots is the boundary.

At 297, offset of 64 links to Mr. JAMES ROGER's schoolhouse.

At 340, offset of 70 links to south corner of do.

The width = 30, set down on the end of do.

At 400, offset to the left of 14 links to a jutting fence.

From 150 to 400, the boundary is on the inside or right, as shown by the characters made by dots and small circles joined. See characters in plates. From this point, 400, the boundary continues to the end of the line, to be on the left side of fence.

At 804, met creek 30 links wide, 5 deep, clear water, running in a southern direction.

At 820, met further bank of do.

At 830, dug a triangular sod out of the ground, making the vertex the point of reference. Here I left a stick 6 inches long, split on top, into which split a folded paper having line 1—830 in pencil marks. This will enable us to know where to begin or close a line for taking the detail.

At 960, offset to the right 20 links.

At 1000, met station F, where I dug 3 triangular sods, whose vertexes meet in the point of reference. This we call *leveling mark*.

The distance, 1000 links, is written lengthwise along the line near the station mark.

The station mark is made in the form of a triangle, with a heavy dot in the centre.

Distances from which lines started or on which lines closed, are marked with a crow's foot or broad arrow, made by 3 short lines meeting in a point.

Along the line write the number of the line and its bearing.

Line 2 may be drawn in the field book as in this figure, or it may be continued in the same line with line 1, observing to make an angle mark on that side of the line to which line 2 turns. This may be seen in lines 4 and 5, where the angle mark is on the right, showing that line 5 turns to the right of line 4.

Line 2, total distance to station G = 1700 links. The distance from the station to the fence, on the continuation of line 2, is 10 links, which is set correctly on the line.

Key offset. See where line 2 starts from end of line 1. At the end of line 1, offset to corner of fence = 10. At 10 links on line 2, offset to corner = 2. This is termed the key offset, and is always required at each station for the computation of offsets and inlets.

Running from one line to another. We mention the distance of the points of beginning and closing as follows:

This shows that the line started from 830, on line 1, and closed on 600, line 5. It also shows, from the manner in which distances 804, 820 and 830 are written, that the line turns to the right of line 1. When we use a distance, as 830, etc., we make 2 broad arrows opposite the distance. This will enable us to mark them off on the plotting lines for future reference.

We take detail on this line—it will serve as a check when the scale is 2, 3, or 4 chains to 1 inch scale.

We number it and enter it on the diagram, which must always be on the first page of the survey. The diagram will show the number of the line; the distances on which it begins and ends; the reference distances. This will enable the surveyor to lay down his plotting or chain lines, and test the accuracy of the survey. Having completed the plotting plan, we then fill in the detail, and take a copy or tracing of it to the field, and then compare it with the locality of the detail.

This comparison is made by seeing where a line from a corner of a building, and through another corner of a fence or building, intersects a fence; then from the intersection we measure to the nearest permanent object. We draw the line in pencil on the tracing, and compare the distance found by scale with the measured distance. Some surveyors can pace distances near enough to detect an error. On the British Ordnance Survey, the *sketchers* or examiners seldom used a chain, unless in filling in omitted detail.

On Supplying Lost Lines or Bearings.

263. It would be unsafe to depend on this method, unless where the line or lines would be so obstructed as to prevent the bearings and distances to be taken. The surveyor seeing these difficulties, will take all the available bearings and measure the distances with the greatest accuracy, leaving no possible doubt of their being correctly taken. Then, *and not till then*, can he proceed to supply the omissions.

Case 1. In fig. 17*b*, we will suppose that all the lines and bearings have been correctly taken, but the distance I K has been obliterated, and that its bearing is given to find the distance I K.

Let the bearing of I K be S. 60 W. From sec. 259, method 2, we have
calculated the departure of K from the line A B = 17,4257
departure of I from do. = 35,6123
consequently the departure of line I K is = K L = 18,1866

We have the angle K I L = 60°, therefore the $\angle$ I K L = 30°, and its departure = ,5000

The product of the last two numbers will give (by sec. 167) I L = 9,0933

By E. I, 47, from having I K and K L we find 10,50 = I K

or I L = 9,1933, divided by the lat. or cos. of 60° or ,86603 = 10,50 = I K

Case 2. *The bearing and distance of the line I K is lost.*

Here we have to find the lines I L and L K. From the above sec., method 2, we have—

Lat. K A = 3,0726 N.	Lat. E F = 8,6610 S.
Lat. A B = 29,1780 N.	Lat. F G = 17,0010 S.
Lat. B C = 6,1280 N.	Lat. H I = 18,7940 S.
Lat. C D = 8,8630 N.	44,456 S.
Lat. D E = 7,7140 N.	
54,9556 N.	
44.4560 S.	

Lat. I L = 10,4996, and from above K L = 18,1866.

Therefore, by E. 1, 47, $K L^2 + L I^2 = K I^2$; consequently K I is found. But $I K \cdot \cos. < K I L = I L$.

Therefore $\frac{I L}{I K} = \cosine < K I L$, which take from table of lat. and dep., and it gives $< K I L = 60^\circ$. Consequently the bearing is S. 60° W., or $\frac{K L}{I K} = \frac{9,0933}{10,50} = ,8662 = \cos. < I K L$; $\therefore$ the $< I K L = 30^\circ$, and the bearing of the line K I = N. 60° E. from station K.

Case 3. *Let there be two lines wanted whose bearings are known to be S. 60° W. and N. 80° W.*

Here the station K may be obstructed by being in a pond, in a building, or that buildings are erected on part of the lines I K and A K (see fig. 17b).

We find from case 2 that A is south of F =	51,8830
I is south of F =	44,4560
A is south of I = t g = A a =	7,4270
We have above, a I = dep. of I = d =	35,6123

Now we have A a and a I, $\therefore$ we find the line A I. And A a divided by a I gives the tangent of $< A I a = ,2085$. And the $< A I a = 11^\circ 47'$.

$\therefore$ I a divided by the cosine $10^\circ = A I = 35,6123 \div ,9789 = 36,38$.

Now we have the $< A I a = 11^\circ 47'$
and the $< A a I = 90^\circ$; $\therefore$ the $< a A I = 78^\circ 13'$
consequently the $< g A I = 11^\circ 47'$
but the $< g A K = 10^\circ 00'$, $\therefore < K A I = 21^\circ 47'$.
Again the $< K I a = 30^\circ 00'$
and the $< A I a = 11^\circ 47'$, $\therefore < A I K = 18^\circ 13'$.
And by Euclid I, 32, we have the $< A K I = 140^\circ 30'$.

By sec. 194, we have $\sin < A K I : A I :: \sin < A I K : A K$.
 $\sin < A K I : A I :: \sin < K A I : K I$.

Case 4. *Let all the sides be given, and all the bearings, except the bearings of I K and A K, to find these bearings.*

By the above methods we can find the departure a i of the point I, east of the meridian A B.

We also have the difference of lat. of the points A and I = t g = A a.
 $\therefore (A a)^2 + (I a)^2 =$ the square of A I; $\therefore$ A I may be found.
Or, $A a \div I a =$ tangent of the $< A I a$; $\therefore < A I a$ may be found.
And $I a \div \cos. < A I a$, will give the side A I.

Now having the sides A I, A K and K I, by sec. 205, we can find the angles K A I and K I A. And the $< A I a$ and $< A I K$ are given; $\therefore$ their sum $< A I K$ is given; $\therefore$ the bearing of the line I K is given.

264.

Calculation of an Extensive Survey (fig. 17c), where the First has been made. Calculated

Line.	Bearing.	Dist. in chains	N. lat.	S. lat.	E. dep.	W. dep.	Equated N. lat.	Equated S. lat.
B C	N. 40° E.	8,00	6,1283		5,1423		6,128	
C D	N. 10° W.	9,00	8,8633			1,5629	8,863	
D E	N. 50° E.	12,00	7,7135		9,1925		7,714	
E F	S. 30° E.	10,00		8,6603	5,0000			8,660
F G	South.	17,00		17,0000				17,000
G H	East.	11,00			11,0000			
H I	S. 20° E.	20,00		18,7938	6,8404			18,794
I K	S. 60° W.	21,00		10,5000		18,1866		10,500
K A	N. 80° W.	17,69	3,0727			17,4260	3,073	
A L	North.	7,00	7,0000				7,000	
L M	West.	8,00				8,0000		
M N	N. 55° W.	9,00	5,1622			7,3724	5,162	
N O	N. 75° W.	7,00	1,8117			6,7615	1,812	
O P	N. 27° W.	6,00	5,3461			2,7239	5,346	
P Q	N. 33° E.	10,00	8,3867		5,4464		8,387	
Q R	N. 77° W.	9,00	8,9330			1,0968	8,933	
R S	N. 37° W.	9,00	7,1878			5,4163	7,188	
S T	N. 43° E.	11,00	8,0449		7,5020		8,045	
T U	S. 52° E.	13,00		8,0036	10,2441			8,003
U B	S. 29° E.	16,80		14,6936	8,1448			14,694
			77,6502	77,6512	68,5125	68,5466	77,651	77,651

Here we find that line K A, which theoretically should close on A, wants but 1,3 links.

To find the Most Westerly Station.

By looking to fig. 17d, it will appear that either the point S or P is the most westerly.

L M = 8,000 west.

M N = 7,370 W.

N O = 6,766 W.

O P = 2,722 W.

Point P = 24,858 west of the assumed point L.

P Q = 5,448 E.

19,410.

Q R = 1,095 W.

R S = 5,414 W.

Point S = 25,919 west of the assumed point L.

Therefore the point S is the most westerly station, through which, if the first meridian be made to pass the area, can be found by the second method.

To Find the Meridian Distances.

When the first mer. passes through the most westerly station, we add the eastings and subtract the westings.

When the first mer. is through the most easterly station, we add the westings and subtract the eastings.

When the first mer. passes through the polygon, we add the eastings in that part east of the first mer., and subtract them in that part west of that mer. We also subtract the westings in that part east of that mer., and add them west of it.

*Meridian is made the Base Line A B, at each of which a Station
by the Third Method.*

Equated E. dep.	Equated W. dep.	A or latitude, and half departure.	B, or Meridian dist.	North area.	South area.
6,145	1,561	N. 6,128	2,572½ E.	15,7643	
		E. 2,572½	5,145		
9,195		N. 8,863	4,364½ E.	38,6826	
5,002		W. 0,780½	3,584		
11,002	18,184 17,423 8,000 7,370 6,760 2,722 5,448 1,095 5,414 7,503 10,246 8,146 68,529	N. 7,714	8,181½ E.	63,1121	
6,842		E. 4,597½	12,779		
		S. 8,660	15,280 E.		132,3248
		E. 2,501	17,781		
		S. 17,000	17,781 E.		302,2770
		0,000	17,781		
		0,000	23,282 E.		
		E. 5,501	28,783		
		S. 18,794	32,204 E.		605,2420
		E. 3,421	35,625		
		S. 10,500	26,533 E.		278,5965
		W. 9,092	17,441		
		N. 3,073	8,729½ E.	26,8258	
		W. 8,711½	0,018		
		N. 7,000	0,018 E.	0,1260	
		0,000	0,018		
	68,529	0,000	3,982 W.		
		W. 4,000	7,982		
		N. 5,162	11,667 W.		60,2251
		W. 3,685	15,352		
		N. 1,812	18,732 W.		33,9424
		W. 3,380	22,112		
		N. 5,346	23,473 W.		125,4867
		W. 1,361	24,834		
		N. 8,387	22,110 W.		185,4366
		E. 2,724	19,386		
		N. 8,933	19,933 W.		178,0660
		W. 0,547½	20,481		
		N. 7,188	23,188 W.		166,6753
		W. 2,707	25,895		
		N. 8,045	22,143 W.		178,1445
		E. 3,751½	18,392		
	68,529	S. 8,003	13,269 W.	106,1915	
		E. 5,123	8,146		
		S. 14,694	4,073 W.	59,8487	
		E. 4,073	0,000		
				310,5513	2246,4179
					310,5513

Required area = $1935\frac{966}{10000}$ chains, or 193,5867 acres.

VARIATION OF THE COMPASS.

264a. In surveying an estate such as that shown in fig. 17c, we run a base line through it, such as A M. We find the magnetic bearing, and its variation from the true meridian. We measure it over carefully, then take a fly-sheet and remeasure the same, then compare, and survey a third time if the two surveys differ. With good care in chaining, it is possible to make two surveys of a mile in length to agree within one foot. With a fifteen feet pole they agree very closely.

We refer the base line A M to permanent objects as follows :

Theodolite at station A, read on station M,	0° 00'
On the S.W. corner of St. Paul's tower,	15° 11'
On the S.E. corner of the Court House (main building),	27° 10'
On the S.W. corner of John Cancannon's Mill,	44° 16'
On the N.E. corner of John Doe's stone house,	276° 15'
On the N.W. corner of Charles Roe's house,	311° 02'

Any two or three of these, if remaining at a future date, would enable us to determine the base A M, to which all the other lines may be referred.

The variation of the compass is to be taken on the line at a station where there is no *local attraction*, the station ought to be at same distance from buildings.

We find the magnetic bearing of A M = N. 64° 10' E., as observed at the hour of 8 A.M., 8th December, 1860, at a point 671 links north of station A, on the base line A M. Thermometer = 40°, and Barometer 29 inches.

Let the latitude of station = 53° 45' 00''

Polar distance of Pole Star (*Polaris*) = 1° 25' 30''

(Declination of *Polaris* being = 88° 34' 30'', ∴ its polar distance is found by taking the declination from 90.)

To Find at what time Polaris will be at its Greatest Azimuth or Elongation.

264b. *Rule.* To the tan. of the polar dist. add the tan. of the lat.; from the sum take 10. The remainder will be the cosine of the hour angle in space, which change into time. The time here means sidereal.

To Find the Greatest Azimuth or Bearing of Polaris.

264c. *Rule.* To radius 10 add sine of the polar distance; from the sum take the cosine of the latitude. The remainder will be the sine of the greatest azimuth.

To Find the Altitude of Polaris when at its Greatest Azimuth.

264d. *Rule.* To the sine of the latitude add 10; from the sum take the cosine of the polar distance. The difference will be the log. sine of the altitude.

In the above example we have lat. = 53° 45', and its tan. = 10,1357596
Polar distance = 1° 25' 30'', and its tangent = 8,3957818
88° 3' 05'' = hour angle in space, whose cosine = 8,5315414
This changed into time gives 5 h., 52 m., 12,3 s. This gives the time from the upper meridian passage to the greatest elongation.

To Find when Polaris will Culminate or Pass the Meridian of the Station on Line A M, being on the Meridian of Greenwich on the 8th Dec., 1860.

264e. From Naut. Almanac, star's right ascension	=	1h. 08m. 43,5s.
Sun's right ascension of mean sun (sidereal time)	=	17 09 59,9
Sidereal time, from noon to upper transit	=	7 58 52,6
Sidereal time, from upper transit to greatest azimuth	=	5 00 01
Sidereal time from noon to greatest eastern azimuth	=	2 58 52

Now, as this is in day time, we cannot take the star at its greatest eastern elongation, but by adding 5h. 52m. 12,3s. to 7h. 58m. 52,6s., we find the time of its greatest western azimuth = 13h. 51m. 4,9s. from the noon of the 8th December, and by reducing this into mean time, by table xii, we have the time by watch or chronometer.

To Find the Altitude and Azimuth in the above.

264f. Lat. 53° 45' N., sine + 10 + 19,906575	cos. =	9,771815
N. polar dist. 1° 25' 30'', cos. = 9,999866	sine + 10 +	18,395648
sine = 9,906709	sine =	8,623833
True altitude = 53° 46' 27''.	Greatest azimuth =	2° 24' 37''.

Alpha and Beta are termed the pointers, or guards, because they point out the Pole star, which is of the same (second) magnitude, and nearly on the same line.

Benetnach *
Alcor *
Alioth * or Epsilon
Delta *
Gamma *
Beta * Alpha * or Dubhe

The distance from Alpha Ursamajor to the Pole star

URSAMAJOR, or DIPPER, or THE PLOUGH,
at its under transit.

is about five times the distance between the two pointers.

When Alioth and Polaris are on the same vertical line, the Pole star is supposed to be on the meridian. Although this is not correct, it would not differ were we to run all the lines by assuming it on the meridian; but as we sometimes take Polaris at its greatest azimuth, both methods would give contradictory results.

264g. *Alioth and Polaris are always on opposite sides of the true pole.* This simple fact enables us to know which way to make the correction for the greatest azimuth. (For more on this subject, see *Sequel Canada Surveying*, where the construction and use of our polar tables will be fully explained.)

Variation of the Compass.

264h. Variation of the compass is the deviation shown by the north end of the needle when pointing on the north end of the mariner's compass and the true north point of the heavens; or, it is the angle which is made by the true and magnetic meridians.

When the magnetic meridian is west of the true meridian, the variation is westerly.

Let S N = true meridian, S = south, and N = north.

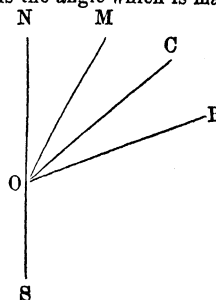
Let M O = magnetic meridian through station O.

Let the true bearing of B = N. 60° 40' E.

Let the magnetic do. = N. 50° 50' E.

Variation east = 9° 50'

In this case, the true bearing is to the right of the magnetic.



Let M = magnetic and N = true North Pole. M N
 Let the true bearing of B = N. $60^{\circ} 50'$ E.
 Let the magnetic do. = N. $70^{\circ} 40'$ E.

Variation west = $9^{\circ} 50'$

Here the true bearing is to the left of the magnetic.

In the first example we protract the $\angle NOC$
 = $\angle MOB$, which show that B is to the right
 of C.

In the second example we make the $\angle NOD$
 = $\angle MOB$, which shows that B is to the left of D.

Hence appears the following rule:

Rule 1. Count the compass and true bearings from the same point
 north or south towards the right.

Take the difference of the given bearings when measured towards the
 east or towards the west; but their sum when one bearing is east and the
 other west.

When the true bearing is to the right of the magnetic, the variation is
 east. When the true bearing is to the left of the magnetic, the variation
 is west.

Example 3. Let the true bearing = N. 60° W. = 300° ,
 and the magnetic bearing = N. 70° W. = 290° .

Variation east = 10° .

Here we have the true bearing at 300° , counting from N. to right, and
 the magnetic bearing at 290° , counting from N. to right.
 10° variation east, because the true
 bearing is to the east of the magnetic.

Example 4. Let true bearing = N. 60° W. = 300° , from N. to right.
 and magnetic bearing = N. 70° W. = 290° , from N. to right.

Variation 10° west, because the
 true bearing is to the right of the magnetic.

Example 5. Let true bearing = N. 5° E. = 5 from N. to right,
 and the magnetic bearing = N. 5° W. = 365 from N. to right.

Variation 10° east, because the true bearing
 is to the right of the magnetic.

Rule 2. From the true bearing subtract the magnetic bearing. If the
 remainder is +, the variation is east; but if the remainder or difference
 is —, the variation is west.

Example 6. True bearing = N. $60^{\circ} 40'$ E.

Magnetic bearing = N. $50^{\circ} 50'$ E.

+ $9^{\circ} 50'$ = variation east.

Example 7. True bearing = N. 5° E. = +,

Magnetic bearing = N. 5° W. = —.

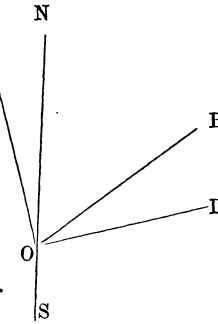
+ 10° east.

Here we call the east +, and the west negative —; and by the method
 of subtracting algebraic quantities, we change the sign of the lower line,
 and add them.

Example 8. Let true bearing = N. 16° W. —,

and magnetic bearing = N. 6° W. —.

— 10° = variation 10° west.



Let us now find the true bearing of the line A M in fig. 17c.

By sec. 264a, we have the magnetic bearing of A M = N. 64° 10' E., < from Polaris, at its greatest western elongation, to the base line A M, as determined = 80° 40'. The work will appear as follows:

On the evening of the 8th December, 1860, we proceeded to the station mentioned in sec. 264a. Set up the theodolite on the line A M. At a distance of 10 chains, I set a picket fast in the ground, whose top was pointed to receive a polished tin tube, half an inch in diameter. Not wishing to calculate the necessary correction of Polaris from the meridian, I preferred to await until it came to its greatest western azimuth, being that time when the star makes the least change in azimuth in 6 minutes, and the greatest change in altitude, this being the time best adapted for finding the *greatest azimuth and true time* of any celestial object.* The station is assumed on the meridian of Greenwich. If on a different meridian, we correct the sun's right ascension. (See our Sequel Spherical Astronomy, and Canada Surveying.)

On the morning of 9th December, 1860, at 1h. 51m. 5s., found the base line A M to bear from Polaris = N. 80° 40' 00'' E.
 Magnetic bearing of line A M = N. 64° 10' 00'' E.
 Polaris at its greatest azimuth = N. 80° 40' 00'' E.
 Greatest azimuth from sec. 264f = 2° 24' 37''
 Bearing of the line A M from true meridian = N. 78° 15' 23'' E.
 Magnetic bearing of line A M = N. 64° 10' 00'' E.

By rule 2, the variation = N. 14° 05' 23'' E.

From sec. 264f, we have the star's altitude when at its greatest azimuth.
 True altitude = 53° 46' 27''
 Correction from table 14 for refraction = 42''
 Apparent altitude = 53° 47' 09''

We had the telescope elevated to the given apparent altitude until the star appeared on the centre, then clamped the lower limb, and caused a man to hold a lamp behind the tin tube on the line A M. Found the < 80° 40', as above. Here the vernier read on Polaris at its greatest western azimuth = 279° 20' 00''
 Read on the tin tube and picket on the line A M = 00° 00' 00''
 On the true meridian = 281° 44' 37''

The last bearing taken from 360° will give the true bearing of A M = N. 78° 15' 23'' E.

After having taken the greatest azimuth, we bring the telescope to bear on A M; if the vernier read zero, or whatever reading we at first assume, the work is correct. If it does not read the same, note the reading on the lower limb, and, without delay, take the bearing of the Pole star, which is yet sufficiently near to be taken as correct, and thus find the angle between it and the base line. The surveyor, having two telescopes, will be in no danger of committing errors by the shifting of the under plate, can have one of the telescopes used as a *tell-tale*, fixed on some permanent object, on which he will throw the light shortly before taking the azimuth of Polaris, to ascertain if the lower limb remained as first adjusted.

264i. A second telescope can be attached to any transit or theodolite, so as to be taken off when not required for tell-tale purposes, as follows: To the under plate is riveted a piece of brass one inch long, three-fourths

inch wide, and two-tenths thick. On this there is laid a collar or washer, about one-eighth inch thick. To these is screwed a right angled piece in the form of L, turned downwards, and projecting one inch outside of the edge of the parallel plates. Into the outer edge of the L piece is fixed a piece having a circular piece three-fourths inch deep, having a screw corresponding to a thread on the telescope of the same depth. This screw piece is fastened on the inside of the L piece by a screw, and has a vertical motion. When we use this as a tell-tale, we bring it to bear on some well defined object, and then clamp the lower plate. We then bring the theodolite telescope to bear on the above named object or tin tube, and note the reading of the limb. After every reading we look through the tell-tale telescope to see if the lower plate or limb is still stationary. If so, our reading is correct; if not, *vice versa*.

The expense of a second telescope so attached will be about twelve dollars, or three pounds sterling. The instrument will be lighter than those now made with two telescopes, such as six or eight inch instruments. This adjustment attached to one of Troughton and Simm's five inch theodolite has answered our purposes very well during the last twenty-two years. We prefer it to a six inch, as we invariably, for long distances, repeat the angles. (See sec. 212.)

265. *To Light the Cross Hairs.* Sir Wm. Logan, Provincial Geologist of Canada, has invented the following appendage: On the end of the telescope next the object is a brass ring, half an inch wide, to which a second piece is adjusted, at an angle of 45° . This second piece is elliptical, two inches by two and three-eighths, in the centre of which is an elliptical hole, one inch by three-eighths. This is put on the telescope. The surface of the second piece may be silvered or polished. Our assistant holds the lamp so as to illuminate the elliptical surface, which then illuminates the cross hairs. He can vary the light as required.

This simple appendage will cost one and a half dollars, and will answer better than if a small lamp had been attached to the axis of the telescope, as in large instruments. Those surveyors who have used a hole in a board, and other contrivances, will find this far more preferable.

We have a reflector on each of our telescopes. The tell-tale being smaller is put into the other, and both kept clean in a small chamois leather bag, in a part of the instrument box. (See sec. 211.)

UNITED STATES SURVEYING.

The following sections are from the Manual of Instructions published by the United States Government in 1858, which are called *New Instructions*, to distinguish them from those issued between 1796 and 1855, which are called the *Old Instructions*. The notes are by M. McDermott.

SYSTEM OF RECTANGULAR SURVEYING.

266. The public lands of the United States are laid off into rectangular tracts, bounded by lines *conforming to the cardinal points*.

These tracts are laid off into townships, containing 23040 acres. These townships are supposed to be square. They contain 36 tracts, called sections, each of which is intended to be 640 acres, or as near that as possible. The sections are one mile square. A continuous number of townships between two base lines constitutes a range.

267. The law requires that the lines of the public surveys shall be *governed by the true meridian, and that the township shall be six miles square*—two things involving a mathematical impossibility, by reason of the convergency of the meridians. The township assumes a trapezoidal form, which unequally develops itself more and more as the latitude is higher.

* In view of these circumstances, the act of 18th May, 1796, sec. 2, enacts that the sections of a mile square shall contain 640 acres, *as nearly as may be*.

* The act 10th May, 1800, sec. 3, enacts "That in all cases where the exterior lines of the townships thus to be subdivided into sections, or half sections, shall *exceed, or shall not extend* six miles, the excess or deficiency shall be specially noted, and added to or deducted from the *western and northern* ranges of sections or half sections in such township, according as the error may be, in *running the lines from east to west or from south to north*.

268. The sections and half sections bounded on the northern and western lines of such townships, shall be sold as containing only the quantity expressed in the returns and plats respectively, and all others as containing the complete legal quantity."

The accompanying diagram, marked A (see sec. 271), will illustrate the method of running out the exterior lines of townships, as well on the *north* as on the south side of the base line.

OF MEASUREMENTS, CHAINING AND MARKING.

269. "Where uniformity in the variation of the needle is not found, the public surveys *must be made* with an instrument operating independently of the magnetic needle. *Burt's Solar Compass*, or other instrument of equal utility, *must* be used of necessity in such cases; and it is deemed best that such instruments should be used under all circumstances. Where the needle can be relied on, however, the ordinary compass may be used in subdividing and meandering."—Note Traversing.

BASE LINES, PRINCIPAL MERIDIANS, AND CORRECTION OR STANDARD LINES.

270. *Base Lines* are lines run due east and west, from some point assumed by the Surveyor General. North and south of this base line, townships are laid off, by lines running east and west.

Standard or Correction Lines are lines run east and west, generally at 24 miles north of the base line, and 30 miles south of it. These lines, like the townships, are numbered from the base line north or south, as the case may be.

Principal Meridians are lines due north and south from certain given points, and are numbered first, second, third, etc. Between these principal meridians the tiers of townships are called ranges, and are numbered 1, 2, 3, 4, etc., east or west of a given principal meridian.

All these lines are supposed to be run astronomically; that is, they are run in reference to the true north pole, without reference to the magnetic pole. In proof of this, it is well to state that the Old Instructions has shown, in the specimen field notes, that the true variation has been found. See pages 13 and 18, and in the New Instructions, pages 28 to 35, both inclusive. Here the method of finding the greatest azimuth is not given, although there is a table of greatest azimuths for the first day of July for the years 1851 to 1861, and for lat. 32° to 44° . At page 30 is given the mean time of greatest elongation for every 6th day of each month, and shows whether it is east or west of the true meridian.

At page 27 are given places near which there is no variation. At page 29 are given places with their latitudes, longitudes, and variation of the compass, with their annual motion.

The method of finding these for other places and dates is not given in either manual. For these, see sequel Canadian method of surveying side lines. For formulas and example, see sections 264*a* and 264*b* of this manual.

Principal Meridians. The 1st principal meridian is in the State of Ohio.

The 2nd principal meridian is a line running due north from the mouth of the Little Blue River, in the State of Indiana.

The 3d principal meridian runs due north from the mouth of the Ohio River to the State line between Illinois and Wisconsin.

The 4th principal meridian commences in the middle of the channel, and at the mouth of the Illinois River; passes through the town of Galena; continues through Illinois and Wisconsin, until it meets Lake Superior, about 10 chains west of the mouth of the Montreal River. For further information, see Old Instructions, page 49.

Ranges are tiers of townships numbered east or west from the established principal meridian, and these lines run north or south from the base line. They serve for the east and west boundary lines of townships. On these lines, section and quarter section corners are established. These corners are for the sections on the west side of the line, but not for those on the east side. (See Old Instructions, page 50, sec. 9.)

Note. This is not always the case. There are many surveys where the same post or corners on the west line of the township have been made common to both sides. This is admitted in the Old Instructions, page 54, sec. 21.

Townships are intended to be six miles square, and to contain 36 sections, each 640 acres. They are numbered north and south, with reference to the base line. Thus, Chicago is in township 39 north of the base line, and in range 14 east of the third principal meridian.

Township lines converge on account of the range lines being run toward the north pole, or due north. This convergency is not allowed to be corrected, but at the end of 4 townships north, and 5 south of the base line, this causes the north line of every township to be 76,15 links less than the south line, or 304,6 links in 4 townships.

The deficiency is thrown into the west half of the west tier of sections in each township, and is corrected at each standard line, where there is a jog or offset made, so as to make the township line on the standard line six miles long. In surveying in the east 5 tiers of sections, each section

is made 80 chains on the township lines. In the east tier of quarter sections of the west tier, each quarter section is 40 chains on the east and west township and section lines.

Example. Let 1, 2, 3 and 4 represent 4 townships north of the base line. Township number 1 will be 6 miles on the base line, and the

North boundary of section 6, in township 1	= 7923,8 links.
North boundary of section 6, in township 2	= 7847,7 links.
North boundary of section 6, in township 3	= 7771,5 links.
North boundary of section 6, in township 4	= 7695,4 links.

Here we make the south line of sec. 30, in township 5 = 8000 links.

271. Townships are subdivided into 36 sections, numbered from east to west and west to east, according to the annexed diagram. Lot 1 invariably begins at the N.E. corner, and lot 6 at the N.W.; lot 30 at S.W., and lot 36 at the S.E. corner.

Surplus or deficiency is to be thrown into the north tier of quarter sections on the north boundary, and in the west tier of quarter sections on the west boundary of the township.

78,477	5	4	3	2	1	T. 2 N.
7	8	9	10	11	12	
18	17	16	15	14	13	
19					24	
30					25	
31					36	
	80	80	80	80		
79,238						
80						T. 1 N.
R. I E.						R. II E.
Base						Line.

Base

Line.

North and South Section Lines How to be Surveyed.

272. Each north and south section line must be made 1 mile, except those which close to the north boundary line of the township, so that the excess or deficiency will be thrown in the north range of quarter sections; viz., in running north between sections 1 and 2, at 40,00 chains, establish the quarter section corner, and note the distance at which you intersect the north boundary of the township, and also the distance you

fall east or west of the corresponding section corner for the township to the north; and at said intersection establish a corner for the sections between which you are surveying.—Old Instructions, p. 9, sec. 28.

East and West Section Lines. Random or Trial Lines.

* 273. All east and west lines, except those closing on the west boundary of the township, or those crossing navigable water courses, will be run from the proper section corners *east* on *random* lines (without blazing), for the corresponding section corners. At 40 chains set *temporary* post, and note the distance at which you intersect the range or section line, and your falling north or south of the corner run for. From which corner you will correct the line west by means of offsets from stakes, or some other marks set up, or made on the random line at convenient distances, and remove the *temporary* post, and place it at average distance on the *true line*, where establish the quarter section corner. The random line is not marked but as little as possible. The brushwood on it may be cut. The true line will be blazed as directed hereafter. The east and west lines in the west tier are by some run from corner to corner, and by others at right angles to the north and south adjacent lines.

East and West Lines Intersecting Navigable Streams.

274. Whenever an east and west section line other than those in the west range of sections crosses a navigable river, or other water course, you will not run a random line and correct it, as in ordinary cases, where there is no obstruction of the kind, but you will run east and west on a *true line* (at right angles to the adjacent north and south line) from the proper section corners to the said river or navigable water, and make an *accurate* connection between the corners established on the opposite banks thereof; and if the error, neither in the length of the line nor in the falling north or south of each other of the fractional corners on the opposite banks, exceeds the limits below specified in these instructions for the closing of a whole section, you will proceed with your operations. If, however, the error exceeds those limits, you will state the amount thereof in your field notes, and proceed forthwith to ascertain which line or lines may have occasioned the excess of error, and reduce it within proper bounds by re-surveying or correcting the line or lines so ascertained to be erroneous, and note in your field book the whole of your operations in determining what line was erroneous, and the correction thereof. (See Old Instructions, p. 10, sec. 32.) Limits in closing = 150 links.

Note. From sec. 272 we find that the north and south lines are intended to be on the true meridian from the south line of the township to its north boundary. This is the intention of the act Feb., 1805. From sec. 273 we find that in the east 5 tiers of sections of every township, a *true* line is that which is run *from post to post*, or from "a corner to the corresponding corner opposite."

But in the west tier of sections, a *true line* is that which is run at right angles to the adjacent north and south line; that is, the north and south line must be run before the east and west line can be established. This agrees with the above act, which requires that certain lines are to be run due east or west, as the case may be.—Old Instructions, p. 10.

Note 2. In many of the old surveys, the field notes show that the surveyor ran from corner to corner on the west and north tiers of sections. This appears to be the case in town. 39 N., range 14 E., of the third principal meridian, surveyed by Mr. John Wall in 1821. This will also appear from the Old Instructions, p. 54, sec. 21, where it is stated as follows: "Previous to 1828, some of the deputies considered, in making the calculations of the area of the north and west tiers of quarter sections in a township, that the quarter section corners on the township and range lines were *common* to the sections on *both sides* of the line, whilst others adopted the method now in use, and particularly explained in diagram numbers 4 and 5." At one time, some of the deputy surveyors, in subdividing a township through which a *navigable stream* passed, ran a random line *east* between the proper sections, and corrected it *west*, making the corner to the fractional sections on *both banks* of the river, and on the *true line*. Others pursued the method as now required.

Note 3. From this appears the necessity of having a copy of the original Field Notes, so as to know which method to pursue in order to re-establish any line or corner, and to know if the surveyor deposited charcoal at the corner or corners.

Insuperable Obstacles. Witness Points.

275. Whenever your course may be obstructed by insuperable obstacles, such as ponds, swamps, marshes, lakes, rivers, creeks, etc., you will *prolong* the line *across* such obstacles, by taking the necessary right angle offsets, or, if this is inconvenient, by a traverse or trigonometrical operation, *until you regain the line on the opposite side*. And in case a north and south, or a *true east* and west line is regained in advance of any obstacle, you will prolong and mark the line back to the obstacle so passed, and state all the particulars in relation thereto in your field notes; and at the intersection of lines with both margins of impassable obstacles, you will establish a witness point (for the purpose of perpetuating the intersections therewith), by setting a post, and giving in your field notes the course and distance therefrom to two trees on opposite sides of the line, each of which trees you will mark with a blaze, and notch facing the post, *except* on the margins of navigable water courses or navigable lakes. In these cases you will mark the trees with the proper number of fractional section, township and range.—O. I., p. 7, sec. 22. This sec. is the same as that in the N. I., p. 3.

Note. In fractional townships it appears that the north and south lines are first established, and afterward the east and west lines are run *due east and west*, or at right angles to the north and south lines. (See the U. S. act 11 Feb., 1805, and sections 272 and 273.)

Note 2. It appears from the manuals that where a north and south line meets a navigable stream, it runs directly across it, and that some surveyors run directly across navigable waters on east and west lines, and others close on it, showing the jog or difference in closing, which must not be more than 150 links from a straight line. Hence it appears that where the official field notes do not show a jog, none exists.

Limits in Closing.

276. Any excess or deficiency in the length of any township boundary

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line, or excess of error in the falling off from the corner to which any closing township line shall be run, that may exceed five chains, or any excess or deficiency exceeding one chain in the length of any section line, or excess of error in falling off from the corner to which any section line shall be run, that shall exceed one chain in closing the lines of a whole section; and at the same rate for the section lines, and at the same rate of one chain and fifty links per mile of the meanders (*traverse survey*), in closing the meanders of a navigable river, or other water course, with the line or lines of a fractional section, must be corrected by you, and reduced within those limits, before leaving the ground, by resurveying the line or lines which may have occasioned the excess or deficiency in the length of such township or section line, or excess of error in closing the lines of a township, or of a whole or fractional section.—O. I., p. 8, sec. 25.

Meandering of Navigable Streams.

277. Standing with the face looking *down* the stream, the bank on the left is termed the "left bank," and that on the right, the "right bank." Both these banks are to be meandered (traversed) by taking the bearings and distances of their sinuosities, and the same entered in the field book.

Where township or section lines cross navigable waters, the line is marked by posts, and bearing trees, and mounds, as the case may require at the time of running those lines.

In meandering, you are to *commence* at one of those corners and meander to the next meander corner, carefully noting your intersections with all meander corners. The opposite bank is to be meandered in like manner.

The crossing distance *between* the *meander corners*, *on the same line*, is to be ascertained by triangulation, in order that the river may be protracted with entire accuracy. The particulars to be given in the field book.

All lakes and deep ponds of the area of 25 acres and upwards, also all bayous, shallow ponds readily to be drained, are not to be meandered.

Notice all streams of water falling into the river, lake or bayou you are surveying, stating the width at their mouth; also all springs, noting their size, depth, and kind of water; also the head and mouth of all bayous, and all islands, rapids and bars are to be noticed, with intersections to their upper and lower points to establish their exact situation. Note the elevation of the banks of rivers and streams, the heights and falls of cascades, and the length of rapids.

Trees are Marked for Line and Bearing Trees.

278. All trees left standing on a line are marked with 3 marks or blazes on front and rear; the distance on the line, the kind of timber, and diameter, is marked in the field book.

Trees are also marked on both sides of the line, making one blaze on the front, another on the rear, and one on the side next the line. By these means the situation of the line may be determined approximately at a future time. (See sec. 211.)

Note. It is hard to expect a true line from corner to corner where trees are allowed to stand on the line. The Canadian plan of cutting everything off the line is preferable.

279. *Township corners* are marked by a post, whose 4 sides are marked

by 6 notches, directed to the 4 cardinal points. Posts to be 3 feet above the ground, not pointed but set in the ground. There are to be, when possible, 4 bearing trees marked, 1 in each township, each having a large blaze facing the post, having marked thereon with a marking iron the range, township and section.

280. *Section corners* have 4 bearing trees, one in each section. *Quarter section corners* have but two bearing trees. The bearings, distances, wood, and diameter of these are entered in the field book.

The section corner post will be notched on two sides.

Example. The post corner sections 9, 10, 15, 16, will have 2 marks or notches on the north side and 4 on the south side, to signify that this is 2 miles from the north boundary and 4 miles from the south boundary. Bearing trees have the sections cut on them.

Note. It would be well in this example to have 3 marks on the east side, to show that it is three miles from the east boundary of the township. These notches would at any future time enable us to know which corner we are at in the absence of bearing trees.

281. *Township corners in a prairie*, or other situation where bearing or witness trees are not at hand, will be perpetuated by depositing in the ground, and at least 3 inches beneath the natural surface thereof, a portion of charcoal (the quantity to be specified in your field notes), not less than two quarts at the place of such corner, over which you will erect a mound of earth 3 feet high, 5 feet square at the base, and 2 feet square at the top, the sides whereof must be revetted or faced with sods laid horizontally and in successive layers on each other, each of said layers having an offset inwards, corresponding to the general slope of the face of the mound; in this there is to be a post which must rise 1 foot above the top of the mound. Or you will deposit at the place of the corner 3 stones, not less than 5 inches square by 3 inches thick, all of which you will describe in the field notes. The top of the uppermost stone to be 3 inches below the natural surface of the ground, and the other 2 successively and beneath the first; and over said stone erect a mound, as if it were made over charcoal or stone. You may perpetuate the corner by inserting endways into the ground, a stone of the dimensions marked, and set in the manner mentioned, over which no mound need be erected.

282. *If a township corner, where bearing or witness trees are not to be found within a reasonable distance therefrom*, shall fall within a ravine, or any other situation unfavorable to the erection of a mound, you will select in the vicinity thereof a suitable plot of ground as a site for a bearing or witness mound, and erect thereon a mound of earth, as if it were at the township corner, with charcoal or stone deposited beneath, as before directed for township corner, and measure, and state in your field notes the distance and course from the position of the true corner, of the bearing or witness mound so placed and erected.

Witness Mounds to Section Corners.

283. *Section corners, where bearing or witness trees cannot be had*, will be made as if for a township corner, except that where mounds are made they need be only 2 feet 6 inches high, by 4 feet square at the base, and 2 feet at the top.

Witness mounds to quarter section corners are 3 feet 6 inches at base, 1 foot 6 inches square at top. Under the post no charcoal or stone is put.—Old Instructions, sec. 18, 19, 20 and 21.

In the above we do not find the manner of digging the pits for the mounds, but the following from the New Instructions shows the manner now adopted:

“At the point or corner dig out a spadeful or two, and in the cavity put a stone or charcoal, which note in the field book. A charred stake is driven 12 inches in the ground, and in the centre of the cavity. Either of these will be a witness for the future. The spot from which the earth for the mound is taken is called a pit. The pits are made at uniform distances from the centre on each side of every mound, so as to indicate hereafter the place of the mound, when it may become obliterated by time or accident. Mounds are to present a conical shape. There is dug around the base a quadrangular trench, 1 spadeful deep of earth. At about 18 inches from this trench, dig pits at the 4 cardinal points; the length of the north and south pits to be east and west, and those of the east and west pits to be north and south.

284. *For corner mounds common to 2 townships, or 2 sections, the quadrangular trenches at north and south are at right angles to the north and south line; but where the mound is common to 4 townships, or 4 sections, the north and south line cuts the quadrangle diagonally.*” (See New Instructions, diagram C.)

Courses and Distances to Witness Trees.

285. You will ascertain and state in your field notes the course and distance from the several section and township corner posts, trees or stones, to a tree in each section for which they stand as a corner. Each of said trees you will mark with a notch and blaze, facing the post, tree or stone; the notch to be at the lower end of the blaze, and on it, which must be neatly made. Mark with a marking iron, in a plain, permanent manner, the letter S, with the number of the section, and over it the letter T, with the number of the township, and above this the letter R, with the number of the range. Where there is no tree within reasonable distance of the corner, note that in the field notes.—O. I., p. 5, sec. 17.

286. From this we infer that at township corners 4 trees are marked, and also at section corners; but at quarter section corners only 2 are marked.

Note. This does not point out the necessity of describing the kind of tree marked, and its diameter, although we find that all the Government field notes give the kind of tree, and its diameter. In the specimen field book, the bearing trees are described with reference to their diameter and wood.

Quarter section corners have simply a blaze, and $\frac{1}{4}$ S marked on it, and two bearing trees marked.

Note 2. Surveyors, in getting out of wood into open land or prairie, are always accustomed to mark line trees, and to describe the kind of wood and their diameters, and, if possible, they mark another tree or trees on the opposite side of the mound, on the same line. By these means the place of the mound can be determined, if these trees are to be found.

287. *Corner Stones.* These stones not to be less than a stone 14 inches long by 12 inches wide and 8 inches thick; are set with the edge facing north and south, for north and south lines, and east and west, for east and west lines; they are to be 7 or 8 inches in the ground. Where stones are placed as range and township lines, as many notches will be distinctly cut with a chisel or pick on the two sides, in direction of the line, as the corner is sections from the nearest section corners. At township corners, 6 notches will be cut on each edge or side toward the cardinal points. At section corners in the interior of a township, as many notches will be cut on the south edge and east side as the corner is sections distant from the south and east boundaries of the township. At the corners of subdivisional intersections with the north boundaries of the townships, 6 notches on the south edge, and at the intersections with the west boundary 6 notches on the east edge, and as many notches on the north or south sides, as the case may require, as the corner is sections distant from the township corner. Quarter section corner stones will have $\frac{1}{4}$ S on the west side, on north and south lines; and on the north side, on east and west lines.—Old Instructions, p. 5, sec. 16 and 17.

288. *Note.* These two sections from the Old Instructions are the same in substance as those given in the New Instructions. 7 or 8 inches without a mound will not be deep enough; better have it at least 1 foot in the ground, and then build a heap of stones around it, and afterwards a mound. Under every stone there ought to be some charcoal, broken delf, glass or slags of iron, or a stone, into which a hole 1 inch deep is cut, to perpetuate the centre or true corner.

The manuals do not mention which part of the stone is to be taken, but the usual custom is to take the middle, unless something to the contrary is mentioned.

The New Manual of Instructions gives a specimen field book of township 25 N., range 2 W., Willamette meridian.

In these it appears that the corners on the north and west boundaries of the township have been made common to the adjacent townships.

That in every case the north and south lines crossed rivers and lakes on the same line, excepting on the line between 4 and 5, where the river came within 4,00 links of the north boundary. Here he ran south as follows:

From the corner to sections 4, 5, 32 and 33, on the north boundary of the township, I run—

South on a true line, between sections 4 and 5.

Chains. Variation 18° east.

2,10. A white oak, 15 inches diameter.

4,00. Set a post on the right bank of Chickeeles River, for corner to fractional sections 4 and 5, from which a bur oak, 16 inches diameter, bears N. 25° E., 34 links distance; a black oak, 20 inches diameter, bears N. 33° W., 21 links.

From these field notes we find that the north and south lines have crossed a navigable stream and lakes on 8 lines, and on straight lines, with the above exception.

That on the east and west lines, 4 lines crossed a navigable river, 2 of which are on a straight line, one makes a jog of 12 links, and the other a jog of 16 links.

Manual of Old Instructions gives specimen field notes, showing that the variation has been taken, and that compared with the east boundary of the township. That the north and south section lines are run straight across navigable water.

That jogs are made on the north and west boundary lines. That the amount of each jog or offset is invariably given.

That on east and west lines crossing navigable rivers, jogs are made, and the amount shown, how determined, and the distance between the corresponding corners on the river.

Both Instructions show that in meandering, the surveyor begins at some fractional corner, and traverses along the stream to the next corner, and so on from section line to section line.

Note 2. Hence it appears that a traverse survey is not to be used in determining where a section line crossed or met a river. (See Re-establishing Lost Corner.)

SUMMARY OF OBJECTS AND DATA REQUIRED TO BE NOTED.

289. Lengths of all lines, with the offsets thereon.

The kind and diameter of all bearing trees, with the courses and distances of the same from their respective corners.

The material of which mounds are constructed; the bearing of the pits from which the earth was dug from the centre; the material put under the post—if charcoal, etc.; how much.

Trees on the line; their names, diameters and distance on the line.

Intersections by the line of *land objects*, and the distance to each settler's claim, and also at leaving it.

Distances to and at leaving woods, prairies, rivers, creeks, lakes, ravines, top and bottom of hills, bottom or swamp lands, underbrush or brushwood.

Describe the *surface* of the land—whether level, rolling, or hilly.

The *soil*—whether first, second, or third rate quality.

The several sorts of *timber* and undergrowth, in the order in which they predominate.

Bottom lands—whether wet, dry, or subject to inundation. If subject to be inundated, show to what depth.

Note. It would be well, as in Europe, to show the high and low water mark.

Springs of water—if fresh, saline, or mineral; with the course of the stream flowing from them.

Note. This supposes that the surveyor has a knowledge of chemistry. To those who have not such knowledge, we refer them to the *Sequel* for our sections on analysis of soils, water, and ashes of plants.

Lakes and ponds—describe their banks; give their height, depth of water, and whether the water is pure or stagnant.

Note. State if there is any inlet or outlet, and the course of these coming to or leaving the lakes or ponds.

Improvements, towns and villages, Indian towns and wigwams, houses, cabins, fields, sugar tree groves, sugar camps, mill seats, forges and factories.

Coal banks or beds, *peat* or *turf* grounds, *minerals* and *ores*; with a description of their quality and extent, and all diggings therefor; also salt springs and lakes.

Roads and trails (or pathways), with their directions—whence they come and whither they go.

Rapids, cataracts, cascades, or falls of water, with the height of their falls in feet.

Precipices, caves, sink holes, ravines, stone quarries, ledges of rocks, with the kind of stone they afford.

Natural curiosities, interesting fossils, petrifications, organic remains, etc.; also all ancient work of art, such as mounds, fortifications, embankments, ditches, or objects of like nature.

The variation of the needle must be noted at all points or places on the lines where there is found any material change of variation, and the positions of such points must be perfectly identified in the field notes.

Besides the ordinary notes taken on the line (and which must always be written down on the spot, leaving *nothing* to be supplied by memory), the deputy will subjoin, at the conclusion of his book, such further description or information touching any matter or thing connected with the township (or other survey), which he may be able to afford, and may deem useful or necessary to be known, with a general description of the township in the aggregate, as respects the face of the country, its soil and geological features, timber, minerals, water, etc.—New Instructions, pp. 17, 18.

Note. Surveyors keep a loose or random field book, where they keep the notes in running lines from corner to corner, and crossing rivers, lakes, etc.; and when they have performed the work, copy the result into the proper field book. That this has been the custom with surveyors will appear by referring to their field books, where the crossing distances on rivers, etc., are given without the necessary notes.

290. *Field Notes of the Subdivision Lines and Meanders of Chickeeles River, in Township 25 N., Range 2 W., Willamette Meridian.*

Chains.	To determine the <i>proper adjustment</i> of my compass for subdividing this township, I commence at the corner to townships 24 and 25 N., range 1 and 2 W., and run <i>north</i> on a blank line along the east boundary of section 36, Variation $17^{\circ} 51' E.$
40,05	To a point 5 links west of the quarter section corner,
80,09	To a point 12 links west of the corner to sections 25 and 36. To retrace this line, or run parallel thereto, my compass must be adjusted to a variation of $17^{\circ} 46' E.$ <i>Note.</i> 12 divided by 8009 gives ,00149, which is the natural sine of 5 minutes; and because the blank line was run west of the true line, the correction is to be taken from $17^{\circ} 51'$, which will give the true magnetic variation of the east side of section 36. The variation here is not that from the true meridian, but is that from the section line assumed as base or meridian. <i>Subdivision Commenced February 1, 1851.</i> 291. From the corner to sections 1, 2, 3, 5 and 36, on the south boundary of the township, I ran—

Chains.	
9,19	A beech, 30 inches diameter (marked 3 blazes on front and rear).
29,97	A beech, 30 inches diameter (on line marked as above).
40,00	Set a post for quarter section corner, from which— A beech, 8 inches diameter, bears N. 23° W., 45 links dist. A beech, 15 inches diameter, bears S. 48° E., 12 links dist.
51,00	A beech, 18 in. diam. (this is a line tree, and marked as above).
71,00	A sugar tree (maple), 30 inches diameter.
80,00	Set a post for corner to sec. 25, 26, 35 and 36, from which— A beech, 28 inches diameter, bears N. 60° E., 45 links dist. A beech, 24 inches diameter, bears N. 62° W., 17 links dist. A poplar, 20 inches diameter, bears S. 70° W., 50 links dist. A poplar, 36 inches diameter, bears S. 66° E., 34 links dist. Land level; second-rate quality. <i>Timber</i> —poplar, beech, sugar tree, and some oak, undergrowth and hazel.
Chains.	
	East on a random line, between sections 25 and 36, Variation 17° 46' E.
9,00	A brook, 20 links wide, runs north,
15,00	To foot of hill; bears north and south.
40,00	Set a post for <i>temporary</i> quarter section corner,
55,00	To opposite foot of hill; bears north and south.
72,00	A brook, 15 links wide, runs north.
80,00	Intersected the east boundary at the corner post to sections 25 and 36, from which corner I ran— <i>West</i> , on a true line, between sections 25 and 36. Variation 17° 46' E.
40,00	Set a post on top of hill; bears north and south; from which— A hickory, 14 inches diameter, bears N. 60° E., 27 links dist. A beech, 15 inches diameter, bears S. 74° W., 9 links dist.
80,00	The corner to sections 25, 26, 35 and 36. <i>Land</i> —east and west parts level; first-rate. Middle part broken, and third-rate. <i>Timber</i> —beech, oak and ash; undergrowth; same and spice in the branch bottoms.
Chains.	
	We proceed as above, N. bet. 25 and 26, then E. bet. 25 and 36. Again, N. between 23 and 24, then E. between 24 and 25. Again, N. between 14 and 13, then E. between 13 and 24. Again, N. between 11 and 12, then E. between 12 and 13. Again, N. between 1 and 2, as follows: North, on a <i>random line</i> . Variation 17° 46' E.
40,00	Set a temporary post for quarter section corner.
80,11	Intersected the north boundary of the township 32 links east of corner to sections 1 and 2, from which corner I ran— South, on a <i>true line</i> , between sections 1 and 2. Variation 18° E.
40,11	Set a post for quarter section corner, from which— A white oak, 20 inches diameter, bears N. 31° W., 65 links dist.

Chains.	A sugar tree, 14 inches diameter, bears S. 49° E., 32 links dist.
80,11	The corner to sections 1, 2, 11 and 12. Land level; good; rich soil. <i>Timber</i> —walnut, sugar tree, beech, and various kinds of oak; open woods. February 2, 1851.
	<i>Note.</i> Here we find that the line between sections 1 and 2 is run from post to post, making no jog or offset on the north boundary of the township; and that the south quarter sections in the north tier of sections are 40 chains, from south to north, leaving the surplus of 11 links in the north tier of quarter sections.
	<i>Field Notes of a Line Crossing a Navigable Stream on an East and West Line.</i>
	292. West, on a <i>true line</i> , between sections 30 and 31, knowing that it will strike the Chickeeles River in less than 80,00 chains. Variation 17° 40' E.
3,41	A white oak, 15 inches diameter.
5,00	Leave upland, and enter creek bottom, bearing N.E. and S.W.
8,00	Elk creek, 200 links wide; gentle current; muddy bottom and banks; runs S.W. Ascertained the distance across the creek on the line as follows: Cause the flag to be set on the right bank of the creek, and in the line between sections 30 and 31. From the station on the left bank of creek, at 8,00 chains, I run <i>south</i> 245 links, to a point from which the flag on the right bank bears N. 45° W., which gives for the distance across the creek, on the line between sections 30 and 31, 245 links.
25,17	A bur oak, 24 inches diameter.
40,00	Set a post for quarter section corner, from which— A buck-eye, 24 inches diameter, bears N. 15° W., 8 links dist. A white oak, 30 inches diameter, bears S. 65° E., 12 links dist.
41,90	Set a post on the left bank of Chickeeles River, a navigable stream, for corner to fractional sections 30 and 31, from which— A buck-eye, 16 inches diameter, bears N. 50° E., 16 links dist. A hackberry, 15 inches diameter, bears S. 79° E., 14 links dist. Land and timber described as above.
	<i>Note.</i> We find this part of the line between sections 30 and 31 in the <i>Manual of New Instructions</i> , page 35, and the other part in page 42, as follows: From the corner to sections 30 and 31, on the west boundary of the township, I ran— East on a <i>true line</i> , between sections 30 and 31. Variation 18° E.
15,10	A white oak, 16 inches diameter.
23,50	Intersected the right bank of Chickeeles River, where I set a post for corner to fractional sections 30 and 31, from which— A black oak, 16 inches diameter, bears N. 60° W., 25 links dist. A white oak, 20 inches diameter, bears S. 35° W., 32 links dist.
	k

Chains.	From this corner I run south 12 links, to a point west of the corner to fractional sections 30 and 31, on the left bank of the river. Thence continue south 314 links, to a point from which the corner to fractional sections 30 and 31, on the left bank of the river, bears N. 72° E., which gives for the distance across the river 9,65 chains. The length of the line between sections 30 and 31, is as follows:
	Part east of the river, 41,90 chains.
	Part across the river, 9,65 "
	Part west of the river, 23,50 "
	Total, 75,05 chains.
	<i>Note.</i> Here the method of finding the distance across the river, and of showing the amount of the jog or deviation from a straight line, is shown.

MEANDERING NOTES. (*New Manual*, p. 42.)

293. Begin at the corner to fractional sections 25 and 30, on the range line. I chain south of the quarter section corner on said line, and run thence down stream, with the meanders of the left bank of Chickeeles River in fractional section 30, as follows:

	Chains.	
S. 41° E.	20,00	At 10 chains discovered a fine mineral spring.
S. 49° E.	15,00	Here appeared the remains of an Indian village.
S. 42° E.	12,00	
S. 12¾° E.	5,30	To the fractional sections 30 and 31.
		Thence in section 31,
S. 12° W.	13,50	To mouth of Elk River, 200 links wide; comes from the east.
S. 41° W.	9,00	At 200 links (on this line) across the creek.
S. 58° W.	11,00	
S. 35° W.	11,00	
S. 20° W.	20,00	At 15 chains, mouth of stream, 25 links wide, comes from S.E.
S. 23¾° W.	8,80	To the corner, to fractional sections 31 and 36, on the range line, and 8,56 chains north of the corner to sections 1, 6, 31 and 36, or S.W. corner to this township.
		Land level, and rich soil; subject to inundation.
		Timber—oak, hickory, beech, elm, etc.

RE-ESTABLISHING LOST CORNERS. (*New Instructions*, p. 27.)

294. Let the annexed diagram represent an east and west line between

Sec. 31.	Sec. 32.	Sec. 33.	Sec. 34.	Sec. 35.	Sec. 36.
	<i>d</i>	<i>a</i>			
	<i>c</i>	<i>b</i>			
Sec. 6.	Sec. 5.	Sec. 4.	Sec. 3.	Sec. 2.	Sec. 1.

two townships, and that all traces of the corner to sections 4, 5, 32 and 33 are lost or have disappeared. I restored and re-established said corner in the following manner:

Begin at the quarter section corner marked *a* on diagram, on the line between sections 4 and 33. One of the witness trees to this corner has fallen, and the post is gone.

The black oak (witness tree), 18 inches diameter, bearing N. 25° E., 32 links distance, is standing, and sound. I find also the black oak station or line tree (marked *b* on diagram), 24 inches diameter, called for at 37,51 chains, and 2,49 chains west of the quarter section corner. Set a new post at the point *a* for quarter section corner, and mark for witness tree. A white oak, 20 inches diameter, bears N. 34° W., 37 links dist. West with the *old marked line*.

Variation 18° 25' E.

At 40,00 chains, set a post for temporary corner to sections 4, 5, 32 and 33.

At 80,06 chains, to a point 7 links south of the quarter section corner (marked *c* on diagram), on line between sections 5 and 32. This corner agrees with its description in the field notes, and from which I run east, on a *true line*, between sections 5 and 32.

Variation 18° 22'.

At 40,03 chains, set a lime stone, 18 inches long, 12 inches wide, and 3 inches thick, for the *re-established corner* to sections 4, 5, 32 and 33, from which—

A white oak, 12 inches diameter, bears N. 21° E., 41 links dist.

A white oak, 16 inches diameter, bears N. 21° W., 21 links dist.

A black oak, 18 inches diameter, bears S. 17° W., 32 links dist.

A bur oak, 20 inches diameter, bears S. 21° E., 37 links dist.

Note 1. The diagram, and letters *a*, *b*, *c*, and that part in parentheses, are not in the Instructions.

Note 2. Hence it appears that the surveyor has run between the nearest undisputed corners, and divided the distance *pro rata*, or in proportion to the original subdivision. Although in this case the line has been found blazed, and one line or station tree found standing, the required section corner is not found by producing the line from *a*, through *b*, to *d*. Although I have met a few surveyors who have endeavored to re-establish corners in this manner, I do not know by what law, theory or practice they could have acted. It is in direct violation of the fundamental act of Congress, 11 Feb., 1805, which says that lines are to be run "*from one corner to the corresponding corner opposite*." (See sequel *Geodetical Jurisprudence*.)

Re-establishing Lost Corners. (From *Old Instructions*, p. 63.)

295. Where old section or township corners have been completely destroyed, the places where they are to be re-established may be found, in *timber*, where the old blazes are tolerably plain, by the *intersections* of the east and west lines with the north and south lines.

If in *prairie*, in the following manner:

Let the annexed diagram represent part of the township. This example is often given: Suppose that the corner to sections 25, 26, 35 and 36 to be missing, and that the quarter section corner on the line between sections 35 and 36 to be found. Begin at the said quarter section corner, and run north on a *random line* to the *first* corner which can be identified, which we will suppose to be that of sections 23, 24, 25 and 26.

15	 1 4	 1 3
22	 2 3	 2 4
27	 2 6	 2 5
34	 3 5	 3 6

At the end of the first 40 chains, set a *temporary* post corner to sections 25, 26, 35 and 36. At 80 chains, set a temporary quarter section corner post, and suppose also that 121,20 chains would be at a point due east or west of said corner 23, 24, 25 and 26. Note the falling or distance from the corner run for, and the distance run. Thence from said corner run south on a *true line, dividing the surplus*, 1,20 chains, equally between the three half miles, viz.: At 40,40 chains, establish a quarter section corner. At 80,80 chains, establish the corner to sections 25, 26, 35 and 36. Thence to the quarter section corner, on the line between sections 35 and 36, would be 40,40 chains.

The last mentioned section corner being established, east or west random or true lines can now be ran therefrom, as the case may require.

This method will in most cases enable the surveyor to renew missing corners, by re-establishing them in the right place.

But it may happen that after having established the north and south line, as in the above case, the corner to sections 26, 27, 34 and 35 can be found; also the quarter section corner on the line between 26 and 35. In this case it might be better to extend the line from the corner 26, 27, 34 and 35, to said quarter section corner, *straight* to its intersection with the *north and south line already established*, and there establish the corner to sections 25, 26, 35 and 36. If this point should differ much from the point where you would place the corner by the first method laid down, it might be well to examine the line between sections 25 and 36.

Note 1. Hence it appears that the north and south lines are first established, in order that the east and west lines may be run therefrom; and that when the east and west lines can be correctly traced to the north and south line, that the point of intersection would be the required corner. It is also to be inferred that where the lines on both sides can be traced to the north and south line, a point equidistant between the points of intersection would be the required corner.

Note 2. It will not do to run from a section or quarter section corner on the west side of a north and south line, to a section corner, or quarter section, on the east side of the line, and make its intersection with the north and south line, the required corner, unless that these two lines were originally run on the same variation, which is seldom the case.

Note 3. Having found approximately the missing corner, we ought to

search diligently for the remains of the old post, mound, bearing trees, or the hole where it stood.

Bearing trees are sometimes so healed as to be difficult to know them. By standing about 2 feet from them, we can see part of the bark cut with an even face. We cut obliquely into the supposed blaze on the tree to the old wound. We count the layers of growth, each of which answers to one year. By these means we find the years since the survey has been made, which, on comparing with the field notes, we will always find not to differ more than one year.

Remains of a post, or where it once stood, may be determined as follows: Take the earth off the suspected place in layers with a sharp spade. By going down to 10 or 12 inches, we will find part of the post, or a *circular surface*, having the *soil black and loose*, being principally composed of vegetable matter. By putting an iron pin or arrow into it, we find it partially hollow. We dig 6 feet or more around the suspected place. Where such remains are found, we make a note of it, and of those present. Put charcoal, glass, delf, or slags of iron, in the hole, and re-establish the corner, noting the circumstances in the field book.

Ditches or lockspitting are sometimes made on the line to perpetuate it. This will be an infallible guide, and we only require to know if the edge or centre of the ditch was the line or boundary, or was it the face or top of the embankment. These answers can be had from the record, or from the persons who have made the ditch, or for whom it has been made. Should this ditch be afterwards ploughed and cultivated, we can see in June a difference in the appearance of the plants that grow thereon, being of a richer green than those adjoining the ditch. Or, we dig a trench across the suspected place. The section will plainly show where the old ditch was, for we will find the black or vegetable mould in the bottom of the old ditch. We may have the line pointed out by the oldest settlers, who are acquainted with the locality. Surveyors ought to spare no pains to have all things so correctly done as to prevent litigation, and to bear in mind that "*where the original line was, there it is, and shall be.*"

ESTABLISHING CORNERS. (*Old Instructions*, p. 62.)

296. In surveying the public lands, the United States Deputy Surveyors are required to *mark only the true lines*, and establish on the ground the corners to townships, and sections, and quarter sections, on the range, township and sectional lines.

There are, no doubt, many cases where the corners are not in the right place, more particularly on *east and west* sectional lines, which, doubtless, is owing to the fact that some deputy surveyors did not always run the random lines the whole distance and close to the section corner, correct the line back, and establish the quarter section corner on the true line, and at average distance between the proper section corner; but only ran east or west (from the proper section corner) 40,00 chains, and there established the quarter section corner.

In all cases where the land has been sold, and the corners can be found and *properly identified*, according to the original approved field notes of the survey, *this office has no authority to remove them.*

RE-ESTABLISHING CORNERS IN FRACTIONAL SECTIONS, AND ALSO THE
INTERIOR CORNER SECTIONS. (*Old Instructions*, p. 55.)

Present Subdivision of Sections.

297. None of the acts of Congress, in relation to the public lands, make any special provision in respect to the manner in which the subdivisions of sections should be made by deputy surveyors.

The following plan may, however, be safely adopted in respect to all sections, excepting those adjoining the north and west boundaries of a township, where the same is to be surveyed:

Let the annexed diagram represent an interior section, as sec. 10. B, D, H and F are quarter section corners. Run a true line from F to D; establish the corner E, making $DE = EF$; then make straight lines from E to B and from E to H, and you have the section divided into quarters.

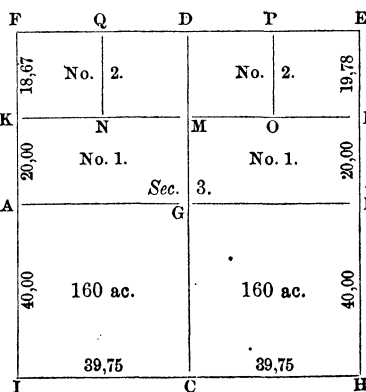
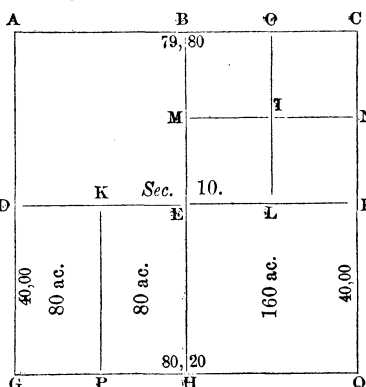
If it is required to subdivide the N. E. quarter into 40 acre tracts, make $EL = LF$, and $BO = OC$, and $GP = PH$, and $DK = KE$; also $EM = MB$, and $FN = NC$. Run from M to N on a true line, and make $MI = IN$. Here the N. E. quarter section is divided into 4 parts, and the S. W. quarter section into two halves.

Note. As the east and west sides of every regular section is 80 chains, and that the quarter section corners on the north and south sides are at average distances, it is evident that the line BH will bisect DF, or any line parallel to GQ. Consequently the method in the section is the same in effect as that in the next.

But if, by a re-survey, we find that AB is not equal to BC, or that GH is not equal to HQ, then we measure the line from D to F, and establish the point E at average distance.

298. Let the annexed diagram represent a subdivision of section 3, adjoining the north boundary of a township, being a fractional section.

In this case, we have on the original map $AF = 38.67$, $BE = 39.78$, $DE = 39.75$, $FD = 39.95$, $IC = 39.75$, and $CH = 39.75$. The S. E. and S. W. quarter sections each equal to 160 acres. Lot No. 1 each equal to 80 acres. In the N. W. quarter section the west half of lot 2 = 37.41 acres, and the east half



of lot 2 = 37,96 acres. These areas are taken from the original survey. In the N.E. quarter section, the west half of lot 2 = 38,23 acres, and the east half of lot 2 = 38,78.

In this example, there can be but *one rule* for the subdivision, to make it agree with the manner in which the several areas are calculated. You will observe that the line I H is 79,50 chains, and that the one half of it, = 39,75, is assumed as the distance from E to D, which last distance, 39,75, is deducted from 79,50, the length of the line E F leaving 39,95 chains between the points F and D. Consequently the line C D must be *exactly* parallel to the line H E, without paying any respect to the quarter section corner near D, which belongs entirely to section 34 of the township on the north. Run the line A B in the same manner as that of D F on diagram sec. 297, except that the corner G is to be established at the point where the line A B intersects the line C D. After surveying thus far, if the S.E. and S.W. quarters are to be subdivided, it can be done as in diagram sec. 297. In this case, to subdivide the N.E. and N.W. quarters, the line K L *must be parallel to* A B. The two lines ought to be 20 chains apart. The corner, M, is made where K L is intersected by C D. But as *two surveyors seldom agree exactly* as to distances, there might be found an excess or deficiency in the contents of the N.E. and N.W. quarters. If so, the line K L should be so far from A B as to *apportion the excess or deficiency* between lots 1 and 2, not equally, but in proportion to the quantities sold in each. If the lots numbered 2 are divided on the township plat by north and south lines, then that of the N.W. quarter must have its south end equidistant between K and M, and its north end equidistant between F and D. The N.E. quarter will be subdivided by a line parallel to M D and L E, *exactly* half way between them.

Note. Here we have the quarter section corners A, B, C and D given, and where the line A B intersects C D, gives the interior quarter section corner.

We find also that $A K = B L = 20$ chains generally, and that $K N = N M$, and $F Q = Q D$. Also $M O = O L$, and $D P = P E$.

Let us suppose that the original map or plat in this example gave the N.E. quarter 157 acres—that is, lot 1 = 80 and lot 2 = 77 acres, and that in surveying this quarter section we find the area = 159 acres, then we say, as 157 : 159 :: 80 to the surplus for lot 1, or, as 157 : 159 :: 77 to surplus in lot 2; and having the corrected area of lot 1, and the lengths of B G and L M, we can easily find the width B L.

Note 2. The above method of establishing the interior corner, M, is according to the statutes of the State of Wisconsin, and appears to be the best, as the original survey contemplates that the lines I F, H E, F E, I H, A B and C D are straight lines.

Government Plats or Maps.

299. The plats are drawn on a scale of 40 chains to one inch. The section lines are drawn with faint lines; the quarter section lines are in dotted lines; the township lines are in heavy lines. The number of the section is above the centre of each section, and its area in acres under it. On the north side of each section is the length thereof, excepting the south section lines of sections 32, 33, 34, 35 and 36. The section corners on the township lines are marked by the letters A, B, C, D, etc., A being at

the N.E. corner, G at the N.W., N at the S.W., and T at the S.E. The quarter section corners are marked by a, b, c, d, etc., a being between A and B, f between G and F, n between N and O, and s between S and T. (See New Instructions, diagram B.)

Note. On the maps or plats which we have seen, A begins at N.W. corner and continues to the right, making F at the S.W. corner of the township. The quarter section corner on the north side of every section is numbered 1, 2, 3, 4, 5 and 6, beginning on the east side, and running to the west line. Number 1 is at the quarter section corner on the north side of each section, 12, 13, 24, 25 and 36. Number 6 is at the quarter section corners on the north side of each, of sections 7, 18, 19, 30 and 31.

There is a large book of field notes, showing only where mounds and trees are made landmarks. The kind of trees marked as witness trees; their diameter, bearing and distances, are given for A, a, B, b, C, c, to X, x, Y, y.

For interior section corners, begin at S.E. corner, showing the notes to sections 25, 26, 35, 36; 23, 24, 25, 26; and two after two to sections 5, 6, 7, 8, at N.W. corner of the township.

For interior quarter section corners, begin at M, the N.E. corner of section 36, and run to U, N.W. corner of section 31, thus:

M to U, at 1, post in mound.

2, bur oak, 18 inches diameter, bears N. 3° E. 80 links.

bur oak, 12 inches diameter, bears S. 89° W. 250 links.

6, post in mound.

Next run L to V, K to W, I to X, and H to Y, giving the witness trees, if any, at quarter section corners numbered 1, 2, etc., as above. Then begin to note from south to north, by beginning at O and noting to F, then P to E, Q to D, R to C, and S to B.

The plats show by whom the outlines and subdivisions have been surveyed; date of contract; total area in acres; total of claims or land exempt from sale; the variation of the township and subdivision lines; and the detail required by section.

SURVEYS OF VILLAGES, TOWNS AND CITIES.

300. A. lays out a village, which may be called after him, as Cleaver-ville, Kilbourntown, Evanston; or it may be named after some river, Indian chief, etc., as Hudson, Chicago. This village is laid out into blocks, streets and alleys. The blocks are numbered 1, 2, 3, etc., generally beginning at the N.E. corner of the village. The lots are laid off fronting on streets, and generally running back to an alley. The lots are numbered 1, 2, 3, etc., and generally lot 1 begins at the N.E. corner of each block. The streets are 80, 66, 50 and 40 feet—generally 66 feet. In places where there is a prospect of the street to be of importance as a place for business, the streets are 80 feet. Although many streets are found 40 feet wide, they are objectionable, as in large cities they are subsequently widened to 60 or 66 feet. This necessarily incurs expenses, and causes litigations.

Sidewalks. The streets are from the side of one building to that of another on the opposite side of the street; that is, the street includes the carriage way and two sidewalks. Where the street is 80 feet wide, each

sidewalk is usually 16 feet. When the street is 66 feet, the width of the sidewalk is usually 14 feet. Where the street is 40 feet, the width of the sidewalk is usually 9 feet.

Corner stones. The statutes of each State generally require corner stones to be put down so as to perpetuate the lines of each village, town, or addition to any town or city.

Maps or plats of such village, town or addition, is certified as correct by the county or city surveyor, as the State law may require. The map or plat is next acknowledged by the owner, before a Justice of the Peace or Notary Public, to be his act and deed.

Plat recorded. The plat is then recorded in a book of maps kept in the Recorder's or Registrar's office, in the county town or seat.

Dimensions on the map. Show the width of streets, alleys and lots; the depths of lots; the angles made by one street with another; the distances from corner or centre stones to some permanent objects, if any. These distances are supposed to be mathematically correct, and according to which the lots are sold.

Lots are sold by their number and block, as, for example: "All that parcel or piece of land known as lot number 6, in block 42; in Matthew Collins' subdivision of the N.E. quarter section 25, in township 6 north, and range 2 east, of the third principal meridian, being in the county of —, and State of —."

All plats are not certified by county or city surveyors. In some States, surveyors are appointed by the courts, whose acts or valid surveys are to be taken as *prima facie* evidence. In other States, any competent surveyor can make the subdivision, and swear to its being correct before a Justice of the Peace.

Lots are also sold and described by metes and bounds, thus giving to the first purchasers the exact quantity of land called for in their deeds, leaving the surplus or deficiency in the lot last conveyed.

Metes and bounds signify that the land begins at an established point, or at a given distance from an established point, and thence describes the several boundaries, with their lengths and courses.

Establishing lost corners. When some posts are lost, the surveyor finds the two nearest undisputed corners, one on each side of the required corners. He measures between these two corners, and divides the distance *pro rata*; that is, he gives each lot a quantity in proportion to the original or recorded distance. Where there is a surplus found, the owners are generally satisfied; but where there is a deficiency, they are frequently dissatisfied, and cause an inquiry to be made whether this deficiency is to be found on either side of the required lots, or in one side of them. As mankind is not entirely composed of honest men, it has frequently happened that *posts, and even boundary stones, have been moved out of their true places* by interested parties or unskilful surveyors.

In subdividing a tract into rectangular blocks, we measure the outlines twice, establish the corners of the blocks on the four sides of the tract, and, by means of intersections, establish the corners of the interior blocks.

Let us suppose a tract to be divided into 36 blocks, and that block 1 begins at the N.E. corner, and continues to be numbered similar to township surveys. We erect poles at the N.W. corners of blocks 1, 2, 3, 4 and 5, and at the N.E. corners of blocks 12, 13, 24, 25 and 36. We set the in-

strument on the south line at S.W. corner of block 36; direct the telescope to the pole at the N.W. corner of block 1. Let the assistant stand at the instrument. We stand at the N.W. angle of 31, and make John move in direction of the pole at the N.W. angle of 36, until the assistant gives the signal that he is on his line. This will give the N.W. angle of 36, where John drives a post, on the top of which he holds his pole again on line, and drives a nail in the true point. We then move to the N.W. angle of 30, and cause John to move until he is on our assistant's line, thereby establishing the N.W. corner of 25, and so on for the N.W. corners of 24, 13 and 12. We move the instrument to the S.W. corner 35, and set the telescope on the pole at N.W. corner of 2, and proceed as before. This method is strictly correct, and will serve to detect any future fraud, and enable us to re-establish any required corner. Where the blocks are large, the lots may be surveyed as above.

Where the ground is uneven, or woodland, this method is not practicable. However, proving lines ought to be run at every three blocks.

CANADA SURVEYING.

301. No person is allowed to practice land surveying until he has obtained license, under a penalty of £10, one-half of which goes to the prosecutor.

Each Province has a Board of Examiners, who meet at the Crown Land Office, on the first Monday of January, April, July and October.

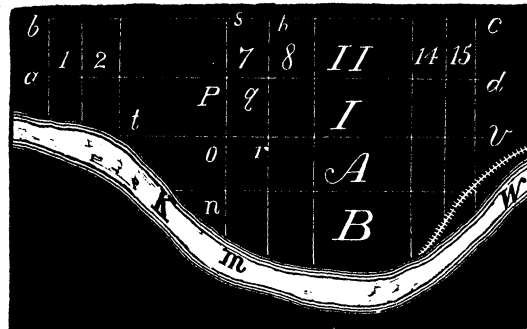
The candidate gives one week's notice to the Secretary of the Board. He must have served as an apprentice during three years. He must have first-rate instruments, (a theodolite, or transit with vertical arch, for finding latitude and the true meridian.) He must know Geometry, (six books of Euclid,) Trigonometry, and the method of measuring superficies, with Astronomy sufficient to enable him to find latitude, longitude, true time, run all necessary boundary lines by infallible methods, and be versed in Geology and Mineralogy, to enable him to state in his reports the rocks and minerals he may have met in his surveys. He must have standard measures, one five links long, and another three feet. He gives bonds to the amount of 1000 dollars. His fees, when attending court, is four dollars per day. He keeps an exact record of all his surveys, which, after his death, is to be filed with the clerk of the court of the county in which he lived. Said clerk is to give copies of these surveys to any person demanding them on paying certain fees, one-half of which is to be paid to the heirs of the surveyor.

The Government have surveyed their townships rectangularly, as in the United States, except where they could make lots front on Government roads, rivers and lakes. This has been a very wise plan, as several persons can settle on a stream; whereas, in the United States, one man's lot may occupy four times as much river front as a man having a similar lot in Canada.

302. Lines are run by the compass in the original survey, but all subsequent side lines are run astronomically. In the United States, lines are run from post to post, which requires to have two undisputed points,

and that a line should be invariably first run and then corrected back for the departure from the rear post. In the Canada system, we find the post in front of the lot, and then run a line truly parallel to the governing line, and drive a post where the line meets the concession in rear.

The annexed Fig. represents a part of the town. of Cox; *b c*, *a d*, etc., are concession lines. Heavy lines are concession roads, 66 feet wide, always between every two concessions. There is an allowance of road generally at every fifth lot.



The front of each concession is that from which the concessions are numbered; that is, the front of concession II is on the line *a d*.

Where posts were planted, or set on the river, the front of concession B is the river, and that of concession A is on the concession line *n f*, etc.

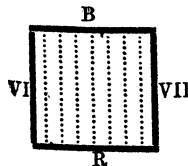
303. Side lines are to be run parallel to the township line from which the lots are numbered.

The line between lots 7 and 8, in concession II, is to be run on the same true bearing as the township line *a b*; but if the line *m*, *n*, *o*, *p*, *s*, etc., be run in the original survey as a proving line, then the line between 7 and 8 is to be run parallel to the line *p s*, and all lines from the line *p s* to the end are to be run parallel to *p s*, and lines from *a* to *p* are to be run parallel to *a b*. When there is no proving or township line where the lots are numbered from, as in con. A, we must run parallel to the line *v w*; but if there is a proving line as *m n*, all lines in that concession shall be run parallel to it.

When there is no town line at either end of the concession, as in con. B, the side lines are ran parallel to the proving line, if any.

When there is neither proving line or township line at either end, as in concession B, we open the concession line *k w*, and with this as base, lay off the original angle.

Example. The original bearing of *k w* is N. 16° W., and that of the side lines N. 66° E. To run the line between lots 14 and 15, in con. B, we lay off from the base *k w* an angle of 82°, and run to the river. The original posts are marked on the four sides thus. This shows that the allowance for road is in rear of con. C; that is, the concession line between concessions B and C is on the west line of allowance of road. The original field notes are kept as in the United States, showing the quality of timber, soil, etc.



If the concessions were numbered from a river or lake, and that no posts were set on the water's edge, then the lines shall be run from the rear to the water.

When concession lines are marked with two rows of posts, and that the land is described in half lots, then the lines shall be drawn from both ends parallel to the governing line, and to the centre of the concession if the lots were intended to be equal, or proportionate to the original depths.

When the line in front of the concession was not run in the original survey, then run from the rear to a proportionate depth between said rear line and the adjacent concession. (See Act, 1849, Sec. XXXVI.)

Example. The line *a d* has not been run, but the lines *b c* and *t v* have been ran.

Let the depth of each concession = 8000 links. Road, on the line *a d*, 100 links. Run the line between 7 and 8, by beginning at the point *h*, and running the line *h q* parallel to *a b*, and equal to half the width of concession I and II. Measure *h q*, and find it 8200 links. Suppose that the allowance for road is in the rear of each concession; that is, the west side of each concession road allowance is the concession line; then 8200 links include 100 links for one road, leaving the mean depth of concession II = to be 8100 links = *h q*. In like manner we find the depth of the line between 8 and 9, and the straight line joining these points is the true concession line. (See Act, May, 1849, Sec. XXXVI.)

304. Maps of towns or villages are to be certified as correct by a land surveyor and the owner or his agent, and shall contain the courses and distances of each line, and must be put on record, as in the United States, within one year, and before any lot is sold. These maps, or certified copies of them, can be produced as evidence in court, provided such copy be certified as a true copy by the County Registrar.

When A got P. L. surveyor S, to run the line between 6 and 7 in concession II, and finds that the line has taken part of his lot 6, on which he has improved; that is, he finds part of B's lot 7 included inside his old boundary fence. The value of his improvements is 400 dollars, belonging to A, and the value of the land to be recovered by B is 100 dollars. Then, if B becomes plaintiff to recover part of his lot 7, worth 100 dollars, he has to pay A the amount of his damages for improvement, viz. 400 dollars, or sell the disputed piece to A for the assessed value. (See Act of 1849, Sec. L.)

305. *In the Seigniories*, fronting on the St. Lawrence, the true bearing of each side line is N. 45° W., with a few exceptions about the vicinity of St. Ignace, below Quebec.

In the Ottawa Seigniories, the true or astronomical bearing is N. 11° 15' E. This makes it easier than in the townships, as there is no occasion to go to the township line for each concession.

306. *Where the original posts or monuments are lost.*

"In all cases when any land surveyor shall be employed in Upper Canada to run any side line or limits between lots, and the original post or monument from which such line should commence cannot be found, he shall in every such case, obtain the best evidence that the nature of the case will admit of, respecting such side line, post or limit; but if the same cannot be satisfactorily ascertained, then the surveyor shall measure the true distance between the nearest undisputed posts, limits or monuments, and divide such distance into such number of lots as the same contained in the original survey, assigning to each a breadth proportionate to that intended in such original survey, as shown on the plan and field-notes thereof, of record in the office of the Commissioner of Crown Lands of this Province; and if any portion of the line in front of the concession in which such lots are situate, or boundary of the township in which such

concession is situate, shall be obliterated or lost, then the surveyor shall run a line between the two nearest points or places where such line can be clearly and satisfactorily ascertained, in the manner provided in this Act, and in the Act first cited in the preamble to this Act, and shall plant all such intermediate posts or monuments as he may be required to plant, in the line so ascertained, having due respect to any allowance for a road or roads, common or commons, set out in such original survey; and the limits of each lot so found shall be taken to be, and are hereby declared to be the true limits thereof; any law or usage to the contrary thereof in any wise notwithstanding."

[This is the same as Sec. XX of the Act of May, 1849, respecting Lower Canada, and of the Act of 1855, Sec. X.]

GEODEDICAL JURISPRUDENCE.

The general method of establishing lines in the United States, may be taken from the United States' Statutes at Large, vol. II, p. 313, passed Feb. 11, 1805.

* CHAP. XIV., Feb. 11, 1805.—An Act concerning the mode of Surveying the Public Lands of the United States.

[See the Act of May 18, 1796, chap. XXIX, vol. I, p. 465.]

Be it enacted by the Senate and House of Representatives of the United States of America, in Congress assembled, That the Surveyor General shall cause all those lands north of the river Ohio which, by virtue of the Act intituled "An Act providing for the sale of the lands of the United States in the territory N.W. of the river Ohio, and above the mouth of the Kentucky River," were subdivided by running through the townships parallel lines each way, at the end of every two miles, and by marking a corner on each of the said lines at the end of every mile, to be subdivided into sections, by running straight lines from those marked to the opposite corresponding corners, and by marking on each of the said lines intermediate corners, as nearly as possible equidistant from the corners of the sections on the same. And the said Surveyor General shall also cause the boundaries of all the half sections which had been purchased previous to the 1st July last, and on which the surveying fees had been paid, according to law, by the purchaser, to be surveyed and marked, by running straight lines, from the half mile corners heretofore marked, to the opposite corresponding corners; and intermediate corners shall, at the same time, be marked on each of the said dividing lines, as nearly as possible equidistant from the corners of the half section on the same line.

Provided, That the whole expense of surveying and marking the lines shall not exceed three dollars for every mile which has not yet been surveyed, and which will be actually run, surveyed and marked by virtue of this section, shall be defrayed out of the moneys appropriated, or which may be hereafter appropriated for completing the surveys of the public lands of the United States.

SEC. 2. And be it further enacted, That the boundaries and contents of the several sections, half sections and quarter sections of the public lands of the United States shall be ascertained in conformity with the following principles, any Act or Acts to the contrary notwithstanding:

1st. All the corners marked in the surveys returned, by the Surveyor General, or by the surveyor of the land south of the State of Tennessee respectively, shall be established as the proper corners of sections or subdivisions of sections which they were intended to designate; and the corners of half and quarter sections, not marked on the said surveys, shall be placed as nearly as possible equidistant from those two corners which stand on the same line.

2nd. The boundary lines, actually run and marked in the surveys re-

turned by the Surveyor General, or by the surveyor of the land south of the State of Tennessee, respectively, shall be established as the proper boundary lines of the sections or subdivisions for which they were intended, and the length of such lines as returned by either of the surveyors aforesaid shall be held and considered as the true length thereof.

And the boundary lines which shall not have been actually run and marked as aforesaid, shall be ascertained by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships where no such corresponding corners have been or can be fixed, the said boundary lines shall be ascertained by running from the established corners due north and south, or east and west lines, as the case may be, to the water course, Indian boundary line, or other external boundary of such fractional township.

An Act passed 24th May, 1824, authorizes the President, if he chooses to cause the survey of lands fronting on rivers, lakes, bayous, or water courses, to be laid out 2 acres front and 40 acres deep. (See United States' Statutes at Large, vol. IV, p. 34.)

An Act passed 29th May, 1830, makes it a misdemeanor to prevent or obstruct a surveyor in the discharge of his duties. Penalties for so doing, from \$50 to \$3000, and imprisonment from 1 to 3 years.

SEC. 2 of this Act authorizes the surveyor to call on the proper authorities for a sufficient force to protect him. (*Ibid*, vol. IV, p. 417.)

The Act for adjusting claims in Louisiana passed 15th Feb., 1811, gave the Surveyor General some discretionary power to lay out lots, fronting on the river, 58 poles front and 65 poles deep. (*Ibid*, vol. II, p. 618.)

FROM THE ALABAMA REPORTS.

307. Decision of the Supreme Court of Alabama in the case of *Lewin v. Smith*.

1. The land system of the United States was designed to provide in advance with mathematical precision the ascertainment of boundaries; and the second section of the Act of Congress of 1805 furnished the rules of construction, by which all the disputes that may arise about boundaries, or the contents of any section or subdivision of a section of land, shall be ascertained.

2. When a survey has been made and returned by the Surveyors, it shall be held to be *mathematically true*, as to the lines run and marked, and the corners established, and the contents returned.

3. Each section, or separate subdivision of a section, is *independent of any other section in the township, and must be governed by its marked and established boundaries*.

4. And should they be obliterated or lost, recourse must be had to the best evidence that can be obtained, showing their former situation and place.

5. The purchaser of land from the United States takes by metes and bounds, whether the actual quantity exceeds or falls short of the amount estimated by the surveyor.

6. Where a navigable stream intervenes in running the lines of a section, the surveyor stops at that point, and does not continue across the river; the fraction thus made is complete, and its contents can be ascertained. Therefore, where there is a discrepancy between the corners of a section, as established by the United States' Surveyor, and the lines as run and marked—the latter does not yield to the former.

7. Whether this would be the case where a navigable stream does not cross the lines.—Query.

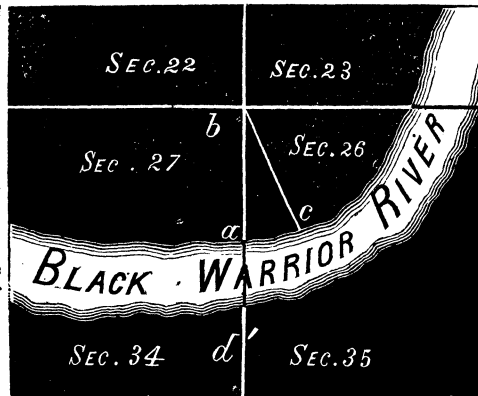
This is the case of *Lewin v. Smith*:

Error to the Circuit Court of Tuscaloosa. Plaintiff—an action of trespass on portion of fractional sec. 26, town. 21, range 11 W., lying north and west of the Black Warrior River.

Line *a b* claimed by Lewin.

Line *b c* claimed by Smith.

Field Notes. Beginning at N.W. corner, south $73^{\circ} 50'$, to a post on N. bank of the river, from which north 80° W. 0.17, box elder—S. 66° E., 0.18, do. Thence with the meander of the river S. 74° E., 7.50. N. 32° E., 10. N. 9° W., 20. N. 10° E., 22. N. 4° W., 24.50,



to a poplar on the south boundary of sec. 23; thence west 13.70 to S.W. corner, containing $100\frac{55}{100}$ acres.

NOTE.—Here the line claimed by Smith was established, by finding the original corners, *b* and *c*. Lewin claimed that, although there was no monument to be found at *a*, that such would be legally established by the intersection of a line from *b* to *d*, *d* being a fractional corner at the stockade fence supposed to be correct. The Court decided that the line *b* to *c* was the true line, as the line and bearing trees corresponded with the field notes, and therefore decided in favor of Smith. The disputed gore or triangle, *a b c*, contained 9 acres, and the jog, *a c* = 207 links.—McD.

FROM THE KENTUCKY REPORTS.

308. From the Kentucky Reports, by Thomas B. Monroe, vol. VII, p. 333. *Baxter v. Evett*. Government survey made in 1803. Patent deed issued in 1812. Ejectment instituted in 1825. Decision in 1830.

The rule is, that visible or actual boundaries, natural or artificial, called for in a certificate of survey, are to be taken as the abutments, so long as they can be found or proved. The legal presumption is, that the surveyor performed the duty of marking and bounding the survey by artificial or natural abutments, either made or adopted at the execution of the survey. And if this presumption could be destroyed by undoubted testimony, yet, as this was the fault of the officer of the Government, and not of the owner of the survey, his right ought not to be injured, when the omission can be supplied by any rational means, and descriptions furnished by the certificate of survey.

In locating a patent, the inquiry first is for the demarkation of boundary, natural or artificial, alluded to by the surveyor. *If these can be found extant, or if not now existing, can be proved to have existed, and their locality can be ascertained, these are to govern.* The courses and distances specified in a plat and certificate of survey, are designed to describe the boundaries as actually run and made by the surveyor, and to assist in preserving the evidence of their local position, to aid in tracing them whilst visible, and in establishing their former position in case of destruction, by time, accident or fraud. As guides for these purposes, the courses and distances named in a plat and certificate of survey are useful; but a line or corner established by a surveyor in making a survey, upon which a grant has issued, cannot be altered because the line is longer or shorter than the distance specified, or because the relative bearings between the abutments vary from the course named in the plat and certificate of survey: so, if the line run by the surveyor be not a right line, as supposed from his description, but be found, by tracing it, to be a curved line, yet the actual line must

govern, the visible actual boundary the thing described, and not the ideal boundary and imperfect description, is to be the guide and rule of property.

These principles are recognized in *Beckley v. Bryan*, prim. dec. 107, and Litt. sel. Cas. 91; *Morrisson v. Coghill*, prin. dec. 382; *Lyon v. Ross*, 1 Bibb. p. 467; *Cowan v. Fauntelroy*, 2 Bibb. p. 261; *Shaw v. Clement*, 1 Call, p. 438, 3d point; *Herbert v. Wise*, 3 Call, p. 239; *Baker v. Glasscocke*, 1 Hen. & Munf., p. 177; *Helm v. Smallhard*, p. 369.

From the same State Reports.

5 Dana, p. 543-4. *Johnson v. Gresham*. Here Gresham found the section to contain 696 acres; had it surveyed into four equal parts, thus embracing 1 to 3 acres of Johnson's land, which extended over the line run, with other improvements. Gresham had purchased that which Johnson had pre-empted.

Opinion of the Court by Judge Ewing, Oct. 19, 1837.

1. Though the Act of 1820, providing for surveying the public lands west of the Tennessee River, directs that it shall be laid off into townships of 36 miles square, and divided into sections of 3600 acres each, yet it is well known, through the unevenness of the ground, the inaccuracy of the instruments, and carelessness of surveyors, that many sections embrace less, and many more, than the quantity directed by the Act. The question therefore occurs, how the excess or deficiency shall be disposed of among the quarters. The statute further directs that in running the lines of townships, and the lines parallel thereto, or the lines of sections, "that trees, posts, or stones, half a mile from the corners of sections, shall be marked as corners of quarter sections." So far, therefore, as the corners or lines of the quarters can be ascertained, *they should be the guides and constituted boundaries* and abutments of each quarter. In the absence of such guides, and of all other *indicia* directing to the place where they were made, the sections should be divided, as near as may be, between the four quarters, observing, as near as practicable, the courses and distances directed by the Act. When laid down according to these rules, the quarter in contest embraces 174 acres, and covers a part of the field of the complainant, as well as his washhouse.

FROM THE ILLINOIS REPORTS.

309. From the Illinois Reports, vol. XI. *Rogers v. McClintock*.

The corners of sections on township lines were made when the township was laid out. They became fixed points, and if their position can now be shown by testimony, these must be retained, although not on a straight line—from A to B. The township line was not run on a straight line from A and B. It was run mile by mile, and these mile points are as sacred as the points A to B. (Land Laws, vol. I, pages 50, 71, 119 and 120.)

Therefore, if the actual survey, as ascertained by the monuments, show a deflected line, it is to be regarded as the true one.—*Baker v. Talbott*, 6 Monroe, 182; *Baxter v. Evett*, 7 Monroe, 333.

Township corners are of no greater authority in fixing the boundary of the survey than the section corners.—*Wishart v. Crosby*, 1 A. R. Marsh, 383.

Where sections are bounded on one side by a township line, and the line cannot be ascertained by the calls of the plat, it seems quite clear that if the corners of the adjacent section corners be found, this is better evidence to locate the township line than a resort to course merely.—1 Greenleaf Evidence, p. 369, sec. 301, note 2; 1 Richardson, p. 497.

Chief Justice Caton's Opinion.

All agree that courses, distances and quantities must always yield to the monuments and marks erected or adopted by the original surveyor, as indicating the lines run by him. Those monuments are facts. The field notes and plats, indicating courses, distances and quantities, are but descriptions which serve to assist in ascertaining those facts. Established

monuments and marked trees not only serve to show the lines of their own tracts, but they are also to be resorted to in connection with the field notes and other evidence, to fix the original location of a monument or line, which has been lost or obliterated by time, accident or design.

The original monuments at each extreme of this line, that is, the one five miles east, and the other one mile west of the corner, sought to be established, are identified, but unfortunately, none of the original monuments and marks, showing the actual line which was run between townships 5 and 6, can be found; and hence we must recur to these two, as well as other original monuments which are established, in connection with the field notes and plats, to ascertain where those monuments were; *for where they were, there the lines are.*

Much of the following is from Putnam's U. S. Digest:

309A. A survey which starts from certain points and lines not recognized as boundaries by the parties themselves, and not shown by the evidence to be *true points* of departure, *cannot be made* the basis of a judgment establishing a boundary. 12 La. An. 689 (18.) See also U. S. Digest, vol. 18, sec. 23, *Martin vs. Breaux*.

a. A party is entitled to the lands actually apportioned, and where the line marked out upon actual survey differs from that laid in the plat, the former controls the latter. 1 Head (Tenn.) 60, *Mayse vs. Lafferty*.

b. When a deed refers to a plat on record, the dimensions on the plat *must govern*; and if the dimension on the plat do not come together, then the *surplus* is to be *divided* in proportion to the dimensions on the plat. *Marsh vs. Stephenson*, 7 Ohio, N. S. 264.

c. Courses and distances on a plat referred to, are to be considered as if they were recited in the deed. *Blaney vs. Rice*, 20 Pick. 62.

d. Where, on the line of the same survey between remote corners, the length varies from the length recorded or called for, in re-establishing intermediate monuments, marking divisional tracts, it is to be presumed that the error was distributed over the whole, and not in any particular division, and the variance *must be* distributed proportionally among the various subdivisions of the whole line according to their respective lengths. 2 Iowa (Clarke) p. 139, *Moreland vs. Page*. *Bailey vs. Chamblin*, 20 Ind. 33.

e. Where the same grantor conveys to two persons, to each one a lot of land, limiting each to a certain number of rods from opposite known bounds, running in direction to meet if extended far enough, and by admeasurement the lots do not adjoin, when it appears from the same deeds that it was the intention they should, *a rule should be* which will divide the surplus over the admeasurement named in the deeds ascertained to exist by actual measurement on the earth, between the grantees in proportion to the length of their respective lines as stated in their deeds. 28 Maine 279, *Lincoln vs. Edgecomb*. *Brown vs. Gay*, 3 Greenl. 118. *Wolf vs. Scarborough*, 2 Ohio St. Rep. 363.

Deficiency to be divided *pro rata*. *Wyatt vs. Savage*, 11 Maine 431.

f. Angel on Water Courses, sec. 57, says of dividing the surplus: "By this process justice will be done, and all interference of lines and titles prevented."

No person can, under different temperatures, measure the same line into divisions a, b, c and d, and make them exactly agree; but if the difference is divided, the points of division will be the same.

When we compare the distance on a map, and find that the paper expanded or contracted, we have to allow a proportionate distance for such variance. (See Table II, p. 165.)

309b. The system of dividing *pro rata* is embodied in the Canada Surveyors' Act, and quoted at sec. 306 of this work. It is also the French system.

By the French Civil Code, Article 646, all proprietors are obliged to have their lines established. In case it may be subsequently found that the survey was incorrect, and that one had too much, if the excess of one would equal the deficit of the other, then no difficulty would occur in dividing the difference.

If the excess in one man's part is greater than the deficit in the other, it ought to be divided *pro rata* to their respective quantities, each participating in the gain as well as the loss, in proportion to their areas. This is the opinion of the most celebrated lawyers.

The following is the French text:

“Le terrain excédant au celui qui manque devra être partagé entre les parties, au *pro rata* de leur quantité respective, en participant au gain comme à la perte, chacun proportionnellement à leur contenance; c'est l'avis de plus célèbres jurisconsultes.”

Adverse possession or prescriptive right, does not interfere when the encroachment was made clandestinely or by gradual anticipation made in cultivating or in mowing it.

For prescriptive right, see the French Civil Code, Article 2262:

“Cependant la prescription ne sera jamais invoquée dans le cas où la possession sera clandestine. C'est-à-dire lorsqu'elle est le résultat d'une anticipation faite graduellement en labourant ou en fauchant.” Cours Complet. D'Arpentage. Paris, 1854. Par. D. Puille, p. 250.

a. No one has a right to establish a boundary without his contiguous owner being present, or satisfied with the surveyor employed.

The expense of survey is paid by the adjacent owners.

The loser in a contested survey has to pay all expenses. In a disputed survey, each appoints a surveyor, and these two appoint a third. If they cannot agree on the third man, the case is taken before a Justice of the Peace, who is to appoint a third surveyor.

The surveyors then read their appointments to one another, and to the parties for whom the survey is made. They examine the respective titles, original or old boundaries, if any exist, all land marks, and then proceed to make the necessary survey, and plant new boundaries. On their plan and report, or *process verbal*, they show all the detail above recited, mark the old boundary stones in black, and the new ones in red.

A stone is put at every angle of the field, and on every line at points which are visible one from another. The stones are in some places set so as to appear four to six inches over ground; but where they would be liable to be damaged, they are set under the ground.

b. Boundary Witnesses. Under each stone is made a hole, filled with delf, slags of iron, lime or broken stones, and on or near this, is a piece of slate on which the surveyor writes with a piece of brass some words called a *mute witness*.

Witness. He then sets the stone and places four other stones around it corresponding to the cardinal points. The *mute witness* or expression can be found after an elapse of one hundred years, provided it has been kept from the atmosphere. Ibid. p. 252 and 253.

The United States take pains in establishing a corner where no witness tree can be made. Under the stake or post is placed charcoal. The mound and pits about it are made in a particular manner. (See sec. 281.)

In Canada, if in wood land, the side lines from each corner is marked or blazed on both sides of the line to a distance of four or five chains, to serve as future witnesses.

309c. When the number of a lot on a plan referred to in the deed, is the only description of the land conveyed, the courses, distances, and other particulars in that plan, are to have the same effect as if recited in the deed. *Thomas vs. Patten*, 1 Shep. 329.

In ascertaining a lost survey or corner, help is to be had by considering the system of survey, and the position of those already ascertained. See *Moreland vs. Page*, 2 Clarke (Iowa) 139.

a. Fixed monuments, control courses and distances. 3 Clarke (Iowa) 143, *Sargent vs. Herod*.

b. *Metes and bounds* control acres; that is, where a deed is given by metes and bounds, which would give an area different from that in the deed, the metes and bounds will control. *Dalton vs. Rust*, 22 Texas 133.

c. Metes and bounds must govern. 1 J. J. Marsh, *Wallace vs. Maxwell*.

d. Marked lines and corners control the courses and distances laid down in a plat. 4 McLean 279.

e. If there are no monuments, courses and distances must govern. U.S. Dig., vol. 1, sec. 47.

f. So frail a witness as a stake is scarcely worthy to be called a monument, or to control the construction of a deed. *Cox vs. Freedley*, 33 Penn. State R. 124.

g. Stakes are not considered monuments in N. Carolina, but regarded as imaginary ones. 3 Dev. 65, *Reed vs. Schenck*.

h. Lines actually marked must be adhered to, though they vary from the course. 2 Overt. 304, and 7 Wheat. 7, *McNairy vs. Hightour*.

i. It is a well settled rule, that where an actual survey is made, and monuments marked or erected, and a plan afterwards made, intended to delineate such survey, and there is a variance between the plan and survey, the survey must govern. 1 Shep. 329, *Thomas vs. Patten*.

j. The actual survey designated by lines marked on the ground, is

the true survey, and will not be affected by subsequent surveys. 7 Watts 91, *Norris vs. Hamilton*.

309*d*. In locating land, the following rules are resorted to, and generally in the order stated:

1. Natural boundaries, as rivers.
2. Artificial marks, as trees, buildings.
3. Adjacent boundaries.
4. Courses and distances.

Neither rule however occupies an inflexible position, for when it is plain that there is a mistake, an inferior means of location may control a higher. 1 Richardson 491, *Fulwood vs. Graham*.

a. Description in a boundary is to be taken strongly against the grantor. 8 Connecticut 369, *Marshall vs. Niles*.

b. *Between*, excludes the termini. 1 Mass. 91, *Reese vs. Leonard*.

b. Where the boundaries mentioned in a deed are inconsistent with one another, those are to be retained which best subserve the prevailing intention manifested on the face of the deed. Ver. 511, *Gates vs. Lewis*.

309*e*. The most material and most certain calls shall control those that are less certain and less material. 7 Wheat. 7, *Newsom vs. Pryor*. *Thomas vs. Godfrey*, 3 Gill & Johnson 142.

a. What is most material and certain controls what is less material. 36 N. H. 569, *Hale vs. Davis*.

b. The least certainty in the description of lands in deeds, must yield to the greater certainty, unless the apparently conflicting description can be reconciled. 11 Conn. 335, *Benedict vs. Gaylord*.

309*f*. Where the boundaries of land are fixed, known and unquestionable monuments, although neither course nor distance, nor the computed contents correspond, the monuments must govern. 6 Mass. 131. 2 Mass. 380. *Pernan vs. Wead*. *Howe vs. Bass*.

a. A mistake in one course does not raise a presumption of a mistake in another course. 6 Litt. 93, *Bryan vs. Beekley*.

b. When there are no monuments and the courses and distances cannot be reconciled, there is no universal rule that requires one of them to yield to the other; but either may be preferred as best comports with the manifest intent of parties, and with the circumstances of the case. U. S. Dig., vol. 1, sec. 18.

c. The lines of an elder survey prevail over that of a junior. *Ib.* 77.

d. Boundaries may be proved on hearsay evidence. *Ibid.* 167.

e. The great principle which runs through all the rules of location is, that where you cannot give effect to every part of the description, that which is more fixed and certain, shall prevail over that which is less. 1 Shobhart 143, *Johnson vs. McMillan*.

309*g*. A line is to be extended to reach a boundary in the direction called for, disregarding the distance. U. S. Dig. vol. 7, 16.

a. Distances may be increased and sometimes courses departed from, in order to preserve the boundary, but the rule authorizes no other departure from the former. *Ibid.* 13.

b. If no principle of location be violated by closing from either of two points, that may be closed from which will be more against the grantor, and enclose the greater quantity of land. *Ibid.* sec. 14.

309*H.* What are boundaries described in a deed, is a question of law, the place of boundaries is a matter of fact. 4 Hawks 64, *Doe vs. Paine*.

a. What are the boundaries of a tract of land, is a mere question of construction, and for the court; but where a line is, and what are facts, must be found by a jury. 13 Ind. 379, *Burnett vs. Thompson*.

b. It is not necessary to prove a boundary by a plat of survey or field notes, but they may be proved by a witness who is acquainted with the corners and old lines, run and established by the surveyor, though he never saw the land surveyed. 17 Miss. 459, *Weaver vs. Robinett*.

c. A fence fronting on a highway for more than twenty years, is not to be the true boundary thereof under Rev. St. C. 2, if the original boundary can be made certain by ancient monuments, although the same are not now in existence. 11 Cush (Mass.) 487, *Wood vs. Quincy*.

d. The marked trees, according to which neighbors hold their distinct land when proved, ought not to be departed from though not exactly agreeing with the description. 3 Call. 239. 7 Monroe, 333. *Herbert vs. Wise*. *Baxter vs. Evett*. *Rockwell vs. Adams*.

e. Where a division line between two adjoining tracts exists at its two extremities, and for the principal part of the distance between the two tracts, and as such is recognized by the parties, it will be considered a continuous line, although on a portion of the distance there is no improvement or division fence. 6 Wendell 467.

f. If the lines were never marked, or were effaced, and their actual position cannot be found, the patent courses so far must govern. 2 Dana 2. 1 Bibb. 466. *Dimmet vs. Lashbrook*. *Lyon vs. Ross*.

g. Or, if the corners are given, a straight line from corner to corner must be pursued. Dig. vol. 1, sec. 33.

h. Abutments are not to be disregarded. *Ibid.* vol. 12, sec. 4.

309*L.* Where there is no testimony on variation, the court ought not to instruct on that subject. *Wilson vs. Inloes*, 6 Gill 121.

a. The beginning corner has no more, or the certificate of survey has no greater, dignity than any other corner. 4 Dan. 332, *Pearson vs. Baker*.

b. Sec. 34. Where no corner was ever made, and no lines appear running from the other corners towards the one desired, the place where the courses and distances will intersect, is the corner. 1 Marsh 382. 4 Monroe 382. *Wishart vs. Crosby*. *Thornberry vs. Churchill*.

c. The land must be bounded by courses and distances in the deed where there are no monuments, or where they are not distinguishable from other monuments. Dig., vol. 1, sec. 47, 48, 49.

d. Seventy acres in the S. W. corner of a section, means that it must be a square. 2 Ham. 327, Walsh vs. Ruger.

309J. The plat is proper evidence. Dig., vol. 1, sec. 61, and Sup. 4, sec. 51.

a. Mistake in the patent may be corrected by the plat on record. The survey is equal dignity with the patent. Dig., vol. 1, sec. 60.

b. A survey returned more than twenty years, is presumed to be correct. 7 Watts 91, Norris vs. Hamilton.

309K. Declaration by a surveyor, chain carrier, or other persons present at a survey, of the acts done by or under the authority of the surveyor, in making the survey, if not made after the case has been entered, and the person is dead, is admissible. U. S. Dig., vol. 12, Boundary, sec. 10. See also English Law Reports, vol. 33, p. 140.

a. An old map, thirty years amongst the records, but no date, and the clerk, owing to his old age, could give no account of it, map admissible. Gibson vs. Poor, 1 Foster (N. H.) 240.

309L. The order of the lines in a deed may be reversed. 4 Dana 322, Pearson vs. Baxter.

a. Trace the boundary in a direct line from one monument to another, whether the distance be greater or less. 41 Maine 601, Melche vs. Merryman.

NOTE. This is the same as the U. S. Act of 11th February, 1805.

b. *Northward* means due north. Haines 293. Dig., vol. 1, sec. 4.

Northerly means north when there is nothing to indicate the inclination to the east or west. 1 John 156, Brandt vs. Ogden.

c. It is a well settled fact, where a line is described as running towards one of the cardinal points, it must run directly in that course, unless it is controlled by some object. 8 Porter 9, Hogan vs. Campbell.

e. A survey made by an owner for his own convenience, is not admissible evidence for him or those claiming under him. 1 Dev. 223, Jones vs. Huggins.

309M. Parties, to establish a conventional boundary, must themselves have good title, or the subsequent owners are not bound by it. 1 Sneeds (Tenn.) 68, Rogers vs. White.

a. Parties are not bound by a consent to boundaries which have been fixed under an evident error, unless, perhaps, by the prescription of thirty years. 12 La. An. 730, Gray vs. Cawvillon.

b. The admission by a party of a mistaken boundary line for a true one, has no effect upon his title, unless occupied by one or both for fifteen years. 10 Vermont 33, Crowell vs. Bebee.

c. A hasty recognition of a line, does not estop the owner. *Overton vs. Cannon*, 2 Humph. 264.

d. In a division of land between two parties, if either was deceived by the innocent or fraudulent misrepresentation of the other, or there was any mistake in regard to their right, the division is not binding on either. 14 Georgia 384, *Bailey vs. Jones*.

e. A division line mistakenly located and agreed on by adjoining proprietors, will not be held binding and conclusive on them, if no injustice would be done by disregarding it. U. S. Digest, vol. 18, sec. 32. See, also, 29 N. Y. 392, *Coon vs. Smith*. English Reports 42, p. 307.

f. A mistaken location of the line between the owners of contiguous lots is not conclusive between the immediate parties to such location, but may be corrected. App. 412, *Colby vs. Norton*.

g. If S surveys for A, A is not estopped from claiming to the true line. 9 Yerg. 455, *Gilchrist vs. McGee*.

h. When owners establish a line and make valuable improvements, they cannot alter it. *Lavery vs. Moore*, 33 N. Y. 650.

309N. A fence between tenants, in common, if taken down by one of them, the others have no cause of action in trespass. 2 Bailey 380, *Gibson vs. Vaughn*.

309o. A line recognized by contiguous owners for thirty years, controls the courses and distances in a deed. 32 Penn. State R. 302, *Dawson vs. Mills*.

a. A line agreed on for thirty years, cannot be altered. 10 Watts 321, *Chew vs. Morton*.

b. Adjacent owners fixed stakes to indicate the boundary of water lots. One filled the part he supposed to belong to him; the other, being cognizant of the progress of the work, held that the other and his grantees were estopped to dispute the boundary. 32 Barb. (N. Y.) 347, *Lavery vs. Moore*.

c. To establish a consentable line between owners of adjoining tracts, knowledge of, and assent to the line as marked, must be shown in both parties. 4 Barr. 234, *Adamson vs. Potts*.

d. When two parties own equal parts of a lot of land, in severalty, but not divided by any visible monuments, if both *are in possession* of their respective parts for fifteen years, acquiescence in an imaginary line of division during that time, that line is thereby established as a divisional line. 9 Vernon 352, *Beecher vs. Parmalee*.

e. Sec. 29. Where parties have, without agreement, and ignorant of their right, occupied up to a division line, they may change it on discovering their mistake. Wright 576, *Avery vs. Baum*.

f. Where A and B and their hired man built a fence without a compass, and acquiesced in the fence for fifteen years, it was held to be the true line in Vermont. 18 Verm. 395, *Ackley vs. Nuck*.

- g.* Quantity generally cannot control a location. Dig. vol. 10, sec. 49.
- h.* Long and notorious possession infer legal possession. *Newcom vs. Leary*, 3 Iredell 49.
- i.* A hasty, ill-advised recognition is not binding. *Norton vs. Cannon*, Dig., vol. 4, sec. 73.
- j.* The line of division must be marked on the ground, to bring it within the bounds of a closed survey. Ibid. sec. 106.
- k.* *Bounded by a water course*, according to English and American decisions, means to the centre of the stream. (See *Angel on Water Courses*, ch. 1, sec. 12.)
- l.* East and north of a certain stream includes to the thread thereof. *Palmer vs. Mulligan*, 3 Caines (N. Y.) 319.
- m.* *Bank and water* are correlative, therefore, to a monument standing on the bank of a river, and running by or along it, or along the shore, includes to the centre. 20 Wend. (N.Y.) 149. 12 John. (N.Y.) 252.
- n.* Where a map shows the lots bounded by a water course, the lots go to the centre of the river. *Newsom vs. Pryor*, 7 Wheat. (U. S.) 7.
- o.* To the bank of a stream, includes the stream itself. *Hatch vs. Dwight*, 17 Mass. 299.
- p.* Up a creek, means to the middle thereof. 12 John. 252.
- q.* Where there are no controlling words in a deed, the bounds go to the centre of the stream. *Herring vs. Fisher*, 1 Sand. Sup. Co. (N.Y.) 344.
- r.* Land bounded by a river, not navigable, goes to the centre, unless otherwise reserved. *Nicholas vs. Siencocks*, 34 N. H. 345. 9 Cushing 492. 3 Kernan (N.Y.) 296. 18 Barb. (N. Y.) 14. *McCullough vs. Wall*, 4 Rich. 68. *Norris vs. Hill*, 1 Mann. (Mich.) 202. *Canal Trustees vs. Havern*, 5 Gilman 548. *Hammond vs. McLaughlin*, 1 Sandford Sup. Ct. R. 323. *Orindorf vs. Steel*, 2 Barb. Sup. Ct. R. 126. 3 Scam. Ill. 510. *State vs. Gilmanton*, 9 N. Hamp. 461. *Luce vs. Cartey*, 24 Wend. 541. *Thomas vs. Hatch*, 3 Sumner 170.
- s.* On, to, by a bank or margin, cannot include the stream. 6 Cow. (N. Y.) 549.
- t.* A water course may sometimes become dry. *Gavett's Administrators vs. Chamber*, 3 Ohio 495. This contains important reasons for going to the centre of the stream.
- u.* Along the bank, excludes the stream. *Child vs. Starr*, 4 Hill 369.
- v.* A corner standing on the bank of a creek; thence down the creek, etc. Boundary is the water's edge. *McCulloch vs. Allen*, 2 Hamp. 309, also *Weakley vs. Legrand*, 1 Overt. 205.
- w.* To a creek, and down the creek, with the meanders, does not convey the channel. *Sanders vs. Kenney*, J. J. Marsh 137. (See next page, which has been printed sometime in advance of this.)

monuments and marked trees not only serve to show with certainty the lines of their own tracts, but they are also to be resorted to in connection with the field notes and other evidence to fix the original location of a monument or line which has been lost, or obliterated by time, accident, or design.

The original monuments at each extreme of this line—that is, the one five miles east, and the other one mile west of the corner—sought to be established, are identified; but, unfortunately, none of the original monuments and marks, showing the actual line which was run between townships 5 and 6, can be found, and hence we must recur to these two, as well as other original monuments, which are established in connection with the field notes and plats, to ascertain where those monuments were, *for where they were, there the lines are.*

WATER COURSES.

309a. *Eminent domain* is the right retained by the government over the estates of owners, and the power to take any part of them for the public use. First paying the value of the property so taken, or the damages sustained to their respective owners. 3 Paige, N. Y. Chancery Rep. 45.

The British Crown has the right of eminent domain over tidal rivers and navigable waters, in her American colonies. Each of the United States have the same. See *Pollard v. Hogan*, 3 Howe, Rep. 223; *Goodtitle v. Kibbe*, 9 Howe Rep. 117; *Stradar v. Graham*, 10 Howe Rep. 95; *Doe v. Beebe*, 13 Howe Rep. 25. From these appear that the State has jurisdiction over navigable waters, provided it does not conflict with any provision of the general government. The Constitution of the U. States reserves the power to regulate commerce—which jurists admit to include the right to regulate navigation, and foreign and domestic intercourse, on navigable waters. On those waters the general government exercises the power to license vessels, and establish ports of entry, consequently it can prevent the construction of any material obstruction to navigation, and declare what rules and regulations are required of vessels navigating them.

Prescriptive right must set forth that the occupier or person claiming any *easement*, has been in an open, peaceable and uninterrupted possession of that which is claimed, during the time prescribed by the statute of limitation of the country, or state in which the *easement* is situated.

In England, the prescribed time is 20 years. *Balston v. Bensted*, 1 Campbell Rep., 463; *Bealey v. Shaw*, 6 East. Rep. 215.

In the United States the time is different—in New Hampshire, 20; Vermont and Connecticut, 15; and South Carolina, 5 years.

Water Course, is a body of water flowing towards the sea or lake, and is either private or public. It consists of bed, bank and water.

Public water course, is a navigable stream formed by nature, or made and dedicated to the public as such by artificial means. Navigable streams may become sometimes dry.

A stream which can be used to transport goods in a boat, or float rafts of timber or saw logs, is deemed a navigable stream, and becomes a public highway. But a stream made navigable by the owners, and not dedicated to the public, is a private water course. See *Wadsworth v. Smith*, 2 Fairfield, Maine Rep. 278.

The owners of the adjoining lands have a title to the bed of the river; each proprietor going to the centre, or thread thereof, when the river is made the boundary.

Should the river become permanently dry on account of being turned off in some other direction; or other cause, then the adjoining *riparian* owners claim to the centre of the bed of the stream, the same as if it were a public highway.

Bounded by a water course—signifies that the boundary goes to the centre of the river. *Morrison v. Keen*, 3 Greenleaf, Maine Rep. 474; 1 Randolph, Va., Rep. 420; *Waterman v. Johnson*, 3 Pickering, Mass. R., 261; *Star v. Child*, 20 Wendell, N. Y. Rep., 149.

To a swamp, means to the middle of the stream or creek, unless described to the edge of the swamp. *Tilder v. Bonnet*, 2 McMull South Carolina Report, 44.

Any unreasonable or material impediment to navigation placed in a navigable stream, is a public nuisance. 12 Peters, U. S. Rep. 91. The legislature cannot grant leave to build an obstruction to navigation. 5 Ohio Rep., 410.

A winter way on the ice, dedicated to the public for 20 years, becomes a highway, and cannot be obstructed. 6 Shepley, Maine Rep., 438.

The legislature cannot declare a river navigable which is not really so, unless they pay the riparian owners for all damages sustained by them. 16 Ohio Rep. 540.

Rivers in which the tide ebbs and flows are public, both their water and bed as far as the water is found to be affected by local influences, but above this, the riparian owners own to the centre of the river, and have the exclusive right of fishing, etc., the public having the right of highway. See 26 Wendell, N. Y. Rep. 404.

Banks of a navigable river are not public highways, unless so dedicated, as the banks of the Mississippi, in Illinois and Tennessee, and the rivers of Missouri for a reasonable time. See 4 Missouri Rep. 343; 3 Scammon 510.

This last decision had reference to a place in an unbroken forest, where it was admitted that the navigators had a right to land and fasten to the shore. It would be unfair to give a captain and crew of any vessel the right to land on a man's wharf, or in his enclosure without his permission; therefore, it would appear "that the public have the privilege to come upon the river bank so long as it is vacant, although the owner may at any time occupy it, and exclude all mankind." *Austin v. Carter*, 1 Mass. Rep. 231.

Obstructing navigation by building bridges without an act of the legislature, sinking impediments or throwing out filth, which would endanger the health of those navigating the river, is a nuisance. See Russel on Crimes 485. Although an obstruction may be built under an act of the legislature in navigable waters, he who maintains it there, is liable for any damage sustained by any vessel or navigator navigating therein. 4 Watts, Pennsylvania Rep. 437.

Bridges can be built over navigable rivers by first obtaining an act of the legislature, *Commonwealth v. Breed*, 4 Pick, Massachusetts R. 460; *Strong v. Dunlap*, 10 Humphrey, Tenn. R. 423. See Angel on Highways, sec. 4.

The State of Virginia, authorized a company to build a bridge at Wheeling, across the eastern channel of the Ohio river, it was suspended so low as to obstruct materially the navigation thereof. The Superior Court ordered its removal, but gave them a limited time to remove it to the other channel, where the company proposed to have sufficient depth of water and a drawbridge of 200 feet wide. The Court did not consider the additional length of channel nor the necessary time in opening the draw a material impediment. Subsequently an act of Congress declared the first bridge built on the eastern channel not to be a *material or unreasonable obstruction*, and ordered that captains and crews of vessels navigating on the river should govern themselves accordingly by lowering their chimneys, etc. 13 Howe Rep. 518; 18 Howe Rep. 421.

If a bridge is built across a river in a reasonable situation, leaving sufficient space for vessels to pass through, and causing no unreasonable delay or obstruction, and is built for the public good, it is not deemed a nuisance. *Rex v. Russel*, 6 Barn. and Cresw. 566; 15 Wendell, 133.

For further, see Judge Caton's decision in the Rock Island Bridge case, delivered in 1862.

Canals. If after being built, a new road is made over it, the canal company is not obliged to erect a bridge. *Morris Canal v. State*, 4 Zab-riskie, N. Y. Rep. 62.

In America, when two boats meet, each turns to the right. They carry lights at the bow. Freight boats must give away to packet or passenger boats. *Farnsworth v. Groot*, 6 Cowen, N. Y. Rep. 698.

In Pennsylvania, the descending boat has preference to the ascending. Act of Pennsylvania, April 10, 1826.

Ferries. The owner of a public ferry ought to own the land on both sides of the river. Savill 11 pl. 29. A ferry cannot land at the terminus of a public highway, without the consent of the riparian owners. *Chambers v. Ferry*, 1 Yeates. A use of twenty years, does not confer the right to land on the opposite side without the consent of the adjacent owners.

If A erects a dam or ditch on his own land, provided it does not overflow the land of his neighbor B, or diverts the water from him, he is justified in so doing. *Colborne v. Richards*, 13 Mass. Rep. 420. But if A injures B, by diverting the water or overflowing his land, B is empowered to enter on A's land and remove the obstructions when finished, but not during the progress of the work, doing no unnecessary damage, or causing no riot. In this case, B cannot recover damages for expense of removal, etc. If B enters suit against A, he recovers damages, and the nuisance is abated. *Gleason v. Gary*, 4 Connecticut Rep. 418; 3 Blackstone Comm. 9 Mass. Rep., 216; 2 Dana, Kentucky Rep. 158.

If B, C and D, as separate owners, cause a nuisance on A's property, A can sue either of the offending party, and the non-joinder of the others cannot be pleaded in abatement. 1 Chitty's Pleadings, 75.

The tenant may sue for a nuisance, even though it be of a temporary nature. *'Angel on Water Courses*, chap. 10, sec. 398.

The reversioners may also have an action where the nuisance is of a permanent one. *Ibid.*

If A and B own land on the same river, one above the other, one of them cannot erect a dam which would prevent the passage of fish to the other. *Weld v. Hornby*, 7 East. R. 195; 5 Pickering, Mass. Rep. 199.

One riparian owner cannot divert any part of the water dividing their estate, without the consent of the other; as each has a right to the use of the whole of the stream. 13 Johnson, N. Y. Rep. 212.

It is not lawful for one riparian owner to erect a dam so as to divert the water in another direction, to the injury of any other owner. 3 Scammon, Illinois Rep. 492.

Where mills are situate on both banks of a river, each having an equal right; one of them, in dry weather, is not allowed to use more than his share of the water. See Angel on Water Courses, chap. 4. p. 105.

One mill cannot detain the water from another lower down the stream, nor lessen the supply in a given time. 13 Connecticut Rep. 303.

One riparian owner cannot overflow land above or below him by means of a dam or sluices, etc., or by retaining water for a time, and then letting it escape suddenly. See 7 Pickering, Massachusetts Rep. 76, and 17 Johnson, N. Y. Rep. 306.

Hence appears the legality of constructing works to protect an owner's land from being overflowed. Such work may be dams or drains leading to the nearest natural outfall; for it is evident, that if by making a drain, ditch or canal, to carry off any overflow to the nearest outlet, such proceedings would be legal, and the party causing the overflow would have no cause of complaint. Merrill v. Parker Coxe, New Jersey Rep. 460.

For the purpose of Irrigation. A man cannot materially diminish the water that would naturally flow in a water course. Hall v. Swift, 6 Scott R. 167. He may use it for motive power, the use of his family, and watering his cattle; also for the purpose of irrigating his land, provided it does not injure his neighbors or deprive a mill of the use of the water. That which is made to pass over his land for irrigation if not absorbed by the soil, is to be returned to its natural bed. Arnold v. Foot, 12 Wendell, N. Y. Rep. 330; Anthony v. Lapham, 5 Pickering, Mass. Rep. 175.

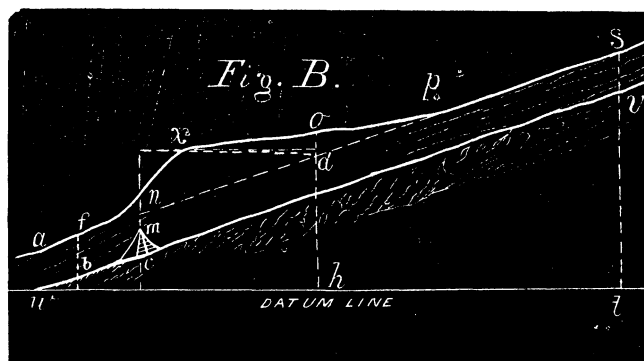
A riparian owner has no right to build any work which would in ordinary flood cause his neighbor's land to be overflowed, even if such was to protect his own property from being destroyed. Angel on Water Courses, chap. 9, p. 334.

In several countries, the law authorizes A to construct a drain or ditch from the nearest outlet of the overflow on his land, along the lowest level through his neighbor's land, to the nearest outfall. This is the law in Canada. Callis on Sewers, 136.

If A raises an obstruction by which B's mill grinds slower than before, A is liable to action. 7 Con. N. Y. Rep. 266, and 1 Rawle, Penn. Rep. 218.

Back water. No person without a grant or license is allowed to raise the water higher than where it is in its natural state, or, unless the so doing has been uninterruptedly done for twenty years. Regina v. North Midland Railway Company, Railway Cases, vol. 2, part 1. p. 1.

No one can raise the level of the water where it enters his land, nor lower it where it leaves it. Hill v. Ward, 2 Gill. Ill. Rep. 285.



Let $a s$ represent the surface of a uniform channel, and $w v$ its bottom. Let $w t = \text{datum line}$, parallel to the horizon; $f b, g m, h d$ and $t s$ the respective heights above datum. Let from a to b belong to A, b to d belong to B, and d to s belong to C. B found that on his land he had 10 feet of a fall from d to n , and the same from n to f . He built a dam $= c m$, making the surface of the water at x the same height as the point d , and claimed that he did no injury to the owner C. If C had a peg or reference mark at d , before B raised his dam, he could prove that B caused back water on him. When this is not the case, recourse must be had to the laws of hydraulics. Mr. Neville, County Surveyor of Louth, Ireland, in his *Hydraulics*, p. 110, shows that (practically) in a uniform channel, when the surface of the water on the top or crest of the dam is on the same level with d , the water will back up to p , making $x p = 1.5$ to 1.9 times $x d$.

The latter is that given by Du Buat, and generally used. See *Encyclopedia Britannica*, vol. 19. The former, 1.5, by Funk. See D'Aubuisson's *Hydraulics* by Bennett, sec. 167.

When the channel is uniform, the surface $x o p$ is nearly that of a hyperbola, whose asymptote is the natural surface; consequently, the dam would take effect on the whole length of the channel. All agree that the effect will be insensible, when the distance, $x p$, from the dam is more than 1.9 times the distance $x d$. Let x be the point behind the dam where the water is apparently still, then $m n$ is half the height of x above m , as the water, in falling from x , assumes the hydraulic curve, which is practically that of a parabola. As we know the quantity of water passing over in a given time, and the length of the dam, we can find the height $m n$, twice of which added to $c m$ gives the height of x above c . Let this height of x above $c = H$. Find where the same level through x , will meet the natural surface as at d , then measure $d p = \text{nine-tenths of } d x$, the point p will be the practical limit of back water, or *remous*. Within this limit we are to confine our inquiries, as to whether B has trespassed on C, and if the dam will cause greater damage in time of high water than when at its ordinary stage. For further, see sections on Hydraulics.

Owners of Islands, own to the thread of the river on each side. *Hendrick v. Johnson*, 6 Porter, Alabama Rep. 472. The main branch or channel is the boundary, if nothing to the contrary is expressed. *Doddridge v. Thompson*, 9 Wheel, U. S. Report, 470. Above the margin goes to the centre. N. Y. Rep. 6 Cow. 518.

Natural and permanent objects are preferred to courses and distances. *Hurley v. Morgan*, 1 Devereaux and Bat. N. Carolina Report, 425.

Boundary may begin at a post or stake on the land, by the river, then run on a given course, a certain distance to a stake standing on the bank of the river, and so along the river.³ The law holds that the centre of the river or water course, is the boundary. 5 New Hampshire Rep. 520; see also *Lowell vs. Robinson*, 4 Maine Rep. 357.

A grant of land extending a given distance from a river, must be laid off by lines equidistant from the nearest points on the river. Therefore a survey of the bank of the river is made, and the rear line run parallel to this at the given distance. *Williams v. Jackson*, N. Y. Rep. 489.

PONDS AND LAKES.

309b. *Land conveyed* on a lake, if it is a natural one, extends only to the margin of the lake. But if the lake or pond is formed by a dam, backing up the water of a stream in a natural valley, then the grant goes to the centre of the stream *in its natural state*. *State v. Gilmanton*, 9 N. Hampshire R. 461.

The beds of lakes, or inland seas with the islands, belong to the public. The riparian owners may claim to low water mark. Land Commissioners v. People, 5 Wend. N. Y. R. 423. Where a pond has been made by a dam across a stream, evidence must be had by parol, or from maps showing where the centre of the river was; for if the land was higher on one side than on the other, the thread of the original stream would be found nearer to the high ground.

Island in the middle of a stream not navigable, is divided between the riparian owners, in proportion to the fronts on the river. 2 Blackstone, 1. But if the island is not in the middle, then the dividing line through it, is by lines drawn in proportion to the respective distances from the adjacent shores. 13 Wendell, N. Y. Rep. 255. If no part of the island is on one side of the middle of the river, then the whole of the island belongs to the riparian owners nearest to the island. See Cooper, Justice, lib. 2, t. 3, and Civil Code of Louisiana, art. 505 to 507.

An island between an island and the shore, is divided as if the island was main land, for if it be nearer the main land than the island, it is divided in proportion as above. Fleta, lib. 3, c. II. § 8.

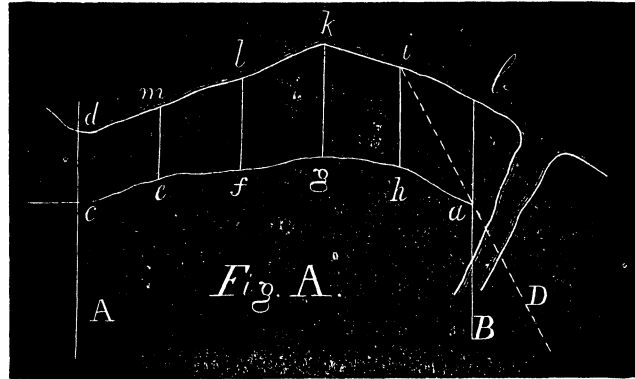
Where there are channels surrounding one or more islands, one has no right to place dams or other obstructions, by which the water of one channel may be diverted into another. 10 Wendell, N. Y. Rep. 260.

If a river or water course divides itself into channels, and cuts through a man's land, forming an island, the owner of the land thus encircled by water can claim his land. 5 Cowen, 216.

ACCRETION OR ALLUVION.

309c. Accretion or alluvion is where land is formed by the accumulation of sand or other deposits on the shore of the sea, lake or river. Such accretions being gradual or imperceptibly formed, so that no one exactly can show how much has been added to the adjacent land in a given time, the adjacent owner is entitled to the accretion. 2 Blackstone Com. 262. See also Cooper Justice, lib. 2, tit. 1.

In subdividing an accretion, find the original front of each of the adjacent lots, between the respective side lines of the estates; then measure the new line of river between the extreme side lines, and divide *pro rata*, then draw lines from point to point, as on the annexed diagram.



The meandered lines are taken from corner to corner of each lot, without regard to the sinuosities of the shore as *b i*.

It is sometimes difficult to determine the position of the lines *c d* and *a b*. As some may contend that *A c* produced in a straight line to the water, would determine the point *d*, also *B a* produced, would determine *b*, from the above diagram appears that by producing *B a* to the water, it would intersect near *i*, thus cutting off one owner from a part of the accretion, and entirely from the water.

The plan adopted in the States of Maine and Massachusetts, in determining *b* and *d*, is as follows: From *a* draw a perpendicular to *B a*, and find its intersection on the water's edge, and call it *Q*. From *a* with *a h* as base, draw a perpendicular, and find its intersection on the water's edge, and call it *P*. Bisect the distance *P Q* in the point *r*, then the line *a r*, determines the point *b*. In like manner we determine the point *d*. Having *b* and *d*, we find *i*, *k*, etc., as above.

In Maine and Massachusetts the point *i*, *k*, *l* and *m* are found as we have found *b* and *d*, erecting two perpendiculars from each abuttal, on the main land, one from each adjacent line and bisecting their distance apart for a new abuttal. 6 Pickering, Mass. Rep. 158; 9 Greenleaf, Maine Rep. 44.

When *A c* and *B a* are township lines, as in the Western States, they are run due East and West, or North and South. In this case, *d* and *b* would be found by producing *A c* and *B a* due East and West, or North and South, as the case may be. Now, let *B a c* be the original shore, and *d, b, n, a* and *B* the present shore, making *c, z, n, d* the accretion or alluvion. It is evident that it would be incorrect to divide the space *a, n, b, d*, between the riparian owners, that only *b d* should be so divided. When *A c* and *B a* are township lines run East and West, or North and South, as in the Western States, they are run on their true courses to the water's edge, intersecting at the points *d* and *b*. Here it would be plain that the space *b d* should be divided in proportion to the fronts *c e*, *e f*, etc., by the above method.

We do not know a case in Wisconsin or Illinois, where a surveyor has adopted this method. They run their lines at right angles to the adjacent section lines, which many of them take for a due East and West, or North and South line, as required by the act of Congress, passed 1805.

The accretion *b, a, n*, in our opinion, would belong to him who owns front *a h*. There is a similar case to this pending for some time in Chicago, where some claim that the water front *a, n, b, d* should be divided; others claim that only *b to d*, as the part *a, b, n* may be washed away, by the same agent which has made it.

"Where land is bounded by water, and allusions are gradually formed, the owner shall still hold to the same boundary, including the accumulated soil. Every proprietor whose land is thus bounded, is subject to a loss by the same means that may add to his territory, and as he is without remedy for his loss in this way, he cannot be held accountable for his gain." *New Orleans v. United States*, laid down as a fundamental law by Judge Drummond, Oct. 1858, in his charge to the jury in the Chicago sand bar case.

When the river or stream changes its course. If it changes suddenly from being between A and B, to be entirely on B, then the whole river belongs to B. But if the recession of a stream or lake be gradual or imperceptible, then the boundary between A and B will be on the water, as if no recession had taken place. 2 Blackstone, Com. 262; 1 Hawkes, North Carolina R. 56.

When a stream suddenly causes A's soil to be joined to B's, A has a right to recover it, by directing the river in its original channel, or by taking back the earth in scows, etc., before the soil so added becomes firmly incorporated with B's land. 2 Blackstone Com. 262.

HIGHWAYS.

309d. *Highway* is a public road, which every citizen has a right to use. 3 Kent Comm. 32. It has been discussed in several States, whether streets in towns and cities are highways; but the general opinion is that they are. *Hobbs v. Lowell*, 19 Pick. Mass. Rep. 405; *City of Cincinnati v. White*, 8 Peters, U. S. Rep. 431. A street or highway ending on a river or sea, cannot be "blocked up" so as to prevent public access to the water. *Woodyer v. Hadden*, 5 Taunton R. 125.

When a road leads between the land of A and B, and that the road becomes temporarily or unexpectedly impassable, the public has a right to go on the adjoining land. *Absor v. French*, 2 Show, 28; *Campbell v. Race*, 7 Cushing, Mass. Rep. 411.

Width of public highways is four rods, if nothing to the contrary is specified, or unless by user for twenty years, the width has been less. *Horlan v. Harriston*, 6 Cow 189.

Twenty years uninterrupted user of a highway is prima facie evidence of a prescriptive right. 1 Saund., 323 a, 10 East 476.

Unenclosed lands adjoining a highway, may be travelled on by the public. *Cleveland v. Cleveland*, 12 Wend. 376.

Owners of the land adjoining a public highway, own the fee in the road, unless the contrary is expressed. The public having only an easement in it. When the road is vacated, it reverts to the original owners. *Comyn digest Dig. tit. Chemin A 2*; *Chatham v. Brand*, 11 Conn. R. 60; *Kennedy v. Jones*, 11 Alabama R. 63; *Jackson v. Hathaway*, 15 Johnson's Rep. 947.

A road is dedicated to the public, when the owners put a map on record showing the lots, streets, roads or alleys. *Manly et al v. Gibson*, 13 Illinois, 308.

In Illinois the courts have decided, that in the county the owners of land adjoining a road have the fee to the centre of it, and that they have only granted an easement, or right to pass over it, to the public. Country roads are styled highways. In incorporated towns and cities, roads are denominated streets, the fees of which are in the corporations or city authorities. The original owner has no further control over that part of his land. *Huntley v. Middleton*, 13 Illinois, 54.

In Chicago, however, the adjacent owners build cellars under the streets, and the corporation rents the ends of unbridged streets on the river, for dock purposes. Where streets are vacated, they revert to the original owners, as in other States. The adjacent owners must grade the streets and build the sidewalks, yet by the above decision they have no claim to the fee therein. It appears strange that Archer road outside the city limits is a highway, and inside the limits, is a street. The road outside and inside is the same. Part of that now inside, was in January, 1863, outside; consequently, what is now a street, was 10 months ago a highway. Then, the fee in the road was in the adjacent owners, now by the above decision, it is in the corporation. It seems difficult to determine the point where a highway becomes a street, and *vice versa*.

Footpaths. *Cul-de-sac* are thoroughfares leading from one road to another, or from one road to a church or buildings. The latter is termed a *cul-de-sac*. These, if used as a highway for 20 years, become a highway. *Wellbeloved on Highways*, page 10. See *Angel on Highways*, sec. 29.

A cannot claim a way over B's land.

A cannot claim a way from his land through B's; but may claim a way from one part of his land to another part thereof, through B's, that is when A's land is on both sides of B's. *Cruises' English Digest*, vol. 3, p. 122.

If A sells part of his land to E, which is surrounded on all sides by A's, or partly by A's and others, a right of way necessarily passes to B. 2 *Roll's Abridgment*, Co. P. L. 17, 18.

If A owned 4 fields, the 3 outer ones enclosing the fourth, if he sells the outer three, he has still a right of way into the fourth. *Cruise*, vol. 3, p. 124; but he cannot go beyond this enclosure. *Ibid*, 126. When a right of way has been extinguished by unity of possessions, it may be revived by severance. *Ibid*, p. 129.

Boundaries on highways, when expressed as bounded by a highway, it means that the fee to the centre of the road is conveyed. 3 *Kent Comm.* 483.

Exceptions to this rule are found in *Canal Trustees v. Haven*, 11 Illinois R. 554, where it is affirmed that the owner cannot claim but the extent of his lot.

By, on, or along, includes the middle of the road. 2 *Metcalf, Mass. R.* 151.

By the line of, by the margin of, by the side of, does not include the fee to any part of the road. 15 *Johnson, N. Y. R.* 447.

The town that suffers its highways to be out of repair, or the party who obstructs the same, is answerable to the public by indictment, but not to an individual, unless he suffers damage by reason thereof in his *person or property*. *Smith v. Smith*, 2 Pick. Mass. Rep. 621; *Forman v. Concord*, 2 New Hampshire Rep. 292. Individuals and private corporations are likewise liable to pay damages. 6 Johnson, N. Y. Rep. 90.

Lord Ellenborough says two things must concur to support this action; an obstruction in the road by the fault of the defendant, and no want of ordinary care to avoid it on the part of the plaintiff. *Butterfield v. Forrester*, 11 East. Rep. 60.

Towns, or corporations, are primarily liable for injuries, caused by an individual placing an obstruction in the highway. The town may be indemnified for the same amount. In Massachusetts the town or corporation is liable to double damages after reasonable notice of the defects had been given, but they can recover of the individual causing it but the single amount. *Merrill v. Hampden*, 26 Maine Rep. 224; *Howard v. Bridgewater*, 16 Pick. Mass. Rep. 189; *Lowell v. Boston and Lowell Railroad corporation*, 23 Pick. Mass. R. 24.

By the extension of a straight line, is to be understood, that it is produced or continued in a straight line. *Woodyer v. Hadden*, 5 Faunl. Rep. 125.

Plankroads, if made on a highway, continue to be highways, the public have the right to pass over them, by paying toll. Angel on Highways, sec. 14.

The Court has the jurisdiction to restrain any unauthorized appropriation of the public property to private uses; which may amount to a public nuisance, or may endanger, or injuriously affect the public interest. Where officers, acting under oath, are intrusted with the protection of such property, private persons are not allowed to interfere. 6 Paige, Chancery Rep. 133.

Railroads may be a public nuisance, when their rails are allowed to be 2 to 3 inches above the level of the streets, as now in Chicago,—thereby requiring an additional force to overcome the resistance. See Manual, 319e, where it has been shown, that the rail was 3 inches above the level of the street, and required a force of 969 pounds to overcome the resistance. This state of things would evidently be a public injury, and be sufficient reasons to prevent a recurrence of it in any place where it had previously existed. It may be a private injury, when the track is so near a man's sidewalk, as to prevent a team standing there for a reasonable time to load or unload.

When a road is dedicated to the public at the time of making a town plat or map, it is held that the street must have the recorded width though the adjoining lots should fall short, because the street has been first conveyed.

When a new street is made, the expense is borne by the adjacent owners or parties benefitted. Subsequent improvements are usually made by a general city or town tax; sometimes by the adjacent owners—the city paying for intersections of streets and sidewalks. In February, 1864, Judges Wilson and Van Higgins, of the Cook County (Illinois) Superior Court, decided that a lot cannot be taxed for more than the actual increase in its value, caused by the improvement in front thereof.

SIR RICHARD GRIFFITH'S SYSTEM OF VALUATION.

NOTE.—All new matter introduced is in italics or enclosed in parenthesis.

309e. *The intention of the General Valuation Act* was, that a valuation of the lands of Ireland, made at distant times and places, should have a relative value, ascertained on the basis of the prices of agricultural produce, and that though made at distant periods, should be the same. The 11th section of the Act, quoted below, gives the standard prices of agricultural produce, according to which the uniform value of any tenement is to be ascertained, and all valuations made as if these prices were the same, at the time of making the valuation.

309f. *Act 15 and 16 Victoria, Cap. 63, Sec. XI.*—Each tenement or rateable hereditament shall be separately valued, taking for basis the net annual value thereof with reference to prices of agricultural produce hereinafter specified; all peculiar local circumstances in each case to be taken into consideration, and all rates, taxes and public charges, if any, (except tithes) being paid by the tenant.

NOTE.—(The articles in italics are not in the above section, but inserted so as to extend the system as much as possible to America and other places.)

General average prices of 100 lbs. of

Wheat,	6s. 9d. or \$1.62	Mutton,	36s. 11d. or \$8.86
Oats,	4s. 4d. “ 1.04	Pork,	28s. 10d. “ 6.91
Barley,	4s. 11d. “ 1.19	Flax,	44s. 1d. “ 10.58
<i>Maize,</i>		<i>Hemp,</i>	
<i>Rice,</i>		<i>Tobacco,</i>	
Butter,	58s. 10d. or 14.11	<i>Cotton,</i>	
Beef,	35s. 3d. or 7.65	<i>Sugar,</i>	
&c.	&c. &c.		

To find the price of live weights.—Deduct one-third for beef and mutton, and one-fifth for pork.

Houses and Buildings shall be valued upon the annual estimated rent which may be reasonably expected from year to year, the tenant paying all incidental charges, except tithes.

Sections 12 to 16, inclusive, of the act, treat of the kind of properties to be valued.

309g. *Lands and Buildings* used for scientific, charitable or other public purposes, are valued at half their annual value, all improvements and mines opened during seven years; all commons, rights of fishing, canals, navigations and rights of navigation, railways and tramways; all right of way and easement over land; all mills and buildings built for manufacturing purposes, together with all water power thereof. But the valuation does not extend to the valuation of machinery in such buildings.

A tenement is any rateable hereditament held for a term of not less than one year.

Every rateable tenement shall be separately valued.

The valuator shall have a map showing the correct boundary of each tenement, which shall be marked or numbered for references. The map shall show if half streets, roads or rivers are included.

The Field Book is to contain a full description of every tenement in the townland (or township), the names of the owners and occupiers, together with references to the corresponding numbers on the plan or map. The book to be headed with the name of the county, parish (or township), each townland (or section).

Gentlemen of property, learning, or the law, should have "Esquire" attached to their names.

Land, is ground used for agricultural purposes.

Houses and Offices, are buildings used for residences.

Other tenements, such as brickfield, brewery, &c.

To determine the value of land, particular attention must be paid to its geological and geographical position, so far as may be necessary to develop the natural and relative power of the soil.

NATURE OF SOILS.

309h. *Examine the soil and subsoil* by digging it up, in order to ascertain its natural capabilities; for if guided by the appearance of the crops, the valuator may put too high a price on bad land highly manured. This would be unjust, as it is the intrinsic and not the temporary value which is to be determined.

To obtain an average value, where the soil differs considerably in short distances; examine and price each tract separately, and take the mean price.

The value of soil depends on its composition and subsoil.

Subsoil may be considered the *regulator* or *governor* of the powers of the soil, for the nature of its composition considerably retards or promotes vegetation.

In *porous or sandy soil*, the necessary nutriment for plants is washed away, or absorbed below the roots of the plants.

In *clayey soils*, the subsoil is impervious, the active or surface soil is cold and late, and produces *aquatic* plants. Hence appears the necessity of strict attention to the subsoil.

Soils are compounded of organic and inorganic matter; the former derived from the disintegration and decomposition of rocks. The proportion in which they are combined is of the utmost importance.

A good soil may contain six to ten per cent. of organic matter; the remainder should have its greater portion silica; the lesser alumina, lime, potash, soda, &c.—(See tables of analysis at the end of these instructions.)

Soils vary considerably in relation to the physical aspect; thus in mountain or hilly districts, where the rocks are exposed to atmospherical influence, the soils of the valleys consist of the disintegrated substance of the rocks, whilst that of the plains is composed of drifted materials, foreign to the subjacent rock. In the former case the soil is characterised by the locality; in the latter it is not.

By reference to the *Geological Map of Ireland*, it will be seen that the mountain soil is referable to the granite, schistose rocks and sandstone.

The fertility of the soil is to some extent dependent on the proportion or combinations which exist between the component minerals of the rocks from which it may have been formed; thus granite in which feldspar is in excess when disintegrated, usually forms a deep and easily improved soil, whilst that in which it is deficient will be comparatively unproductive.

The detritus of mica slate and the schistose rocks form moderately friable soils fit for tillage and pasture.

Sandstone soils derived from sandstone, are generally poor.

*The most productive lands in Ireland are situate in the carboniferous limestone plain, which, as shown on the Geological Map, occupies nearly two-thirds of that country. When to the naturally fertile calcareous soils of this great district, foreign matters are added, derived from the disintegration of granite and trappean igneous rocks, as well as from mica slate, clay slate, and other sedimentary rocks, soils of an unusually fertile character are produced. Thus the proverbially rich soil of the *Golden-valle*, situate in the limestone district extending between Limerick and Tipperary, is the result of the intermixture of disintegrated trap derived from the numerous igneous protusions which are dispersed through that district, with the calcareous soil of the valley.*

Lands of superior fertility occur near the contacts of the upper series of the carboniferous limestone and the shales of the millstone grit, or lower coal series; important examples of this kind will be found in the valley of the Barrow and Nore, etc., etc.

For geological arrangement the carboniferous limestone of Ireland has been divided into four series.

1st Series beginning from below the yellow sandstone and carboniferous slate.

2d Series, the lower limestone.

3d Series, the calp series.

4th Series, the upper limestone.

Soil derived from 1st Series is usually cold and unproductive, except where beds of moderately pure limestone are interstratified with the ordinary strata, consisting of sandstone and slate-shale.

The 2d Series, when not converted by drift, consisting chiefly of limestone-gravel intermixed with clay, usually presents a friable loam fit for producing all kinds of cereal and green crops, likewise dairy and feeding pastures for heavy cattle, and superior sheep-walks.

The 3d Series consists of alternations of dark grey shale, and dark grey impure argillo-siliceous limestone, producing soil usually cold, sour, and unfit for cereal crops; but in many districts naturally dry, or which has been drained and laid down for pasture. This soil produces superior feeding grasses, particularly the cock's foot grass. These pastures improve annually, and are seldom cultivated, because they are considered the best for fattening heavy cattle.

The 4th Series produces admirable sheep pasture, and, in some localities, superior feeding grounds for heavy cattle, and produces every variety of cereal and green crops.

309i. It is of the utmost importance that the valuator should carefully attend to the mineral composition of the soil in each case, and a reference to the Geological Map will frequently assist his judgment in this respect, the relative position of the subjacent rocks having been determined upon sectional and fossiliferous evidence. He should carefully observe the changes in the quality and fertility of the soil near to the boundaries of different rock formations, and should expect and look for sudden transitions from cold, sterile, clayey soils, as in the millstone grit districts, into the rich unctuous loams of the adjoining limestone districts, which

usually commence close to the line of boundary; and similar rapid changes will be observed from barrenness to fertility, along the boundaries of our granite, trap, and schistose districts, and likewise on the border of schistose and limestone districts, the principle being that every change in the composition of the subjacent rocks tends to an alteration in the quality both of the active and subsoils.

As it appears from the foregoing that the detritus of rocks enters largely into the composition of soils and other formations, the most trustworthy analysis is supplied, which, compared with the crops usually cultivated, will show their relative value and deficiencies.

NOTE.—(The table of analysis given by Sir Richard Griffith is less than one page. Those given by us in the following pages of these instructions are compiled from the most authentic sources, and will enable the valuator or surveyor to make a correct valuation. The surveyor will be able, in any part of the world, to give valuable instructions to those agriculturists with whom he may come in contact. We also give the method of making an approximate analysis of the rocks, minerals and soils which he may be required to value. Where a more minute analysis is required, he may give a specimen of that required to be analysed to some practical chemist—such as Jackson, of Boston; Hunt, of Montreal; Blaney, Mariner, or Mahla, of Chicago; Kane, or Cameron, of Dublin; Muspratt, or Way, of England, etc. etc.

Table in section 310 contains the analysis of rocks and grasses.

Section 310*a*, analysis of trees and grasses.

Section 310*b*, analysis of grains, hemp and flax.

Section 310*c*, analysis of vegetables and fruit.

Section 310*d*, analysis of manures.

Section 310*e*, comparative value of manures; the whole series making several pages of valuable information.

In Canada, the law requires that Provincial Land Surveyors should know a sufficient share of mineralogy, so as to enable them to assist in developing the resources of that country. In Europe, all valuations of lands are generally made by surveyors, or those thoroughly versed in that science; but in the United States a political tinsmith may be an assessor or valuator, although not knowing the difference between a solid and a square. This state of things ought not to be so, and points out the necessity of forming a Civil Engineers' and Surveyors' Institute, similar to those in other countries.)

From these tables it will appear what materials are in the formation of the soil, and the requirements of the plants cultivated; thus, in corn and grasses, silica predominates. Seeds and grain require phosphoric acid. Beans and leguminous plants require lime and alkalies. Turnips, beets and potatoes require potash and soda.

The soils of loamy, low lands, particularly those on the margins of rivers and lakes, usually consist of finely comminuted detrital matter, derived from various rocks; such frequently, in Ireland, contain much calcareous matter, and are very fertile when well drained and tilled. The rich, low-lying lands which border the lower Shannon, etc., are alluvial, and highly productive.

It is necessary that the valuator should enter into his book a short, accurate description of the nature of the soil and subsoil of every

tenement which may come under his consideration, and that all valuers may attach the same meaning or descriptive words to them. The following classification will render this description as uniform as possible :

Classification of soils, with reference to their composition, may be comprehended under the following heads, viz :

Argillaceous or clayey—clayey, clayey loam, argillaceous, alluvial.

Silicious or sandy—sandy, gravelly, slaty or rocky.

Calcareous—limey, limestone gravel, marl.

Peat soil—moor, peat.

The color of soils is derived from different admixtures of oxide or rust of iron.

Argillaceous earths, or those in which alumina is abundant, as brick and pipe clays.

The soil in which alumina predominates is termed clay.

When a soil consists chiefly of *blue* or *yellow* tenacious clay upon a retentive subsoil, it is nearly unfit for tillage ; but on an open subsoil it may be easily improved. Clayey soils containing a due admixture of sand, lime and vegetable matter, are well adapted to the growth of wheat, and are classed amongst the most productive soils, where the climate is favorable. Soils of this description will, therefore, graduate from cold, stiff clay soils to open clay soils, in proportion as the admixture of sand and vegetable matter is more or less abundant, and the subsoil more or less retentive of moisture.

Loams are friable soils of fine earth, which, if plowed in wet weather, will not form clods.

A *strong clayey loam* contains about one-third part of clay, the remainder consisting of sand or gravel, lime, vegetable and animal matters, the sand being the predominating ingredient.

A *friable clayey loam* differs from the latter by containing less clay and more sand. In this case the clay is more perfectly intermixed with the sand, so as to produce a finer tilth, the soil being less retentive of moisture, and easier cultivated in wet weather.

Sandy or gravelly loams is that where sand or gravel predominates, and the soil is open and free, and not sufficiently retentive of moisture.

A *stiff clay soil* may become a rich loam by a judicious admixture of sand, peat, lime and stable manure, but after numerous plowings and exposure to winter frosts in order to pulverize the clay, and to mix with it the lime, peat, sand, etc.

Alluvial soils are generally situated in flats, on the banks of rivers, lakes, or the sea shore, and are depositions from water, the depositions being fine argillaceous loam, with layers of clay, shells, sand, etc. The subsoil may be different.

On the sea shore and margin of lakes, the the clay subsoils usually contain much calcareous matter in the form of broken shells, and sometimes thick beds of white marl.

The value of the soil and subsoil depend on the proportion of lime it may contain. This may be found by an analysis. (See sequel for analysis.)

Rich alluvial soils are the most productive when out of the influence of floods. These soils are classed as clayey, loamy, sandy, etc., according to their nature.

Flat lands or holms, on banks of rivers, are occasionally open and sandy, but frequently they are composed of most productive loams.

SILICEOUS SOILS.

309j. *Sandy soils* vary very much in their grade, color and value, according to the quality of the sand. White shelly sands, which are usually situated near the sea shore, are sometimes very productive, though they contain but a very small portion of earthy matter.

Gravelly soils are those in which coarse sand or gravel predominates; these, if sufficiently mixed with loam, produce excellent crops.

Slatey soils occur in mountains composed of slate rock, either coarse or fine grained. In plowing or digging the shallow soils on the declivities of such places, a portion of the substratum of slate intermixes with the soil, which thus becomes slatey.

Rocky soils. Soil may be denominated rocky where it is composed of a number of fragments of rock intermixed with mould. Such soils are usually shallow, and the substratum consists of loose broken rock, presenting angular fragments.

CALCAREOUS SOILS.

309k. *Calcareous or limestone soils*, are those which contain an unusual quantity of lime, and are on a substratum of limestone. These lands form the best sheepwalks.

Limestone gravel soil, is where we find calcareous or limestone gravel forming a predominant ingredient in soils.

Marly soils are of two kinds, clayey marl, or calcareous matter combined with clay and white marl, which is a deposition from water, and is only found on the margins of lakes, sluggish rivers and small bogs.

On the banks of the River Shannon, beds of white marl are found 20 feet deep. When either clayey or white marl enters into the composition of soils, so as to form an important ingredient, such soils may be denominated *marly*.

PEATY SOILS.

309l. *Flat, moory soils* are such as contain more or less peaty matter, assuming the appearance of a black or dark friable earth. When the peat amounts to one-fourth, and the remainder a clayey loam, the soil is productive, especially when the substratum is clay or clayey gravel.

When the peat amounts to one-half, the soil is less valuable.

When the peat amounts to three-fourths of the whole, the soil becomes very light, and decreases in value in proportion to the increase of the peat in the soil.

Peaty or boggy soils are composed of peat or bog, which, when first brought into cultivation, present a fibrous texture and contain no earthy matter beyond that which is produced by burning the peat.

The quantity of ashes left by burning is red or yellow ashes, about one-eighth of the peat, generally one-tenth or one-twelfth in shallow bogs.

In deep bogs the ashes are generally white, and weigh about one-eightieth of the peat. Such land is of little value unless covered with a heavy coat of loamy earth or clay. Hence it appears that the value of peaty soil depends on the amount of red ashes it contains. For this reason peaty soils are valued at a low price.

NOTE.—(Bousingault, in his "Rural Economy," says: "*The quality of an arable land depends essentially on the association of its clay and sand or gravel.*")

Sand, whether it be siliceous, calcareous or felspathic, always renders a soil friable, permeable and loose; it facilitates the access of the air and the drainage of the water, and its influence depends more or less on the minute division of its particles.)

The following table, given by Sir Richard Griffith, is from Von Thaer's Chemistry, as found by him and Einhoff:

309m.	CLASS.	Clay, per cent	Sand, or Gravel, per cent	Carbonate of Lime, per cent	Humus per cent	Compa- rative Value.
1	First class strong wheat land..	74	10	4.5	11.5	100
2	Do.....	81	6	4	8.7	98
3	Do.....	79	10	4	6.5	96
4	Do.....	40	22	36	4	90
5	Rich light land in natural grass	14	49	10	27	—
6	Rich barley land.....	20	67	3	10	78
7	Good wheat land.....	58	36	2	4	77
8	Wheat land.....	56	30	12	2	75
9	Do.....	60	38	In small proportions.	2	70
10	Do.....	48	50		2	65
11	Do.....	68	30		2	60
12	Good barley land.....	38	60		2	60
13	Do. second quality.....	33	65		2	
14	Do. Do.....	28	70		2	
15	Oat lands.....	23½	75		1.5	
16	Do.	18½	80		1.5	
17	Rye land.....	14	85		1	
18	Do do	9	90		1	
19	Do do	4	95		0.75	
20	Do do	2	97.5		0.5	

Under the head clay, has been included alkalies, chlorides, and supposed to be in fair proportions. The soil in each case supposed to be uniform to the depth of six inches.

In the *Field Book* the following explanatory terms may be used as occasion may require:

Stiff.—Where a soil contains a large proportion (say one-half or even more) of tenacious clay; this cracks in dry weather, forming into lumps.

Friable.—Where it is loose and open, as in sandy, gravelly or moory lands.

Strong.—Where it has a tendency to form into clods.

Deep.—Where the depth is less than 8 inches.

Dry.—No springs. Friable soil, and porous subsoil.

Wet.—Numerous springs; soil and subsoil tenacious.

Sharp.—A moderate share of gravel or small stones.

Fine or soft.—No gravel: chiefly composed of very fine sand, or soft, light earth, without gravel.

Cold.—Parts on a tenacious clay subsoil, and has a tendency, when in pasture, to produce rushes and other aquatic plants.

Sandy or gravelly.—A large proportion of sand or gravel.

Slaty.—Where the slaty substratum is much mixed with the soil.

Worn.—Where it has been a long time cultivated without rest or manure.

Poor.—When of a bad quality.

Hungry.—When consisting of a great proportion of gravel or coarse sand resting on a gravelly subsoil. On such land manure does not produce the usual effect.

The color of the soil and the features of the land ought to be mentioned, such as steep, level, rocky, shrubby, etc., etc.

Indigenous plants should be observed, as they sometimes assist to indicate particular circumstances of soil and subsoil.

<i>Name of Plant.</i>	<i>Indicates</i>
Thistle.....	Strong, good soil.
Dockweed and nettle.....	Rich, dairy land.
Sheep sorrell.....	Gravelly soil.
Trefoil and vetch.....	Good dry vegetable soil.
Wild thyme.....	Thinness of soil.
Ragweed...	Deep soil.
Mouse-ear hawk-weed.....	Dryness of soil.
Iris, rush and lady's smock.....	Moisture of soil.
Purple red nettle and naked horsetail.....	Retentive subsoil.
Great Ox-eye.....	Poverty of soil.

CLASSIFICATION OF SOILS WITH REFERENCE TO THEIR VALUE.

309n. All lands to be valued may be classed under arable and pasture. Arable land may be divided into three classes, viz :

Prime soils, rich, loamy earth.

Medium soils, rather shallow, or mixed.

Poor soils, including cultivated moors.

Pasture, as fattening, dairy and stone land pastures.

The prices set forth in the Act (see sec. 309f) is the basis on which the relative and uniform valuation of all lands used for agricultural purposes must be founded. It is incumbent on the valuator to ascertain the depth of soil and nature of subsoil, to calculate the annual outlay per acre. He should calculate the value per acre of the produce, according to the scale of the Act, and from these data deduce the net annual value of the tenement.

309o. *Tables of produce, etc.*, formulæ for calculation, and an acreable scale of prices, supplied in the following sections, are given as auxiliaries with a view to produce uniformity among the valuers employed. Thus, if the valuator finds it necessary to test his scale of prices for a certain quality of land, he may select one or more farms characteristic of the average of the neighborhood. Their value should be correctly calculated and an average price per acre obtained, from which he deduces the standard field price of such description of land. The farms (or fields) thus examined will serve as points of comparison for the remainder of the district.

SCALE FOR ARABLE.

<i>Class and Description.</i>		<i>Average price per acre.</i>	
FINE SOILS.	1. Very superior, friable clayey loam, deep and rich, lying well, neatly fielded, on good, sound clayey subsoil, having all the properties that constitute a superior subsoil, average produce 9 barrels (or stones = lbs. = bushels) per acre.....	<i>From. To.</i>	<i>s. d. s. d.</i>
			30 0 26 0
	2. Superior, strong, deep and rich, with inferior spots deducted, lying well on good clay subsoil.....		27 0 24 0
MEDIUM SOILS.	3. Superior, not so deep as the foregoing, or good alluvial soils—surface a little uneven.....		25 0 22 0
	4 Good medium loams, or inferior alluvial land of an even quality.....		21 0 18 0
	5. Good loams, with inferior spots deducted.....		17 6 15 0
	6. Medium land, even in quality, rather shallow, deep and rocky.....		14 0 10 0

CULTIVATED MOORS OR BOGS.	POOR SOILS.	7. Cold soil, rather shallow and mixed, lying steep on cold clayey, or cold, wet, sandy subsoil.....	9 0 7 0
		8. Poor, dry, worn, clayey or sandy soil, on gravelly or sandy subsoil.....	6 6 5 0
		9. Very poor, cold, worn, clayey, or poor, dry, shallow, sandy soil, or high, steep, rocky, bad land.....	4 0 1 0
		10. Good, heavy moor, well drained, on good, clayey subsoil.....	12 0 10 0
		11. Medium moory soil, drained, and in good condition.....	9 0 6 0
		12. Poor moory, or boggy arable, wet, and unmixed with earth.....	5 6 1 0

The above prices opposite each class is what the valuator's field price should be in an ordinary situation, subject to be increased or decreased for local circumstances, together with deductions for rates and taxes.

309*p.* *Of Arable land.*—The amount of crop raised depends on the system of tillage, and the crops raised. The system of cultivation should be such as would maintain an adequate number of stock to manure the farm, and the crops should be suited to the soil; thus, lands on which oats or rye could be profitably grown, may not repay the cost of cultivating it for wheat.

The following tables show the average maximum cost, produce and value of crops in ordinary cultivation for one statute acre.

TABLE OF PRODUCE.

	Potatoes	Mangel Wurzel		Turnips	Vetches Cabbage (Green.) (Kale.)		Beans
		Longrad or Orange.	Leaves.				
Total produce in tons.....	7	22	1	20	4	20	cwt. 20
Price per ton.....	s. d. 40 0	s. d. 10 0	s. d. 5 0	s. d. 8 0	s. d. 30 0	s. d. 5 0	s. d. 8 9
Total val. of produce pr. acr.	£ s. 14 0	£ s. 11 5	£ s. 8 0	£ s. 6 0	£ s. 5 0	£ s. 8 0	
Total cost of culture pr. acr.	8 10	6 15	7 0	3 3	1 18	5 10	

	Wheat.		Barley.		Oats.		Rye.		Flax.	Meadow.		Clover.
	Grain.	Straw.	Grain.	Straw.	Grain.	Straw.	Grain.	Straw.		Hay.	After-grass.	
Total produce pr. acre.....	Brls. 8	Tns. 2	Brls. 10	Tns. 1½	Brls. 11	Tns. 1½	Brls. 10	Tns. 2	Cwt. 5	Tns. 2½	Tns. 2½	Tns. 3
Price per.....	s. d. 18 9	s. 15	s. 11	s. 13	s. d. 8 5½	s. 17	s. 6	s. 14	45	s. 30	s. 5	s. 30
Total val of pr. duce	£ s. d. 9 0 0	£ s. d. 6 10 0	£ s. d. 6 3 0	£ s. d. 4 8 0	£ s. d. 11 5 4	£ s. d. 7 6 10	£ s. d. 4 8 0	£ s. d. 11 5 4	£ s. d. 7 6 10	£ s. d. 4 8 0	£ s. d. 7 6 10	£ s. d. 4 8 0
Total cost of culture	3 9 0	3 2 0	3 11 0	3 0 0	7 8 1	9 6 2	0					

NOTE.—The barrel is pounds, and the ton = 2,240 pounds.

From this table it appears that the cost of cultivating turnips, and other broad-leaved plants, is greater than that for grain crops.

300*g*.

SCALE OF PRICES FOR PASTURE.

Classes and Description.	Acres, Sauts.	Average.			Price per Acre.	Observations.
		Cattle.	Sheep.	Swine		
FEEDING LAND.					s. s.	
1. Very superior fattening land, soil composed of finely comminuted loam, producing the most succulent qualities of grass, exclusively used for finishing heavy cattle and sheep,	10	Two sets of cattle of six each set, from 1st April to 1st Sept. finished.	One set of sheep 5 to each acre for 8 months.	—	35 to 31	This soil being used for "finishing" cattle and sheep, the latter replace the former when finished for market.
2. Superior dairy pasture or fattening land, with verges of prime heavy moors, all having a grassy tendency,	15	Six and 3 calves.	—	3	30 to 24	This land is calculated at 3½ firkins of butter to each cow.
3. Good dairy pasture on clay or sandy soils, or good rocky pasture, each adapted to dairy purposes or fattening sheep,	20	Six and 3 calves.	—	3	23 to 17	This soil is calculated at 2½ firkins of butter to each cow.
4. Tolerable mixed clayey or moory pastures, or good rocky pasture, adapted to dairy purposes or the rearing of young cattle or sheep,	25	Six and 3 calves.	—	3	16 to 11	This description of soil is calculated at 2½ firkins of butter to each cow.
DAIRY LAND.						
5. Coarse sour rushy pasture on shallow clayey or moory soil, or dry rocky shrubby pasture, adapted to the rearing of young cattle or store sheep,	30	The different qualities included in this brace are calculated at nine heifers, equal to 6 collop top for each subdivision.	—	—	10 to 5	This description of soil is calculated for the purpose of rearing young cattle or sheep.
6. Inferior coarse sour pasture on cold shallow clayey or shallow moory soil, or dry rocky shrubby pasture, adapted chiefly to winterage for young cattle or store sheep,	35		—	—	6 to 4	The description of land that this brace includes ranges from coarse sour verges, inferior dry rocky pastures, and mixed green and heathy pastures, chiefly adapted and generally used for the rearing of young cattle of an inferior description.
7. Good mixed green and heathy pasture in the homestead of mountains or inferior dry rocky shrubby pasture, adapted to the rearing of light dry cattle or sheep,	40		—	3	0	
8. Mixed green and heathy mountain pasture, or inferior close rocky or shrubby pasture, adapted to the rearing of young cattle or sheep,	45		—	2	0	
9. Mixed brown heathy pastures with spots of green intermixed, or very inferior bare rocky pastures, or steep shrubby banks near homestead,	50		—	1s. to 9d		
10. Heathy pastures high and remote, or cut away bog, partly pasturable,	—	—	—	—	8d to 4d	
11. Red bog or coarse high remote mountain tops,	—	—	—	—	3d to ½d	
12. Precipitous cliffs,	—	—	—	—	—	
STORE PASTURE.						

NOTE.—The price inserted opposite each class of lands, according to its respective produce, is what the valuator's field price should be in an ordinary situation, subject to be increased or reduced for particular local circumstances, together with deductions for rates and taxes.

In the calculations for testing his scale price, the valuator should tabulate, as above, at the prices per ton or barrel, the average produce per acre of the district under consideration. These values he will again tabulate according to the system of farming adopted.

The following may serve as a formula :

ONE HUNDRED STATUTE ACRES UNDER FIVE YEARS' ROTATION
AS FOLLOWS:

	Acre. Stat.	Cost of Tillage.	Value of Tillage.
		£ s. d.	£ s. d.
1st Year, $\frac{1}{5}$ or 20 acres,	{ Potatoes, 3	25 10 0	42 0 0
	{ Vetches, 2	6 6 0	12 0 0
	{ Mangel Wurtzel, 3	20 5 0	33 15 0
	{ Turnips, 12	84 0 0	96 0 0
2d Year, $\frac{1}{5}$ or 20 acres,	{ Winter Wheat, 12	41 0 0	108 0 0
	{ Spring Wheat, 8	24 17 0	52 0 0
	{ Barley, 6	8 17 0	26 5 0
3d Year, $\frac{1}{5}$ or 20 acres,	{ Hay, 1	2 0 0	4 10 0
	{ Clover, 13	41 0 0	95 0 0
4th Year, $\frac{1}{5}$ or 20 acres,	Pasture, 20		
5th Year, $\frac{1}{5}$ or 20 acres,	{ Potato Oats, 20	70 13 0	123 0 0
	{ Common Oats, 100	324 16 0	592 10 0
Allow for wear and tear of implements,		10 0 0	
“ Five per cent. on £500 capital,		25 0 0	
Deduct Expenses,			359 16 0
Nett Annual Value of Produce,			232 14 0

FATTENING LANDS.

309r. It has been ascertained that the fat in an ox is one-eighth of the lean, and is in proportion of the fatty matter to the saccharine and protein compounds in the herbage. The method of grazing, too, has some influence. The best lands will produce about ten tons of grass per acre, in one year. One beast will eat from seven to nine stones in one day. Six sheep will eat as much as one ox. One Irish statute acre of prime pasture will finish for the market two sets of oxen from April to September. From September until December it is fed by sheep. The general formulæ may be as follows:

SUPERIOR FINISHING LAND.

Mode of Farming and Description of Stock.	Nett Increase.	Act. Price.	Am't.
	cwt.qrs.lbs	s. d. £	s. d.
Two sets of cattle to be finished in the season, the lands preserved during the months of January, February and March.			
A four-year old heifer, weighing about 5 cwt., well wintered, and coming on in good condition, in the first two months of April and May, will increase,	1 2 0	35 6	2 13 3
A heifer in the same condition, in the months of June, July and August, will increase,	1 2 0	“	2 13 3
On the same land, 5 sheep to the Irish acre will increase at the rate of 2 lb. per week, for October, November and December,	1 1 0	41 0	2 11 3
Gross produce on one Irish acre, or 1A. 2R. 19P. statute measure,			7 17 9

Expenses.	£	s.	d.
Interest on capital for one beast to the Irish acre, at 5 per cent. for £10,	0	10	0
Herd, per Irish acre, (a herd will care 150 Irish acres,) at 2s. per acre,	0	2	0
Contingencies,	1	10	0
Commission on the sale of 2 beasts and 7 sheep, at 2½ per cent.,	1	9	0
Extra expenses,	0	8	0
Deduct expenses,	3	19	0
Nett produce per Irish acre, or 1A. 2R. 10P., statute measure,	3	18	9

Cattle in good condition will fatten quicker on this description of land during the early months than under the system of stall-feeding.

DAIRY PASTURE.

309s. *Dairy pastures* are more succulent than fattening lands. The average quantity of butter which a good cow will give in the year may be taken at 3¼ firkins = 218 lbs.; or, allowing nine quarts to the pound of butter, the milk will be 1,960 quarts. If the stock be good, under similar circumstances its produce may be considered to vary with the quantity and quality of the herbage. This and the quality and suitability of the stock must be carefully discriminated and considered.

The general formula is as follows:

In column A, set the cows and produce; the hogs, and increase in weight; the calves, when reared; the milk used by the family. In column B, set the weight of the produce. In column C, set the Act price. And in column D, the amount. The sum of column D will be the gross receipts, from which deduct the sum of all the expenses, rent of land under tillage, and the difference will be the nett annual produce for that part used as a dairy pasture.

STORE PASTURE.

309t. The value of store pasture depends on the amount of stock it can feed. The valuator will estimate the number of acres which would feed a three years beast for the season, from which the number of stock for the whole tenement may be ascertained, which, calculated at an average rate for their increase or improvement, will give the gross value. This valuation must be checked for all incidental expenses and local circumstances—in general, *two-thirds of the gross produce* may be considered as a fair value.

In mountain districts, it is divided into inside and remote grazing. The inside is allotted for milch cattle and winter grass. The remote or outside pasture is for summer grazing for dry cattle and sheep.

The annual value of these pastures is to be obtained from the herds or persons living on or adjacent to them, taking for *basis* the number of *sums* grazed and the rate per sum.

The following will enable the valuator to estimate the number of sums on any tenement:

One three years old heifer is called a "*sum*" or *collop*; one sum is = to three yearlings = one two years old and one, one year old = four

ewes and four lambs = five two years old sheep = six *hoggets* (one year old sheep) = to two-thirds of a horse.

LAND IN MEDIUM SITUATION.

309u. The above classifications, scales of prices, etc., for different kinds of land, have been calculated with reference to the quality of the soil and its productive capabilities, arising from the composition, depth and nature of the subsoil, without taking into consideration the extremes of position in which each particular kind may occasionally be found. The value thus considered may be defined as *the value of land in medium or ordinary situation*.

Land in an ordinary or medium situation. Should not be distant more than five or six miles from a principal market town, having a fair road to it, not particularly sheltered or exposed, not very conveniently or very inconveniently circumstanced as to fuel, lime and manures; not remarkably hilly or level, the greatest elevation of which shall not exceed 300 feet above the level of the sea.

When the valuation of the property is made, he will enter in the first column the valuations obtained, and in the second column the valuations corrected for local circumstances.

LOCAL CIRCUMSTANCES.

309v. The local circumstances may be divided into two classes, viz: natural and artificial.

Natural, is that which aids or retards the natural powers of the soil in bringing the crop to maturity.

Artificial, is that which afford or deny facilities to maintain or increase the fertility of the soil, and such as involve the consideration of remunerations for labor of cultivation. Local circumstances may, therefore, be classed under—*climate*, *manure*, and *market*.

309w. *Climate* includes all the phenomena which affect vegetation, such as temperature, quantity of atmospheric moisture, elevation, prevailing winds, and aspect. Various combinations of these, and other external causes, are what cause diversity of climate.

The germination of plants, and the amount of atmospheric moisture, are considerably dependent on temperature; hence the advantage of a locality in which its mean is greatest. Its average in Ireland varies from 48° (Fahrenheit) in the north to 51° in the south, the corresponding atmospheric moisture being from 4.27 to 4.83 grains to the cubic foot. These are considerably modified by elevation, which produces nearly the same effect as latitude, every 350 feet in height being equivalent to one degree of temperature.

309x. The average depth of rain which falls in one year in Ireland, varies from 40 inches on the west coast to 33 on the east. The proportion of the rain fall is greater for the mountain districts than for the low lands. The general effect of elevation on arable lands in this case are, that the soluble and fine parts of the soil are washed out, and ultimately carried down by the streams. Such elevated districts are also frequently exposed to high winds, etc. The prevailing winds, and how modified, are to be taken into consideration.

309y. *In Ireland, on land exposed to westerly winds*, the crops are fre-

quently injured in the months of August and September. A suitable deduction should therefore be made for such lands, although the intrinsic value may be similar to land in a more sheltered situation.

To determine the influence of climate requires considerable care and extensive comparison. Thus, the soil which in an elevated district is worth 10s. per acre, will be worth 15s. if placed in an ordinary situation, about 300 feet above the level of the sea, and not particularly sheltered or exposed. The same description of lands, however, in a more favorable situation, say from 50 to 100 feet above the sea, distant from mountains, and having a south-east aspect, may be worth 20s. per acre.

In making deductions from cultivated lands, in mountainous districts, the following table will be found useful, and may be applied in connection with heights given in Ordnance Survey maps:

Altitude in feet.	Deduct per £.
800 to 900 feet.....	5 shillings.
700 " 800 "	4 " "
600 " 700 "	3 " "
500 " 600 "	2 " "
400 " 500 "	1 " "

Arable land in the interior of mountains, may be considered 100 feet of altitude, worse than on the exterior declivities on the same height; so also those on the north may be taken 100 worse than those having a southern aspect, both having the same height.

In mountain districts, take the homestead pasture at 3, the outer at 2, and the remote at 1.

Deduct for steepness in proportion to the inconvenience sustained by the farmer in plowing and manuring.

Deduct for bad roads, fences, and for difference in the soils of a field where it is of unequal quality.

MANURE.

309z. *Manures* are that which improve the nature of the soil, or restore the elements which have been annually consumed by the crops. The most important of these, in addition to stable manure and that produced from towns, consist of limestone, coal turbary, sea weed, sea sand, etc.

In a limestone country, where the soil usually contains a sufficient quantity of calcareous matter, the value of lime as a manure is trifling when compared to its striking effects in a drained clayey or loamy argillaceous soil. It promotes the decomposition of vegetable or animal matter existing in the soil, and renders stiff clay friable when drained, and more susceptible of benefit from the atmosphere, by facilitating the absorption of ammonia, carbonic acid gas, etc.; decomposes salts injurious to vegetation, such as sulphate of iron, (which it converts into sulphate of lime and oxide of iron, and known here as gypsum or plaster of Paris,) and further it improves the filtering power of soils, and enables them to retain what fertilizing matter may be contained in a fluid state.

Lime may therefore be used in due proportion, either on moory arenaceous or argillaceous soils; hence the vicinity of limestone quarries is to be considered relatively to the value of lime as a manure to the lands

under consideration: say from sixpence to two shillings STERLING per pound to be added according to circumstances.

The vicinity of coal mines and turf bogs are likewise an important consideration affecting the value of land, for the expense of hauling fuel, for burning lime and domestic purposes, must be considered. The percentage should vary from sixpence to two shillings and sixpence per pound.

Sea manure includes sea weed and sea sand, containing shells, both of which are highly valuable, especially the former.

Where sea weed of good quality is plentiful and easy of access, the land within one mile of the strand is increased in value 4s. in the pound at least. Where the soil is a strong clay or clayey loam, shelly sea sand, when abundant, will increase the value of the land 2s. 6d. in the pound, for the distance of one mile.

The valuator will consider whether sea weed is cast on the shore or brought in boats, and the nature of the road. If hilly, reduce them to level by table at p. 72315. The following will enable the valuator to ascertain the value at any distance from the strand:

Supply rather scarce at one mile, 2s.	For every one-half mile
“ middling “ 3s.	deduct 6d.
“ plentiful “ 4s.	

The proximity to towns, as a source of manure and market farm, garden and dairy produce, is to be considered.

MARKET.

310. To this head may be referred the influence of cities, towns and fairs; these possess a topical influence in proportion to their wealth and population. The following is a classification of towns:

Villages, from 250 to 500 inhabitants.

Small market towns, from 500 to 2000.

Large market towns, from 2000 to 19,000.

Cities, from 19,000 to 75,000, and upwards.

Small villages, of from 250 to 500 inhabitants, do not influence the value of land in the neighborhood beyond the gardens or fields immediately behind the houses. The increase in such cases above the ordinary value of the lands will rarely exceed 2s. in the pound.

Large villages and small towns, having from 500 to 1000 inhabitants, usually increase the value of land around the town to a distance of three miles. For the first half mile, the increase is 3s. in the pound; for the next half mile, 2s.; next, 16d. etc., deducting one-third for each half mile, making, for three miles distant, 6d. in the pound, or one-fortieth.

Market towns, having from 8000 to 75,000 inhabitants, town parks, or land within one mile, is 10s. in the pound higher than in ordinary situations. Beyond this the value decreases proportionately to 6s. at the distance of three miles from the town. Thence, in like manner, to a distance of seven miles, where the influence of such town terminates.

Cities and large towns, having a population of from 19,000 to 75,000 inhabitants. The annual value of town parks will exceed by about 14s. in the pound the price of similar land in ordinary situations; and this increased value will extend about two miles in every direction from the houses of the town, beyond which the adventitious value will gradually decrease for the next mile to 12s. in the pound; at the termination of four miles, to 6s.; at seven miles, to 4s.; and at nine and a half miles, its influence may be considered to end.

Its increase to be made for the vicinity of towns, is tabulated as follows:

Class.	Population.	Distance in Miles.																							
		Town lots.	1.		2.		3.		4.		5.		6.		7.		8.		9.		10.				
			s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.			
9	From 250 to 500,	—	2	0	1	0	0	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—			
8	“ 500 “ 1,000,	—	3	0	2	0	1	0	6	—	—	—	—	—	—	—	—	—	—	—	—	—			
7	“ 1,000 “ 2,000,	—	4	0	3	0	2	0	1	0	6	—	—	—	—	—	—	—	—	—	—	—			
6	“ 2,000 “ 4,000,	—	6	0	5	0	3	0	2	0	1	0	6	—	—	—	—	—	—	—	—	—			
5	“ 4,000 “ 8,000,	—	—	8	0	6	0	4	0	2	0	1	0	6	—	—	—	—	—	—	—	—			
4	“ 8,000 “ 15,000,	—	—	10	0	8	0	6	0	4	0	2	0	1	0	6	—	—	—	—	—	—			
3	“ 15,000 “ 19,000,	—	—	12	0	10	0	8	0	6	0	4	0	2	0	1	0	6	—	—	—	—			
2	“ 19,000 “ 75,000,	—	—	—	14	0	12	0	10	0	8	0	6	0	4	0	2	0	1	0	6	—			
1	“ 75,000 and upwards,	—	—	—	—	22	0	20	0	18	0	15	0	10	0	6	0	3	0	2	0	1	0		

In applying the above table, *the population must be used only for a general index*, as it is the wealth and commercial influence which principally fixes the class; the valuator must use his judgment, combining the comparative wealth with the population, and raise it one class in the tables, or even more. If there be a large poor class, he should take a class lower.

The general influence of markets and towns includes the effects of railways, canals, navigable rivers, and highways; thus, of two districts equally distant from a market, and equal in other respects, that which is intersected by or lies nearer to the best and cheapest mode of communication for sale of produce, is the most valuable.

Bleach greens, fair greens, orchards, osieries, etc., should be valued according to the agricultural value of the land which they occupy.

Plantations and woods, are valued according to their agricultural value.

(NOTE.—We have made up the following section from Sir Richard Griffith's instructions, and Brown on American Forest Trees. The latter is a very valuable work.)

310a. The condition of trees is worthy of attention, as indicating the nature of the soil, thus:

Acer. Maple. Requires a deep, rich, moist soil, free from stagnant water; some species will thrive in a drier soil.

Alnus. Alder. A moist damp soil.

Betula. Birch. In every description—from the wettest to the driest, generally rocky, dry, sandy, and at great elevation.

Carpinus. Ironwood and Hornbeam. Poor clayey loams, incumbent on sand and chalky gravels.

Castanea. Chestnut. Deep loam, not in exposed situations. A rich, sandy loam and clayey soils, free from stagnant water.

Cupressus. Cypress. A sandy loam, also clayey soil.

Chamerops. Cabbage Tree. A warm, rich, garden mould.

Gleditsia. Locust. A sandy loam.

Juglens. Hickory. Grows to perfection in rich, loamy soils. Also succeeds in light siliceous, sandy soils, as also in clayey ones.

Larix. Larch. A moist, cool loam, in shaded localities.

Laurus. Sassafras. A soil composed of sand, peat and loam.

Lyriodendron. Poplar, or Tulip Tree. A sandy loam.

Pinus. Pine. Siliceous, sandy soils; rocky, and barren ones.

Platanus. Buttonwood, or Sycamore. Moist loam, free from stagnant moisture.

Quercus. Oak. A rich loam, with a dry, clayey subsoil. It also thrives on almost every soil excepting boggy or peat.

Robinia. Locust. Will grow in almost any soil; but attains to most perfection in light and sandy ones.

Tilia. Lime Tree. Will thrive in almost any soil provided it is moderately damp.

(For further, see Brown on Forest Trees, Boston: 1832.)

It would be well, in every instance, to make sublots of plantations.

In some instances, plantations may be a direct inconvenience or injury to the occupying tenant. In such cases, the circumstances should be noted, and a corresponding deduction be made for the valuation of the farm so affected.

Bogs and turbary should be valued as pasture. The vicinity of turf, as well as coal, is one of the local circumstances to be considered as increasing the value of the neighboring arable land.

Where the turf is sold, the bog is valued as arable, and the expense of cutting, saving, etc. of turf deducted from the gross proceeds, will give the net value.

Bogs, swamps, and morasses, included within the limits of a farm, should be made into sublots, if of sufficient extent.

Mines, quarries, potteries, etc. The expense of working, proceeds of sales, etc., should be ascertained from three or four yearly returns.

Mines, not worked during seven years previous, are not to be rated.

Tolls. The rent paid for tolls of roads, fairs, etc., should be ascertained, and also the several circumstances of the tolls. If no rent be paid, the value must be ascertained from the best local information.

Fisheries and ferries. From the gross annual receipts deduct the annual expenses for net proceeds. It will be necessary to state if the whole or part of a fishery or ferry is in one township, or in two, etc., and to apportion the proceeds of each.

Railways and canals. "The rateable hereditament," in the case of railways, is the land which is to be valued in its existing state, as part of a railway, and at the rent it would bring under the conditions stated in the Act. The profits are not strictly rateable themselves, but they enter materially into the question of the amount of the rate upon the lands by affecting the rent which it would bring, or which a tenant would give for the railway, etc., not simply as land, but as a railway, etc., with its peculiar adaptation to the production of profit; and that rent must be ascertained by reference to the uses of it (with engines, carriages, etc., the trading stock), in the same way as the rent of a farm would be calculated, by reference to the use of it, with cattle, crops, etc. (likewise trading stock). In neither cases would the rent be calculated on the dry possession of the land, without the power of using it; and in both cases, the profits are derived not only from the stock, but from the land so used and occupied.

It will be necessary, therefore, to ascertain the gross receipts for a

year or two, taken at each station along the line; also the amount of receipts arising from the intermediate traffic between the several stations. From the total amount of such receipts, the following deductions are to be made, viz.: interest on capital: tenants' profits; working expenses; value of stations: depreciation of stock.

It is to be observed, that the valuation of railway station houses, etc., should be returned separately.

WASTE.

The value of the ground under houses, yards, streets, and small gardens, is included in their respective tenements. So also in the country, roads, stackyards, etc., are included in the tenements. The area of ground occupied by these roads should be entered as a deduction at the foot of the lot in which they occur.

When a farm is intersected by more roads than is necessary to its wants, the surplus may be considered waste. Also deduct small ponds, barren cliffs, beaches along lakes, and seashores.

OF THE VALUATION OF BUILDINGS.

310b. By a system analogous to that pursued in ascertaining the value of land, the value of buildings may be worked out; the one being based on the scale of agricultural prices, and modified by local circumstances; the other, on an estimate of the intrinsic or absolute value, modified by the circumstances which govern house letting.

The absolute value of a building is equivalent to a fair percentage on the amount of money expended in its construction, and it varies directly in proportion to the solidity of structure, combined with age, state of repair, and capacity, as shown in the following classification:

Buildings are divided into two classes: those used as houses, and those used as offices. In addition to the distinction of tenements already noticed in sec. 309g, it may here be observed that houses and offices, together with land, frequently constituted but one tenement. All out-buildings, barns, stables, warehouses, yards, etc., belonging or contiguous to any house, and occupied therewith by one and the same person or persons, or by his or their servants, as one entire concern, are to be considered parts of the same tenement, and should be accounted for separately in the house book, such as herd's house, steward's house, farm house, porter's house, gate house, etc.

A part of a house given up to a father, mother, or other person, without rent, does not form a separate tenement.

Country flour mills, with miller's house and kiln, form one tenement.

310c. CLASSIFICATION OF BUILDINGS WITH REFERENCE TO THEIR SOLIDITY.

Buildings,	Slated,	House or office (1st class),	Built with stone or brick, and lime mortar.
		Basements to do. (4th),	
		House or office (2nd),	
	Thatched,	House or office (3rd),	Stone walls with mud mortar.
			Dry stone walls, pointed.
		Offices (5th),	Good mud walls.
			Dry stone walls.

The above table comprises four classes of houses and five of offices, of each of which there may be three conditions, viz., new, medium, and old, which may also be classified and subdivided, as follows:

CLASSIFICATION OF BUILDINGS WITH REFERENCE TO AGE AND REPAIR.

Quality.	Description.
NEW,	A. + { Built or ornamented with cut stone, or of superior solidity and finish.
	A. { Very substantial building, and finished without cut stone ornament.
	A. — { Ordinary building and finish, or either of the above, when built twenty years.
MEDIUM,	B. + { Not new, but in sound order and good repair.
	B. { Slightly decayed, but in good repair.
	B. — { Deteriorated in age, and not in perfect repair.
OLD,	C. + { Old, but in repair.
	C. { Old, out of repair.
	C. — { Old, dilapidated, scarcely habitable.

The remaining circumstance to be considered is capacity or cubical content, from which, in connexion with the foregoing classifications, tables have been made for computing the value of all buildings used either as houses or offices. (See sequel for tables.)

Houses of one story are more valuable, in proportion to their cubical contents, than those of two stories. Those more than two stories diminish in value, as ascertained by their cubical contents, in proportion to their height.

Tables are calculated and so arranged on a portion of a house 10 feet square and 10 feet high, = 100 cubic feet, so that a proportionate price given for a measure of 100 cubic feet, as above, is greater than for a similar content 20 feet high, or for 10 square feet and 30 or 40 feet high. For example, in an ordinary new dwelling house, the price given by the table for a measure containing 10 square feet and 10 feet high, is 7½ pence; for the same area and 20 feet high, the price is 1s. 0¾d.; for the same area and 30 feet high, 1s. 4½d.; and for the same area and 40 feet high, the price is 1s. 6½d.

OF THE MEASUREMENT OF BUILDINGS.

310d. Ascertain the number of measures (each 100 square feet) contained in each part of the building. Measure the height of each part, and examine the building with care. Enter in the field book the quality letter, which, according to the tables, determines the price at which each measure containing 10 square feet is to be calculated.

The houses are to be carefully lettered as to their age and quality. Addition or deduction is to be made on account of unusual finish or want of finish, etc. Such addition or deduction is to be made by adding or deducting one or more shillings in the pound to meet the peculiarity, taking care to enter in the field book the cause of such addition or deduction.

Enter also the rent it would bring in one year in an ordinary situation.

If any doubts remain as to the quality letter, examine the interior of the building.

In measuring buildings, the external dimensions are taken—length, breadth and height—from the level of the lower floor to the eaves. In

attic stories formed in the roof, half the height between the eaves and ceiling is to be taken as the height.

Basement stories or cellars, both as dwellings and offices, are to be measured separately from the rest of the building.

Main house is measured first, then its several parts in due form.

Extensive or complicated buildings should have a sketch of the ground plan on the margin of the field book, with reference numbers from the plan to the field book.

If a town land boundary passes through a building, measure the part in each.

MODIFYING CIRCUMSTANCES.

310e. The chief circumstances which modify the tabular value are deficiencies, unsuitableness, locality, or unusual solidity.

Deficiencies.—In large public buildings, such as for internal improvements, an allowance of 10 to 30 per cent. is made; also in stables and fuel houses. When the walls of farm houses exceed 8 or 12 feet in height, but have no upper flooring, they should not be computed at more than 8 feet, except in the cases of grain houses, factories, barns, foundries, etc. The full height is, however, to be registered in each case.

Unsuitableness.—Houses found too large, or superior to the farm and locality—where there are too many offices or too few.

All buildings are to be valued at the sum or rent they would reasonably rent for by the year.

Buildings erected near bleach greens, or manufactories which are now discontinued, or if they were built in injudicious situations, should be considered an incumbrance rather than a benefit to the land; consequently, only a nominal value should be placed on them.

The tabular amount for large country houses, occupied by gentlemen, usually exceeds the sum they could be let for, and this difference increases with the age of the building. The following is to correct this defect:

Houses amounting from	to	Reduction per Pound.	Reduction per cent.
£10	£35	None.	None.
35	40	0s. 6d.	0.025
40	50	1 0	0.05
50	60	1 6	0.075
60	70	2 0	0.10
70	80	2 6	0.125
80	90	3 0	0.150
90	100	3 6	0.175
100	110	4 0	0.200
110	120	4 6	0.225
120	140	5 0	0.250
140	160	5 6	0.275
160	200	6 0	0.300
200	300	7 0	0.350
300 and upwards,		8 0	0.400

Where any improvements have been made to gentlemen's houses, care should be taken to ascertain whether any part of the original house was made useless, or of less value. If so, deduct from the price given by the table as the case may require.

Locality includes aspect, elevation, exposure to winds, means of access, abundance or scarcity of water, town influence, etc., each of which is to be carefully considered on the ground.

In determining the value of buildings immediately adjoining large towns, ascertain the percentage which the town valuator has added to the tabular value of these on the limits of the town lot. Those in the town lot are referred to another heading, as will appear from sec. 310f.

Solidity.—In large mills, storehouses, factories, etc., well built with stone or brick, and well bonded with timber, a proportional percentage should be added to the tabular value for unusual solidity and finish, which will range from 30 to 50 per cent. The value thus found may be checked by calculating the tabular value of the ground floor, and multiplying this amount by the number of floors, not including the attic.

VALUATION OF HOUSES IN CITIES AND TOWNS.

310f. In valuing houses in cities and towns, there are circumstances for consideration in addition to those already enumerated, viz., arrangement of streets, measurement, comparative value, gateways, yards, gardens, etc. To effect this object, each town should be measured according to a regular system; and the following appears to be a convenient arrangement for the purpose:

Arrangement of streets.—The valuator should commence at the main street or market square, and work from the centre of the town towards the suburbs, keeping the work next to be done on his right hand side, measuring the first house in the street, and marking it No. 1 on his field map and in his field book. Afterwards proceed to the next house on the same side, marking it No. 2, and so on till he completes the measurement of the whole of the houses on that side of the street. He is then to turn back, proceeding on the other side, keeping the work to be done still at his right hand. The main street being finished, he proceeds to measure the cross streets, lanes or courts that may branch from it, commencing with that which he first met on his right hand in his progress through the main street. This street is measured in the same manner as the main street; and all lanes, courts, etc., branching from it are measured in like manner, observing the same rule of measurement throughout.

Having finished the first main street, with all its branches, he is to take the next principal street to his right hand, from the first side of the first main street, and proceed as in the first, measuring all its branches as above.

(NOTE.—Let Clark and Lake streets, in the city of Chicago, be the two principal streets, and their intersection one block north of the Court House, the principal or central point of business. Clark street runs north and south; Lake street, east and west. Nearly all the other principal streets run parallel to these. We begin at the west side of Clark and north side of Lake, and run west to the city limits, and return on the south side of the street, keeping the buildings on the right, to Clark street. We continue along the south side of Lake, east to the city limits, and then return on the north side of Lake, keeping the buildings on the right, to the place of beginning. Having finished all the branches leading into this, we take the next street north of Lake, and measure on the north side of it west to the city limits, and so proceed as in the first main street. Having finished all the east and west streets north of the first or Lake street, we proceed to measure those east and west streets south of the first or Lake street, as above. We now proceed to measure the

north and south streets, taking first the one next west of Clark, and run north to city limits; then return on the west side of the street to Lake, and continue south to the city limits; return on the east side of the street to the place of beginning. Thus continue through the whole city.)

In measuring buildings, the front dimensions, and that of returns, is set in the first column of his book, the line from front to rear is placed in the second column, and the height in its own place.

In offices, the front is that on which the door into the yard is situated.

In houses with garrets, measure the height to the eave, and set in the field book, under which set the addition made on account of the attic, and add both together for the whole height.

Every house having but one outside door of entrance, is to be numbered as one tenement. Where there are two doors, one leading to a shop or *store*, to which there is internal access from the house, the whole is to be considered as one tenement; but if the shop and other part of the house be held by different persons, the value of each part should be returned.

Where a number of houses belonging to one person are let from year to year to a number of families, each house is to be returned as one tenement.

Buildings in the rear of others in towns are to be valued separately from those in front.

COMPARATIVE VALUE.

310*g*. In towns, a shop for the sale of goods is the most valuable part of a house; and any house having much front, and affords room for two or three shops, is much more valuable than the same bulk of house with only one shop.

When a large house and a small one have each a shop equally good, the smaller one is more valuable in proportion to its cubical contents, as ascertained by measurement, and a proportionate percentage should be added to the lesser building to suit the circumstances of the case.

Where large houses and small mean ones are situated close to each other, the value of the small ones are advanced, and that of the large ones lessened. In such cases, a proportionate allowance should be made.

Stores (warehouses) in large towns do not admit of so great a difference for situation as shops—a store of nearly equal value, in proportion to its bulk, in any part of a town, unless where it is adjoining to a quay, railway depot or market; then a proportionate additional value should be added.

Gateways.—In stores or warehouses in a commercial street, where there is a gateway underneath, no deduction is made.

In shops or private dwellings, a gateway under the front of the house is a disadvantage, compared to a stable entrance from the rear. In such cases, a proportionate deduction should be made on account of the gateway.

In measuring gateways, take the height the same as that of the story of which it is a part.

Passages in common are treated similar to gateways.

Where any addition or deduction is made on account of gateways, it should be written in full at the end of the other dimensions, so as to be added or subtracted as the case may be.

Where deductions are made on account of want of finish in any house, state the nature of the wants, and where required.

Stores do not want the reductions for large amount, which has been directed in the case of gentlemen's country seats.

OF TOWN GARDENS AND YARDS.

310*h*. In large towns, the open yard is equal to half the area covered by the buildings; if more, an additional value is added, but subtracted if less. Allowance is made if the yard is detached or difficult of access.

The quantity of land occupied by the streets, houses, offices, warehouses, or other back buildings belonging to the tenements, together with the yards, is to be entered separately at the end of the town lots in which they occur, the value of such land being one of the elements considered in determining the value of the houses, etc.

A timber yard, or commercial yard, is to be valued. If large, state the area, and if paved, etc., the kind of wall or enclosure, and if any offices are in it, their value is to be added to that of the yard.

Gardens in towns.—In towns, the yards attached to the houses are to be considered as one tenement; but the garden, in each case, is to be surveyed separately, and not included in the value of the tenement. The gardens in towns are to be valued as farming lands under the most favorable circumstances.

OF THE SCALE FOR INCREASING THE TABULAR VALUE OF HOUSES FOR TOWN INFLUENCE.

310*i*. Ascertain the rents paid for some of the houses in different parts of the city. This will enable one to determine the tabular increase or decrease.

As it is better to have a house rented by a lease than by the year or half year, therefore a difference is made between a yearly rent and a lease rent: for a new house, two shillings in the pound in favor of the lease rent; for a medium house, about three shillings in the pound; and for an old house, about four shillings in the pound.

In all houses whose annual value is under ten pounds, the rent from year to year is higher in proportion to the cubical contents than in larger houses let in the same manner, but the risk of losing by bad tenants is greater for small houses, therefore in reducing such small houses, when let by the year or half year, to lease rents, five shillings in the pound at least should be deducted.

In villages and small market towns, an addition of twenty-five per cent. to the prices of the tables will generally be found sufficient.

In moderate sized market towns, the prices given in the tables may be trebled for the best situations in the main street, near the market or principal business part of the town; and in the second and third classes, the prices will vary from one hundred to fifty per cent. above the tables; and in large market towns, the prices for houses of the first class, in the best situations, will be about three and one-half times those of the tables.

In dividing the streets or houses of any town into classes, the valuator is, in the first instance, to fix on a medium situation or street, and having ascertained the rents of a number of houses in it, he is, by measurement, to determine what percentage, in addition to the country tables, should

be made, so as to produce results similar to the average of the ascertained rents.

Having determined the percentage to be added to the price given in the tables for houses in medium situations, the standard for the town about to be valued may be considered as formed; and from this standard, percentages in addition are to be made for better and best situations, or for any number of superior classes of houses, or of situations which the size of the town may render necessary.

In towns, the front is the most invaluable, therefore value the front and rear of the building separately, so as to make one gross amount. It is impossible to determine accurately the proportion between the value of the front and rear buildings; but it has been found that in revising the valuations of several towns, that the proportion of five to three was applicable to the greater number of houses in good situations; that is, the country price given by the tables should be multiplied by five for the front, and three for the back buildings, stores and offices.

WATER-POWER.

310j. Ascertain the value of the water power, to which add that of the buildings.

A horse-power is that which is capable of raising 33,000 pounds one foot high in one minute.

The horse-power of a stream is determined by having the mean velocity of the stream, the sectional area, and the fall per mile.

The fall, is the height from the centre of the column of water to the level of the wheel's lower periphery. The weight of a cubic foot of water is 62.25 pounds.

Total weight discharged per minute = $V \cdot A \cdot 62.25$. Here A = sectional area, and V = mean velocity in feet per minute.

A body falling through a given space acquires a momentum capable of raising another body of equal weight to a similar height; therefore, the total weight discharged per minute, multiplied by the *modulus* of the wheel, and this product divided by 33,000 pounds, will give the required horse-power.

<i>Modulus for overshot wheel</i>	0.75
“ “ breast wheel, No. 1, with buckets.....	.66
“ “ “ “ No. 2, with float boards....	.55
“ “ turbine	.65 to 78
“ “ undershot wheel.....	.33

NOTE.—James Francis, Esq., C.E., has found at Lowell, Massachusetts, as high as 90 to 94, from Boyden's turbines.

Fourneyron and D'Auibuson give the modulus for turbine of ordinary construction and well run = 0.70.

To measure the velocity of a stream. Assume two points, as A and B, 528 feet apart; take a sphere of wax, or tin, partly filled and then sealed, so as to sink about one-third in the water; drop the sphere in the centre of the water, and note when it comes on the line A-A, and on the line B-B. A and A may be on opposite sides of the river, or on the river, or on the same side at right angles to the thread of the stream. Let the time in passing from the line AA to the line BB be six minutes. Then as six min. : 528 ft. : : 60 min. to 5280 ft.; that is, the measured surface velocity is one mile per hour.

M. Prony gives $V =$ surface, $W =$ bottom, and $U =$ mean velocity, and $U = 0.80 V =$ mean velocity,
 $W = 0.60 V =$ bottom velocity;
 therefore, as 6 minutes gives a surface velocity of 88 ft.; this multiplied by 0.80, gives 70.4 ft. per minute as the mean velocity.

310k. The following may serve as an example for entry of data and calculation:

Data.	Ft.	In.	
			36 8
			288 = 2 feet = Sectional area = A.
A Breast Wheel, No. 1.			144
Mean velocity of stream per min- ute,	144		288 = Cubic content per minute. 62.25 = Weight of one foot.
Breadth of stream in trough,	-	36	18000 lbs. = Weight discharged. 12 = Fall of water.
Depth of do.	-	8	216000 = Total available power. 66 = Modulus.
Fall of water,	12	-	1425600
			This divided by 33000, gives 4.32 = effective horse-power.

Otherwise:

Data.	Ft.	In.	
Breast wheel No. 1. Revolutions per minute, 6.6.			36 x 8.5 = 2.12 feet = sectional area of bucket. 14 x 12 = 168, and 168 - 8.5 = 159.5 = 13.29 = reduced diameter at centre of buckets. 13.29 x 3.1416 = circumference at centre of buckets = 41.751, and $\frac{41.751 \times 6.6}{2} = 292$ cub. ft. in buckets half full.
Diameter of wheel,	14	-	292 x 62.25 = 18250
Breadth of do.		36	12 = fall of water.
Depth of shroud- ing,		8.5	219000
Fall of water,	12	-	66 = modulus. 33000) 144540.00 (= 4.38 effective horse-power.

For undershot wheels, the data are as follow:

Data.	Ft.	In.	
			Ft. In. 4 6 = Breadth of float boards. 10 = Depth of do. acted on.
Revolutions per minute, 5.2.			3.75 = Area of float boards. 798 = Velocity of stream.
Diameter of wheel,	16	-	2992
Breadth of float board,	4	6	62.25 = Weight of one cubic foot.
Depth of do.,	2	-	187031.25
Velocity of stream per minute,	798	-	2.75 = Height of fall due to velocity.
Height of fall due to vel- ocity,	2	9	514335.9
Depth of do. under wheel,	-	-	33 = Modulus. 169730 33000 = 5.14 horse-power.

310l. It is to be observed that the horse-power deduced from measure-
 ment of a bucket-wheel may be found in some instances rather greater
 than that from the velocity and fall of water, as it is necessary that space
 should be left in the buckets for the escape of air, and also to economize
 the water.

When a bucket-wheel is well constructed, multiply the cubic content
 of water discharged per minute by .001325, and by the fall; the product
 will be the effective horse-power approximately.

For turbines, the effective cubical content of water discharged per min-
 ute multiplied by the height of the fall, and divided by 700, will be equal
 to the effective horse-power.

In practice, twelve cubic feet of water falling one foot per second, is considered equal to a horse-power.

When the water is supplied from a reservoir, and discharged through a sluice, measure from the centre of the orifice to the surface of the water, and note the dimensions of the orifice.

Head of water.—The velocity due to a head of water is equal to that which a heavy body would acquire in falling through a space equal to the depth of the orifice below the free surface of the fluid; that is, if V = velocity, and $M = 16\frac{1}{12}$ feet, or the space fallen through in one second, and H = the height, the velocity may be represented thus: $V = 2 \sqrt{M H}$; thus the natural velocity for .09 feet head of water will be $V = 2 \sqrt{16\frac{1}{12} \times .09} = 2.4$ feet per second. In practice, $V = 8 \sqrt{H}$. The effective velocity = five times the square root of the height. (See sec. 312.)

VALUE OF WATER-POWER.

310*m*. The water-power is to be valued in proportion* as it is used, and the time the mill works.

One horse running twenty-two hours per day during the year, is valued at £1 15*s*. This amount multiplied by the number of horses' power, will give the value of the water-power.

The annexed table is calculated with reference to class of machinery and time of working.

Quality of Machinery.	Number of Working Hours.							
	8	10	12	14	16	18	20	22
	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>
New,	13 3	18 6	23 3	26 9	28 9	30 9	33 0	35 0
Medium, ...	12 0	16 9	21 0	24 3	26 0	27 9	29 6	31 6
Old,	10 6	15 0	18 9	21 6	23 3	24 9	26 6	28 0

In this, two hours are allowed for contingencies and change of men.

The highest proportionate value is set on 14 hours' work, as during that time sufficient water can be had, and one set of men can be sufficient.

Where the supply of water throughout the year is not the same, the valuator is to determine for each period by the annexed table.

Description of Mill,..... Class of Machinery,.....A.				
Horses' Power.	Working Time.		Value of Water-power.	Observations.
	Number of Months per Year.	Number of Hours per Day.		
			<i>£ s. d.</i>	
9	8	22	10 10 0	For 8 months the full power of the wheel is used, but for the remaining 4, not more than two-thirds of the water-power can be calculated on.
6	4	12	2 6 6	
			12 16 6	

Where a mill is worked part of the year by water and another part by steam, care must be taken to determine that part worked by water, and also to value the machinery, as it sometimes happens that the mill may be one quality letter and the machinery another—higher or lower.

MODIFYING CIRCUMSTANCES.

310*n*. The wheel may be unsuitable and ill-contrived; the power may be injudiciously applied; the supply may be scarce, may overflow, or have backwater.

In gravity wheels, the water should act by its own weight—the principle upon which its maximum action depends being that the water should enter the wheel without impulse, and should leave it without velocity. The water should, therefore, be allowed to fall through such a space as will give it a velocity equal to that of the periphery of the wheel when in full work, thus: if the wheel move at the rate of five feet per second, the water must fall on it through not less than two-fifths of a foot; for the space through which a falling body must move to acquire a given

velocity is expressed thus: $\frac{V^2}{4M} = \frac{V^2}{64.333}$

For mills situate in inland towns of considerable importance, such as Armagh, Carlow, Navan, Kilkenny, etc., in a good wheat country, where wheat can be bought at the mill, and the flour sold there also, five shillings in the pound may be added on the water-power for the advantage of situation.

The vicinity of such towns, say within three to four miles, may be called an ordinary situation. Beyond this distance, where the wheat has to be carried from, and flour to, the market, the water-power gradually decreases in value; and from such a town to ten miles distance from it, the water-power may be rated according to the following table.

		s.	d.
Add to water-power,	{	10 0	per pound within the town lot.
		8 0	when distant from 0 to 1 mile.
		6 0	“ “ “ 1 to 3 “
		4 0	“ “ “ 3 to 5 “
		2 0	“ “ “ 5 to 8 “
		1 0	“ “ “ 8 to 10 “
		0 0	“ “ 10 and upwards.

Beyond ten miles from a good local market, a flour mill can rarely require percentage for market.

But this rule of increase does not apply to small mills, such as flour mills, where only one pair of millstones is used; in this case, only half the above percentage is to be added within three miles of a large town; beyond that distance, no addition is to be made.

In the case of bleach mills, they should be as near to their purchasing or export market as flour or corn mills, and the valuator should make deductions for a remote situation, especially where the chief markets for buying linen are distant, or add a percentage to the water-power where the situation has unusual advantages in these respects.

3100. HORSE-POWER DETERMINED FROM THE MACHINERY DRIVEN.

In a flax mill, each stock is equivalent to one horse-power. The bruising machine of three rollers = $1\frac{1}{2}$ stocks.

The numbering of horse-power in the mill may thus be counted, and the value ascertained from the table for horse-power from sec. 3107.

In spinning mills, the horse-power may be determined from the number of spindles driven, and the degree of fineness spun, for in every spinning mill the machinery is constructed to spin within certain range of fineness. Therefore ascertain the range of fineness and number of spindles.

Yarn is distinguished by the degree of fineness to which it is spun, and known by the number of leas or cuts which it yields to the pound.

One lea or cut = 300 lineal yards.

12 leas = 1 hank; 200 leas = 16 hanks; and 8 leas = 1 bundle = 60000 yards.

Leas to the pound.	No. of Spindles.
From 2 to 3,	40 throstles require one horse-power.
From 12 to 30,	60 " " " "
From 70 to 120,	120 " " " "

In cotton mills, the throstle spindle is used for the coarser yarns, and for the finer kinds the mule spindle.

Leas to the pound.	No. of Spindles.
From 10 to 30,	180 throstles equal one horse-power.
From 10 to 50,	500 mules " " "

In bleaching mills, ascertain the number of beetling engines; measure the length of the wiper beam in each, together with the length of beetles, and their depth, taken across the direction of the beam; also the height the beetles are raised in each stroke.

From these data, the horse-power of such engine can be found by inspection of the table calculated for this purpose. Ascertain the number of pairs of washing feet, and if of the ordinary kind; the pairs of rub-boards, starching mangle, squeezing machine, calender, or any other machine worked by water, and state the horse-power necessary to work each.

The standard for a horse-power in a beetling mill is taken as follows: Beam, furnished with cogs for lifting the beetles, 10 feet long. The wiper beam makes 30 revolutions in a minute; and being furnished with two sets of cogs on its circumference, raises the beetle 60 times per minute, working beetles 4 feet 4 inches in length, and 3 inches in depth, from front to rear, making 30 revolutions per minute, or lifting the beetles 60 times in a minute one foot high, is equal to *one horse-power*. This includes the power necessary to work the traverse beam and guide slips, which retain the beetle in a perpendicular position.

Taking the wiper beam at 10 feet long, and height lifted as 1 foot, making 30 revolutions per minute, the following table will show, by inspection, the proportionate horse-power required to raise beetles of other dimensions 60 feet in one minute, assuming the weight of a cubic foot of dry beach wood = 712 ounces.

When the engine goes faster or slower, a proportionate allowance must be made.

Inches from front to rear.	LENGTH OF BEETLES.																					
	Ft. In		Ft. In.		Ft. In.		Ft. In.		Ft. In.		Ft. In.											
	4	4	4	6	4	8	4	10	5	0	5	2	5	4	5	6	5	8	5	10	6	0
Number of Horse Power.																						
3	1.00	1.03	1.06	1.10	1.13	1.16	1.20	1.24	1.28	1.32	1.36											
3 $\frac{1}{4}$	1.07	1.10	1.14	1.18	1.22	1.26	1.30	1.34	1.38	1.42	1.46											
3 $\frac{1}{2}$	1.15	1.19	1.23	1.27	1.32	1.36	1.40	1.45	1.49	1.53	1.58											
3 $\frac{3}{4}$	1.23	1.27	1.32	1.37	1.41	1.45	1.49	1.54	1.58	1.63	1.69											
4	1.31	1.36	1.41	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80											
4 $\frac{1}{4}$	1.40	1.44	1.49	1.54	1.59	1.64	1.70	1.75	1.80	1.85	1.91											
4 $\frac{1}{2}$	1.48	1.53	1.58	1.64	1.69	1.75	1.80	1.85	1.91	1.97	2.03											

From this table it appears that a ten feet wiper beam, having its beetles four inches in depth, five feet long, and to lift those beetles one foot high sixty times in a minute, would require the power of one and one-half horses.

If the wiper beam be more or less than ten feet in length, or if the lift of the beetles be more or less than one foot, a proportionate addition or deduction should be made.

The following is given to assist the valuator in determining the value of the other machinery in a bleaching mill:

One pair of rub-boards,	= 0.5 to 0.7 horse-power.
“ starching mill,	1 “
“ drying and squeezing machine,	1 “
“ pair of wash-feet,	1.5 to 2 “
“ calender (various),	3 to 8 “

In *beetling mills*, the long engine, with a ten feet wiper beam, is considered the most eligible standard for computing the water-power. Such a beam, having beetles four inches long and three inches deep, is equal to one horse-power. On these principles, the value of water-power may be ascertained from the table, sec. 310l.

310p. In *flour mills*, the power necessary to drive the machinery night and day for the year round, has been determined as follows:

The grinding portion, or flour millstones, have been considered to require, for each pair, four horses-power. The flour dressing machine of ordinary kind, together with the screens, sifters, etc., or cleansing machinery, require, on an average, four horses-power. Some machines, however, from their size and feed with which they are supplied, will require more or less than four horses-power, and should be noted by the valuator.

Every dressing, screening and cleansing machine is equal to one pair of stones.

(NOTE.—In Chicago, ten horses power is estimated for one pair of stones, together with all the elevating and cleansing machinery.—M. M'D.)

The following table has been made for one pair of millstones, four feet four inches diameter, for one year:

Quality of Machine.	Number of Working Hours per Day.															
	8.		10.		12.		14.		16.		18.		20.		22.	
	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.
New, A	2	13 0 3	4	13 0 4	4	13 0 5	7	0 5	15	0 6	3	0 6	12	0 7	0	0
Medium, B	2	8 0 3	7	0 4	4	0 4	17	0 5	4	0 5	11	0 5	18	0 6	6	0
Old, C	2	2 0 3	0	0 3	15	0 4	6	0 4	13	0 4	19	0 5	6	0 5	12	0

If more than one pair of millstones be used in the mill, multiply the above by the number of pairs usually worked, and if they are more or less than four feet four inches in diameter, make a proportional increase or decrease.

In flour mills, the valuator will state the kind of stones, how many French burrs, their diameter, the number worked at one time, the number of months they are worked, the number of months that there is a good supply, a moderate one, and a scarcity of supply.

FORM FOR FLOUR MILLS.—No. 1.

Description of Mill,.....Flour Mill. Class of Machinery,.....A.					
Dressing Machine, Screens, etc.	Millstones, No. of Pairs Worked.	Working Time.		Value of Water-power.	Observations.
		No. of Months per Year.	No. of Hours per Day.		
-	4	6	22	£ 14 0 0	In this mill there are five pairs of stones, one pair always up, being dressed; machine and screens and sifters only used when one or two pairs of stones are stopped, and not worked in summer, except one or two days in the week. Two sets of elevators used along with the millstones.
-	2	3	16	2 18 0	
-	1	3	10	0 18 0	
1	Only used when one or two pairs of stones are thrown out.				
-				17 16 0	

No. 2.

Description of Mill,.....Flour Mill. Class of Machinery,.....B.					
Dressing Machine, Screens, etc.	Millstones, No. of Pairs Worked.	Working Time.		Value of Water-power.	Observations.
		No. of Months per Year.	No. of Hours per Day.		
-	2	4	22	£ 4 4 0	In this mill there are three pairs of stones—one pair generally up, two driven for four months along with machines, screens and sifters, and one for one month with them also; during three months the machines and one pair of millstones must be worked alternate days, and during the other four months there is no work done. One set of elevators used along with the millstones.
-	1	1	22	0 11 0	
-	1	3	9	0 14 0	
1	-	5	22	2 13 0	
				8 2 0	

310*q*. In oatmeal mills, one pair of grinding stones require three horse-power; one pair of shelling stones, fans and sifters, require two horse-power. Elevator is taken at one-eighth of the power of the stones.

The following table, for one pair of millstones for one year, is to be used as the table for flour mills:

Quality of Machinery.	Number of Working Hours per Day.																	
	8		10		12		14		16		18		20		22			
	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.		
New, A.....	2	0 0	2	16 0	3	10 0	4	0 0	4	6 0	4	12 0	4	19 0	5	5 0		
Medium, B.	1	16 0	2	10 0	3	3 0	3	13 0	3	18 0	4	3 0	4	9 0	4	15 0		
Old, C.....	1	12 0	2	5 0	2	16 0	3	4 0	3	10 0	3	14 0	3	19 0	4	4 0		

310r. In corn mills, ascertain the number of pairs of grinding and shelling millstones and other machinery, and note the time each is worked. Where there are two pairs—one of which is used for grinding and the other for shelling; if there be fans and sifters, the shelling and sifters is = to two horses' power = two-thirds of a pair of grinding stones. Where one pair is used to shell and grind alternately, it is reckoned at three-fourths pair of grinding stones, unless the fans and sifters be used at the same time. In this case they will be counted as seven-eighths pair of stones. Where there are two pairs of grinding, with one pair of shelling with fans and sifters, the water power is equal to two and two-thirds pairs of millstones; but if one pair is idle, then the power = one and two-thirds pairs of grinding millstones, etc.

FORM No. 1.

Description of Mill,.....Corn Mill.									
Class of Machinery,.....A.									
Millstones, No. of Pairs Worked.				Working Time.		Value of Water-power.		Observations.	
Grind'g	Shelling	Grind'g and Shelling	Equivalent in Grinding Stones.	No. of Months per Year.	No. of Hours per Day.	£	s.		
2	1	—	2½	8	22	9	6	0	In this mill there are three pairs of stones, with elevators, fans, and sifters. Horse-power for 8 months equal to 8, or 2½ grinding stones; and for 4 months 5 horse power, or 1½ grinding stones.
1	1	—	1½	4	12	1	19	0	
					Add ¼ for Ele- vators, .	11	5	0	
						1	8	0	
						12	13	0	

FORM No. 2.

Description of Mill,.....Corn Mill.									
Class of Machinery,.....B.									
Millstones, No. of Pairs Worked.				Working Time.		Value of Water-power.		Observations.	
Grind'g	Shelling	Grind'g and Shelling	Equivalent in Grinding Stones.	No. of Months per Year.	No. of Hours per Day.	£	s.		
1	1	—	1½	6	16	2	18	6	In this mill there are two pairs of stones, but no fans, sifters, or elevators.
1	1	—	1½	3	7	0	12	0	

FORM No. 3.

Description of Mill,.....Corn Mill. Class of Machinery.....C.							
Millstones, No. of Pairs Worked.			Equivalent Grinding Stones.	Working Time.		Value of Water- power.	Observations.
Grind'g	Shelling	Grind'g and Shelling		No. of Months per Year	No. of Hours per Day.		
-	-	1	$\frac{7}{8}$	4	16	£ 1 0	In this mill there are two pairs of stones, only one pair can be worked at a time: there are fans and sifters in use, but no elevators. This mill works merely for the supply of the neighborhood, and is distant four miles from a market town.
-	-	1	$\frac{7}{8}$	4	8	0 9	

When there are two or more mills in a district, compare the value of one with the other.

Three stocks in a flax mill is equal to the power necessary to work a pair of millstones in a corn mill. Note the quantity ground annually as a further check, for it has been ascertained that a bushel of corn requires a force of 31,500 lbs. to grind, the stones being about 5 feet in diameter, and making 95 revolutions per minute.

310s. *In fine*, it should be borne in mind, that for each separate tenement a similar conclusion is ultimately to be arrived at, viz., that the value of land, buildings, etc., as the case may be, when set forth in the column for totals, is the rent which a liberal landlord would obtain from a solvent tenant for a term of years, (*rates, taxes, etc., being paid by the tenant;*) and that this rent has been so adjusted with reference to those of surrounding tenements that the assessment of rates may be borne equably and relatively by all.

The valuator, therefore, should endeavor to carry out fairly the spirit of the foregoing instructions, which have been arranged with a view to promote similarity of system in cases which require similarity of judgment.

As it may appear difficult to apply Griffith's System of Valuation to American cities, on account of the number of frame or wooden buildings, we give a table at p. 72B53, showing the comparative value of frame and brick houses. All the surveyors and land agents, to whom we have shown and explained this system of valuation, have approved of it, and expressed a hope of seeing such a system take the place of the present hit or miss valuations, too often made by men who are unskilled in the first rudiments of surveying and architecture.

(310c.)

FORM OF FIELD BOOK.

County of Clare, Barony of Bunratty Lower, Parish of St. Patrick, Townland of Shanakyle.—Ordnance Survey, 63.

Original Map.	Names.		Description.		Area.	Value per Acre.				Nett Annual Value.			Observations.
	Occupiers.	Immediate Lessors.	Of Tenement.	Of Soil.		In ordi- nary situa- tions.	s.	d.	With al- lowance for local circum- stances.	Land.	Buildings.	Totl.	
Part 7	Michael Mason, .	Hamilton Jackson, Esq.,	House, .	.	A. R. P.	s.	d.	s.	d.	£	s.	d.	} Improved. Old deduction.
"	Patrick Connolly, .	"	Garden, .	.	0 0 32	—	—	—	13 0	—	0 13 4	—	
"	William Connors, .	"	Gardens, .	.	0 1 10	—	—	—	14 0	—	—	—	
"	James Dalton, .	William Connors, .	House, .	.	—	—	—	—	—	—	0 12 6	—	
"	John Costelloe, .	Hamilton Jackson, Esq.,	House and Land, .	.	—	—	—	—	27 0	—	1 5 0	—	} Improved. Old deduction.
Part 2	{ Mattw. Hanrahan, Patk. Kelly, (Pat.) Pat. Kelly, (John,) }	Hamilton Jackson, Esq.,	{ House, Office & Land, House, Office & Land, House, Offices & Land, }	{ . . .	—	—	—	—	—	—	1 5 9	—	
	"	"	Land, .	.	9 1 33	—	—	—	0 6	—	—	—	
	"	"	Land, .	.	4 0 24	—	—	—	6 0	—	—	—	
	"	"	Land, .	.	3 0 0	—	—	—	9 0	0 9 9	—	—	
Part 8	"	"	Land, .	.	—	—	—	—	16 0	—	—	—	} Too high. Worn. Rather low. Bad ; part of deduction.
"	"	"	Land, .	.	1 2 0	—	—	—	11 0	—	—	—	

(310a.)

FORM OF TOWN BOOK.

Townland of Killarney.

Parish of Killarney.

Town of Killarney.

High Street.

No.	Name and Description.	Quality	Letter	Height	Length	Breadth	No. of Measure	Rate per Measure	Amount of Items.	Area of Garden.	Value of Garden or Yard.	Gross tabular Amount with Multiplier.	Value for Public Use.	Yearly Rent.	Valuation Estimate.	Immediate Lessee.	Observations.
				Ft. In	Ft. In	Ft. In		s. d.	£. s. d.	A. R. P.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.		
1	Daniel Mahony, House, offices, and yard	MB	27	634	0 23	0 73	1 1	4 4 6				Good front, Multiplier, 4 4 6		40 0 0	Two shops, goodsitu ation, 25 0 0	Christ. Galwey,	Has back gate-way to yard. Built some offices lately, and otherwise improved the concern.
	House, offices, and yard	1B	27	630	0 22	0 65	0 11	3 4 7				£14 1 8					
	Do., do.,	1B	12	020	0 8	0 16	0 6	0 8 8									
	Do., do.,	1B	10	020	0 2	0 4	0 0	0 2 0									
	Kitchen,	1B	14	614	0 15	0 21	0 7	0 13 1		Good							
	Office (Bake-house),	1B	14	615	0 15	0 22	0 3	0 6 10		yard to							
	Do.,	1B	7	0 9	0 5	0 4	0 2	0 0 8		this concern							
	Addition to Bake-house,	1B	7	0 23	0 15	0 43	0 2	0 7 2		worth							
	Sheds,	2B	10	0 18	0 17	0 30	0 2	0 6 3		about							
	Stable,	2B	10	0 18	0 17	0 30	0 2	0 6 3				£25 4 2	26 0 0				New portions.
3	Richard Murphy, House, yard, and small garden.							9 15 9									
	House, do.,	1C	17	613	6 23	0 31	0 7	1 0 0		Small garden, add 3s. 6d. to house.							
	House, do.,	1B	6	6 9	6 16	0 15	0 3	0 4 8									
	Office, do.,	1C	6	0 14	0 17	6 24	0 1	0 2 0									
	Do., do.,	1C	6	6 10	0 5	0 5	0 1	0 0 6									
	Old office, do.,	—	—	—	—	—	—	0 2 0									
Part of adjoining house.																	
	Deduct shop,	1C	17	6 9	6 23	0 21	0 7	0 13 6									
								2 2 8									
								0 7 0									
								1 15 8									
								£8 7 7									

Note.—The Length multiplied by the Breadth and divided by 10 = Measures.

TABLES

FOR ASCERTAINING THE

ANNUAL VALUE OF HOUSES IN THE COUNTRY.

(310v.)

I.—SLATED HOUSES,

WALLS BUILT WITH STONE, OR BRICK, AND LIME MORTAR.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft. Inch	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	d.	d.
6 0	0 5 $\frac{1}{4}$	0 5	0 4 $\frac{3}{4}$	0 4 $\frac{1}{2}$	0 3 $\frac{3}{4}$	0 3 $\frac{1}{2}$	0 3	2 $\frac{1}{4}$	1
3	0 5 $\frac{1}{2}$	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{2}$	0 4	0 3 $\frac{3}{4}$	0 3 $\frac{1}{2}$	2 $\frac{1}{4}$	1
6	0 5 $\frac{1}{2}$	0 5 $\frac{1}{2}$	0 5	0 4 $\frac{1}{2}$	0 4	0 3 $\frac{3}{4}$	0 3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
9	0 6	0 5 $\frac{1}{2}$	0 5	0 4 $\frac{1}{2}$	0 4 $\frac{1}{2}$	0 3 $\frac{3}{4}$	0 3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
7 0	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{4}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{2}$	0 4	0 3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
3	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{2}$	0 4	0 3 $\frac{3}{4}$	2 $\frac{3}{4}$	1 $\frac{1}{4}$
6	0 6 $\frac{1}{2}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{2}$	0 4 $\frac{1}{2}$	0 3 $\frac{3}{4}$	3	1 $\frac{1}{4}$
9	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{2}$	0 3 $\frac{3}{4}$	3	1 $\frac{1}{2}$
8 0	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{2}$	0 4	3	1 $\frac{1}{2}$
3	0 7	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{2}$	0 4	3 $\frac{1}{4}$	1 $\frac{1}{2}$
6	0 7 $\frac{1}{2}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4	3 $\frac{1}{4}$	1 $\frac{1}{2}$
9	0 7 $\frac{1}{2}$	0 7	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{2}$	3 $\frac{1}{2}$	1 $\frac{3}{4}$
9 0	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{3}{4}$	3 $\frac{1}{2}$	1 $\frac{3}{4}$
3	0 7 $\frac{3}{4}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{3}{4}$	3 $\frac{1}{2}$	1 $\frac{3}{4}$
6	0 8	0 7 $\frac{3}{4}$	0 7	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	3 $\frac{1}{2}$	1 $\frac{3}{4}$
9	0 8 $\frac{1}{4}$	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	3 $\frac{1}{2}$	1 $\frac{3}{4}$
10 0	0 8 $\frac{1}{2}$	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{2}$	0 5	3 $\frac{1}{2}$	1 $\frac{3}{4}$
3	0 8 $\frac{1}{2}$	0 8	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5	3 $\frac{3}{4}$	1 $\frac{3}{4}$
6	0 8 $\frac{1}{2}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5	3 $\frac{3}{4}$	2
9	0 9	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{2}$	3 $\frac{3}{4}$	2
11 0	0 9 $\frac{1}{4}$	0 8 $\frac{3}{4}$	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{2}$	4	2
3	0 9 $\frac{1}{4}$	0 8 $\frac{3}{4}$	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{2}$	4	2
6	0 9 $\frac{1}{4}$	0 9	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	4	2
9	0 9 $\frac{1}{4}$	0 9 $\frac{1}{4}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	4	2
12 0	0 10	0 9 $\frac{1}{4}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	4 $\frac{1}{4}$	2
6	0 10 $\frac{1}{4}$	0 9 $\frac{3}{4}$	0 8 $\frac{3}{4}$	0 8	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6	4 $\frac{1}{2}$	2 $\frac{1}{4}$
13 0	0 10 $\frac{3}{4}$	0 10	0 9	0 8 $\frac{1}{4}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{4}$	4 $\frac{3}{4}$	2 $\frac{1}{4}$
6	0 11	0 10 $\frac{1}{4}$	0 9 $\frac{1}{4}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7	0 6 $\frac{1}{4}$	4 $\frac{3}{4}$	2 $\frac{1}{4}$
14 0	0 11 $\frac{1}{4}$	0 10 $\frac{3}{4}$	0 9 $\frac{3}{4}$	0 9	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	4 $\frac{3}{4}$	2 $\frac{1}{4}$
6	0 11 $\frac{1}{4}$	0 10 $\frac{3}{4}$	0 10	0 9 $\frac{1}{4}$	0 8 $\frac{1}{4}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	5	2 $\frac{1}{4}$
15 0	1 0	0 11	0 10 $\frac{1}{4}$	0 9 $\frac{1}{2}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	5	2 $\frac{1}{4}$
6	1 0 $\frac{1}{4}$	0 11 $\frac{1}{4}$	0 10 $\frac{1}{2}$	0 9 $\frac{3}{4}$	0 8 $\frac{3}{4}$	0 8	0 7	5 $\frac{1}{2}$	2 $\frac{1}{4}$
16 0	1 0 $\frac{3}{4}$	0 11 $\frac{3}{4}$	0 10 $\frac{3}{4}$	0 10	0 9	0 8	0 7 $\frac{1}{4}$	5 $\frac{1}{4}$	2 $\frac{1}{2}$
6	1 1	0 11 $\frac{3}{4}$	0 11	0 10 $\frac{1}{2}$	0 9 $\frac{1}{2}$	0 8 $\frac{1}{2}$	0 7 $\frac{1}{4}$	5 $\frac{1}{2}$	2 $\frac{1}{2}$
17 0	1 1 $\frac{1}{4}$	1 0	0 11 $\frac{1}{4}$	0 10 $\frac{3}{4}$	0 9 $\frac{3}{4}$	0 8 $\frac{3}{4}$	0 7 $\frac{3}{4}$	5 $\frac{3}{4}$	2 $\frac{3}{4}$
6	1 1 $\frac{1}{2}$	1 0 $\frac{1}{4}$	0 11 $\frac{1}{2}$	0 10 $\frac{3}{4}$	0 9 $\frac{3}{4}$	0 8 $\frac{3}{4}$	0 7 $\frac{3}{4}$	5 $\frac{3}{4}$	3

I.—SLATED HOUSES,
WALLS BUILT WITH STONE, OR BRICK, AND LIME MORTAR—*continued*.

Height.	A+		A		A—		B+		B		B—		C+		C	C—
Ft Inch	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	d.	d.
18 0	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	0	10	0	9	0	7 $\frac{3}{4}$	6	3 $\frac{3}{4}$
6	1	2	1	1	1	0	0	11	0	10	0	9	0	8	6	3
19 0	1	2 $\frac{1}{2}$	1	1 $\frac{1}{4}$	1	0 $\frac{1}{4}$	0	11 $\frac{1}{4}$	0	10 $\frac{1}{4}$	0	9 $\frac{1}{4}$	0	8 $\frac{1}{4}$	6 $\frac{1}{4}$	3
6	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	$\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	0	8 $\frac{1}{2}$	6 $\frac{1}{2}$	3
20 0	1	2 $\frac{3}{4}$	1	1 $\frac{3}{4}$	1	0 $\frac{3}{4}$	0	11 $\frac{3}{4}$	9	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	0	8 $\frac{1}{2}$	6 $\frac{1}{4}$	3 $\frac{1}{4}$
6	1	3	1	2	1	1	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	0	8 $\frac{3}{4}$	6 $\frac{1}{2}$	3 $\frac{1}{2}$
21 6	1	3 $\frac{1}{4}$	1	2 $\frac{1}{4}$	1	1 $\frac{1}{4}$	1	0	0	11	0	10	0	8 $\frac{3}{4}$	6 $\frac{3}{4}$	3 $\frac{1}{4}$
6	1	3 $\frac{1}{2}$	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0 $\frac{1}{4}$	0	11 $\frac{1}{4}$	0	10	0	9	6 $\frac{1}{2}$	3 $\frac{1}{4}$
22 0	1	3 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{3}{4}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9	6 $\frac{3}{4}$	3 $\frac{1}{2}$
6	1	4	1	3	1	1 $\frac{3}{4}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9	6 $\frac{3}{4}$	3 $\frac{1}{2}$
23 0	1	4 $\frac{1}{4}$	1	3 $\frac{1}{4}$	1	2	1	0 $\frac{3}{4}$	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	6 $\frac{3}{4}$	3 $\frac{3}{4}$
6	1	4 $\frac{1}{2}$	1	3 $\frac{1}{2}$	1	2 $\frac{1}{4}$	1	1	0	11 $\frac{1}{4}$	0	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	7	3 $\frac{1}{2}$
24 0	1	4 $\frac{3}{4}$	1	3 $\frac{3}{4}$	1	2 $\frac{1}{4}$	1	1	1	0	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	7 $\frac{1}{4}$	3 $\frac{1}{2}$
6	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{1}{2}$	1	1 $\frac{1}{4}$	1	0	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	7 $\frac{1}{4}$	3 $\frac{1}{2}$
25 0	1	5	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0 $\frac{1}{4}$	0	11	0	10	7 $\frac{1}{2}$	3 $\frac{3}{4}$
6	1	5 $\frac{1}{4}$	1	4 $\frac{1}{4}$	1	3	1	1 $\frac{3}{4}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10	7 $\frac{1}{2}$	3 $\frac{3}{4}$
26 0	1	5 $\frac{3}{4}$	1	4 $\frac{3}{4}$	1	3	1	2	1	0 $\frac{3}{4}$	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	7 $\frac{3}{4}$	3 $\frac{3}{4}$
6	1	5 $\frac{3}{4}$	1	4 $\frac{3}{4}$	1	3 $\frac{1}{2}$	1	2	1	0 $\frac{3}{4}$	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	7 $\frac{3}{4}$	3 $\frac{3}{4}$
27 0	1	6	1	4 $\frac{3}{4}$	1	3 $\frac{1}{2}$	1	2 $\frac{1}{4}$	1	1	0	11 $\frac{1}{4}$	0	10 $\frac{1}{4}$	7 $\frac{3}{4}$	3 $\frac{3}{4}$
6	1	6 $\frac{1}{4}$	1	5	1	3 $\frac{1}{2}$	1	2 $\frac{1}{4}$	1	1	0	11 $\frac{1}{4}$	0	10 $\frac{1}{4}$	7 $\frac{3}{4}$	3 $\frac{3}{4}$
28 0	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{1}{2}$	1	1 $\frac{1}{4}$	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	7 $\frac{3}{4}$	3 $\frac{3}{4}$
6	1	6 $\frac{1}{2}$	1	5 $\frac{1}{4}$	1	4	1	2 $\frac{1}{2}$	1	1 $\frac{1}{4}$	1	0	0	10 $\frac{3}{4}$	8	4
29 0	1	6 $\frac{3}{4}$	1	5 $\frac{1}{4}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0	0	10 $\frac{3}{4}$	8	4
6	1	6 $\frac{3}{4}$	1	5 $\frac{1}{2}$	1	4 $\frac{1}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0	0	10 $\frac{3}{4}$	8	4
30 0	1	7	1	5 $\frac{1}{2}$	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{2}$	1	0 $\frac{1}{4}$	0	10 $\frac{3}{4}$	8	4
6	1	7	1	5 $\frac{3}{4}$	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{3}{4}$	1	0 $\frac{1}{4}$	0	11	8 $\frac{1}{4}$	4
31 0	1	7 $\frac{1}{4}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{2}$	1	3 $\frac{1}{4}$	1	1	1	0 $\frac{1}{2}$	0	11	8 $\frac{1}{4}$	4
6	1	7 $\frac{1}{2}$	1	6	1	4 $\frac{3}{4}$	1	3 $\frac{1}{4}$	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{1}{4}$	8 $\frac{1}{4}$	4
32 0	1	7 $\frac{1}{2}$	1	6 $\frac{1}{4}$	1	4 $\frac{3}{4}$	1	3 $\frac{1}{2}$	1	2	1	0 $\frac{3}{4}$	0	11 $\frac{1}{4}$	8 $\frac{1}{2}$	4
6	1	7 $\frac{3}{4}$	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{1}{2}$	1	2	1	0 $\frac{3}{4}$	0	11 $\frac{1}{4}$	8 $\frac{1}{2}$	4
33 0	1	7 $\frac{3}{4}$	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{1}{2}$	1	2 $\frac{1}{4}$	1	0 $\frac{3}{4}$	0	11 $\frac{1}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{4}$
6	1	8	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{1}{4}$	1	1	0	11 $\frac{1}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{4}$
34 0	1	8	1	6 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	3 $\frac{3}{4}$	1	2 $\frac{1}{4}$	1	1	0	11 $\frac{1}{4}$	8 $\frac{3}{4}$	4 $\frac{1}{4}$
6	1	8 $\frac{1}{4}$	1	6 $\frac{3}{4}$	1	5 $\frac{1}{2}$	1	3 $\frac{3}{4}$	1	2 $\frac{1}{2}$	1	1	0	11 $\frac{1}{4}$	8 $\frac{3}{4}$	4 $\frac{1}{4}$
35 0	1	8 $\frac{1}{2}$	1	6 $\frac{3}{4}$	1	5 $\frac{1}{2}$	1	4	1	2 $\frac{1}{2}$	1	1	0	11 $\frac{1}{4}$	8 $\frac{3}{4}$	4 $\frac{1}{2}$
6	1	8 $\frac{1}{4}$	1	7	1	5 $\frac{1}{4}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{4}$	0	11 $\frac{3}{4}$	8 $\frac{3}{4}$	4 $\frac{1}{2}$
36 0	1	8 $\frac{1}{2}$	1	7	1	5 $\frac{1}{2}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{4}$	0	11 $\frac{3}{4}$	8 $\frac{3}{4}$	4 $\frac{1}{2}$
6	1	8 $\frac{3}{4}$	1	7 $\frac{1}{4}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{4}$	0	11 $\frac{3}{4}$	9	4 $\frac{1}{2}$
37 0	1	8 $\frac{3}{4}$	1	7 $\frac{1}{4}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{4}$	0	11 $\frac{3}{4}$	9	4 $\frac{1}{2}$
6	1	9	1	7 $\frac{1}{4}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{4}$	0	11 $\frac{3}{4}$	9	4 $\frac{1}{2}$
38 0	1	9	1	7 $\frac{1}{2}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{2}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{4}$	1	0	9	4 $\frac{1}{2}$
6	1	9	1	7 $\frac{1}{2}$	1	6	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{4}$	1	0	9	4 $\frac{1}{2}$
39 0	1	9	1	7 $\frac{1}{2}$	1	6	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
6	1	9	1	7 $\frac{1}{2}$	1	6 $\frac{1}{2}$	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
40 0	1	9 $\frac{1}{4}$	1	7 $\frac{3}{4}$	1	6 $\frac{1}{4}$	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{4}$
6	1	9 $\frac{1}{4}$	1	7 $\frac{3}{4}$	1	6 $\frac{1}{4}$	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{4}$

(310w.)

II.—THATCHED HOUSES,

BRICK OR STONE WALLS, BUILT WITH LIME MORTAR.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft Inch		d.	d.	d.	d.	d.	d.	d.	d.
6 0	—	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1
3	—	4 $\frac{3}{4}$	4	3 $\frac{3}{4}$	3 $\frac{3}{4}$	2 $\frac{3}{4}$	2 $\frac{3}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
6	—	4 $\frac{3}{4}$	4	3 $\frac{3}{4}$	3 $\frac{3}{4}$	3	2 $\frac{3}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
9	—	4 $\frac{3}{4}$	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3	2 $\frac{3}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
7 0	—	5	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2	1 $\frac{3}{4}$
3	—	5	4 $\frac{3}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3	2	1 $\frac{3}{4}$
6	—	5 $\frac{1}{4}$	4 $\frac{3}{4}$	4 $\frac{3}{4}$	4	3 $\frac{1}{4}$	3	2	1 $\frac{3}{4}$
9	—	5 $\frac{1}{4}$	5	4 $\frac{1}{2}$	4	3 $\frac{1}{2}$	3 $\frac{1}{4}$	2	1 $\frac{3}{4}$
8 0	—	5 $\frac{1}{2}$	5	4 $\frac{3}{4}$	4 $\frac{1}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	2	1 $\frac{3}{4}$
3	—	5 $\frac{3}{4}$	5 $\frac{1}{4}$	4 $\frac{3}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$
6	—	5 $\frac{3}{4}$	5 $\frac{1}{4}$	5	4 $\frac{3}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$
9	—	6	5 $\frac{1}{2}$	5	4 $\frac{3}{4}$	4	3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$
9 0	—	6	5 $\frac{1}{2}$	5	4 $\frac{3}{4}$	4	3 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$
3	—	6 $\frac{1}{4}$	5 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{3}{4}$	4	3 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$
6	—	6 $\frac{1}{4}$	5 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{3}{4}$	4	3 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$
9	—	6 $\frac{1}{2}$	6	5 $\frac{1}{2}$	5	4 $\frac{1}{4}$	3 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$
10 0	—	6 $\frac{3}{4}$	6 $\frac{1}{4}$	5 $\frac{1}{2}$	5	4 $\frac{1}{4}$	4	2 $\frac{1}{2}$	1 $\frac{3}{4}$
3	—	6 $\frac{3}{4}$	6 $\frac{1}{4}$	5 $\frac{3}{4}$	5 $\frac{1}{4}$	4 $\frac{1}{2}$	4	2 $\frac{1}{2}$	1 $\frac{3}{4}$
6	—	7	6 $\frac{1}{2}$	6	5 $\frac{1}{4}$	4 $\frac{3}{4}$	4 $\frac{1}{4}$	2 $\frac{3}{4}$	2
9	—	7 $\frac{1}{4}$	6 $\frac{1}{2}$	6	5 $\frac{1}{2}$	4 $\frac{3}{4}$	4 $\frac{1}{4}$	2 $\frac{3}{4}$	2
11 0	—	7 $\frac{1}{2}$	6 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{3}{4}$	4 $\frac{1}{2}$	2 $\frac{3}{4}$	2
3	—	7 $\frac{1}{2}$	6 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{3}{4}$	5	4 $\frac{1}{2}$	2 $\frac{3}{4}$	2
6	—	7 $\frac{3}{4}$	7	6 $\frac{1}{2}$	5 $\frac{3}{4}$	5	4 $\frac{3}{4}$	2 $\frac{3}{4}$	2
9	—	7 $\frac{3}{4}$	7 $\frac{1}{4}$	6 $\frac{1}{2}$	6	5	4 $\frac{3}{4}$	3	2
12 0	—	7 $\frac{3}{4}$	7 $\frac{1}{4}$	6 $\frac{1}{2}$	6	5 $\frac{1}{4}$	4 $\frac{3}{4}$	3	2 $\frac{1}{4}$
6	—	8	7 $\frac{1}{2}$	6 $\frac{3}{4}$	6 $\frac{1}{4}$	5 $\frac{1}{4}$	4 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{1}{4}$
13 0	—	8 $\frac{1}{4}$	7 $\frac{3}{4}$	7	6 $\frac{1}{2}$	5 $\frac{1}{2}$	5	3 $\frac{1}{4}$	2 $\frac{1}{4}$
6	—	8 $\frac{1}{2}$	7 $\frac{3}{4}$	7 $\frac{1}{4}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	5	3 $\frac{1}{4}$	2 $\frac{1}{4}$
14 0	—	8 $\frac{3}{4}$	8	7 $\frac{1}{2}$	6 $\frac{3}{4}$	5 $\frac{3}{4}$	5 $\frac{1}{4}$	3 $\frac{1}{2}$	2 $\frac{1}{4}$
6	—	9	8 $\frac{1}{4}$	7 $\frac{3}{4}$	7	6	6 $\frac{1}{4}$	3 $\frac{1}{2}$	2 $\frac{1}{4}$
15 0	—	9 $\frac{1}{4}$	8 $\frac{3}{4}$	8	7 $\frac{1}{4}$	6	6 $\frac{1}{2}$	3 $\frac{3}{4}$	2 $\frac{1}{4}$
6	—	9 $\frac{1}{2}$	8 $\frac{3}{4}$	8 $\frac{1}{4}$	7 $\frac{1}{2}$	6 $\frac{1}{4}$	6 $\frac{1}{2}$	3 $\frac{3}{4}$	2 $\frac{1}{4}$
16 0	—	10	9	8 $\frac{1}{2}$	7 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{3}{4}$	3 $\frac{3}{4}$	2 $\frac{3}{4}$
6	—	10 $\frac{1}{4}$	9 $\frac{1}{4}$	8 $\frac{1}{2}$	7 $\frac{3}{4}$	6 $\frac{1}{2}$	6	2 $\frac{3}{4}$	2 $\frac{3}{4}$
17 0	—	10 $\frac{1}{2}$	9 $\frac{1}{2}$	8 $\frac{3}{4}$	8	6 $\frac{3}{4}$	6	2 $\frac{3}{4}$	2 $\frac{3}{4}$
6	—	10 $\frac{1}{2}$	9 $\frac{3}{4}$	9	8	6 $\frac{3}{4}$	6 $\frac{1}{4}$	4	2 $\frac{3}{4}$
18 0	—	10 $\frac{3}{4}$	10	9 $\frac{1}{4}$	8 $\frac{1}{2}$	7	6 $\frac{1}{4}$	4	3
6	—	11	10 $\frac{1}{4}$	9 $\frac{1}{4}$	8 $\frac{1}{2}$	7 $\frac{1}{4}$	6 $\frac{1}{2}$	4 $\frac{1}{4}$	3
19 0	—	11 $\frac{1}{4}$	10 $\frac{1}{2}$	9 $\frac{1}{2}$	8 $\frac{3}{4}$	7 $\frac{1}{2}$	6 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{1}{4}$
6	—	11 $\frac{1}{2}$	10 $\frac{3}{4}$	9 $\frac{3}{4}$	8 $\frac{3}{4}$	7 $\frac{3}{4}$	6 $\frac{3}{4}$	4 $\frac{3}{4}$	3 $\frac{1}{4}$
20 0	—	11 $\frac{3}{4}$	10 $\frac{3}{4}$	10	9	7 $\frac{3}{4}$	6 $\frac{3}{4}$	4 $\frac{3}{4}$	3 $\frac{1}{4}$

(310x.)

III.—THATCHED HOUSES,

PUDDLE MORTAR WALLS,—DRY WALLS, POINTED,—MUD WALLS OF A GOOD KIND.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft. Inch.			<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>
6 0	—	—	3	2 $\frac{1}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{1}{4}$	3 $\frac{3}{4}$
3	—	—	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{4}$	2	1 $\frac{1}{4}$	3 $\frac{3}{4}$
6	—	—	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2	1 $\frac{1}{4}$	3 $\frac{3}{4}$
9	—	—	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{1}{2}$	2	1 $\frac{1}{2}$	3 $\frac{3}{4}$
7 0	—	—	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{1}{2}$	3 $\frac{3}{4}$
3	—	—	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{2}$	3 $\frac{3}{4}$
6	—	—	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{2}$	1
9	—	—	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{2}$	1
8 0	—	—	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1
3	—	—	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1
6	—	—	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1
9	—	—	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{1}{2}$	2	1
9 0	—	—	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2	1
3	—	—	4 $\frac{1}{2}$	4 $\frac{1}{4}$	4	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1
6	—	—	4 $\frac{3}{4}$	4 $\frac{1}{4}$	4	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1
9	—	—	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
10 0	—	—	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3	2	1 $\frac{1}{4}$
3	—	—	5	4 $\frac{3}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3	2 $\frac{1}{4}$	1 $\frac{1}{4}$
6	—	—	5	4 $\frac{3}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3	2 $\frac{1}{4}$	1 $\frac{1}{4}$
9	—	—	5 $\frac{1}{4}$	4 $\frac{3}{4}$	4 $\frac{1}{2}$	3 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
11 0	—	—	5 $\frac{1}{4}$	5	4 $\frac{3}{4}$	4	3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
3	—	—	5 $\frac{1}{4}$	5	4 $\frac{3}{4}$	4	3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
6	—	—	5 $\frac{1}{2}$	5 $\frac{1}{4}$	5	4	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{4}$
9	—	—	5 $\frac{1}{2}$	5 $\frac{1}{4}$	5	4	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{4}$
12 0	—	—	5 $\frac{3}{4}$	5 $\frac{1}{4}$	5 $\frac{1}{4}$	4 $\frac{1}{4}$	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
6	—	—	6	5 $\frac{3}{4}$	5 $\frac{1}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
13 0	—	—	6	5 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{3}{4}$	2 $\frac{3}{4}$	1 $\frac{1}{2}$
6	—	—	6 $\frac{1}{4}$	6	5 $\frac{3}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	2 $\frac{3}{4}$	1 $\frac{1}{2}$
14 0	—	—	6 $\frac{1}{2}$	6	5 $\frac{3}{4}$	4 $\frac{3}{4}$	4	2 $\frac{3}{4}$	1 $\frac{1}{2}$
6	—	—	6 $\frac{1}{2}$	6 $\frac{1}{4}$	6	5	4	2 $\frac{3}{4}$	1 $\frac{1}{2}$
15 0	—	—	6 $\frac{3}{4}$	6 $\frac{1}{4}$	6	5	4 $\frac{1}{4}$	3	1 $\frac{1}{2}$
6	—	—	7	6 $\frac{1}{2}$	6 $\frac{1}{4}$	5 $\frac{1}{4}$	4 $\frac{1}{4}$	3	1 $\frac{1}{4}$
16 0	—	—	7 $\frac{1}{4}$	6 $\frac{3}{4}$	6 $\frac{1}{4}$	5 $\frac{1}{4}$	4 $\frac{1}{2}$	3	1 $\frac{3}{4}$
6	—	—	7 $\frac{1}{4}$	6 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{1}{4}$	1 $\frac{3}{4}$
17 0	—	—	7 $\frac{1}{2}$	7	6 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{3}{4}$	3 $\frac{1}{4}$	2
6	—	—	7 $\frac{3}{4}$	7 $\frac{1}{4}$	6 $\frac{3}{4}$	5 $\frac{3}{4}$	4 $\frac{3}{4}$	3 $\frac{1}{4}$	2
18 0	—	—	7 $\frac{3}{4}$	7 $\frac{1}{2}$	7	6	5	3 $\frac{1}{2}$	2
6	—	—	8	7 $\frac{3}{4}$	7	6	5	3 $\frac{1}{2}$	2
19 0	—	—	8 $\frac{1}{4}$	7 $\frac{3}{4}$	7 $\frac{1}{2}$	6 $\frac{1}{4}$	5	3 $\frac{1}{2}$	2
6	—	—	8 $\frac{1}{2}$	7 $\frac{3}{4}$	7 $\frac{1}{2}$	6 $\frac{1}{4}$	5 $\frac{1}{4}$	3 $\frac{1}{2}$	2
20 0	—	—	8 $\frac{1}{2}$	8	7 $\frac{1}{2}$	6 $\frac{1}{4}$	5 $\frac{1}{4}$	3 $\frac{1}{2}$	2

310y.

IV.—BASEMENT STORIES,

OF DWELLING HOUSES, OR CELLARS, USED AS DWELLINGS.

Height.	A- $\frac{1}{2}$	A	A—	B- $\frac{1}{2}$	B	B—	C- $\frac{1}{2}$	C	C—
Ft. Inch.	d.	d.	d.	d.	d.	d.	d.	d.	d.
6 0.	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{1}{2}$	1 $\frac{1}{2}$	3 $\frac{3}{4}$
3	2 $\frac{3}{4}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{2}$	3 $\frac{3}{4}$
6	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{3}{4}$	1 $\frac{1}{2}$	3 $\frac{3}{4}$
9	3	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{3}{4}$	1 $\frac{1}{2}$	1
7 0	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{3}{4}$	1 $\frac{1}{2}$	1
3	3 $\frac{1}{4}$	3	3	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{1}{2}$	1
6	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{1}{2}$	1
9	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{1}{2}$	1
8 0	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1
3	3 $\frac{3}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1
6	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
9	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
9 0	4 $\frac{1}{4}$	4	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
3	4 $\frac{1}{4}$	4	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
6	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2	1 $\frac{1}{4}$
9	4 $\frac{1}{2}$	4 $\frac{1}{2}$	4	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{1}{2}$	2	1 $\frac{1}{4}$
10 0	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
3	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
6	5	4 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
9	5	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{4}$
11 0	5	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{1}{2}$	1 $\frac{1}{4}$

Where houses are built of wood, as in America, we deduct 10 per cent. from the value of a brick house of the same size and location, where the winters are cold. In the Southern States, where the winters are warm, we deduct 20 per cent. from the value of a brick house similarly situated. We value a first-class frame or wooden house as if it was built of brick, and then make the above deductions, or that which local modifying circumstances will point out, such as climate, scarcity of timber, brick, lime, etc.

OFFICES.

The rate per square for offices of the I., II., III. and IV. Classes, is half that supplied in the foregoing Tables; Offices of the V. Class have the rate per square as follows:

310x.

V.—OFFICES THATCHED,

WITH DRY STONE WALLS.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft. Inch.			<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>
5 0	—	—	$1\frac{1}{2}$	$1\frac{1}{4}$	1	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
3	—	—	$1\frac{1}{2}$	$1\frac{1}{4}$	1	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
6	—	—	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
9	—	—	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
6 0	—	—	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
3	—	—	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
6	—	—	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{1}{2}$
9	—	—	2	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{1}{2}$
7 0	—	—	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$
3	—	—	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$
6	—	—	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
9	—	—	$2\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
8 0	—	—	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
3	—	—	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
6	—	—	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
9	—	—	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
9 0	—	—	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
3	—	—	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
6	—	—	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
9	—	—	$2\frac{3}{4}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	1	$\frac{1}{2}$
10 0	—	—	$2\frac{3}{4}$	$2\frac{1}{4}$	2	2	$1\frac{1}{2}$	1	$\frac{3}{4}$
6	—	—	$2\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	2	$1\frac{1}{2}$	1	$\frac{3}{4}$
11 0	—	—	$2\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	2	$1\frac{1}{2}$	1	$\frac{3}{4}$
6	—	—	3	$2\frac{3}{4}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	1	$\frac{3}{4}$
12 0	—	—	3	$2\frac{3}{4}$	$2\frac{1}{2}$	2	$1\frac{3}{4}$	1	$\frac{3}{4}$
6	—	—	$3\frac{1}{4}$	3	$2\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{4}$	1	$\frac{3}{4}$
13 0	—	—	$3\frac{1}{4}$	3	$2\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{4}$	1	$\frac{3}{4}$
6	—	—	$3\frac{1}{2}$	3	$2\frac{3}{4}$	$2\frac{1}{4}$	2	1	$\frac{3}{4}$
14 0	—	—	$3\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{3}{4}$	$2\frac{1}{4}$	2	1	$\frac{3}{4}$
6	—	—	$3\frac{1}{2}$	$3\frac{1}{4}$	3	$2\frac{1}{2}$	2	1	$\frac{3}{4}$
15 0	—	—	$3\frac{3}{4}$	$3\frac{1}{4}$	3	$2\frac{1}{2}$	2	$1\frac{1}{4}$	$\frac{3}{4}$

310A.

HOUSES IN TOWNS.

TABLES for ascertaining, by inspection, the relative value of any portion of a Building (nine square feet, or one yard,) and of any height, from I to V stories.

SIGNIFICATION OF THE LETTERS.

1ST CLASS.	A+	Built or ornamented with cut stone, of superior solidity and finish.
	A	Very substantial building and finish, without cut stone ornament.
	A—	Ordinary building and finish, or either of the above, when built 25 or 30 years.
2ND CLASS.	B+	Medium, in sound order, and in good repair.
	B	Medium, slightly decayed, but in repair.
	B—	Medium, deteriorated by age, and not in good repair.
3RD CLASS.	C+	Old, but in repair.
	C	Old, and out of repair.
	C—	Old, and dilapidated—scarcely habitable.

TABLE PRICES FOR HOUSES, AS DWELLINGS, SLATED.

Stories	FIRST CLASS.			SECOND CLASS.			THIRD CLASS.		
	A+	A	A—	B+	B	B—	C+	C	C—
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
I	1 6	1 5	1 4	1 2	1 0	0 10	0 8	0 6	0 4
II	2 6	2 4	2 2	2 0	1 9	1 6	1 3	1 0	0 8
III	3 0	2 10	2 8	2 6	2 3	2 0	1 9	1 4	0 10
IV	3 4	3 3	3 0	2 9	2 6	2 4	2 0	1 7	1 0
V	3 7	3 6	3 3	2 9	2 9	2 6	2 2	1 9	1 1

BASEMENTS AS DWELLINGS.

0 10	0 9	0 8	0 7	0 6	0 5	0 4	0 3	0 2
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TABLE PRICES FOR OFFICES, SLATED.

Stories	FIRST CLASS.			SECOND CLASS.			THIRD CLASS.		
	A+	A	A—	B+	B	B—	C+	C	C—
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
I	0 9	0 8½	0 8	0 7	0 6	0 5	0 4	0 3	0 2
II	1 3	1 2	1 1	1 0	0 10	0 8	0 6	0 5	0 4
III	1 6	1 5	1 4	1 3	1 0	0 10	0 8	0 6	0 5
IV	1 8	1 7	1 6	1 4	1 2	1 0	0 9	0 7	0 5½
V	1 9	1 8	1 7	1 6	1 4	1 1	0 10	0 8	0 6

CELLARS AS OFFICES.

0 6	0 5	0 5	0 4	0 3½	0 3	0 2	0 1½	0 1
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GEOLOGICAL FORMATION OF THE EARTH.

810b. Rocks originally horizontal, are now, by subsequent changes, inclined to the horizon: some, are found contorted and vertical; often inclined both ways from a summit, and forming basins, which God has ordained to be great reservoirs for water, coal and oil, from which man draws water by artesian wells, to fertilize the sandy soil of Algiers, and to supply him with fuel and light, on the almost woodless prairies of Illinois.

Unstratified rocks, are those which do not lie in beds, as granite.

Stratified rocks, lie in beds, as limestones, etc.

Dykes, are where fissures in the rocks are filled with igneous rocks, such as lava, trap rocks. Dykes seldom have branches; they cross one another, and are sometimes several yards wide, and extend from sixty to seventy miles in England and Ireland.

Veins, fenders or lodes, are fissures in the rocks, and are of various thicknesses; are parallel to one another in alternate bands, or, cross one another as net work.

Metallic veins, are principally found in the primary rocks in parallel bands, and seldom isolated, as several veins or lodes are in the same locality. These lodes or veins which intersect others, contain a different mineral.

Gangue or matrix, is the stony mineral which separates the metal from the adjoining rock.

Metallic indications, are the gangue and numerous cavities in the ground, or holes on the surface, corresponding to those formed underneath by the action of the water.

The crust of the earth, is supposed to be four and one-fourth miles, and arranged as follows by Rognault and others:

Formations Group.	Recent.	Tertiary Formation.	Secondary Formation.	Transition Rocks.	Primary.
I.	Latest Formation.				
II.	Upper Tertiary or Pleistocene and Miocene.	Modern volcanoes, both extinct and burning. Strata of recent sand, alluvium, loess, pebbles, drift, tufa, containing fossil bones.			
III.	Middle Tertiary.	Freshwater limestones, burrstones, sometimes containing ligures.			
IV.	Lower Tertiary.	Marls with gypsum, fossils of the mammifera. Coarse limestone. Plastic clay with lignite.			
V.	Upper Cretaceous.	Extensive limestone stratum called chalk, with interposing layers of silex.			
VI.	Lower Cretaceous.	Tuffaceous chalk of Touraine sand, or sandstone, generally green. Ferruginous sands.			
VII.	Oolitic or Jurassic and Lias.	Calcareous strata, more or less compact and marly, alternating with layers of clay. The upper strata of this group is termed Oolite, and the other, Lias.			
VIII.	Trias.	Variegated marls, often containing masses of gypsum and rock salt. Limestone very fossiliferous. Sandstone of various colors.			
IX.	Sandstone.	Conglomerate and sandstone.			
X.	Permian.	Limestone mixed with slate. Limestone conglomerate and sandstone, termed the new red sandstone.			
XI.	Carboniferous.	Sandstone, slates with seams of coal and carbonate of iron, (clay iron stone.) Carboniferous or mountain limestone, with seams of coal.			
XII.	Devonian.	Heavy beds of old red sandstone, with small seams of anthracite (or hard coal.)			
XIII.	Silurian.	Limestone, roofing slate, coarse grained sandstone called greywacke.			
XIV.	Cambrian.	Compact limestone, argillaceous shale or slate rocks having often a crystalline texture.			
XV.	Primary rocks.	Granite and gneiss forming the principal base of the interior of the globe, accessible to our observations.			

310c. *Quartz, silica* or silicic acid, is of various forms, color and transparency, and is generally colorless, but often reddish, brownish, yellowish and black. It is the principal constituent in flint, sea and lake shore gravel, and sandstones. It scratches glass: is insoluble, infusible, and not acted on by acids. If fused with caustic potash or soda, it melts into a glass.

Vitreous quartz, in its purest state, is rock-crystal, which is transparent and colorless.

Calcædonic quartz, resembles rock-crystal, but if calcined it becomes white. It is more tenacious than vitreous quartz, and has a conchoidal fracture.

Sand, is quartz in minute grains, generally colored reddish or yellowish brown, by oxide of iron, but often found white.

Sandstone, is where the grains of quartz are cemented together with calcareous, siliceous or argillaceous matter.

Alumina. Pure alumina is rarely found in nature. It is composed of two equivalents of the metal aluminum and three of oxygen, and is often found of brilliant colors and used by jewellers as precious stones. The sapphire is blue, the ruby is red, topaz when yellow, emerald when green, amethyst when violet, and adamantite when brown. On account of its hardness, it is used as *emery* in polishing precious stones and glass. It is infusible before the blowpipe with soda.

Potash or *Potassa*, is the protoxide of the metal potassium, and when pure = $K O$ or one equivalent of each.

Soda = $Na O$ = protoxide of the metal sodium.

Lime = $Ca O$ = protoxide of the metal calcium.

Magnesia = $Mg O$ = protoxide of the metal magnesium.

Felspar, is widely distributed and of various colors and crystallization. In granite, it has a perfect crystalline structure. As the base of porphyries, it is compact, of a close even texture. In granite felspar, the crystals of it is found in groups, cavities or veins, often with other substances. In porphyry, the crystals are embedded separately, as in a paste. It has a clear edge in two directions, and is nearly as hard as quartz. It is composed of silica, alumina and potash.

Common Felspar, is composed of silica, alumina and potassa. (See table of analysis of rocks.)

Albite—soda felspar, differs from felspar in having about eleven per cent. of soda in place of the potash, and in its crystallization, which belongs to the sixth series of solids, the three cleavages all meeting at oblique angles; yet the appearance of felspar and albite are very similar, and difficult to distinguish one from another. Their hardness and chemical characters are the same except the albite, which tinges the blowpipe-flame yellow. It forms the basis of granite in many countries: especially in North America, and is characterized by its almost constant whiteness.

Labradorite, a kind of felspar, contains lime, and about four per cent. of soda. It reflects brilliant colors in certain positions, particularly shades of green and blue; but its general color is dark grey. It is less infusible than felspar or albite, and may be dissolved in hydrochloric acid. It is abundant in Labrador and the State of New York.

Mica. It cleaves into very thin transparent, tough, elastic plates, commonly whitish, like transparent horn, sometimes brown or black. It

is principally composed of silica and alumina, combined with potassa, lime, magnesia, or oxyde of iron.

Quartz or *silica*, has no cleavage—glassy lustre.

Felspar, has a cleavage, but more opaque than silica.

Mica, is transparent and easily cleaved.

Granite, is of various shades and colors, and composed of quartz, (silica) felspar and mica. It forms the greater portion of the primary rocks. In the common granite, the felspar is lamellar or in plates, and the texture granular.

Porphyritic, is where crystals of felspar is imbedded in fine grained granite. It is red, green, brownish and sometimes gray.

Hornblende, is of various colors. That which forms a part of the basalts and syenites, is of a dark green or brownish color. It does not split in layers like mica when heated in the flame of a candle. Its color distinguishes it from quartz and felspar. It has no cleavage, and is composed of silica, lime, magnesia and protoxide of iron.

Augite, is nearly the same as hornblende, but is more compact. When found in the trap-rocks, it is of a dark green, approaching to black.

Gneiss, resembles granite; the mica is more abundant, and arranged in lines producing a lamellar or schistose appearance; the felspar also lamellar. It has a banded appearance on the face of fracture, the bands being black when the color of rock is dark gray. It breaks easily into slabs which are sometimes used for flagging.

Porphyritic gneiss, is where crystals of felspar appear in the rock, so as to give it a spotted appearance.

Protogine, is where talc takes the place of mica in gneiss.

Serpentine, is chiefly found with the older stratified rocks, but also found in the secondary and trap-rocks. It is mottled, of a massive green color, intermixed with black, and sometimes with red or brown; has a fine grained texture lighter than hornblende; may be cut with a knife, sometimes in a brittle, foliated mass. It is composed of about silica 44, magnesia 43, and water 13. Sometimes protoxide of iron, amounting to ten per cent., replaces the same amount of magnesia.

Syenite, resembles granite, excepting that hornblende, which takes the place of mica. It is not so cleavable as mica, and its laminae are more brittle. It is composed of felspar, quartz and hornblende. The felspar is lamellar and predominates. There are various kinds of syenites, as the

Porphyritic, where large crystals of felspar are imbedded in fine grained syenites.

Granitoid, is where small quantities of mica occur.

Talc, has a soft, greasy feeling, often in foliated plates, like mica, but the leaves or plates are not elastic. The color is usually pale green, sometimes greenish white, translucent, and in slaty masses. The last description from the township of Patton in Canada, and analyzed by Dr. Hunt, for Sir William Logan, Director of the Geological Survey of Canada, gives in the report for 1853 to 1856, the following:

Silica, 59.50; magnesia, 29.15; protoxide of iron, 4.5; oxyde of nickel, traces; alumina, 0.40; and loss by ignition, 4.40; total = 97.95.

A soft silvery white talcose schist from the same township, gave silica, 51.50; magnesia, 22.86; protoxide of iron, 7.38; oxyde of nickel, traces; lime, 1.25; alumina, 8.50; water, 3.60; total = 99.59.

Soapstone or steatite, is a granular, whitish or grayish talc.

Chlorite, is a dark or blackish green mineral, and is abundant in the altered silurian rocks, sometimes intermingled with grains of quartz and fespatic matters, forming chlorite sand, stones and schists or slates, which frequently contains epidote, magnetic and specular iron ores. Massive beds of chlorite or potstone, are met with, which, being free from harder minerals, may be sawed and wrought with great facility. A specimen from the above named township (Patton) was of a pale greenish, gray color, oily to the touch, and composed of lamellæ of chlorite in such a way as to give a schistose structure to the mass. Dr. Hunt, in the above report, gives its analysis: silica, 39.60; magnesia, 25.95; protoxide of iron, 14.49; alumina, 19.70; water, 11.30; total = 101.04.

Green sand, has a brighter color than chlorite, without any crystallization.

Limestones, are of various colors and hardness, from the friable chalk to the compact marble, and from being earthy and opaque, to the vitreous and transparent.

Carbonate of lime, when pure, is *calc spar*, and is composed of lime, 56.3; and carbonic acid, 43.7.

Impure carbonate of lime, is lime, carbonic acid, silica, alumina, iron, bitumen, etc.

Fontainebleau limestone, contains a large portion of sand.

Tufa, is lime deposited from lime water.

Stalactite, resembles long cones or icicles found in caverns.

Satin spar, is fibrous, and has a satin lustre.

Carbonate of magnesia or dolomite, is of a yellowish color, and contains lime, magnesia and carbonic acid, and makes good building and mortar stone.

Carbonate of magnesia, (pure) is composed of carbonic acid, 51.7, and magnesia, 48.3. Magnesian limestone, dolomite, (pure) is composed of carbonate of lime, 54.2, and carbonate of magnesia, 45.8. The following is the analysis from Sir W. Logan's report above quoted, of six specimens from different parts of Canada.

No. I. From Loughborough, is made up of large, cleavable grains, weathers reddish, with small disseminated particles, probably serpentine, and which, when the rock is dissolved in hydrochloric acid, remains undissolved, intermingled with quartz.

No. II. Is from a different place of said township. It is a coarse, crystalline limestone, but very coherent, snow-white, vitreous and translucent, in an unusual degree. It holds small grains disseminated, tremolite, quartz and sometimes rose-colored, bluish and greenish apatite and yellowish-brown mica, but all in small quantities.

No. III. From Sheffield, is nearly pure dolomite. It is pure, white in color, coarsely crystalline.

No. IV. From Madoc, is grayish-white, fine grained veins of quartz, which intersect the rock.

No. V. From Madoc, fine grained, grayish-white, silicious, magnesian limestone.

No. VI. From the village of Madoc, is a reddish, granular dolomite.

The following table shows the analysis of these specimens:

	I.	II.	IV.	V.	VI.
Specific gravity.....		7.860	2.849	2.757	2.834
Carbonate of Lime.....	55.79	52.57	46.47	51.90	57.37
“ Magnesia.....	37.11	45.97	40.17	11.39	34.06
“ Iron.....				4.71	
Peroxyde of Iron.....		0.21	1.21		1.32
Oxyde of Iron and Phosphates (traces).....					
Quartz and Mica.....		0.60			
Insoluble Quartz.....	7.10				
Quartz.....			12.16	32.00	7.10

MAGNESIAN MORTARS.

Limestones, containing 10 to 25 per cent. of clay, are more and more hydraulic. That which contains 33 per cent. of clay, hardens or sets immediately. Good cement mixed with two parts of clear sand and made into small balls as large as a hen's egg, should set in from one and a half to two hours. If the ball crumbles in water, too much quick-lime is present. Where the ground is wet, it is usually mixed—one part of sand to one of cement, but where the work is submerged in water, then the best cement is required and used in equal parts, and often more, as in the case of Roman cement.

By taking carbonate of lime and clay in the required proportions and calcining them, we have an artificial cement. Example: Let the carbonate of lime produce 45 per cent. of lime, then is it evident that by adding 15 lbs. of pure dry clay to every 100 lbs. of carbonate of lime, and laying the materials in alternate layers and calcining that, we produce a cement of the required strength. The limestones should be broken as small as possible; the whole, when calcined, to be ground together.

Cement used in Paris, is made by mixing fat lime and clay in proper proportions.

Artificial cement, is made in France, by mixing 4 parts of chalk with one of clay. The whole is ground into a pulp, and when nearly dry, it is made into bricks, which are dried in the air and then calcined in furnaces at a proper degree of heat. The temperature must not be too elevated. (See Regnault's Chemistry, Vol. I, p. 617.)

Plaster of Paris, is composed of lime, 26.5, sulphuric acid, 37.5, and water, 17. It is granular, sulphate of lime, slakes without swelling, sets hard in a short time, but being partially soluble in water, should be only used for outside or dry work.

Water lime, is composed of carbonate of lime, alumina, silica and oxyde of iron. It sets under water.

Water cements, differ from water lime in having more silica and alumina. It must be finely reduced. The English engineers use this and fine sharp sand in equal parts.

Building stones. Felspathic rocks, such as green stone, porphyry and syenite, in which the felspar is uniformly disseminated, are well adapted for structures requiring durability and strength. *Syenite*, in which potash abounds, is not fit for structures exposed to the weather. *Granite*, in which quartz is in excess, is brittle and hard, and difficult to work. An excess of mica makes it friable. The best granite is that in which all its constituents are uniformly disseminated, and is free from oxides of iron. Gneiss makes good building and flag stones. *Limestones*, should be free from clay and oxides of iron, and have a fine, granular appearance.

Sand, is quartz, frequently mixed with felspar.

Coarse sand, is that whose grains are from one-eighth to one-sixteenth of an inch in diameter.

Fine sand, is where the diameter of the grains are from one-sixteenth to one twenty-fourth of an inch.

Mixed sand, is where the fine and coarse are together.

Pit sand, is more angular than sea or river sand, and is therefore preferred by many builders in France and America, for making mortar; but in England and Ireland, river sand, when it can be procured, is generally used. Pit sand should be so well washed as not to soil the fingers. By these means, any clay or dirt present in it is removed.

Sand for casting, must be free from lime, be of a fine, siliceous quality, and contain a little clay to enable the mould to keep its form.

Sand for polishing, has about 80 per cent. of silica; is white or grayish, and has a hard feeling.

Sand for glass, must be pure silica, free from iron. Its purity is known by its white color or the clearness of the grains, when viewed through a magnifying glass.

Fuller's earth, has a soapy feeling, and is white, greenish-white or grayish. It crumbles in water, and does not become plastic. Its composition is, silica, 44; alumina, 23; lime, 4; magnesia, 2; protoxide of iron, 2; specific gravity, about two and one-half.

Clay, is plastic earth, and generally composed of one part of alumina and two parts of quartz or silica.

Clay for bricks, should be free or nearly so from lime, slightly plastic, and when moulded and spread out, to have an even appearance, smooth and free from pebbles. Clay free from iron, burns white, but that which contains iron, has a reddish color, the protoxide of iron in the clay becoming peroxidized by burning.

Pipe and potters' clay, has no iron, and therefore burns white.

Fire brick clay, should contain no iron, lime or magnesia.

Marl, is an unctuous, clayey, chalky or sandy earth, of calcareous nature, containing clay or sand and lime, in variable proportions.

Clay marl, resembles ordinary soil, but is more unctuous. It contains potash, and is therefore the best kind for agricultural purposes.

Chalk marl, is of a dull, white or yellowish color, and resembles impure chalk; is found in powder or friable masses.

Shelly marl, consists of the remains of infusorial animals, mixed with the broken shells of small fish. It resembles Fuller's earth, usually of a bluish or whitish color, feels soft, and readily crumbles under the fingers. It is found in the bottom of morasses, drained ponds, etc.

Slaty or stony marl, is generally red or brown, owing to the oxyde of iron it contains; some have a gravelly appearance, but generally resembles hard clay.

TABLE SHOWING THE COMPOSITION OF ROCKS.

NAME OF SUBJECT.		EARTHS.		OXIDES.		ALKALI.		ACIDS.		MISCELLANEOUS.
	Silica.	Alumina.	Lime.	Magnesia.	Iron.	Potash & Soda.	Potash.	Soda.	Sulphuric.	Fluoric.
Quartz rock.....	82	9	0.4		0.4					
Granite.....	75	13	0.4	1	2	7				
Felspar.....	64	19	0.7	0.5	0.7	13				
" Common.....	65	18.13								
" Labrador.....	55.8	26.5	11.0		per. 1.3			4		Water, 0.5
" Albite.....	69.1	19.2						11.7		
Mica.....	46	23	0.3	5	8	10				1
" Slate.....	73	13	0.2	2	4	5				0.5
" ".....	46.3	35.8			4.5					0.7
" Potash.....	40.1	31.6			8.7					1.12
" Magnesian.....	46.0	12.7		15.7	19	0.63				1.9
Hornblend, basaltic.....	42.24	13.92	12.34	13.74	14.59	.33		9.2		2.10
" syenitic.....	45.7	12.18	13.83	18.79	7.32	.22		5.61		
Schorl, Devonshire.....	35.2	35.5	0.58	0.70	m17.86	.43				
Tourmaline, Sweden.....	37.65	33.46	0.25	10.98	m 9.38	2.53		2.09		1.50
Augite, Sweden.....	53.36		22.19	4.99	17.38	.09				Boracic Acid, 4.11
Greenstone.....	55.0	15	7	9						" 3.83
Basalt.....	53.0	19	5	2	15					0.7
Chlorite slate.....	64.0	9	0.2	7		0.8				
Limestone, carbonate of lime.....	5.0	0.7	40.00						2	Plaster of Paris, 20.3
" sulphate of, or gypsum.....			32.9						46.3	
" calcined.....			41.5						58.5	
" magnesian.....		3.40	25.09	12.89	3.27	0.61		2.18	0.51	Cement, 1.18
" dolomite or carbonate magnesia.....			30.5	22.1					47.4	
" hydraultic, N. Y.....			18.80	18.80	1.25				39.8	Moisture, 1.40
" unburnt, ".....	13.50	2.73	25.0	17.83	alumina				39.33	" 1.50
" burnt, ".....	11.76	10.77	39.50	22.27	alumina				30.9	" 1.20
" unburnt, Ulster, N. Y.....	15.37	9.13	25.50	12.35	2.25				34.2	" 1.30
" burnt, ".....	22.75	13.40	37.6	16.65	3.30				5.0	
" good French cement stone.....	21.0		40.0	21					23	
" moderately hydraulic.....	5.0	2	22.2	20					28.4	Water and loss, 10.6
" highly hydraulic.....	clay 10.1		carb. 86 carb. 1.8	carb. 2.1						
" puzzolana of Italy.....	clay 19.0		carb. " 3.0	carb. " 1.0						Phospho. Acid, 25.
" phosphatc of lime.....	44.5	15	8.8	4.7	12	1.4	3.0			

310D. TABLE SHOWING THE COMPOSITION OF GRASSES AND TREES.

NAME OF SUBJECT.	EARTHS.			OXIDE.		ALKALIES.		ACIDS.			PHOSPHATES.		CHLORIDES.		Miscellaneous.
	Silica.	Lime.	Magnesesia.	Iron.	Soda.	Potash.	Sulphuric.	Carbonic.	Phosphoric.	Iron.	Magnesia.	Chlorine.	Chloride of Chlorate Sodium.	Potash.	
Lucerne	3.46	50.57	3.64	0.63	6.44	14.03	4.23	0.55	13.68				3.3		
Upright brome grass.....	38.48	10.38	4.99	0.26		20.33	5.46	0.55	7.53				3.11		Ash, 5.11.
Crested dogtail.....	40.11	10.16	2.43	0.18		24.99	3.20	0.55	7.24						" 6.38.
Orchard grass, seed ripe.....	32.18	8.14	3.47	0.23		33.06	3.96	2.88	6.41						" 5.51.
Meadow soft grass.....	28.31	8.31	3.41	0.31		34.83	4.41	1.82	8.02						" 6.37.
Meadow barley grass.....	56.23	5.04	2.42	0.66		20.26	4.29		6.04						" 5.67.
Timothy grass.....	31.09	14.94	5.30	0.27		24.25	4.86	4.02	11.29						" 5.29.
Yellow oat grass.....	35.20	7.98	3.07	2.40		36.06	4		9.31						" 5.28.
June grass.....	32.93	5.63	2.71	0.28		31.17	4.26	0.40	10.02						" 5.94.
Hard Fescue.....	28.53	10.31	2.83	0.78		31.84	3.41	1.38	12.07						" 5.42.
Downy oat grass.....	36.28	4.72	3.17	0.72		31.21	3.37		10.82						" 5.22.
White Clover.....	3.68	26.41	8.15	1.96		14.83	7.21	18.03	11.53						
Red Clover.....	7	33	8	0.4		36	3	15.20	8.02			3			Ash, 8.
Sanfoin, in seed.....	3.49	29.67	4.59	0.58		29.61	2.33		7.97						" 6.50.
Italian rye grass, in flower.....	59.18	9.95	2.23	0.78		12.45	2.82		6.34						" 6.97.
Meadow hay.....	38	23	7	1		1.9	2		6			2			" 6.
Apple tree, root.....	1.46	12.13	0.16			15.07	1.84		0.42						Org. mat. 1.20
" bark.....	1.26	1.86	2.56			0.44	38.39	49.56							to & coal 4.61
" outside wood.....	0.45	15.56	3.52			3.29	12.21	15.79				0.30			Do 3.55
" heart wood.....	0.20	2.06	2.93			2.75	22.17	38.98				0.51			Do 3.61
Cherry tree, wood.....	2.06	28.69	9.19	0.07		20.78	3.29		7.73						
" bark.....	19.98	41.95	5.10	0.20		7.46	0.80		3.26						
Peach tree, bark of trunk.....	4.15	42.17	2.16			0.40	14.53								Org. mat. 3.30
" wood of trunk.....	1.35	23.26	6.40			7.11	4.19								Do 5.20
Pear tree, bark.....	0.40	30.33	9.40			6.20	1.51								Loss, 3.81
" sapwood.....	0.30	12.64	3.00	0.31		22.25	1.84								

ANALYSIS OF TREES

NAMES.	Silica.	Alumina.	Line.	Magnesia.	Oxyde of Iron.	Manganese.	Potash.	Soda.
Plum tree, outside wood.	.45	...	15.56	3.52	...	...	3.29	3.33
“ heart “	.20	...	2.66	2.93	...	...	2.75	1.66
“ root “	1.46	...	11.64	.16	...	...	15.07	22.00
Chestnut, bark.....	1.20	...	51.60	.60	...	...	1.36	.33
“ outside wood..	1.43	...	40.76	5.77	...	...	4.56	1.41
“ inside “	1.73	...	38.20	.51	...	...	2.73	1.98
Beech, red, bark.....	3.30	2.23	52.29	.82	...	3.85	.13	...
“ outside wood.....	1.45	...	31.56	5.44	...	...	12.13	15.58
“ heart “	1.60	...	31.82	1.44	...	...	4.04	25.53
Butternut, bark.....	.30	...	37.68	10.08	...	...	1.00	11.27
“ outside wood.	4.80	...	38.98	3.52	...	...	4.42	5.61
“ heart “	5.40	...	43.02	4	...	...	1.00	11.82
Basswood, bark	4.60	...	41.92	2.24	...	...	1.26	12.77
“ outside wood..	2.10	...	38.86	7.36	...	...	10.12	2.88
“ heart “	1.40	...	45.24	7.44	...	...	4.05	10.41
Elm, (white) bark	1.75	...	27.46	18.10	...	...	3.79	1.65
“ outside wood	...	...	...	...	...	...	...	...
Maple, bark.....	.15	...	49.33	3.64	...	...	.88	7.75
“ outside wood.....	.50	...	31.86	8.40	...	...	.87	...
“ heart “	.55	...	43.14	7.24	...	...	4.21	...
Oak, (white) sapwood.....	1.01	...	30.35	.36	...	...	13.41	...
“ heart wood	1.18	...	43.21	.25	...	...	9.68	...
“ twigs “	1.15	...	34.10	.50	...	...	9.74	6.89
“ (white swamp) bark	2.00	...	52.26	.25	...	...	.46	trace
“ outside wood	1.50	...	30.22	.50	...	...	20.49	3.15
“ heart “	.50	...	35.57	.51	...	...	14.79	3.69
Hickory, outside wood ...	4.48	...	...	6.20	...	...	7.40	.08
“ inside “ ...	6.15	...	...	8.60	...	...	20.19	.09
“ heart “ ...	1.30	...	...	4.	...	...	12.21	.06
Pine, pitch.....	7.50	...	13.60	4.35	...	...	14.10	20.75
“ scotch fir.....	6.59	...	23.18	5.02	...	...	2.20	2.22
Rose bush, bark.....	3.30	...	22.56	2.86	...	...	5.12	8.52
Birch, soluble compound	1.00	...	...	...	...	...	...	...
“ insoluble “	5.50	...	52.2	...	.5	3.5	...	...
Lime tree, bark.....	2.27	...	60.81	...	1.24	...	16.14	4.53
“ wood.....	5.26	...	29.93	...	7.97	...	35.80	5.23
Mulberry, (white) soluble	...	...	...	...	...	...	52	11.5
“ insoluble	2.9	...	4.6	...	.5	1.3	...	...
“ chinese, soluble	1.0	...	...	...	...	...	...	68
“ insoluble	13.1	...	7.2	...	...	...	...	...
<i>Datura stramonium</i>	5.21	...	4.11	17.56	pr 3.94	...	20.22	14.24
Sweet Flag.....	2.39	...	7.70	...	pr 1.91	...	6.90	32.93
Common Chamomile.....	6.80	...	3.67	...	pr 3.28	...	9.66	30.58
Cockle.....	2.39	...	6.14	...	pr 1.21	...	13.20	22.86
Foxglove.....	12.78	...	6.53	3.70	3.19	...	10.89	43.53
Hemlock	2.62	...	8.39	9.64	2.40	...	12.80	21.69
Blue Bottle.....	3.29	...	...	4.56	1.61	...	7.32	36.54
Strawberries	12.05	...	14.21	trace	11.12	...	21.07	27.01
Poppy.....	1.41	...	5.06	...	1.21	...	6.85	33.11

AND WEEDS, ETC.

Phosphate of Lime.	Phosphate of Iron.	Phosphate of Magnesia.	Sulphuric Acid.	Carbonic Acid.	Phosphoric Acid.	Chlorine.	Chloride of Sodium.	MISCELLANEOUS.
...	...	...	12.21	15.79	...	...	.33	Org. mat. 3.20, coal, .35
trace	trace	trace	22.17	38.98	...	...	.51	" 3.60.
"	"	"	1.84	...	...	...	.11	" 1.20.
2.90	.20	"	.31	39.90	...	...	...	" 5.
17.44	1.30	"	.50	23.84	...	...	...	" 1.74.
8.60	.30	"	...	29.52	...	...	...	" 3.20.
phos-	1.96	phates	...	40.41	5.62	...	...	" 1.50, coal 1.50
17.23	.85	.93	.47	24.39	...	.05	...	" 1.86.
22.04	.40	.02	.62	24.59	...	.24	...	" 2.80.
2.25	.30	.15	.74	32.12	...	...	.15	" 2.80.
2.20	3.40	.06	13.73	20.02	...	...	.16	" 3.40.
.59	3.41	.28	21.43	4.48	...	...	.18	" 3.20.
8.50	.20	.30	.72	25.88	8.50	...	.24	" 1.70.
17.95	1.20	2.60	.88	16.64	17.95	...	.50	" 2.53.
8.96	1.30	.04	.27	17.96	8.96	...	.52	" 2.
...	...	...	.14	39.44	...	...	...	" 2.
...	...	...	12.02	...	...	...	...	" ...
3.13	.32	.02	1.50	37.12	...	...	.08	" 1.50.
5.70	.73	1.80	1.17	87.25	...	...	.08	" 2.40.
5.09	1.34	.22	1.03	33.33	...	...	2.78	" 1.93.
Phosphates.	32.25	...	4.24	8.95	...	...	.39	" 5.70.
	13.30	...	.47	19.29	...	...	.16	" 7.10.
	23.60	...	.25	17.55	...	...	.08	" 5.90.
	...	...	.30	40.34	...	...	...	" 2.13.
	...	...	...	32.92	...	...	...	" ...
	...	...	...	34.41	...	...	...	" 2.70.
	14.44	...	.89	29.57	...	.10	...	
	11.45	...	4.64	21.41	...	.09	...	
	6.34	...	5.26	33.63	...	.07	...	
	11.10	.90	3.45	17.50	...	2.30	...	
...	17.03	2.75	2.23	36.48	...	...	...	
...	15.30	...	5.00	28.70	...	...	3.20	Water, 4.
...	...	...	2.00	17.00	...	...	...	
...	...	...	...	31	4	...	...	
...	...	...	.75	...	4.02	...	2.21	
...	...	...	5.30	...	4.85	...	1.49	
...	...	...	8.00	2.3	...	...	...	Hydrochloric acid, 4.
...	...	...	...	...	...	...	...	
...	...	...	8.00	22.6	...	...	...	" " 2.04.
...	...	...	...	18.7	5.4	...	...	
...	...	...	...	...	34.72	...	...	Iodide of Sodium, .34.
...	...	...	5.06	...	11.48	...	2.84	Chlor. of Potass'm 14.66.
...	...	...	4.60	...	16.01	...	...	" " 7.15.
...	...	...	2.39	...	29.27	...	...	" " 7.55.
...	...	...	3.91	...	15.65	...	9.03	
...	...	...	3.43	...	24.96	...	16.61	
...	...	...	2.69	...	15.49	...	...	" " 11.88.
...	...	...	8.15	6	8.59	...	2.78	" " 3.40.
...	...	...	2.26	...	23.37	...	...	

ANALYSIS OF GRAINS AND STRAWS,

NAMES.	Silica.	Lime.	Magnesia.	Oxyde of Iron.	Potash.	Soda.
Barley, grain, mean of 10	26.49	2.58	8.55	1.43	19.77	3.93
“ straw, mean of 3	54.56	8.08	4.13	1.33	18.40	.68
“ grain, at Cleves.	21.99	3.36	10.05	1.93	3.91	16.79
“ grain, at Leipsic	29.10	1.67	6.91	2.10	20.91	...
Buckwheat, grain.....	.69	6.66	10.38	1.05	8.74	30.10
“ grain.....	7.06	.14	2.66	...	23.33	2.04
“ straw.....	4.37	.22	40.37	1	10.37	1.94
Maize or Indian Corn, grain	1.44	1.44	16.22	.30	32.48	...
“ straw, mean of 2	26.97	7.97	6.64	.81	9.62	26.30
Millet, grain, (Giessen)	59.63	.86	7.66	.63	9.58	1.31
Oats, grain, mean of 7	47.08	3.92	7.70	.64	16.76	2.49
“ straw, mean of 2	48.42	8.07	3.78	1.83	19.14	9.69
“ potato, grain,.....	50.03	1.31	8.25	.27	19.70	1.35
Rice, grain	3.35	1.27	11.69	.45	18.48	10.67
“ straw	74.09	.73	4.49	.67	10.27	3.82
Rye, grain, <i>by Way and Ogden</i>	9.22	2.61	12.81	1.04	33.83	.39
“ grain, mean of 3	3.36	4.19	11.17	1.25	26	7.91
“ grain, <i>by Liebig</i>	.69	9.06	2.41	.40	11.43	18.89
“ straw, “	64.50	6	10	2	17.19	...
Wheat, grain, mean of 32	3.35	3.40	12.30	.79	29.97	3.90
“ straw, mean of 10	67.88	6.23	2.74	.74	12.14	.60
Flax, whole plant in Ireland	21.35	12.83	7.79	6.08	9.78	9.82
“ best in Belgium	2.68	18.52	3.93	1.10	22.30	14.11
Hemp, whole plant, mean of 4	8.20	42.91	5.47	2.71	9.93	.50
Linseed	.92	25.98	.22	3.67	25.85	.71
Rape, seed	1.11	12.91	11.39	2.56	25.18	...
“ straw	.89	20.95	.62	...	8.13	19.82
Beet, Mangel Wurzel, (yellow)	2.22	1.78	1.78	3	23.54	19.08
“ “ “ long red	1.40	1.90	1.79	.52	21.68	3.13
“ mean of 4	4.44	3.65	2.97	1.24	30.80	12.19
“ long blood root	1.85	1.50	1.15	...	13.10	53.65
“ tops	1.99	8.65	8.66	.96	21.36	7.01
Carrot, (white Belgian root,)	1.19	8.83	3.96	1.10	32.44	13.52
“ tops	4.56	32.64	2.92	2.40	7.12	10.97
“ fresh root, (New York report.) ..	.65	3.65	1.60	...	8.50	40.25
Artichoke, Jerusalem	15.97	2.82	2.81	6.39	54.67	...
Cauliflower, heart	1.92	2.96	2.38	1.69	34.39	14.77
Parsnip	4.10	11.43	9.94	...	36.12	3.12
Potato, mean of.....	4.23	2.07	5.28	.52	57.75	1.86
“ tops	3.85	16.96	7.09	1.05	28.02	16.24
Tomato	.01	trace	0.10	...	.07	.09
Turnip, white globe	1	8.69	4.56	1.44	42.83	2.66
“ swede	.28	10.67	4.65	.38	47.46	3.93
“ mean of 10.....	3.43	11.14	3.61	1.09	36.98	6.76
“ tops	.86	23.27	3.09	.86	28.65	5.41
Beans, mean of 6	2.55	19.30	5.91	2	28.87	6.64
“ straw	7.05	19.99	6.69	.22	53.08	1.60
Peas, mean of 4	.52	5.36	8.54	.98	36.30	7.11
“ straw	20.03	54.91	6.88	.40	4.73	...
Lentils	1.07	5.07	1.98	1.61	27.84	6.65
Vetch or tare	2.01	4.79	8.49	.75	30.57	9.56
“ “ straw	8.66	38.33	6.36	.17	35.49	1.02

VEGETABLE AND LEGUMINOUS PLANTS.

Sulphuric Acid.	Carbonic Acid.	Phosphoric Acid.	Chlorine.	Chloride of Sodium.	MISCELLANEOUS.
1.03	...	35.20	...	.47	
2.13	...	3.26	...	6.95	
.26	...	40.63	...	...	
...	...	33.48	...	...	
2.16	...	50.07	...	...	
7.30	...	57.60	.20	...	
6.78	...	9	2.99	...	Oxyde Mang. and Alumina, .8.
2.77	...	44.87	...	...	
1.19	...	17.08	...	3.42	
.35	...	18.19	...	1.43	
1.29	...	18.19	...	.20	Chloride potassium, .14.
3.26	...	2.56	...	...	
.10	...	18.87	...	.07	
...	...	53.36	...	...	
3.56	...	1.09	...	...	
.17	...	39.92	...	...	
.71	...	46.34	...	...	
.51	...	51.81	...	...	
.83	...	3.82	...	.57	Chloride potassium, .26.
.33	...	46	...	.09	
3.88	...	5.43	...	.22	
2.65	...	10.84	...	6	
6.83	...	8.81	4.58	...	
1.28	...	5.26	...	1.41	
.91	...	40.11	...	1.65	
.53	2.20	45.96	...	...	
7.60	16.31	4.76	...	...	
3.68	18.14	4.49	...	24.55	
3.14	...	1.65	...	49.51	
3.03	...	4.19	...	24.55	
1.65	16.27	9.85	.81	...	Phosphate of Iron, 1.15.
5.80	...	5.15	...	33.96	
6.55	17.30	8.55	...	6.50	
6.20	17.82	1.67	...	13.67	
4.30	28.2	10.55	...	...	" " .70.
2.70	...	13.27	...	3.3	Carbonic acid deducted.
11.16	...	27.85	...	2.86	
6.50	...	18.66	...	5.54	Phosphate of Iron, 3.71.
13.64	deduct	12.57	...	7.10	
6.88	...	7.62	12.33	...	
.01	.04	.08	.01	...	
12.6	...	8.61	...	...	
12.16	...	13.07	...	...	
12.43	...	9.74	...	7.85	Chloride potassium, .59.
12.52	...	9.29	16.05	...	
1.91	...	21.66	...	1.35	" " .36.
1.09	...	7.24	...	4.26	
4.39	...	33.52	...	2.16	
6.77	...	4.83	...	...	
...	...	29.07	...	6.13	
4.10	...	38.08	...	2	
2.39	...	5.49	...	2.75	

310a. ANALYSIS OF THE ASHES OF MISCELLANEOUS ARTICLES.

NAME OF SUBJECT.	EARTHS.		OXIDE.	ALKALIES.		ACIDS.		CHLORIDES.		MISCELLANEOUS.
	Silica.	Lime.		Potash.	Soda.	Sulphuric.	Carbonic.	Chlorine.	Chloride Sodium.	
Onion, bulb.....	3.04	12.66	*12.29	32.35	8.04	8.34	...	...	4.49	Note.—In the Iron * signifies Phosphate.
" stalk.....	10.77	25.10	*10.61	13.98	14.43	10.50	...	...	trace	
Coffee, bean.....	2.95	3.58	9.01	...	11.07	...	...	...	1.67	
Cabbage, leaves.....	.75	20.97	5.94	.60	11.70	21.48	...	12.37	5.77	U. S. Patent Office. Cameron. Chloride potassium, 6.22.
" cow.....	1.66	15.01	2.39	.70	40.86	2.43	16.08	12.52	...	
" stalk.....	1.04	10.61	3.85	.41	40.93	4.05	7.27	19.57	...	
Brocoli, heart.....	.69	4.70	3.93	*2.12	47.16	...	11.11	24.53	trace	Chloride potassium, 6.22.
" leaves.....	1.83	26.44	3.43	6.21	22.10	7.55	...	16.62	...	
Cucumber.....	7.12	6.31	4.26	*2.06	47.42	4.60	...	14.97	...	
Radish.....	8.17	8.78	3.53	*2.19	21.16	...	...	40.09	...	"
Tobacco, leaves, mean of 10.....	...	41.80	12.18	4.41	17.42	.25	4.06	...	...	"
Hops.....	21.50	15.98	5.77	5.12	25.18	...	5.41	...	...	"
" whole plant.....	17.88	14.15	5.34	2.71	19.41	.70	8.28	14.64	3	"
Vine, grape, mean of 3.....	.88	36.26	6.91	2.12	27.88	8.96	...	13.18	...	1.39
Sugar-cane, stalk.....	17.64	2.34	3.93	...	32.93	...	7.97	...	17.12	"
" whole plant, mean of 2.....	45.68	7.61	9.87	...	15.12	.77	7.76	...	4.40	"
Sorgho, Bagasse.....	14.40	11.80	9.80	trace	8.10	9.60	trace	3.70	...	Jackson.
Orange, whole fruit.....	5.24	22.99	6.55	1.74	38.72	7.64	2.95	...	...	
Cherry, whole fruit.....	9.04	7.47	5.46	3.74	51.85	1.12	5.09	...	2.02	
Apple.....	4.32	4.08	8.75	2.65	35.68	26.09	6.09	...	...	Oxyde manganese, 1.92
Gooseberry.....	2.58	12.20	5.85	8.65	38.65	9.27	5.89	...	1.23	
Pear.....	1.49	7.98	5.22	*1.96	54.69	8.52	5.69	...	14.28	
Chestnut.....	2.32	7.84	7.84	1.95	39.36	19.18	...	7.83	...	4.82
Celery.....	3.85	13.11	13.11	*2.66	43.17	...	5.58	11.58	1.58	...

310r. PERCENTAGE VALUE OF MANURES.

SUBSTANCES.	Water.	Nitrogen dry state	Nitrogen natural state	ph's ac'd dry state	MISCELLANEOUS.
Farm yard manure....	68.2	1.96	.61	1.08	Bechelburn.
" "	70.5	2.45	.72	2.00	Grignon, France.
Wheat straw	12.3	.41	.36	.22	
Rye straw.....	12.4	.35	.30		Alsace.
Oat straw.....	21.0	.36	.28	.21	"
Barley straw.....	11.0	.26	.23	.20	"
Pea straw.....	8.5	1.95	1.79		"
Buckwheat straw.....	11.6	.54	.48		"
Leaves of rape.....	12.8	.86	.75	.30	
" potato.....	76.0	2.30	.55		
" carrot.....	70.9	2.94	.85		Recently collected.
" oak.....	25.0	1.57	1.18		"
" beech.....	39.3	1.91	1.18		
Saw dust fir.....	24.0	.31	.23	.03	"
" oak.....	26.0	.72	.54	.04	
Malt dust.....	6.0	4.90	4.51		
Apple refuse.....	6.4	.63	.59		
Hop ".....	73.0	2.23	.56		Air dried.
Beet root refuse.....	70.0	1.26	.38	.40	
Linseed cake.....	13.4	6.	5.20	3.83	
Rape cake.....	10.5	5.50	4.92	4.34	
Hempseed cake.....	5.0	4.78	4.21	1.08	
Cotton seed cake.....	11.0	4.52	4.02		
Cow dung.....	85.9	2.30	.32	.74	Solid excrements.
" urine.....	88.3	3.80	.44		"
" excrements.....	84.3	2.59	.41	.55	Solid and liquid.
Horse excrements.....	75.3	2.21	.55	1.22	"
" urine.....	85.0	14.47	2.04		"
" excrements.....	75.4	3.02	.74	1.12	"
Pigs' urine.....	97.9	11.	.23	2.09	"
Pigs' excrements.....	91.4	5.17	.54	3.65	
Sheeps' excrements.....	57.6	1.70	.72	1.52	
" urine.....	86.5	9.70	1.31	.03	
" dung.....	67.1	2.79	.91	1.32	"
Pigeons' dung.....	61.8	9.12	3.48	5.88	Fresh excrements.
Human urine.....	93.3	21.64	1.46	3.88	
" excrements.....	91.0	14.67	1.33	2.85	Solid and liquid.
Flemish manure.....			.20		Liquid manure.
Poudrette from Belloni	12.5	4.40	3.85		Sauburan.
Do. from Berry in 1847	13.6	2.29	1.98	2.55	"
Do. from Montfaucon..	41.4	2.67	1.56	1.08	"
Do. in 1847.....	28.0	2.47	1.78	4.80	"
Blood, liquid.....	81.0	15.58	2.95	1.63	Slaughter house.
" dry.....	21.4	15.50	12.18	1.68	Commercial.
" coag. & pressed	73.5	17.	4.51		From the press.
Blood, steamed.....		5.59			Wahl's, Chicago.
Bones boiled.....	7.5	7.58	7.02	24.	
" unboiled.....	8.	8.89	6.22	22.20	
" dust.....		7.92		24.	
Glue refuse.....	37.8	3.27	2.13		
Sugar refineries.....		2.44		26.	
Ox hairs.....	8.9	15.12	13.78		
Woolen rags.....	11.3	20.26	17.98		
Guano, Peruvian.....	25.6	6.31	4.71	18.93	
" African.....	25.	8.25	6.19	17.	
Soot of wood.....	5.6	1.31	1.15	1.	
" coal.....	15.6	1.59	1.35		
Oyster shells.....	17.9	0.40	.32	.65	

SEWAGE MANURE.

310r. The value of this manure is now fully established. Dr. Cameron, Professor to the Dublin Chemical Society, has recently shown that "100 tons of the sewage water of Dublin contain—

Nitrogen,	16 lbs.,	worth 10s. 8d.
Phosphoric Acid,	4.2	" 1s. 4 $\frac{3}{4}$ d.
Salts of Potash,	5.1	" 7 $\frac{1}{2}$ d.
Salts of Soda,	14.2	" 2 $\frac{1}{4}$ d.
Organic matter,	75	" 4d.

Taking the population of Dublin at 300,000, the value of the sewage is worth more than £100,000, or two-thirds of the local taxation of the city."

He calculates the value of the night soil at £3000, and the urine at £85,000, showing one to be thirty times as valuable as the other.

Those who have seen the river Thames or the Chicago river made the receptacle of city sewage, will admit that God never intended that liquid manure should pass into these streams causing disease and death, but that they should be made available in fertilizing the neighboring fields, as in Edinburgh and various other places.

We recommended a plan of intercepting sewers for Chicago in 1854, by which the sewage could be collected at certain places, and from thence wasted into Lake Michigan far from the city, or used for irrigating the adjacent level prairies. The plan was rejected, but the consequence has been that an Act passed the Legislature of Illinois in 1865, creating a commission for cleansing the Chicago river, at an expense of two MILLIONS OF DOLLARS. The commissioners have now (30th June, 1865,) commenced their preparatory survey. In Chicago the people are obliged to connect their water-closets with the main sewers, thereby making the sewers gas generators on a large scale. Public water-closets are built at the crossings of some of the bridges, and private ones without traps or syphons are built under the sidewalks. This system of sewerage begins to show its bad effects, and will have to be abandoned at some future day.

To any person who has spent one hour in a chemical laboratory, it will appear that noxious gases will soon saturate any amount of water that can be held in a trap or syphon, and that no contrivance can be adopted to exclude permanently the poisonous effluvia of sulphide of ammonium and sulphuretted hydrogen.

It will cost London thirty millions of dollars to build the intercepting sewers commenced in 1858. Paris commenced a similar work in 1857, and Dublin is now about to do the same. About April, 1865, an Act passed the English House of Lords for the utilization of town sewage, which was supported by the first vote of the PRINCE OF WALES. The great LIEBEG has commenced operation on the London sewage. He has it free of charge for ten years; so that in a few years the value of sewage will be as well known to the Americans and Europeans as it is now to the Chinese. Then there will not be a scientific engineer who will advocate the converting of currentless streams and neighboring waters into cess-pools. The sanitary and agricultural conditions of the world will forbid it. (*See also sections on Drainage and Irrigation.*)

DESCRIPTION OF MINERALS.

310K. **ANTIMONY.** *Stibnite*, or gray sulphuret of antimony. Comp. Sb73, S27. Found chiefly in granite, gneiss and mica, with galena, blende, iron, copper, silver, zinc and arsenic. Found columnar, massive, granular, and in delicate threads. Fusible. Gravity, 4.5. Lustre, shining. Fracture, perfect and brittle. Color, lead to steel gray; tarnishes when exposed.

White Antimony. Contains antimony, 84. Found in rectangular crystals, whose color is white, grayish and reddish, of a pearly lustre. H—2.5. Gravity, 5 to 6.

Sulphuret of Antimony and Lead. Found rhombic, fibrous and columnar. Color, lead to steel gray. H = 2 to 4. Specific gravity, 5 to 6.

ARSENIC, White. Sometimes found in primary rocks with Co. Cu. Ag. and Pb. Color, tin white. Is soluble. G., 3.7. Fracture, conchoidal. Lustre, vitreous.

Native Arsenic. Found in Hungary, Bohemia, and in New Hampshire with lead and silver. Color, tin white to dark gray. A = 3.5. Gravity, 5.7. F = imperfect.

Orpiment or Yellow Sulphuret of Antimony. Found in Europe, Asia and New York. Foliated masses and prismatic crystals. Color, fine yellow. H = 1.5 to 2. Gr., 3 to 3.5. F = perfect. Lustre, pearly.

Realger or Red Sulphuret of. Found in Europe, with Cu. and Pb. Color, red to orange. H = 1.5 to 2. Gr., 3 to 4. Lustre, resinous. F = imperfect. Massive and acicular.

BISMUTH. Native. Found in quartz, gneiss, mica, with Co. As., Ag. and Fe. Color, silver white. Found amorphous, crystallized, lamellar. H = 2 to 2.5. Gr. = 9. F = perfect. Lustre. Metallic.

Sulphuret of Bismuth. Comp., Bi. 81, S19. Found as above. Massive acicular crystals. H = 2.3. Gr., 6.6. Color, lead gray.

COBALT. Smalline. Found in primary rocks, with As. Ag. and Fe. Massive, cubes and octohedrons. H = 5. Gr., 6 to 7. Color, tin white to steel gray. L = metallic. Fracture uneven.

Arsenical Cobalt. Found, as in the latter, massive, staelectical and dentrical. Comp., Co. + As. + S. Color, tinge of copper red. Gr., 7.3. F = brittle.

Bloom or Peach Cobalt. Found in oblique crystals. Foliated like mica. Color, red, gray, greenish. H = 1.5 to 2. Gr., 3. Lustre, pearly. Fracture, like mica.

COPPER. Native. Nearly pure. Found in veins in primary rocks, and as high as the new red sandstone, in masses or plates. Aborescent, filiform. Color, copper red. H = 2.5 to 3. Gr., 8.6.

Sulphuret of. Comp., Cu. 76.5, S22 + Fe. 50. Found in great rocks, especially the primary and secondary ones. In double, six-sided pyramids, lamellar, tissular, long tabular, six-sided prisms. Color, blackish steel gray. Gr., 5.5. Fracture, brittle and brilliant.

Sulphuret of Copper and Iron. (Copper pyrites.) Comp., Cu. 36, S32, Fe. 32. Found in veins in granite and allied rocks, graywacks, and with iron pyrites, carbonates of Cu. blende, galena. Color, brass yellow when hammered. H = 3 to 4. Gr., 4. Found in various shapes, Tetrahedral, octohedral, massive, like native and iron pyrites.

Gray Sulphuret of Cu. and Iron. Comp., Cu. 52., Fe. 23. The same location and associates as the last. It is not magnetic like oxide of iron, nor so hard as arsenate of iron. Color, steel gray to black. Lustre, metallic. F = brittle. Found amorphous, disseminated, crystallized in small tetrahedral crystals.

Copper Pyrites, most prevalent. Comp., Cu. 76.5, S22, Fe. .5. Found similar to sulphuret of copper. Color, brass yellow. Found in small, imperfect crystals in concretion and crystallized lamellar. F = uneven. Lustre, metallic. Gr., 4.3.

Red Oxide of Copper. Contains 88 to 91 of copper. Found with other copper ores. It is fusible and effervesces with nitric acid, but not with hydrochloric acid. Color, red. F = generally uneven. H = soft. Found amorphous, crystallized, in cubes and octohedrons.

Blue Carbonate of Cu. Comp., Cu. 70, CO₂ 24, HO₆. Found in primary and secondary rocks. Is infusible without a flux, and gives a green bead with borax in the blow pipe flame. It is massive, incrusting and stalactical. Color, blue. F = imperfectly foliated.

Green Carbonate of Copper. Found with other copper ores, in incrustations and other forms. Color, light green. L = adamantine. H = 2 to 4. Gr., 4.

NICKEL. *Arsenical.* Comp., As. 54, Ni. 4.4. Found in secondary rocks, as gneiss, with cobalt, arsenic, Fe., sulphur and lead, and is massive, reticulated, botryoidal. Gives out garlic odor when heated. Color, copper red, which tarnishes in air. H = 5. Gr., 7 to 8. L = metallic.

Nickel Glance. Found with arsenic and sulphur, massive and in cubes. Comp., Ni. 28 to 38. Color, silver white to steel gray. H = 5. Gr., 6.

White Nickel. Comp., Ni. 20 to 28, As. 70 to 78. Color, tin white, found as cubic crystals.

Placodine. Ni. 57. Color, bronze yellow. Found tabular, oblique and in rhombic prisms. H = 5 to 6. Gr., 8.

Antimonial Nickel. Ni. 29. Found in hexagonal crystals. Color, pale copper red, inclined to violet.

Nickel Pyrites. Contain Ni. 64. Color, brass yellow to light bronze. Found capillary and in rhombohedral crystals.

Green Nickel. Contain 36 per cent. of oxide of nickel. Found with copper and other ores of nickel. Color, apple green.

ZINC. *Blende. Mock-lead. Block Jack.* Found in veins in primary and secondary rocks, with Fe. Pb. and Cu. Comp., zinc 67, Pb. 33. Found massive, lamellar, granular and crystallized. It decrepitates if heated, and is infusible. Color, yellow, brown or black. Lustre, shining and adamantine. F = brittle and foliated. Gr., 3 to 4.

Carbonate of Zinc. (Calamine.) Comp., zinc 64.5, carbonic acid, 35.5. Found in beds or nests in secondary limestones, and in veins, with oxides of iron and sometimes lead. Crystallized, compact, amorphous, cupreiferous and pseudomorphous. Color, gray, greenish, brown, yellow and whitish. L = vitreous and pearly. F., brittle. Gr., 4 to 4.5.

Red Oxide of Zinc. Comp., zinc 94, protoxide of manganese 6. Found in iron mines and limestones. Massive and disseminated. Cleavage like mica. Color, deep or light red with a streak of orange yellow. Lustre, subadamantine and brilliant.

Sulphate of Zinc. Found in rhombic prisms. Color, white. L = vitreous. F., perfect. Gr., 20.4.

MANGANESE. Binoxide of. Comp., $\text{MnO}_2 = \text{Mn } 64 + \text{O } 36$. Found in veins and masses in primary rocks, with iron. Forms a purple glass with borax in the blow pipe flame. Color, dark steel gray, with a black streak. L = metallic. F., conchoidal and earthy. H = 2 to 2.5. Gr., 4 to 5. Found massive, and in fibrous concretions. Crystallized. Infusible alone.

Phosphate of Manganese. (Triplite.) Protoxide Mn. 33, protoxide of Fe. 32, and phosphoric acid 33. Gives a violet gloss with borax. Color, yellowish, streak of gray or black. L = resinous and opaque. H5 to 5.5. Gr., 3 to 4.

Bog Ore of Mn., or Wad. Found in low places, formed from minerals, containing manganese. Comp., Mn. 30 to 70, protoxide of iron 20 to 25. Color, brownish black. Lustre, dull and earthy. H = 1. Gr., 4.

TIN. Oxide of. Comp., tin, 77.5, O21.5, oxide of iron .25, and silver .75. Found in the crystalline rocks with Cu. and iron pyrites. Found in various places, especially in Cornwall in England. Color, brown or black, with a pale gray streak. Found lamellar, in grains and massive. Decrepitates on charcoal. L = adamantine. F., indistinct and brittle. H = 6 to 7. Gr., 6.5 to 7.

Sulphuret of Tin, or Pyrites. Color, steel gray or yellowish. Streak, black. F = brittle. H4. Gr., 4. Comp., tin 34, S25, Cu. 36 and Fe. 2.

PLATINUM. Found only in the metallic state, with various metals, such as gold, silver, iron, copper and lead, and disseminated in rocks of igneous origin, as the primary. Often found in syenite with gold, but it is principally found in alluvium or drift. Color, very light steel gray to silver white. Lustre, glistening. It is found in grains and rolled pieces, seldom larger than a pea. Resembles coarse iron filings. It is malleable; infusible, excepting in the flame of the oxyhydrogen blowpipe.

GOLD. Found in granite, quartz, slate, hornstone, sandstone, limestone, clay slate, gneiss, mica slate, and especially in talcose slate, rarely in graywack and tertiary slate, but never in serpentine. Associated with Cu., Zn., Fe., Pb., Baryta., antimony, platinum. Where it is found in primary rocks, it is frequently in schiste. Color, yellow. Seldom found massive; often disseminated, capillary, amorphous, dendritic, crystallized in cubes, octohedrons, rhomboidal, dodecahedron and tetrahedron. Lustre, glistening and metallic. Fracture, hackly and tessular. H = 2.5 to 3. Gr., 19.4. It is malleable and unaltered by exposure, and is easily cut and flattened under the hammer, which distinguishes it from copper and iron pyrites, which crumble under the hammer.

SILVER. Sulphuret of. Comp., Ag. 87, S13. It is soluble in nitric acid. Found in primary and secondary rocks, with other ores of silver. Gives off sulphurous odor when heated in the flame of a blow pipe flame. Found in cubes and octohedrons, reticulated. Imperfect at cleavage, is malleable, amorphous and in plates. Color, blackish, lead gray, with a shining streak. L = metallic, F. flat and conchoidal. H2.3. Gr., 7.

Silver, native. Usually alloyed with gold, bismuth and copper. Found in primary and secondary rocks, often in penetrating crystals, or amorphous in common quartz, with copper and cobalt. It is fusible into a globule. Color, silver white, but often gray or reddish. It is seldom found massive, but often in plates and spangles, dentiform, filiform and

aborescent. Crystallized in cubes, octohedrons, lamellar and ramose, with no cleavage. L=splendent to shining. F., fine hackly. H2.5 to 3. Gr., 10.4

Sulphuret of Silver and Antimony. Comp. S16, Sb. 14.7, Ag. 68.5, Cu. 6. Found in the primary rocks, such as granite and clay slate, with native silver and copper. It is found massive and in compound crystals, having an imperfect cleavage. Color, iron black. L=metallic. F., conchoidal. H2.2. Gr., 6.3.

Chloride of Silver. Comp., Ag. 75, chlorine 25. Found in the primary rocks with other ores of silver. Massive, seldom columnar, often incrusting, in cubes, with no distinct cleavage, also reniform and acicular. Color, pearly gray, greenish, blue or reddish, with a shining streak. Lustre, resinous to adamantine.

Mercury, native. Found in Austria, Spain, Peru, Hungary and California. Found in fluid globules. Color, tin white. Gr., 13.6.

Sulphuret of Mercury, or Cinnabar. Comp., mercury s. 14.75. Found chiefly in the new red sandstone, sometimes in mica slate, limestone, gneiss, graywack, beds of bituminous shale of coal formation. In California, at the Almaden mines, it is found in greenish talcose rock. Color, brownish black to bright red, cochineal red, lead gray, sometimes a tinge of yellow. Found massive, six-sided prisms, sometimes fibrous, with a streak of scarlet red. It evaporates before the blow pipe and does not give off allicaceous fumes. L=metallic to unmetallic. Fracture, perfect, fibrous, granular or in thin plates. H2.3. Gr., 7 to 8.

LEAD. Native. Rarely met. It has been found in the County of Kerry in Ireland, Carthagea in Spain, and Alston moor, in the County of Cumberland, England.

Sulphuret of Lead, or Galena. Comp., Pb. 86.5, S13.8. Found in veins, beds and imbedded masses, in primary and secondary mountains, but more frequently in the latter, particularly in limestone. The indications are calc spar, mineral-blossom, red color of the soil, crumbling of magnesian limestone and sink-hole appearance of the surface. Color, leaden or blackish gray. Found amorphous, reticulated and crystallized in cubes and octohedrons, with a perfect cleavage, parallel to the planes of the cubes. L=metallic. F., lamellar and brittle. Gr., 7.6.

Sulphate of Lead. Comp., Pb. 73, sulphuric acid, 27. It is produced from the decomposition of galena, and found associated with galena. Color, white, sometimes green or light gray. Found massive, granular, lamellar, and often in slender crystals. L=vitreous or resinous. F., brittle. H2.8 to 3. Gr., 6.3 to 6.5.

Minium or Red Lead. Found with galena in pulverulent state. Color, bright red and yellow. Gr., 4.6.

Phosphate of Lead. Comp., Pb. 78.6, phosphoric acid 19.7, hydrochloric acid 1.7. Color, bright green or orange brown. Found in hexagonal prisms, reniform, globular and radiated. Streak, white. H3.8 to 4. Gr. 6.5 to 7.

Chromate of Lead. Found in gneiss. Color, bright red, with a streak of orange yellow. Found massive and in oblique rhombic prisms.

Black Lead, Plumbago, or Graphite. Found in gneiss, mica, granular limestone, clay slate, and generally in the coal formation. Color, iron

black. Lustre, metallic. In six-sided prisms, foliated and massive. $H = 1$ to 2. Gr., 2.

IRON. Native. Is found in meteorites, alloyed with nickel. It is massive, magnetic, malleable and ductile. $F =$ hackley. $H4.5$. Gr. 7.3 to 7.8. A specimen in Yale College contains Fe. 9.1 and Ni. 9.

Iron Pyrites, or Bisulphuret of Iron. Occurs in rocks of all ages and in lavas. Found usually in cubes, pentagonal, dodecahedrons or octohedrons. Also massive. Color, bronze yellow, with a brownish streak. Lustre, metallic and splendid. Brittle. $H = 6$ to 6.5. Gr. 4.8 to 5.1. Strikes fire with steel, and is not magnetic. Comp., Fe. 45.74, S54.26.

Auriferous Iron Pyrites. Is that which contains gold.

Magnetic Pyrites, or Sulphuret of Iron. Found massive, and sometimes in hexagonal, tabular prisms. Color, bronze yellow to copper red, with a dark streak. $F =$ brittle. $H3.5$ to 4.5. Gr. 4.6 to 4.65. Slightly magnetic. Comp., Fe. 59.6, S40.4. This ore is not so hard as the bisulphuret of iron, and is of a paler color than copper pyrites.

Magnetic Iron Ore. Found in granular masses, octohedrons, dodecahedrons, granite, gneiss, mica, clay slate, hornblende, syenite, chlorite, slate and limestone. Color, iron black, with a black streak. $F =$ brittle. $H5.5$ to 6.5. Gr., 5 to 5.1. Highly magnetic. Comp., Fe. 71.8, oxygen 28.2. This is the most useful and diffused iron ore.

Specular Iron Ore, Peroxide of Iron. Found massive, granular, micaceous, sometimes in thin, tabular prisms. Color, dark steel gray or iron black. Lustre, often splendid, passing into an earthy ore of a red color, yielding a deep red color without lustre. $H = 5.5$ to 6.5. Gr., 4.5 to 5.3. Slightly magnetic.

The Specular Variety. Has a highly, metallic lustre.

Micaceous, Specular Iron Ore. Has a foliated structure.

Red Ochre. Often contains clay, is soft and earthy. It is more compact than red chalk.

Bog Iron Ore. Occurs in low ground; is loose and earthy; of a brownish, black color.

Clay Iron Stone. Has a brownish red, jaspery and compact appearance. Comp. of specular iron are Fe. 69.3, oxygen 30.7. The celebrated iron mountains of Missouri are composed of specular iron ore. One of the mountains is 700 feet high. There, the massive, micaceous and ochreous varieties are combined.

Chromate of Iron. Found massive and octohedral crystals, in serpentine rocks, imbedded in veins or masses. Color, iron and brownish black, with a dark streak. $L =$ sub-metallic. $H5.5$. Gr., 4.3 to 4.5. When reduced to small fragments, it is magnetic. Comp., chromium 60, protoxide of iron 20.1, alumina 11.8, and magnesia 7.5.

Carbonate of Iron. Found principally in gneiss and graywack, also in rocks of all ages. Found massive, with a foliated structure, in rhombohedrons and hexagonal prisms. Color, light gray to dark brown red; blackens by exposure. $L =$ pearly to vitreous. $H3$ to 4.5. Gr., 3.7 to 3.8. Comp., protoxide of iron 61.4, carbonic acid 38.6. This ore is extensively used in the manufacture of iron and steel. These, with the magnetic, specular, bog ore and clay ironstone, are the principal sources of the iron commerce.

EXAMINATION OF A SOLID BODY.

310L. Note its state of aggregate, hardness, specific gravity, fracture, lustre, color, locality and associates. Heat a portion of the substance, (reduced to a fine powder) in a test tube; if no change of color appears, it is free from *organic matter*.

It is free from water, if there is no change of weight.

If organic matter is present, it blackens first, then reddens.

No organic matter is present, if it entirely volatilizes.

It is a compound of two or more substances, when only a portion volatilizes.

It is an alkali or alkaline earth, if it fuses without any other change.

Is it soluble, insoluble, or partially soluble in water?

Is it soluble with boiling dilute hydrochloric acid?

Take two portions of the substance, burn one part, and to the other, add dilute hydrochloric acid; if no effervescence takes place until we put dilute acid on the burnt substance, it shows the presence of an *organic acid*.

The substance may be either a borate, carbonate, chlorate, nitrate, phosphate or sulphate.

Borates. The alkaline borates are soluble in water, the others are nearly insoluble. They are decomposed in the wet way by sulphuric, nitric and hydrochloric acids, thus liberating boracic acid. If the mixture of any borate and fluorspar be heated with sulphuric acid, fluoride of boron is disengaged, recognized by the dense, white fumes it gives off in the air, and its mode of decomposition by contact with water.—*Regnaults*.

Otherwise. From moderately, dilute solutions of borates. Mineral acids separate boracic acid, which crystallizes in scales.

Otherwise. Heat the solution of a borate with one-half its volume of concentrated sulphuric acid and the same of alcohol. Kindle the latter. The boracic acid imparts a fine green color to the flame. Stir the mixture whilst burning. Melt the borate with two parts of fluorspar and one of bisulphuret of potash in a dark place; the flame at the instant of fusion is tinged green.

Carbonates. Dissolved in cold or heated acids, disengage carbonic acid with a lively effervescence, which, if conducted through a tube into lime water, gives the latter a milk-white appearance. This gas will also slightly redden blue litmus paper previously moistened; but heat restores the blue color. If the gas is collected in a tube, and a small lighted taper let down into it, it will be extinguished.

An *engineer* constructing tunnels or subterranean works, will find the above tests sufficient to warn him of approaching danger from "foul air" or "choke damp." Water absorbs an equal bulk of this gas, hence the benefit of workmen throwing down a few buckets of water into a well, previous to going down into it after recess. Although the above tests will detect the presence of carbonic acid in subterranean work, where the air may be impure, it requires the greatest caution on the part of the engineer to preserve the health of the workmen.

Carbonic acid, is inodorous and tasteless. Sulphuretted hydrogen has the odor of rotten eggs, and is often found in subterranean works.

Alkaline carbonates are soluble, the other carbonates are not.

Nitrates. All nitrates, excepting a few sub-nitrates, are soluble in water.

A solid nitrate, heated with concentrated sulphuric acid, evolves fumes of nitrous acid, sometimes accompanied by red-brown vapors of peroxide of nitrogen.

Otherwise, heat the nitrate with concentrated sulphuric acid, then put in a slip of clean metallic copper, red vapors of peroxide of nitrogen are evolved.

Otherwise, to a solution of a nitrate, add its bulk of concentrated sulphuric acid. When cool, suspend a crystal of protosulphate of iron, (green copperas.) After sometime, a brown ring will appear about the crystal. The liquid in this case must not be stirred or heated.

Phosphates. Generally dissolve in nitric and hydrochloric acids. Sulphuric acid does not give any reaction, but generally decomposes them. With phosphates soluble in water, nitrate of silver gives a lemon-yellow phosphate of silver. Is soluble, with difficulty, in acetic acid.

Phosphates. Insoluble in water, are dissolved in nitric acid, then this solution is neutralized by ammonia; to this neutral mixture, the nitrate of silver test gives the above yellow color.

Sesquioxide of Iron. In an alkaline solution of a phosphate, gives an almost white gelatinous precipitate of phosphate of sesquioxide of iron. Insoluble in acetic acid.

Molybdate of Ammonia, added to any phosphate solution, and then nitric or hydrochloric acid added in excess, a yellow color soon appears, and subsequently a yellow precipitate.

This is a very characteristic test. The substance ought to be first dissolved in nitric acid, and then nearly neutralized before adding the molybdate of ammonia.

Sulphates. Nearly all the sulphates are soluble in water. They do not effervesce with acids. This distinguishes them from carbonates. The sulphates of baryta, strontia and lead, are nearly insoluble; that of lime is slightly soluble.

From all the soluble sulphates, nitrate of baryta or chloride of barium, throws down a white precipitate insoluble in nitric acid, which is a characteristic property of the sulphates. In applying this test, the solution ought to be neutral or nearly so. This can be done by adding Magnesia to the solution so as to render it equal to sulphate of magnesia, MgO, SO_3 .

BLOW PIPE EXAMINATIONS.

310m. Heat a portion of the substance on charcoal, in the inner flame of the blow pipe.

If potash or soda, the flame is tinged yellow.

If an alkaline earth, (barium, calcium, strontium, magnesium,) it will radiate a white light, and is infusible. Now moisten this infusible mass with nitrate of cobalt and heat again.

If the flame becomes blue, alumina is present.

If green, oxide of zinc.

If pale pink, magnesia; but if silica, it will fuse into a colorless bead, on the addition of carbonate of soda.

If a bead, or colored infusible residue is formed, mix it with carbonate of soda, and heat on charcoal in the inner flame of the blow pipe.

If tin, copper, silver or gold, are present, a bead of the metal will be formed, without any incrustation on the charcoal.

If iron, cobalt or nickel, are present, the metal will be mixed up with the carbonate of soda, giving the bead a gray opaque appearance.

If zinc or antimony, it will give a white deposit around the bead.

If lead, bismuth or cadmium, a yellow or brown deposit.

QUALITATIVE ANALYSES OF METALLIC SUBSTANCES.

310x. Let M = equal the mass or substance to be analyzed. We reduce it to a fine powder and boil with hydrochloric acid, so as to reduce it to a chloride, but if we suspect the presence of a metal not soluble by the above, we boil it with aqua regia (= nitro-hydrochloric acid) until it is dissolved; then we evaporate and boil again with dilute hydrochloric acid and evaporate to dryness, and so continue till every trace of nitric acid disappears. We have the metals now reduced to chlorides, which are soluble in distilled water. The solution is now set aside for analysis, which is to be acid, neutral or alkaline, as the nature of the reagent may require.

The solution is acid if it changes blue litmus paper red, and alkaline, if it changes red litmus paper blue, or turmeric paper brown.

Taylor gives nitro-prusside of sodium as a very delicate test for alkali. He "passes a little hydrosulphuric acid into the solution to be examined, and then adds the solution of the nitro-prusside of sodium, which gives a magnificent rose, purple, blue, or crimson color, according to the strength of the alkaline. This will indicate an alkali in borates, phosphates, carbonates, and in the least oxideable oxides, as lime and magnesia."

The metals are divided into groups or classes.

CLASS I. Potash = KO , soda = NaO , and ammonia NH_3 . None of these, in an acidified solution, gives a precipitate with hydrosulphuric acid, hydrosulphate of ammonia, or carbonate of soda.

CLASS II. Magnesia, MgO . Lime, CaO . Baryta, BaO . Strontia, SrO . None of these gives a precipitate with hydrosulphuric acid, or hydrosulphate of ammonia.

Carbonate, or phosphate of soda, with either of this class, gives a copious white precipitate insoluble in excess.

CLASS III. Alumina = Al_2O_3 .	Oxide of nickel NiO .
Oxide of zinc ZnO .	Oxide of cobalt CoO .
Oxide of chromium.	Protoxide of iron FeO .
Protoxide of manganese MnO .	Per oxide of iron Fe_2O_3 .

In neutral solutions these metals are precipitated by hydrosulphate of ammonia.

In a slightly acid solution, hydrosulphuric acid gives no precipitate excepting with peroxide of iron, with which it gives a yellowish white precipitate.

CLASS IV. Arsenious acid AsO_3 , arsenic acid AsO_5 , teroxide of antimony SbO_3 , oxide of mercury HgO , peroxide of mercury HgO_2 , oxide of lead, copper, silver, tin, bismuth, gold and platinum.

All of this class are precipitated from their acid solution by hydrosul

phuric acid. We can thus determine to which of the four classes of metals the substance under examination belongs.

POTASH, in a solution of chloride of potassium.

* *Bichloride of platinum*, in a neutral or slightly acid solution, gives a fine yellow crystalline prec., = KCl. Pt. Cl_2 , slightly soluble in water, but insoluble in alcohol; somewhat soluble in dilute acids. When the solution is dilute, evaporate it with the reagent on a water bath, and then digest the residue with alcohol, when the above yellow crystals will appear.

Tartaric acid. Let the solution be concentrated, then add the reagent, and agitate the mixture with a glass rod for some time, and let it remain, when a white prec, slightly soluble in water, will appear, the prec = $\text{KO. HO. C}_8\text{H}_4\text{O}_{10}$.

Blow Pipe flame. Wash the platinum wire in distilled water, then place a piece of the salt to be examined on the wire, which will give a violet color to the outer flame.

Alcohol flame, having a potash salt in solution, gives the same reaction as the last.

SODA, in a solution of sulphate of soda.

Bichloride of platinum, added as for potassa, then evaporated, will give yellow needle-shaped crystals different from that by potassa. The prec. is readily soluble in water and alcohol.

Antimoniate of potash. Let the solution and the reagent be concentrated, and the solution under examination slightly alkaline or neutral; then apply the reagent, which, if soda is present, will produce a white crystalline prec. of antimoniate of soda.

Blow Pipe. Hold the salt on the platinum wire in the inner or reducing flame, it will impart a golden yellow color to the outer, or oxidizing flame.

OXIDE OF AMMONIUM, NH_4O , in a solution of chloride of ammonium.

Bichloride of platinum gives the same reaction as for potassa. If we have a doubt whether it is potassa or ammonia, ignite the precipitate and digest the residue with water, then, if nitrate of silver be added, and gives a precipitate, it shows the presence of potassa. In this case we must take care that all traces of hydrochloric acid are removed.

Heated in a test tube. If the substance be heated in a test tube with some hydrate of lime, or caustic potassa or soda, it will give off the peculiar odor of ammonia, and changes moistened turmeric paper brown and red litmus paper blue. If this does not happen, we say ammonia is absent.

BARYTA, = BaO , in a solution of chloride of barium.

Sulphuric acid. White prec. in very dilute solution, insoluble in dilute acids.

Sulphate of lime, in solution, gives an immediate prec., requiring 500 times its weight of water to dissolve it.

Oxalate of ammonia. White prec. readily sol. in free acids. This is the same reaction as for lime, but it requires a stronger solution of baryta than of lime.

Flame of alcohol, containing baryta, is yellowish, and is different from that of lime, which has a reddish tinge, and strontia, which is carmine.

Blow Pipe, in the inner flame, the substance strongly heated on plati-

* Those marked with an asterisk are the most delicate tests.

num wire, imparts a light green color to the outer flame. If the substance be insoluble, first moisten it with dilute hydrochloric acid.

LIME, = CaO , in a solution of chloride of calcium.

Oxalate of ammonia. Let the solution be neutralized with muriate of ammonia; then add the reagent, which will give a copious white prec. of oxalate of lime, soluble in hydrochloric acid, but insoluble in acetic acid. This detects lime in a highly diluted solution.

Sulphuric acid, dilute. In concentrated solution gives an immediate prec. soluble in much water, which is not the case with baryta.

Blow Pipe. Heated in the inner flame, gives an orange red color to the outer flame. Moisten an insoluble compound with dilute hydrochloric acid before this test.

Burnt with alcohol, the flame will be a reddish tint, but not so red as that given by strontia.

STRONTIA = SrO . In a solution of chloride of strontium.

Oxalate of ammonia, in concentrated solution, a white prec., but not in dilute solution. This distinguishes strontia from lime.

Sulphate of lime. The prec. will be formed after some time even in a concentrated solution. This distinguishes strontia from baryta. (See above.)

Sulphuric acid gives an immediate prec. in a concentrated solution, but only after some time in a dilute one, where the prec. will be minute crystals.

In the flame of alcohol, stir the mixture, and a beautiful carmine color is produced.

Blow Pipe, in the inner flame, an intense carmine red. Moisten the insoluble compound with dilute H.Cl as above for lime and baryta.

NOTE. *Sulphuric acid* gives, with a weak solution of lime, no precipitate; with chloride of barium, an immediate white p.; with a weak solution of strontia, a prec. after some time. The prec. from baryta and strontia are insol. in nitric acid, but that from lime is sol.

MAGNESIA MgO ., in a solution of sulphate of magnesia MgO. SO_3 .

Phosphate of soda, a white, highly crystalline prec. of phosphate of magnesia = 2MgO. HO. PO_5 . In this case the solution must not be very dilute. By boiling the solution and reagent together the prec. is more easily produced.

Phosphate of soda and ammonia. In using this reagent, add ammonia or its carbonate, which makes the prec. less soluble. Agitate with a glass rod, which, if it touches the side of the test tube, will cause the prec. there to appear first. The prec. is crystalline, slightly soluble in water, less in ammonia, but readily in dilute acids; $\therefore$ the solution must be ammoniacal. Ignite this prec., the ammonia is driven off, and the residue = phosphate of magnesia = 2MgO. PO_5 .

Blow Pipe. Moisten the substance with nitrate of cobalt, and heat in the blow-pipe, the compound assumes a pale flesh or rose color.

NOTE. Sulphate of lime gives a prec. with baryta and strontia.

Oxalate of ammonia gives a prec. with a very dilute solution of lime, but only with a concentrated solution of magnesia and strontia, and in a much stronger sol. of baryta than lime.

Phosphate of soda, with lime, a gelatinous precipitate.

do do with magnesia.

Hydrofluosilic acid, in a solution of baryta, gives a white, transparent prec. By evaporating the prec. to dryness, and washing the residue with alcohol, we obtain all of the silico-fluoride of barium undissolved. If the sol. is dilute, the prec. will be after some time.

ALUMINA, (Al_2O_3 .) in a sol. of sulphate of alumina.

Caustic Ammonia, (NH_3 .) gives a semi-transparent, gelatinous, bulky prec. nearly insol. in excess of the ammonia.

Caustic Potash, (KO .) gives a similar prec. soluble in an excess of the reagent, but if we add chlorate of ammonia to the solution, the alumina is again precipitated.

Hydrosulphate of Ammonia, added to a neutral solution, gives a white prec. of hydrate of alumina, (Al_2O_3, HO) and hydrosulphuric acid is liberated.

Phosphate of Soda, white prec., sol. in mineral acids, nearly insol. in acetic acid.

Lime Water, precipitates alumina.

NOTE. Ammonia in excess precipitates alumina, but not magnesia or the other alkaline earths.

CHROMIUM, (Cr_2O_3 .) in a sol. of sulphate of chrom.

Hydrosulph. Acid, in neither acid or neutral solutions, gives no prec.

Hydrosulphate of Ammonia, in a neutral solution, gives a dark green prec. insol. in excess of the reagent.

Caustic Ammonia, if boiled with the solution, will produce the same as the last. If not boiled, a portion of the prec. will re-dissolve, giving the liquid a pink color.

Blow Pipe. Reduce the substance to a sesquioxide of chromium, which will give in the inner flame a yellowish green glass, and in the outer flame a bright emerald green.

Heat with a mixture of nitrate of potash and carbonate of soda; a yellow bead is formed. Dissolve this bead in water acidulated with nitric acid, and add acetate of lead; a bright yellow prec. of chromate of lead is formed.

PEROXIDE OF IRON. In a solution of sulphate of iron, $FeO. SO_3$. The compound is boiled with nitric acid to oxidize the metal, and then evaporated to dryness.

Hydrosulphuric Acid, gives no precipitate.

Sulphide of Ammonium, precipitates the iron completely as a black precipitate of sulphide of iron, FeS , which is insoluble in an excess of the precipitant.

The above precipitate when exposed for some time to the air, becomes brown sesquioxide of iron.

Ferrocyanide of Potassium, (prussiate of potassæ,) light blue precipitate of KFe_3Cfy_2 . The precipitate is insoluble in dilute acids. *This is the most delicate test for iron.*

Sulphocyanide of Potassium. A red solution, but no precipitate.

Tincture of Galls. Bluish black in the most dilute solution.

Caustic Potash. Reddish prec. sol. in excess.

Caustic Ammonia the same, insol. in excess.

Blow Pipe, heated on a platinum wire with borax in the outer flame, gives a brownish red glass, which assumes a dirty green color in the inner or reducing flame.

OXIDE OF COBALT. CoO , in a solution of nitrate or chloride of cobalt.

Ammonia, when the solution does not contain free acid, or much ammoniacal salt, the metal is partially precipitated as a bluish precipitate, readily soluble in excess of the reagent, giving a reddish brown solution.

Sulphide of Ammonium. A black precipitate of sulphide of cobalt, CoS , soluble in nitric acid, but sparingly in hydrochloric acid.

Sesquicarbonate of Ammonia. A pink prec. CoO , CO_2 readily soluble in excess, giving a red solution.

Solution of Potassa. Blue prec. changing by heat to violet and red.

Ferrocyanide of Potassium. A grayish green prec.

Blow Pipe. In both flames with borax, a beautiful blue glass whose color is scarcely affected by other oxides. In this reaction the cobalt must be used in a small quantity.

OXIDE OF NICKEL, NiO in a sol. of sulphate of nickel, NiO , $\text{SO}_3 + 7\text{HO}$.

Hydrosulphate of Ammonia. Black prec. from neutral solution, slightly sol. in excess of the reagent, if the ammonia is yellow. The prec. is sol. in NO_5 and sparingly in HCl .

Hydrosulphuric Acid in acidified sol., no prec., but in neutral sol., it gives a partial prec.

* *Caustic Ammonia*. A light green prec. sol. in excess, giving a purplish blue solution. In this case any salt of ammonia must be absent.

Caustic Potash. Apple green prec. insol. in excess.

Ferrocyanide of Potassium, greenish white prec. Cyanide of potassium, yellowish green prec. sol. in excess, forming a dull yellow sol. From this last sol., SO_3 precipitates the nickel.

Blow Pipe. Any compound of nickel with carbonate of soda or borax in the inner flame, is reduced to the metallic state, forming a dusky gray or brown beads. In the outer flame the bead is violet while hot, becoming brown or yellow on cooling.

OXIDE OF MANGANESE = MnO in a solution of sulphate of manganese = MnO , $\text{SO}_3 + 7\text{HO}$.

* *Hydrosulphate of Ammonia* in neutral sol. gives a bright flesh colored gelatinous prec. becoming dark on exposure to the air. It is insoluble in excess of the reagent, but sol. in HCl and NO_5 .

* *Caustic Ammonia*, if free from muriate of ammonia, gives a white or pale flesh colored = MnO , HO , becomes brown in air.

* *Caustic Potash*, the same as the last, but muriate of ammonia does not entirely prevent the precipitate.

Carbonate of Potash, or Ammonia, white prec. which does not darken so readily as the above. It is slightly soluble in chloride of ammonium.

Blow Pipe. Mix the substance with carbonate of soda and a little nitrate or potash, and heat in the outer flame; it will give a green color, and produce manganate of soda, which will color water green.

If the substance is heated with borax in the outer flame, it will produce a bead of a purple color; this if heated in the inner flame will cause the color to disappear.

OXIDE OF ZINC, ZnO in a solution of sulphate of zinc, Zn , $\text{SO} + 7\text{HO}$.

* *Hydrosulphate of Ammonia*, in neutral or alkaline solution, gives a copious white curdy prec. if the zinc is pure. If iron is present it will be colored in proportion to the iron present in the sol.

Hydrosulphuric Acid in acid sol. no prec.

Caustic Ammonia, or *Potash*, a white gelatinous prec. soluble in excess. From either solution in excess, hyd. sulph. acid (HS) throws down the white prec. of sulphide of zinc.

Carbonate of Potash, when no other salt of potash is present, gives a white prec. = $3(\text{ZnO}, \text{HO}) + 2(\text{ZnO}, \text{CO}_2)$ insol. in excess of the reagent.

Blow Pipe, moistened with nitrate of cobalt and heated in the outer flame, gives a pale green color which is a delicate test to distinguish it from manganese, alumina and cobalt.

ARSENIC ACID = AsO_5 . Boil the compound with HCl, and at the boiling point, add nitric acid as long as red flames of nitrous vapor appear, then evaporate slowly so as not to redden the powder, and expel the acid; then dissolve in distilled water for examination. HS, added to the above sol. slightly acidified with HCl, gives no immediate prec., but if allowed to stand for some time, or if heated to boiling point, a yellow prec. is obtained. Apply the gas several times, always heating to boiling point each time.

Hyd. Sulph. of Ammonia, as in the above solution, but a little more acid gives the same prec. but of a lighter color.

Ammonia nitrate of Silver. In a neutral solution as first made, add nitrate of silver which gives but a faint cloudy appearance; now add ammonia drop by drop till it gives a yellow prec. of arsenite of silver, which is very soluble in alkali.

NOTE. The same prec. is obtained from the presence of phosphate of soda.

Reinsch's test, in a solution acidified by adding a few drops of hydrochloric acid is a very delicate test, and considered nearly as delicate as Marsh's.

Boil with the acidified liquid in a test tube, a clean strip of copper foil; the arsenic will be prec. on the copper as a metallic deposit. Antimony, bismuth, mercury and silver, give the same reduction as arsenic.

In order to determine which is present, take out the copper foil and dry it between folds of filtering paper, or before a gentle heat; place it in a dry test tube and apply heat; the arsenic being volatile, will be deposited in the upper end of the tube as a crystalline deposit, using but gentle heat. If it were antimony it would not be volatile, and would be deposited as a white sublimate, insol. in water, amorphous, and requiring more heat than arsenic. If it were mercury, it would be in small metallic globules.

Marsh's test, is dangerous, excepting in the hands of an experienced chemist. Those who wish to apply it, will find the method of using it in Sir Robert Kane's Chemistry, or in those of Graham, Fowne, Bowman, and others.

TROXIDE OF ANTIMONY = SbO_3 , in a solution of chloride of antimony = SbCl_3 . This solution is made by dissolving the gray ore, or bisulphide of antimony in hydrochloric acid; the solution then diluted with water, acidified with HCl, is examined.

Hydrosulphuric Acid, gives an orange red prec. of SbS_3 , insol. in cold dilute acids, soluble in potassa and sulphide of ammonia.

Hydrosulphate of Ammonia. Add the reagent in small quantities; it will give an orange prec. of SbS_3 , soluble in excess.

Caustic Ammonia, or Potassa. Add slowly, and it will give a white prec. of teroxide of antimony = SbO_3 , soluble in excess.

Water in excess. A white prec. which crystallises after some time, and is sol. in tartaric acid.

NOTE. The same reaction is had with bismuth, but the prec. is not soluble in tartaric acid.

A piece of zinc, in a dilute solution made with *aqua regia*, precipitates both antimony and tin.

A piece of tin, in the above sol., prec. the antimony.

TEROXIDE OF BISMUTH, in a solution of nitrate of teroxide of bismuth = $\text{BiO}_3, 3\text{NO}_5$.

Hyd. Sulph. Acid. A black prec. insol. in cold dilute acids, but sol. in hot dilute nitric acid.

Chromate, or Bichromate of Potash, yellow prec. very sol. in dilute nitric acid.

Water in excess, added to a solution of sesquichloride of bismuth, slightly acidified with hydrochloric acid, produces a white prec. insol. in tartaric acid, which distinguishes it from antimony.

Heat a salt of Bismuth. It turns yellow, but on cooling off, becomes again colorless.

Blow Pipe. In the inner flame with carbonate of soda, it forms small metallic globules, easily broken.

Blow Pipe. In the outer flame with borax, gives a yellowish bead, becoming nearly colorless when cool.

OXIDE OF TIN = SnO , in a sol. of chloride of tin, SnCl .

Hydrosulphuric Acid, dark brown prec. in neutral or acid solutions. Insol. in cold dilute acids. If the prec. is boiled with nitric acid, it is converted into the insoluble binoxide of tin.

Hydrosulphate of Ammonia, brown prec. sol. in excess if the reagent is yellow.

Chloride of Mercury. First a white prec. then a gray prec. of metallic mercury, even in very dilute solution and in the presence of much HCl .

Caustic Ammonia, white bulky prec. insol. in excess.

Caustic Potash, do. = SnOHO , sol. in excess.

Terchloride of Gold = (AuCl_3) very dilute. In dilute solutions, gives a dark purple prec. known as the purple of Cassius. If this mixture is now heated, it is resolved into metallic gold and binoxide of tin.

PEROXIDE OF TIN = SnO_2 , in a sol. of bichloride of tin = SnCl_2 .

Hyd. Sulph. Acid, bright yellow prec. insol. in dilute. SO_3 , or HCl , made insoluble by boiling with NC_5 , soluble in HCl added to a little NO_5 . Sol. in alkalies.

Caustic Potassa, or Ammonia, white bulky prec. sol. in excess, especially with potassa. The prec. with ammonia is SnO_2, HO , and with potassa = KO, SnO_2 .

Blow Pipe. In the outer flame with borax, it will give a colorless bead, but if there is much tin, the bead will be opaque.

A piece of clean zinc, in a sol. of perchloride of tin, will precipitate the tin in the metallic state in beautiful feathery crystals; which under the microscope appear as brilliant crystalline tufts.

OXIDE OF MERCURY = HgO , in a solution of bichloride of mercury, (corrosive sublimate) = HgCl_2 .

Hydrosulphuric Acid, added slowly, gives a white or yellow prec. If added in excess, it gives a black prec. of HgS , insol. in dilute SO_3 , HCl or NO_5 . It is soluble in aqua regia with the aid of heat. If the precipitate be dried and heated in a test tube, metallic mercury is produced.

Chloride of Tin, add slowly, a white prec. of Hg_2Cl_2 = subchloride of mercury will appear, this prec. becomes gray with an excess of the reagent. If we boil this precipitate in its solution, the mercury is reduced to the metallic state.

* *Iodide of Potassium*, add drop by drop, gives a beautiful red prec. soluble in an excess of either the solution or reagent.

Heat a strip of copper, the mercury will be deposited on it which when rubbed will appear like silver. If the strip be heated in a test tube, the mercury will appear in minute globules in the cool part of the tube.

OXIDE OF LEAD = PbO , in a solution of nitrate of lead, = PbO , NO_5 , made by dissolving the substance in nitric acid, and allowing it to crystallise. We may also use a solution of acetate of lead. Acetate of lead is formed by dissolving oxide of lead in an excess of acetic acid, then evaporate to dryness, the salt is acetate, or sugar of lead.

The following reactions take place with *nitrate of lead*:

Hydrosulphuric acid, in neutral or slightly acid solution, gives a black prec. of sulphide of lead = PbS , but if boiled with nitric acid, it becomes $\text{PbO} + \text{SO}_3$.

Caustic Ammonia, a white prec. insol. in excess. Other ammoniacal salts must not be present.

Dilute, SO_3 , a white heavy prec. nearly insol. in acids, but soluble in potassa. Now collect the prec. and moisten it with a little hydrosulphate of ammonia, it will become black. This distinguishes lead from baryta and strontia, which are insoluble.

Carbonate of Potassa, white prec. insol. in excess. Prec. = PbO , CO_2 .

Iodide of Potassium, beautiful yellow prec. If this is boiled with water and allowed to cool, beautiful yellow scales are formed.

Chromate of Potassa, fine yellow prec. insol. in dilute acids, but sol. in potassa.

Hydrochloric Acid, a white prec. Boil the solution and let it cool, then needle-shaped crystals will be formed.

OXIDE OF SILVER, AgO , in a solution of nitrate of silver.

Hydrochloric Acid, or any soluble chloride, a white curdy prec. of chloride of silver, insol. in water and nitric acid, sol. in ammonia. This becomes violet on exposure to light, and is sparingly sol. in HCl .

Common Table Salt, gives the same prec.

Hyd. Sulph. Acid, and *Hyd. Sulphate of Ammonia*, gives a black prec. insol. in dilute acids, but sol. in boiling nitric acid.

Caustic Ammonia, brown prec. sol. in excess.

Caustic Potassa, brown prec. insol. in excess.

Phosphate of Soda, a pale yellow prec. sol. in NO_5 and ammonia.

Chromate of Potassa, dark crimson prec.

NOTE. With lead, the prec. would be yellow.

Slip of clean copper, iron or zinc, suspended in the liquid, precipitates the silver in the metallic state.

NOTE. Silver is precipitated by other metals more electro-negative, such as tin, lead, manganese, mercury, bismuth, antimony, and arsenic.

OXIDE OF COPPER. CuO , in a solution of sulphate of copper = CuO , $\text{SO}_3 + 5\text{HO}$.

Hyd. Sulph. Acid, in a neutral, acid or alkaline solution, gives a black prec. = CuS , insol. in dilute SO_3 , or HCl , but sol. in moderately dilute nitric acid. Insol. in excess of the reagent.

Hyd. Sulphate of Ammonia. The same as the last, excepting that the reagent in excess dissolves the prec.

Caustic Ammonia, added slowly, precipitates any iron as a greenish or red brown mud, and the supernatant liquid is of a fine blue color. With nickel, ammonia gives a blue but of a pale sapphire color, whilst that of copper gives a deep ultramarine.

Caustic Potassa, blue prec. insol. in excess. If the potassa be added in excess and then boiled, the prec. will be black oxide of copper = CuO .

Ferrocyanide of Potassium = *Prussiate of Potassa*, gives a chocolate colored prec. = Cu_2FeCy_3 , insoluble in dilute acids. This is a very delicate test. The prec. is soluble in ammonia. Potassa decomposes it. Before adding this test, acidify the solution with acetic acid or acetate of potassa.

If but a small quantity of copper is present, no prec. will be produced, but the solution will have a pink color.

Iron or Steel perfectly cleansed in a neutral sol. or one slightly acidified with SO_3 , will become coated with metallic copper, thus enabling us to detect a minute quantity of copper, which is sometimes entirely precipitated from its solution. This detects 1 of copper in 180,000 of solution.

Blow Pipe. In the outer flame with borax while hot, the copper salt is green, but becomes blue on cooling.

TEROXIDE OF GOLD = AuO_3 in a solution of terchloride of gold.

Hydrosulphuric acid, black prec. of tersulphide of gold = AuS_3 , insol. in mineral acids, but sol. in *aqua regia*.

Sulphate of Iron, bluish black prec. becomes yellow when burnished.

Oxalic acid, if boiled, a prec. of a purple powder, which will afterwards cohere in yellow flakes of metallic gold when burnished.

Chloride of Tin, with a little bichloride of tin, gives a purple tint, whose color varies with the quantity of gold in the solution, and is insol. in dilute acids. In using this test, first add the golden solution to the chloride of tin, and then add the solution of bichloride of tin, drop by drop. If only a small quantity of gold is present, the solution will have but a pink tinge.

Tin-iron Solution. This reagent is made by adding sesquichloride of iron to chloride of tin, till a permanent yellow is formed.

Pour the golden solution, much diluted in a beaker, and set it on white paper. Now dilute the tin-iron reagent, and dip a glass rod into it, which remove and put into the gold solution, when, if a trace of gold is present, a purple or bluish streak will be in the track of the rod. This may be used in acid solutions.

BINOXIDE OF PLATINUM = PtO_2 , in a solution of bichloride of platinum.

Hyd. Sulphuric Acid, black prec. when boiled. Insol. in dilute acids.

Chloride of Ammonium. After several hours, a yellow crystalline prec. slightly sol. in water, but insol. in alcohol.

Chloride of Tin, in the presence of hydrochloric acid, is a dark brown color; but if the solution is dilute, the color is yellow.

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QUANTITATIVE ANALYSES.

The mineral is finely pulverized, in an agate or steel mortar. The pestle is to have a rotary motion so as not to waste any part of the mineral. When pulverized, wash and decant the fine part held in the solution, and again pulverize the coarse part remaining after decantation.

If the mineral is malleable, we file off enough for analysis.

Digesting the mineral, is to keep it in contact with water or acid in a beaker, and kept for some time at a gentle heat. If the mineral is insol. in water or HCl, we use aqua regia, (nitro-hydrochloric acid) composed of four parts of hydrochloric acid and one part of nitric acid. Aqua regia will dissolve all the metals but silica and alumina.

Filtering papers, are made of a uniform size, and the weight of the ash of one of them marked on the back of the parcel.

Filtering.—One of the filtering papers is placed in a glass funnel which is put into a large test tube or beaker, and then the above solution poured gently on the side of the filtering paper, wash the filter with distilled water. The filter now holds silica and alumina. Burn the filter and precipitate or insoluble residue, the increase of weight will be the siliceous matter in the amount analyzed, which may be twenty-five, fifty or one hundred grains, perhaps fifty grains will be the most convenient; therefore, the increase of weight found for siliceous matter if multiplied by two, will give the amount per cent.

Decanting, is to remove the supernatant liquid from vessel A to vessel B, and may be easily done by rubbing a little tallow on the outside of the edge of A, over which the liquid is to pass, and holding a glass rod in B, and bringing the oiled lip of A to the rod, then decant the liquid.

The Engineer is supposed to have seen some elementary work on Chemistry or Pharmacy. Fowne's, Bowman's and Lieber's are very good ones; from either of which he can learn the first rudiments.

The following table shows the substances treated of in this work, showing their symbols, equivalents or atomic weights and compounds.

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TABLE OF SYMBOLS AND EQUIVALENTS.

Name.	Sym- bol.	Equi- val't.	Compound.	Equi- val't.
Aluminum	Al	14	Al ₂ O ₃ , Alumina.....	52
"	"	14	Al ₂ Cl ₃ , Chloride of Aluminum.....	136
"	"	14	Al ₂ O ₃ , 3SO ₃ , Sulphate of Alumina...	172
Antimony	Sb	129	Sb ₂ O ₃ , Oxide of Antimony.....	153
Arsenic	As	75	As ₂ O ₃ , Arsenious Acid	99
"	"	75	As ₂ O ₅ , Arsenic Acid.....	115
Barium	Ba	69	BaO, Baryta.....	77
"	"	69	BaCl, Chloride of Barium.....	105
Bismuth	Bi	107	Bi ₂ O ₃ , Sesquioxide of Bismuth.....	238
"	"	107	Bi ₂ O ₃ , 3NO ₅ , Nitrate of Bismuth....	400
"	"	107	Bi ₂ Cl ₃ , Sesquichloride of Bismuth.	322
Cadmium	Cd	56	CdO, Oxide of Cadmium.....	64
"	"	56	CdS, Sulphide of Cadmium.....	72
Calcium	Ca	20	CaO, Lime.....	28
"	"	20	CaCl, Chloride of Lime.....	56
Carbon	C	6	CO ₂ , Carbonic Acid.....	22
"	"	6	CO, Carbonic Oxide.....	14
"	"	6	CS ₂ , Sulphide of Carbon.....	38
Chlorine	Cl	36	ClO ₅ , Chloric Acid.....	76
"	"	36	HCl, Hydrochloric Acid.....	*37

Name.	Sym- bol.	Equi- val't.	Compound.	Equi- val't.
Chromium	Cr	28	Cr ₂ O ₃ , Sesquichloride of Chromium.	80
“	“	28	Cr ₂ O ₃ , 3S ₂ O ₃ , Sulphate of Chromium.	200
Cobalt	Co	30	CoO, Protoxide of Cobalt.....	38
“	“	30	Co ₂ O ₃ , Sesquioxide of Cobalt.....	84
Copper, (Cuprum)...	Cu	32	Cu ₂ O, Suboxide of Copper.....	72
“	“	32	CuO, Black Oxide of Copper.....	40
“	“	32	CuO, SO ₃ , Sulphate of Copper.....	80
Fluorine	F	18	HF, Hydrofluoric Acid.....	19
Gold, (Aurum).....	Au	200	AuO, Oxide of Gold.....	208
“	“	200	AuO ₃ , Ter oxide of Gold.....	224
“	“	200	AuCl ₃ , Ter chloride of Gold.....	308
Hydrogen	H	1	HO, Water.....	9
“	“	1	HO ₂ , Binoxide of Hydrogen.....	17
Iodine	I	126	IO ₅ , Iodic Acid.....	166
“	“	126	HI, Hydriodic Acid.....	127
Iron, (Ferum).....	Fe	28	FeO, Protoxide of Iron.....	36
“	“	28	Fe ₂ O ₃ , Sesquioxide of Iron.....	80
Lead, (Plumbum)....	Pb	104	PbO, Protoxide of Lead.....	112
“	“	104	Pb ₃ O ₄ , Red Oxide of Lead.....	344
“	“	104	PbCl, Chloride of Lead.....	140
Magnesium	Mg	12	MgO, Magnesia.....	20
“	“	12	MgCl, Chloride of Magnesium.....	48
Manganese.....	Mn	28	MnO, Protoxide of Manganese.....	36
“	“	28	MnO ₂ , Binoxide or Black Oxide of Manganese.....	44
“	“	28	MnO ₃ , Manganic Acid.....	52
“	“	28	Mn ₂ O ₇ , Permanganic acid.....	112
Mercury	Hg	202	HgO, Protoxide of Mercury.....	210
“	“	202	HgO ₂ , Red or Binoxide of Mercury.	218
“	“	202	HgCl, Chloride of Mercury.....	238
“	“	202	HgCl ₂ , Perchloride of Mercury.....	274
Nickel	Ni	30	NiO, Oxide of Nickel.....	38
“	“	30	Ni ₂ O ₃ , Sesquioxide of Nickel.....	84
Nitrogen	N	14	NO ₅ , Nitric Acid.....	54
“	“	14	NO ₂ , Binoxide of Nitrogen.....	30
“	“	14	NH ₃ , Ammonia.....	17
Oxygen.....	O	8	Air = 23.10 of O, and 76.9 per cent of N.....	72
Phosphorous	P	32	PO ₅ , Phosphoric Acid.....	56
“	“	32	PO ₃ , Phosphorous Acid.....	
“	“	32	PH ₃ , Phosphoretted Hydrogen.....	35
Platinum	Pt	99	PtO, Protoxide of Platinum.....	107
“	“	99	PtO ₂ , Binoxide of Platinum.....	115
Potassium, (Rorium)	K	40	KO, Potash.....	48
“	“	40	KCl, Chloride of Potassium.....	76
Silicon	Si	22	SiO ₃ , Silicic Acid or Silica.....	46
Silver, (Argentum)..	Ag	108	AgO, Oxide of Silver.....	116
“	“	108	AgCl, Chloride of Silver.....	144
Sodium, (Natronium)	Na	24	NaO, Soda.....	32
“	“	24	NaCl, Chloride of Sodium.....	60
Strontium	Sr	44	SrO, Strontia.....	52
“	“	44	SrCl,	80
Sulphur	S	16	SO ₃ , Sulphuric Acid.....	40
“	“	16	HS, Hydrosulphuric Acid.....	17
Tin, (Stannum).....	Sn	59	SnO, Protoxide of Tin.....	67
“	“	59	SnO ₂ , Peroxide of Tin.....	75
Zinc	Zn	32	ZnO, Oxide of Zinc.....	40
“	“	32	ZnCl, Chloride of Zinc.....	68

TABLE, SHOWING THE ACTION OF REAGENTS ON METALLIC OXIDES.

Name of Base or Metal	Hvd. Sulph. Acid. HS, in acidified sol.	Hyd. Sulph. of Am- monia, NH ₄ S, HS.	Carbonate of Soda, Na ₂ CO ₃ .	Carb. of Ammonia, 2NH ₄ CO ₃ .	Caustic Potash, KO.	Caustic Ammonia, NH ₃ .	Ferroc- of Potass- ium, K ₂ FeC ₂ .
Alumina.....	No reaction.....	White, insol. in xs.	W. insol xs.	W. insol.....	W. sol. in xs.....	W. insol. xs.....	No reaction.
Antimony.....	Orange red SbS ₃ ...	Or red sol. xs.....	W. sparingly sol.	W. spar. sol.....	W. spar. sol.....	W. insol.....	White. 0
Baryta.....	0	0	W. insol. xs.	W. insol.....	W. insol.....	W. insol.....	White.
Bismuth.....	Brn to blk BiS ₃ ...	Brown to black.....	W. insol. xs.	W. insol.....	W. insol.....	White sol. xs.....	White.
Cadmium.....	Bright yellow CdS	Br't yel. insol. xs.	W. insol. xs.	W. insol.....	W. insol.....	0	0
Calcium, (lime).....	0	0	Green	Green	Green sol xs.....	Gr'n'h blue sp. sol.	0
Chromium.....	0	Greenish blue.....	Pink insol.....	Pink spar sol.....	Blue insol.....	Blue sol. xs ..	Pale green
Cobalt.....	0	Black, CoS.....	Bluish if boiled.....	In excess.....	Boiled, a dirty red	Brownish red.....	Or gray.
".....	0	0	Greenish blue.....	Greenish blue.....	Dark br'n if boiled	Pale to fine blue...	Chocolate
Copper, oxide of.....	Black CuS.....	Black.....	Dark br'n if boiled	Sol. in xs.....	In excess of KO...	Sol. in xs.....	Color. 0
".....	0	0	0	Yellow insol.....	Yellowish brown...	Yellow insol.....	0
Gold.....	Black AuCl ₃	Br'n-black sol' xs.	Rust colored.....	Yellow insol.....	Rust colored insol.	Rust colored insol.	Deep blue.
Iron, peroxide of.....	Yellow white.....	Black.....	White insol.....	Rust col. insol.....	White sol.....	White insol.....	White. 0
Lead, oxide of.....	Black, PbS.....	Black insol.....	White insol.....	White insol.....	White insol.....	White insol.....	0
Magnesia.....	0	0	White insol.....	0	White insol.....	White insol.....	White.
Mercury, per oxide.....	W becomes blk.....	W. becomes blk.....	Reddish br'n insol.	W. insol.....	Yellow insol.....	White to brown...	White.
Manganese.....	0	Flesh col. insol.....	White insol.....	White insol.....	White to brown...	White to brown...	White.
Nickel.....	0	Black NiS.....	Pale green insol...	Pale green sol.....	Pale green insol...	Pale green sol.....	Pale green.
Platinum.....	Br'h blk PtCl ₂ ...	Br'o'h blk sol.....	Yellow white.....	Yellow.....	Yellow.....	Yellow.....	0
".....	0	0	With carb potassa	0	0	0	0
Silver.....	Black.....	Black insol.....	White insol.....	White sol.....	Pale br'n insol.....	Pale brown sol.....	White. 0
Strontia.....	0	0	White insol.....	White insol.....	White sol.....	White sol.....	White.
Tin, per oxide.....	Yellow.....	Yellow sol.....	White insol.....	White insol.....	White sol.....	White sol.....	White.
Zinc, oxide of.....	0	White.....	White insol.....	White sol.....	White sol.....	White sol.....	White.

xs. = in excess.

For Potassa, Soda and Ammonia, see p. 72B75.

ANALYSIS OF VARIOUS SOILS.

Name of Soil.	EARTHS.		OXIDES.		Potash & Soda.		ACIDS.		Chloride Sodium.	Humic acid.	Organic matter.
	Water.	Alu- mina.	Silica.	Lime.	Mag- nesia.	Iron.	Manga- nese.	Sul- phuric.	Car- bonic.	Phos- phoric.	
Clay, potters'.....	11.	27.	61.			1.					
" porcelain.....	15.	27.	55.	2.		1.					
" Stourbridge.....	12.6	30.4	57.								
" pure.....		40.	60.								
" for tiles.....		38 to 34	62 to 66								
" loam.....		34 to 28	66 to 72								
Loamy soil.....		28 to 16	72 to 84								
Sandy loam.....		16 to 4	84 to 96								
Sandy soil.....		4.	96.								
Marly soil.....				5 to 20							
Fertile pasture, Hanover.....	1.	9.35	71.85	.99	.25	5.41	.93	.17		.13	8.82
" arable, Ohio.....		5.67	87.14	.66	.31	2.22	.36	.08	.15	.06	2.37
Barren soil.....		.45	61.58	.32	.13	.52					38.01
Calcareous soil.....				20.							
Vegetable mould.....											
Fertile alluvial pasture.....	.03	6.44	84.51	.74	.53	5.41	.45	.05	.01		5 to 10
Good red clover land.....		1.74	98.72	.12	.70	2.06	.32	.01	.17	.10	.89
Bad ".....		2.65	92.01	.24	.70	3.19	.48	trace	.16	.08	.15
Very fertile arable in Moravia.....		8.51	77.21	.96	1.96	6.59	1.52	.78			1.11
Good cotton soil, Mississippi.....		7.82*	80.55		1.81	1.45	.66	.60	.21	.04	4.74
Cotton soil exhausted.....		24.*	85.92		.76	.67		.39	.09	trace	6.29
Green sand.....	11.5	33.93	48.03		1.30			5.61			
Shell marl.....		.60	3.10	45.75							14.6
Clay marl.....	1.40	3.20	84.9	4.7							2.8

NOTE.—Those marked * contain iron not determined.

Barren soil, includes insol. humus.

ANALYSIS OF SOILS.

310s. The fertility of soils is composed of their siliceous matter, phosphoric acid and alkalies. The latter ought to be abundant. The surveyor may judge of the soil by the crops—as follows:

If the straw or stalks lodge, it shows a want of silica, or that it is in an insoluble condition, and requires lime and potash to render it soluble.

If the seeds or heads does not fill, it shows the want of phosphoric acid.

If the leaves are green, it shows the presence of ammonia; but if the leaves are brown, it shows the want of it.

Chemical analysis. By qualitative analysis, we determine the simple bodies which form any compound substance, and in what state or combination.

Quantitative analysis, points out in what proportion these simple bodies are combined.

A body is organic, inorganic, or both.

The body is organic, if when heated on a platinum foil, or clean sheet of iron over a spirit lamp, it blackens and takes fire. And if by continuing the heat the whole is burnt away, we conclude that the substance was entirely organic, or some salt of ammonia.

Soluble in water.—The substance is reduced to powder, and a few grains of it is put with distilled water in a test tube or porcelain capsule; if it does not dissolve on stirring with a glass rod, apply gentle heat. If there is a doubt whether any part of it dissolved, evaporate a portion of the solution on platinum foil; if it leaves a residue, it proves that the substance is partially soluble in water. Hence we determine if it is soluble, insoluble or partially so in distilled water.

Substances soluble in water, are as follows:

Potassa. All the salts of potassa.

Soda. Do. do. do.

Ammonia. (Caustic,) and all the ordinary salts of it.

Lime. Nitrate, muriate, (chloride of calcium.)

Magnesia. Sulphate and muriate.

Alumina. Sulphate.

Iron. Sulphates and muriates of both oxides.

Substances, insoluble, or slightly soluble in water, are as follows:

Lime. Carbonate, phosphate and sulphate of.

Magnesian. Phosphate of ammonia and magnesian.

Magnesia. Carbonate, phosphate of.

Alumina, and its phosphate.

Iron, oxides, carbonate, phosphate of.

Inorganic substances found in plants, as bases, are—alumina, lime, magnesia, potash, soda, oxide of iron, oxides of manganese.

As acids—sulphuric, phosphoric, chlorine, fluorine, and iodine and bromine in sea plants.

Take a wheelbarrowful of the soil from various parts of the field, to the depth of one foot. Mix the whole, and take a portion to analyze.

Proportion of clay and sand in a soil. Take two hundred grains of well dried soil, and boil it in distilled water, until the sand appears to be divided. Let it stand for some time, and decant the liquid. Add a fresh supply of water, and boil, and decant as above, and so continue until the

clay is entirely carried off. The sand is then collected, dried and weighed. For the relative proportion of sand in fertile soils, (see sec. 309.)

Organic matter in the soil. Take two hundred grains of the dry soil, and heat it in a platinum crucible over a spirit lamp, until the black color first produced is destroyed; the soil will then appear reddish, the difference or loss in weight, will be the organic matter.

Estimation of ammonia. Put one thousand grains of the unburnt soil in a retort, cover it with caustic potash. Let the neck of the retort dip into a receiver containing dilute hydrochloric acid, (one part of pure hydrochloric acid to three parts of distilled water;) bring the neck of the retort near the liquid in the receiver, and distill off about a fourth part; then evaporate the contents of the receiver in a water bath; the salt produced will be sal ammoniac, or muriate of ammonia, of which every one hundred grains contains 32.22 grains of ammonia.

Estimation of silica, alumina, peroxide of iron, lime and magnesia.

Put two hundred grains of the dry soil in a florence flask or beaker, then add of dilute hydrochloric acid four ounces, and gently boil for two hours, adding some of the dilute acid from time to time as may be required, on account of the evaporation. Filter the liquid and wash the undissolved soil, and add the water of this washing to the above filtrate. Collect the undissolved in a filter, heat to redness and weigh; this will give clay and siliceous sand insoluble in hydrochloric acid.

Estimation of silica. Evaporate the above solution to dryness, then add dilute hydrochloric acid, the white gritty substance remaining insoluble is silica, which collect on a weighed filter, burn and weigh.

Estimation of alumina and peroxide of iron. The solution filtered from the silica is divided into two parts. One part is neutralized by ammonia, the precipitate contains alumina and peroxide of iron, and possibly phosphoric acid. It is thrown on a filter and washed, strongly dried, (not burnt) and weighed; it is now dissolved in hydrochloric acid, and the oxide of iron is precipitated by caustic potash in excess; the precipitate is washed, dried and burnt, its weight gives the oxide of iron, which taken from the above united weight of iron and alumina, will give the weight of the alumina.

The phosphoric acid here is considered too small and is neglected.

Estimation of lime. The liquid filtered from the precipitate by the ammonia, contains lime and magnesia. The lime may be entirely precipitated by oxalate of ammonia. Collect the precipitate and burn it gently and weigh. In every one hundred grains of the weight, there will be 56.29 grains of lime.

Estimation of magnesia. Take the filtered liquid from the oxalate of ammonia, and evaporate to a concentrated liquid, and when cold, add phosphate of soda and stir the solution. Let it stand for some time. Phosphate of magnesia and ammonia will separate as a white crystalline powder. Collect on a filter, and wash with cold water, and burn. In one hundred grains, there are 36.67 grains of magnesia.

Estimation of potash and soda. Take the half of the liquid. Set aside in examining for silica, (see above,) and render it alkaline to test paper by adding caustic barytes, and separate the precipitate. Again, add carbonate of ammonia, and separate this second precipitate, and evapor-

ate the liquid to dryness in a weighed platinum dish; heat the residue gently to expel the ammoniacal salts. Weigh the vessel with its contents; the excess will be the alkaline chlorides, which may be separated if required, by bi-chloride of platinum, which precipitates the potassa as chloride of potassium; one hundred parts of which contain 63.26 of potassa, and one hundred parts of chloride of sodium contain 53.29 of soda.

Estimation of Phosphoric Acid. For this we will use Berthier's method, which is founded on the strong affinity which phosphoric acid has for iron. Let the fluid to be examined contain, at the same time, phosphoric acid, lime, alumina, magnesia, and peroxide of iron. Let the oxide of iron be in excess—to the fluid add ammonia, the precipitate will contain the whole of the phosphoric acid, and principally combined as phosphate of iron. Collect the precipitate, and wash, and then treat with dilute acetic acid, which will dissolve the lime, magnesia, and excess of iron, and alumina, and there will remain the phosphate of iron or phosphate of alumina, because alumina is as insoluble as the iron in acetic acid. Collect the residue and calcine them. In every one hundred grains of the calcined matter, fifty will be phosphoric acid.

Estimation of Chlorine and Sulphuric Acid. These are found but in small quantities in soils, unless gypsum or common salt has been previously applied. Boil four hundred grains of the burnt soil in half a pint of water, filter the solution, and wash the insoluble residue with hot water, then burn, dry, weigh, and compare it with the former weight; this will give an approximate value of the constituents soluble in water. Now acidulate the filtered liquid with nitric acid, and add nitrate of silver; if chlorine is present, it will give a white curdy precipitate, which collect on a filter, wash, dry and burn in a porcelain crucible; the resulting salt, chloride of silver, contains 24.67 grains, in one hundred of chlorine.

Estimation of Sulphuric Acid. To the filtered solution, add nitrate of barytes; a white cloudiness will be produced, showing the presence of sulphuric acid. The precipitate will be sulphate of barytes, which collect, wash, and weigh as above. In one hundred grains of this precipitate, there will be 34.37 of sulphuric acid.

Estimation of Manganese. Heat the solution to near boiling, then mix with excess of carbonate of soda. Apply heat for some time. Filter the precipitate, and wash it with hot water, dry, and strongly ignite with care. The resulting salt, carbonate of manganese = MnO_2 . In every one hundred grains of this salt, there are 62.07 of protoxide of manganese.

Analysis of Magnesian Limestone.

310r. Supposed to contain carbonate of lime, carbonate of magnesia, silica, carbonic acid, iron and moisture.

Weigh one hundred grains of the mineral finely powdered, and dry it in a dish on a sand-bath or stove. Weigh it every fifteen minutes until the weight becomes constant, the loss in weight will be the hygroscopic moisture.

Otherwise. Pulverize the mineral, and calcine it in a platinum or porcelain crucible, to drive off the carbonic acid and moisture.

To determine the Silica. Take one hundred grains. Moisten it with water, and then gradually with dilute hydrochloric acid. When it

appears to be dissolved, add some of the acid and heat it, which will dissolve everything but the silica, which is filtered, washed and weighed.

To determine the Iron. Take the filtrate last used for silica. Neutralize it with ammonia, then add sulphide of ammonium, which precipitates the iron as sulphide of iron, FeS . The solution is boiled with sulphate of soda to reduce the iron to the state of protoxide. Boil so long as any odor is perceptible; then pass a current of H_2S , which will precipitate the metals of class IV. Collect the filtrate and boil it to expel the hydrosulphuric acid gas, then boil with caustic soda in excess, until the precipitate is converted into a powder.

Collect the precipitate and reduce it to the state of peroxide, by adding dilute nitric acid; then add caustic ammonia, which precipitates the iron as Fe_2O_3 , then collect and dry at a moderate heat. In every 100 parts of the dried precipitate, there are 70 of metallic iron.

To determine the Lime. Boil the last filtrate from the iron, having made it slightly acid with hydrochloric acid. When the smell of sulphide of ammonium is entirely removed, filter the solution and neutralize the clear solution with ammonia, then add oxalate of ammonia in solution, as long as it will give a white precipitate. We now have all the lime as an oxalate. Boil this solution, and filter the precipitate, and ignite; when cool, add a solution of carbonate of ammonia, and again gently heat to expel the excess of carbonate of ammonia. We now have the whole of the lime converted into carbonate of lime, which has 56 per cent. of lime. Or, dry the *oxalate* at 212° . When dry, it contains 38.4 per cent. of lime.

NOTE. If we have not oxalate of ammonia, we use a solution of oxalic acid, and add caustic ammonia to the liquid containing the lime and reagent till it smells strong of the ammonia; then we have the lime precipitated as an oxalate, as above.

If we suspect Alumina, the liquid is boiled with NO^5 to reduce the iron to a sesquioxide, (peroxide.) Then boil it with caustic potassa for some time, which will precipitate the iron as Fe_2O_3 , which collect as above.

To determine the Alumina, supersaturate the last filtrate with HCl , and add carbonate of ammonia in excess, which will precipitate the alumina as hydrate of alumina, which collect, dry and ignite; the result is Al_2O_3 = sesquioxide of alumina, which has 53.85 per cent. of alumina.

To determine the Magnesia. In determining the lime, we had in the solution, hydrochloric acid and ammonia, which held the magnesia in solution; we now concentrate the solution by evaporation, and then add caustic ammonia in excess. Phosphate of soda is then added as long as it gives a precipitate. Stir the liquid frequently with a glass rod, and let it rest for some hours. The precipitate is the double phosphate of ammonia and magnesia. Wash the precipitate with water, containing a little free ammonia, because the double phosphate is slightly soluble in water. When the prec. is dried, ignite it in a porcelain crucible, and then weigh it as phosphate of magnesia = 2MgO , PO^5 . By igniting as above, the water and ammonia are driven off, and the double phosphate is reduced to phosphate of magnesia. In every 100 grains are 17.86 of magnesia. (NOTE. This simple method is from Bowman's Chemistry.)

To determine the Carbonic Acid. Take 100 grains and put them into a bottle with about 4 ounces of water. Put about 60 grains of hydro-

chloric acid into a small test tube and suspend it by a hair through the cork in the bottle, and so arranged that the mouth of the test tube will be above the water. Let a quill glass tube pass through the cork to near the surface of the liquid in the bottle. Weigh the whole apparatus, and then let the test tube and acid be upset, so that the acid will be mixed with the water and mineral. The carbonic acid will now pass off; but as it is heavier than air, a portion will remain in the bottle, which has to be drawn out by an India-rubber tube applied to the mouth, when effervescence ceases. The whole apparatus is again weighed; the difference of the weights will be the carbonic acid.

Analysis of Iron Pyrites.

310v. This may contain gold, copper, nickel, arsenic, besides its principal ingredients, sulphur and iron, and sometimes manganese.

To determine the Arsenic. Reduce a portion of the pyrites to fine powder; heat it in a test tube in the flame of a spirit lamp. The sulphur first appears as a white amorphous powder, which becomes gradually a lemon yellow, then to tulip red, if arsenic is present.

To determine the Sulphur. One hundred grains of the pyrites are digested in nitric acid, to convert the sulphur into sulphuric acid; dilute the solution, and decant it from the insoluble residue, which consists in part of gold. If any is in the mineral, it is readily seen through a lens.

This decanted solution will contain the iron, together with oxides of copper, if any is present, and the sulphur as sulphuric acid. Evaporate the solution to expel the greater part of the nitric acid, now dilute with three volumes of water, and add chloride of barium as long as it causes a precipitate. Boil the mixture; filter, wash and ignite the precipitate, which is now sulphate of baryta, in every 100 parts of which there are 13.67 of sulphur. To this sulphur, must be added the sulphur that was found on top of the liquid as a yellow porous lump when digested with the nitric acid.

To determine the Iron. Add sulphide of ammonium as long as it will cause a precipitate of sulphide of iron = FeS , whose equivalent is 44; that is, iron 28 and sulphur 16; therefore every one hundred parts of FeS contain 63.63 of iron. But heat to redness and weigh as per oxide of iron = Fe_2O_3 . In every 100 grains there are 70 of iron.

NOTE. Sulphide of ammonia precipitates manganese.

To determine the Manganese and Iron separately. Take a weighed portion and dissolve it in aqua regia as above, evaporate most of the acid, and then dilute, leaving the solution slightly acid; pass HS through it, which will precipitate the gold, copper and arsenic, and leave the iron and manganese in solution. Collect the filtrate, to which add chlorate of potassa to peroxide of iron; now add acetate of soda, and then heat to a boiling point; this precipitates the iron, and that alone as peroxide of iron, which collect, wash, dry, weigh, and heat to redness; the result is Fe_2O_3 , having 70 per cent. of iron.

To find the Manganese, neutralize the last filtrate, and add hypochlorite of soda, let it stand for one day, then the manganese will be precipitated as binocide of manganese = MnO_2 ; collect, dry, etc. In every 100 grains of it, there are 63.63 of manganese.

Analysis of Copper Pyrites.

310v. *The moisture* is determined as in sec. 310t.

To determine the Sulphur. Proceed as in sec. 310v, by reducing 100 grains to powder, then boil in aqua regia until the sulphur that remains insoluble collects into a yellowish porous lump. Dilute the acid with three volumes of water, filter and wash the insoluble residue (consisting of sulphur and silica) until the whole of the soluble matter is separated from it. Reserve the insoluble residue for further examination.

Now evaporate the filtered solution so as to expel the nitric acid, and add some hydrochloric acid from time to time, so as to have HCl in a slight excess. From this solution precipitate the sulphur, as sulphuric acid, by chloride of barium, (as in 310v.) Collect the precipitate, wash, dry and weigh, as has been done for iron pyrites.

To determine the Copper. To the filtered solution add hydrosulphuric acid, which precipitates the copper as sulphide of copper = CuS . This precipitate is washed with water, saturated with HS. The precipitate and ash of the filter is poured into a test tube or beaker, and a little aqua regia added to oxidize the copper. Then boil and add caustic potassa, which will precipitate the copper, as black oxide of copper, CuO , having 79.84 per cent. of copper.

To determine the sulphur and siliceous matter in the above residue. Let the residue be well dried and weighed, then ignited to expel the sulphur; now weighed, the difference in weight will be the sulphur, which, added to the weight of sulphur found from the sulphate of baryta, will give the whole of the sulphur.

The siliceous matter is equal to the weight of the above residue after being ignited.

To determine the Iron. The solution filtered from the sulphide of copper is now boiled to expel the hydrosulphuric acid, filtered, and then heated with a little nitric acid to reduce the iron to a state of peroxide. To this add ammonia in slight excess, which precipitates the iron as a peroxide. This filtered, dried and weighed, will contain, in every 100 grains, 70 grains of iron; because $40 : 28 :: 100$. Here 28 is the atomic weight of iron, and 40 that of sesquioxide of iron = $\text{Fe}_2\text{O}_3 = 56 + 24 = 80$, but 80 and 56 are to one another as 40 is to 28.

Those marked with an asterisk (*) are the most delicate tests.

310w. *Sulphuret of Zinc*, (blende) may contain Iron, Cadmium, Lead, Copper, Cobalt and Nickel.

The mineral is dissolved in aqua regia. Collect the sulphur as in sec. 310t, and expel the NO_5 by adding HCl and evaporating the solution, which dilute with water, and again render slightly acid by HCl. To this acid solution (free from nitric acid) add HS, which precipitates all the copper, lead and cadmium, and leaves the iron, manganese and zinc in solution. Let the precipitate = A.

To determine the Iron, neutralize the solution with ammonia, and precipitate the iron by caustic ammonia, or better by succinate of ammonia. Collect the precipitate, and heat to redness in the open air, which will give peroxide of iron = Fe_2O_3 , which has 70 per cent. of iron.

To determine the Zinc. The last filtrate is to be made neutral, to which add sulphide of ammonium, which precipitates the zinc from magnesia,

lime, strontia or baryta, as sulphide of zinc. Pour the filtrate first on the filter, then the precipitate. Collect, dry and heat to redness, gives oxide of zinc = ZnO , having 80.26 per cent. of zinc.

We may have in the reserved precipitate A, copper, lead and cadmium.

To determine the Cadmium. Dissolve A, in NO_5 , and add carbonate of ammonia in excess, which will precipitate the cadmium. Collect the precipitate and call it B. To the filtrate add a little carbonate of ammonia, and heat the solution when any cadmium will be precipitated, which collect and add to B, and heat the whole to redness to obtain oxide of cadmium, which has 87.45 per cent. of cadmium.

To determine the Copper, make the last filtrate slightly acid. Boil the solution now left with caustic ammonia, collect and heat to redness, the result will be oxide of copper CuO , having 80 per cent. of Cu.

To determine the Lead. The lead is now held in solution, render it slightly acid and pass a current of HS , which will precipitate black sulphide of lead; if any = PbS , which collect and heat to redness to determine as oxide of lead = PbO , which has 92.85 per cent. of lead.

To separate Zinc from Cobalt and Nickel. The mineral is oxidized as above, and then precipitated from the acid solution by carbonate of soda. The precipitate is collected and washed with the same reagent, so as to remove all inorganic acids. The oxides are now dissolved in acetic acid, from which HS will precipitate the zinc as sulphide of zinc = ZnS , which oxidize as above and weigh.

To separate the oxides of Nickel and Cobalt. Let the oxides of nickel and cobalt be dissolved in HCl , and let the solution be highly diluted with water; about a pound of water to every 15 grains of the oxide. Let this be kept in a large vessel, and let it be filled permanently with chlorine gas for several hours, then add carbonate of baryta in excess; let it stand for 18 hours, and be shaken from time to time. Collect the precipitate and wash with cold water; this contains the cobalt as a sesquioxide, and the baryta as carbonate. Reserve the filtrate B. Boil the precipitate with HCl , and add SO_3 , which will precipitate the baryta and leave the cobalt in solution, which precipitate by caustic potassa, which dry and collect as oxide of nickel.

The nickel is precipitated from the filtrate B, by caustic potassa, as oxide of nickel, which wash, dry and collect as usual.

To separate Gold, Silver, Copper, Lead and Antimony.

310x. The mineral is pulverized and dissolved in aqua regia, composed of one part of nitric acid and four parts of hydrochloric acid. Decant the liquid to remove any siliceous matter. Heat the solution and add hydrochloric acid which will precipitate the silver as a chloride, which wash with much water, dry and put in a porcelain crucible. Now add the ash of the filter to the above chloride of silver, on which pour a few drops of NO_5 , then warm the solution and add a very few drops of HCl to convert the nitrate of silver into chloride of silver. Expel the acid by evaporation. Melt the chloride of silver and weigh when cooled. When washed with water any chloride of lead is dissolved; but if we suspect lead, we make a concentrated solution, and precipitate both lead and silver as chlorides by HCl ; then dissolve in NO_5 and precipitate the lead by caustic potassa as oxide of lead, leaving the silver in solution, which if acidified,

and HS passed through it, will precipitate the silver as sulphide of silver which heat to redness, and weigh as oxide of silver.

To determine the Gold. We suppose that every trace of NO_5 is removed from the last filtrate and that it is diluted. Then boil it with oxalic acid, and let it remain warm for two days, when the gold will be precipitated, which collect and wash with a little ammonia to remove any oxalate of copper that may adhere to the gold. Heat the dried precipitate with the ash of the filter to redness, and weigh as oxide of gold AuO , which has 96.15 per cent. of gold.

To determine the Copper. To the last filtrate diluted, add caustic potassa at the boiling point, which will precipitate the copper. Wash the precipitate with boiling water, dry, heat to redness, and weigh as protoxide of copper = CuO . In every 100 grains there are 79.84 grains of copper.

To separate Lead and Bismuth.

The mineral is first dissolved in NO_5 , then add SO_3 in excess, and evaporate until the NO_5 is expelled. Add water, then the lead is precipitated as sulphate of lead, which collect, etc. In every 100 grains there are 68.28 of lead.

The bismuth is precipitated from the filtrate by carbonate of ammonia. The precipitate is peroxide of bismuth = Bi_2O_3 , which collect, etc. This precipitate has 89.91 per cent. of bismuth.

To determine the Antimony. Let a weighed portion be dissolved in NO_5 . Add much water and evaporate to remove the acid, leaving the solution neutral. Now add sulphide of ammonium, which precipitates the alumina, cobalt, nickel, copper, iron and lead. Collect the filtrate, to which add the solution used in washing the precipitate. Concentrate the amount by evaporation and render it slightly acid. Then add hydrochloric acid, which precipitates the silver as a chloride, leaving the antimony in solution, which is precipitated by caustic ammonia as a white insoluble precipitate. SbO_3 , which, when dried, etc., contains 84.31 per cent. of antimony.

NOTE. The caustic ammonia must be added gradually.

For the difference between antimony and arsenic, see p. 72B79.

To determine Mercury.

310x. Mercury is determined in the metallic state as follows: There is a combustion furnace made of sheet iron about 8 inches long, 5 inches deep, and 4 inches wide. There is an aperture in one end from top to within 2 inches of the bottom, and a rest corresponding within 1 inch of the other end. A tube of Bohemian glass is opened at one end, and bent and drawn out nearly to a point at the other. The bent part is to be of such length as to reach half the depth of a glass or tumbler full of water and ice, into which the fine point of the reducing tube must be kept immersed during the distillation of the mercury. Fill the next inch to the bottom or thick end with pulverized limestone and bicarbonate of soda; then put in the mineral or mercury. Next 2 inches of quick or caustic lime, then a plug of abestoes. The tube is now in the sheet-iron box and heated with charcoal, first heating the quick lime, next the mineral, and lastly the limestone and soda. Allow the process to go on some time, until the mercury will be found condensed in the glass of water, which collect, dry on blotting paper, and weigh.—*Graham's Chemistry.*

Otherwise. Dissolve the mineral in HCl. Add a solution of protochloride of tin in Cl in excess, and boil the mixture. The mercury is now reduced to the metallic state, which collect as above.

To determine Tin.

Dissolve in HCl and precipitate with HS in excess, letting it remain warm for some hours. Collect the precipitate and roast it in an open crucible, adding a little NO₅ so as to oxidize the tin and the other metals that may be present. To a solution of the last oxide, add ammonia and then sulphide of ammonium, which will hold the tin in solution and precipitate the other metals of class 3. See p. 72B74:

If we suspect antimony in the solution, the reagent last used must be added slowly, as antimony is soluble in excess of the reagent.

W A T E R.

310z. Distilled water is chemically pure. Ice and rain water are nearly pure. Distilled water at a temperature of 60° has a specific gravity of 1000. That is, one cubic foot weighs 1000 ounces = 62½ lbs., containing 6.232 imperial gallons = 7.48 United States gallons.

NOTE. *Engineers in estimating* for public works, take one cubic foot of water = 6½ imperial gallons, and one cubic foot of steam for every inch of water.

Water, at the boiling point, generates a volume of steam = to 1689 times the volume of water used. The volume of steam generated from one inch of water will fill a vessel holding 7 gallons.

Water presses in all directions. Its greatest pressure is at two-thirds of the depth of the reservoir, measured from the top. The same point is that of percussion.

Greatest density of water is at 39° 30', from which point it expands both ways. Ice has a specific gravity of 0.918 to 0.950. The water of the Atlantic Ocean has a specific gravity of 1.027; the Pacific Ocean = 1.026; the Mediterranean (mean) = 1.0285; Red Sea, at the Gulf of Suez = 1.039.

MINERAL WATERS, are carbonated, saline, sulphurous and chalybeate.

Carbonated, is that which contains an abundance of carbonic acid, with some of the alkalis. This water reddens blue litmus, and is sparkling.

Saline, is that in which chloride of sodium predominates, and contains soda, potassa and magnesia.

Sulphurous, is known by its odor of rotten eggs, or sulphuretted hydrogen, and is caused by the decomposition of iron pyrites, through which the water passes. The vegetation near sulphur springs has a purple color.

Chalybeate, is that which holds iron in solution, and is called *carbonated* when there is but a small quantity of saline matter. It has an inky taste, and gives with tincture of galls, a pink or purple color. It is called *sulphated* when the iron held in solution is derived from iron pyrites, and is found in abundance with the smell of sulphuretted hydrogen. The chalybeate waters of Tunbridge and Bath in England, derive their strong chalybeate taste from one part of iron in 35,000 parts of water, or two grains of iron in one gallon of the water. Water travers-

ing a mineral country, is found to contain arsenic, to which, when found in chalybeate, chemists attribute the tonic properties of this water. Hoffman finds one grain of arsenic per gallon in the chalybeate well of Weisbaden. Mr. Church finds one grain of arsenic in 250 gallons of the river Whitbeck in Cumberland, England, which water is made to supply a large town.

Arsenic has been found in 46 rivers in France. The springs of Vichy, of Mont d'Or and Plombiers, contain the 125th part of a grain of arsenic in the gallon.

If lime is present, oxalate of ammonia gives a white prec.

If chloride of sodium, nitrate of silver gives a prec. not entirely dissolved in nitric acid.

If an alkaline carbonate, such as bicarbonate of lime.

Arsenic nitrate of silver gives a primrose yellow prec.

An alkaline solution of logwood, gives a violet color to the water if lime is present. The solution of logwood gives the same reaction with bicarbonate of potassa and soda. To distinguish whether lime or potassa and soda are present, we add a solution of chloride of calcium, which gives no precipitate with bicarbonate of lime.

Sulphuric acid, is present, if, after sometime, nitrate of baryta gives a prec. insol. in nitric acid.

Carbonate of lime is present, if the water when boiled appears milky. Lime water as a test, gives it a milky appearance.

Organic matter is precipitated by terchloride of gold, or a solution of acetate of copper, having twenty grains to one ounce of water. After applying the acetate of copper, let it rest for 12 hours; at the end of which time all the organic matter will be precipitated.

Organic matter may be determined by adding a solution of permanganate of potassa, which will remain colored if no organic matter is present; but when any organic substance is held in solution, the permanganate solution is immediately discolored. We make a permanganate solution by adding some permanganate of potassa to distilled water, till it has a deep amethyst red tint. We now can compare one water with another by the measures of the test, sufficient to be discolored by equal volumes of the waters thus compared.

Carbonates of lime and magnesia, also sulphate of lime, act injuriously on boilers by forming incrustations.

The presence of chloride of sodium and carbonate of lime in small quantities, as generally found in rivers, is not unhealthy.

M. Boussingault has proved that calcareous salts of potable water, in conjunction with those contained in food, aid in the development of the bony skeleton of animals. *Taylor* says that the search for noncalcareous water is a fallacy, and that if lime were not freely taken in our daily food, either in solids or liquids, the bones would be destitute of the proper amount of mineral matter for their normal development.

Where the water is pure, lead pipes should not be used, as the purest water acts the most on lead. Let there be a slip of clean lead about six to eight inches square immersed in the water for 48 hours, and exposed to the air. Let the weight before and after immersion be determined, and then a stream of sulphuretted hydrogen made to pass through the

water and then into the supposed lead solution, which will precipitate the lead as a black sulphide of lead.

Taylor says, that water containing nitrates or chlorides in unusual quantity, generally acts upon lead.

Water in passing through an iron pipe, loses some if not all of its carbonic acid, thereby forming a bulky prec. of iron, which is carried on to meet the lead where it yields up its oxygen to the lead, forming oxide of lead, to be carried over and supplied with the water, producing lead disease.

It is to be hoped that iron supply pipes or some others not oxidizable, will be used.

HYDRAULICS.

SUPPLY OF TOWNS WITH WATER.

310z. Water is brought from large lakes, rivers or wells. That from small lakes is found to be impure, also that from many rivers. A supply from a large lake taken from a point beyond the possibility of being rendered impure is preferable, provided it is not deficient in the mineral matter required to render it fit for culinary purposes. The water must be free from an excess of mineral, or organic matter, and be such as not to oxidize lead.

Solid matter in grains per gallon, are as follows in some of the principal places:

Loch Katrine in Scotland,	2	Danube at Vienna,	10.15
Loch Ness in Annandale,	2	Scheldt, Belgium,	20.88
River Thames at London,	23.36	Schuylkill, Philadelphia.	4.49
“ “ Greenwich,	27.79	Croton, N. Y.,	4.16
“ “ Hampton,	15	Chicago river,	20.75
Mean of 4 English rivers,	20.75	Lake Michigan 2 miles out,	8.01
Rhone at Lyons, France,	12.88	Cochituate at Boston,	3.12
Seine at Paris,	20	St. Lawrence, near Montreal,	11.04
Garonne at Toulon,	9.56	Ottawa, “ “	4.21
Rhine at Basle,	11.97	Hydrant at Quebec,	2.5

Water drawn from wells contains variable quantities of mineral matter, which, according to *Taylor*, is from 130 to 140 grains in wells from 40 to 60 feet deep. The artesian wells which penetrate the London clay, contain from 50 to 70 grains in the imperial gallon.

Catch basin, or water shed, is that district area whose water can be impounded and made available for water supply. One-half the rain-fall may be taken as an approximate quantity to be impounded, which is to be modified for the nature of the soil and local evaporation.

Mr. Hawkesly in England collects 48 per cent. of the rain-fall.

Mr. Stirrat in Scotland, finds 67 “ “

In Albany, U. States, 40 to 60 per cent. may be annually collected.

The engineer will consult the nearest meteorological observations.

ANNUAL RAIN-FALL.

310A*. The following table of mean annual rain-fall is compiled from authentic sources. That for the United States is from the Army Meteorological Register for 1855.

Penzance, England,	43.1	Santa Fe, New Mexico,	19.8
Plymouth, "	35.7	Ft. Deroloco, "	16.6
Greenwich, "	23.9	Ft. Yuma, "	10.4
Manchester, "	27.3	San Diego, "	12.2
Keswick, Westmoreland,	60	Monterey, "	24.5
Applegate, Scotland,	33.8	San Francisco, California,	23.5
Glasgow, "	33.6	Hancock Barracks, Maine,	37
Edinburgh, "	25.6	Ft. Independence, Mass.,	35.3
Glencose, Pentlands, Scotland,	36.1	Ft. Adams, Rhode Island,	52.5
Dublin, Ireland,	30.9	Ft. Trumbull, Connecticut,	45.6
Belfast, "	35	Ft. Hamilton, N. Y.,	43.7
Cork, "	36	West Point, "	54.2
Derry, "	31.1	Plattsburgh, "	33.4
St. Petersburg, Russia,	16	Ft. Ontario, "	30.9
Rome, Italy,	36	Ft. Niagara, "	31.8
Pisa, "	37	Buffalo, "	38.9
Zurich, Switzerland,	32.4	Ft. Mifflin, Penn.,	45.3
Paris, France,	21	Ft. McHenry, Maryland,	42
Grenada, Central America,	126	Washington City,	41.2
Calcutta, E. Indies,	77	Ft. Monroe, Virginia,	50.9
Detroit, Michigan,	30.1	Ft. Johnston, N. Carolina,	46
Ft. Gratiot, "	32.6	Ft. Moultrie, South Carolina,	44.9
Ft. Mackinaw, Michigan,	23.9	Oglethorpe, Georgia,	53.3
Milwaukee, Wis.,	30.3	Key West, Florida,	47.7
Ft. Atkinson, Iowa,	39.7	Ft. Pierce, "	63
Ft. Des Moines, "	26.6	Mt. Vernon, Alabama,	63.5
Ft. Snelling, Minnesota,	25.4	Ft. Wood, Louisiana,	60
Ft. Dodge, "	27.3	Ft. Pike, "	71.9
Ft. Kearney, Nebraska,	28	New Orleans, "	50.9
Ft. Laramie, "	35	Ft. Jessup, "	45.9
Ft. Belknap, Texas,	22	Ft. Town, Indian Territory,	51.1
Brazos Fork, "	17.2	Ft. Gibson, "	36.5
Ft. Graham, "	40.6	Ft. Smith, Arkansas,	42.1
Ft. Croghan, "	36.6	Ft. Scott, Kansas,	42.1
Corpus Christi, Texas,	41.1	Ft. Leavenworth, Kansas,	30.3
Ft. McIntosh, "	18.7	Jefferson, Missouri,	37.8
Ft. Filmore, New Mexico,	9.2	St Louis, "	42
Ft. Webster, "	14.6		

Daily supply of water to each person in the following cities:

New York, 52 gallons. Boston, 97. Philadelphia, 36. Baltimore, 25.
St. Louis, 40. Cincinnati, 30. Chicago, 43. Buffalo, 48. Albany, 69.
Jersey City, 59. Detroit, 31. Washington, 19. London, 30.

Reservoirs. The following is a list of some of the principal reservoirs with their contents in cubic feet and days' supply:

Rivington Pike, near Liverpool, 504,960,000 cubic feet, holds 150 days' supply.

Bolton, 21 millions cubic feet = 146 days' supply.

Belmont, 75 million cubic feet = 136 days' supply.

Bateman's Compensation, near Manchester, has 155 million cubic feet.

Bateman's Crowdon, near Manchester, 18,493,600 cubic feet.

Bateman's Armfield, near Manchester, 38,755,556 cubic feet.

Longendale, 292 million cubic feet = 74 days' supply.

Preston, 4 reservoirs, 26,720,000 cubic feet = 180 days' supply.

Compensation, Glasgow, 12 millions cubic feet.

Croton, New York, 2 divisions, 24 millions cubic feet.

Chicago, Illinois, the water will be, in 1867, taken from a point two miles from the shore of Lake Michigan, in a five-foot tunnel, thirty-two feet under the bottom of the Lake, thus giving an exhaustless supply of

pure water. The water now supplied is taken from a point forty-five feet from the shore, and half a mile north of where the Chicago River enters Lake Michigan, consequently the supply is a mixture of sewage, animal matter and decomposed fish, with myriads of small fish as unwelcome visitors.

CONDUITS OR SUPPLY MAINS.

310B*. Best forms for open conduits, are semi-circle, half a square, or a rectangle whose width = twice the depth, half a hexagon, and parabolic when intended for sewerage. (See sec. 133.)

Covered conduits ought not to be less than 3 feet wide and $3\frac{1}{2}$ high, so as to allow a workman to make any repairs. A conduit 4 feet square with a fall of 2 feet per mile, will discharge 660,000 imperial gallons in one hour. The conduit may be a combination of masonry on the elevated grounds, and iron pipes in the valleys; the pipes to be used as syphons.

The ancients carried their aqueducts over valleys, on arches, and sometimes on tiers of arches. They sometimes had one part covered and others open. Open ones are objectionable, owing to frost, evaporation and surface drainage.

DISCHARGE THROUGH PIPES AND ORIFICES.

310C*. *Pipes under pressure.* Pipes of potter's clay, can bear but a light pressure, and therefore are not adapted for conveying water.

Wooden Pipes, bear great pressure, but being liable to decay, are not to be recommended.

Cast Iron Pipes, should have a thickness as follows: $t = 0.03289 + 0.015 D$. Here d = diameter, and t = thickness of the metal. *D'Aubisson's Hydraulics.* $t = 0.0238, d + 0.33$. According to Weisbach.

Claudel gives the following, which agrees well with Beardmore's table of weight and strength of pipes. $t = 0.00025 h d$ for French metres. $t = 0.00008 h d$ for English feet. Here t = thickness, h = total height due to the velocity, and d = diameter.

Lead Pipes, will not bear but about one-ninth the pressure of cast iron, and are so dangerous to health, as to render them unfit to be used for drawing off rain water, or that which is deficient in mineral matter.

The pressure on the pipe at any given point, is equal to the weight of a column of water whose height is equal to that of the effective height, which is the height, h diminished by the height due to the velocity in the pipe.

$Pressure = h - 0.15,536 v^2$. Here v is the theoretical velocity.

Torricellis' Fundamental Formula, is

$V = \sqrt{2 g h}$ for theoretical velocity.

$v = m \sqrt{2 g h}$ for practical or effective velocity.

The value of $2g$ is taken at 64.403 as a mean from which it varies with the latitude and altitude.

The value of g can be found for latitude L , and altitude A , assuming the earth's radius = R .

$$g = 32.17 \left(1.0029 \cos. 2 L \right) \times \left(1 - \frac{2 A}{R} \right)$$

$$g = 20887600 \text{ (1.0016 Cos. } 2 \text{ L)}$$

$$v = m \sqrt{2gh} = 8.025 m \sqrt{h} = \text{mean velocity.}$$

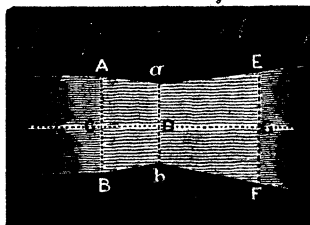
$$Q = 8.025 A m \sqrt{h} = \text{discharge in cubic feet per second.}$$

$$A = \frac{Q}{8.025 m \sqrt{h}} = \text{sectional area.}$$

$$\sqrt{h} = \frac{Q}{8.025 m A} \text{ from which } h \text{ is found.}$$

The value of m , the coefficient of efflux is due to the *vena contracta*. Its value has been sought for by eminent philosophers with the following result: As the prism of water approaches an outlet, it forms a contracted vein, (*vena contracta*) making the diameter of the prism discharge less than that of the orifice, and the quantity discharged consequently less by a multiplier or coefficient, m . The value of m is variable according to the orifice and head, or charge on its centre.

Vena Contracta. The annexed figure shows the proportions of the contracted vein for circular orifices, as found by Michellotti's latest experiments. A B is the entrance, and $a b$ the corresponding diameter at outlet; that is the theoretical orifice, A B, is reduced to the practical or actual one, $a b$. When A B = 1, then C D = 0.50, and $a b = 0.787$; therefore the area of the orifice at the side A B = $1 \times .785$ and that at $a b = .787^2 \times 0.7854$; that is the theoretical is to the actual as 1 is to 0.619. $\therefore m = 0.619$.



The values of m have been given by the following:

Dr. Bryan Robinson, Ireland, in 1739,	gives	$m = 0.774$.
Dr. Mathew Young, do.	1788,	" .623.
Venturi, Italy,		" .622.
Abbe Bossuet, France,		" .618.
Michellotti, Italy,		" .616.
Eytelwein, Germany,		" .618.
Castel, France,	1836,	" .644.
Marriot, do		" .692.
Rennie, England,		" .625.
Xavier, France,		" .615.

NOTE. It is supposed that Dr. Robinson used thick plates, chamfered or rounded on the inside, thereby making it approach the *vena contracta*, and consequently increasing the value of m or coefficient of discharge.

Rejecting Robinson and Marriot's, we have a mean value of $m = 0.622$, which is frequently used by Engineers.

Taking a mean of Bossuet, Michellotti, Eytelwein and Xavier, we find the value of $m = 0.617$, which appears to have been that used by Neville in the following formulas, where $A =$ sectional area of orifice, $r =$ radius, Q discharge in cubic feet per second, $h =$ height of water on the centre of the orifice, and $m = 0.617 =$ coefficient of discharge.

When $h = r$, then $Q = 8.025 m \sqrt{H} \times .960 A$.

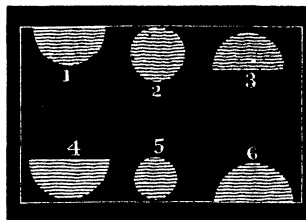
Do.	1.25 r,	do.	do.	.978 A.
Do.	1.5 r,	do.	do.	.978 A.
Do.	1.75 r,	do.	do.	.989 A.
Do.	2 r,	do.	do.	.992 A.
Do.	3 r,	do.	do.	.996 A.
Do.	4 r,	do.	do.	.998 A.
Do.	5 r,	do.	do.	.9987 A.
Do.	6 r,	do.	do.	.9991 A.

Hence it appears, that when $h = r$, the top of the orifice comes to the surface, and that when h becomes greater or equal to $8 r$, that the general equation $Q = 8.03 m \sqrt{H} \times A$, requires no modification.

The following 6 formulas are compiled from Neville's Hydraulics.

In the annexed figure, 1, 3, 4 and 6 are semi-circular, and 2 and 5 are circular orifices.

The value of Q may be found from the following simple formulas, where A is the area of each orifice, and $m = 0.617$ = the coefficient of efflux.



$$1. Q = 3.0218 A \sqrt{r.}$$

$$2. Q = 4.7553 A \sqrt{r.}$$

$$3. Q = 3.6264 A \sqrt{r.}$$

$$4. Q = 4.9514 \sqrt{h} \times A \cdot \left(1 + \frac{1}{4.712} \cdot \frac{r}{h} - \frac{1}{32} \cdot \frac{r^2}{h^2}\right)$$

$$5. Q = 4.9514 \sqrt{h} \times A \cdot \left(1 - \frac{1}{32} \cdot \frac{r^2}{h^2} - \frac{5}{1024} \cdot \frac{r^4}{h^4}\right)$$

$$6. Q = 4.9514 \sqrt{h} \times A \cdot \left(1 - \frac{1}{4.712} \cdot \frac{r}{h} - \frac{1}{32} \cdot \frac{r^2}{h^2}\right)$$

Adjutages, with cylindrical tubes, whose lengths = $2\frac{1}{2}$ times their diameters, give $m = 0.815$.

Michellotti, with tubes $\frac{1}{2}$ an inch to 3 inches diameter and head over centre of 8 to 20 feet, found $m = 0.813$.

The same result has been found by Bidone, Eytelwein and D'Aubisson.

Weisbach, from his experiments, gives $m = 0.815$. Hence it appears that cylindrical tubes will give 1.325 times as much as orifices of the same diameter in a thin plate.

For tubes in the form of the contracted vein, $m = 1.00$.

For conical tubes converging on the exterior, making a converging $<$ of $18\frac{1}{2}^\circ$, $m = 0.95$.

For conical diverging the narrow end toward the reservoir and making the diverging $< = 5^\circ 6'$, $m = 1.46$, and the inner diameter to the outer as 1 is to 1.27.

NOTE. The *adjutage* or tube, must exceed half the diameter (that length being due to the contracted vein) so as to exceed the quantity discharged through a thin plate.

$$\text{Circular Orifices. } Q = 3.908 d^2 \sqrt{h.}$$

$$\text{Cylindrical adjutage as above. } Q = 5.168 d^2 \sqrt{h.}$$

Tube in the form of vena contracta. $Q = 5.673 d^2 \sqrt{h}$.

In a compound tube, (see fig., sec. 310c*) the part A a b B is in the form of the contracted vein, and a b E F a truncated cone in which D G = 9 times a b and E F = 1.8 times a b. This will make the discharge 2.4 times greater than that through the simple orifice. (See Byrne's Modern Calculator, p. 321.)

Orifices Accompanied by Cylindrical Adjutages.

When the length of the adjutage is not more than the diameter of the orifice, then $m = 0.62$.

Length 2 to 3 times the diameter, $m = 0.82$.	36 times $m = 68$.
Do. 12 do. $m = .77$.	43 " $m = 63$.
Do. 24 do. $m = .73$.	60 " $m = 60$.

310b. Orifices Accompanied with Conical Converging Adjutages.*

When the adjutage converges towards the extremity, we find the area of the orifice at the extremity of the adjutage the height h of the water in the reservoir above the same orifice. Then multiply the theoretical discharge by the following tabular coefficients or values of m :

Let A = sectional area, then $Q = m A \sqrt{2gh} = 8.03 m A \sqrt{H}$.

Angle of Convergence	Coefficients of the		Angle of Convergence	Coefficients of the	
	Discharge.	Velocity.		Discharge.	Velocity.
0° 0'	.829	.830	13° 24'	.946	.962
1 36	.866	.866	14 28	.941	.966
3 10	.895	.894	16 36	.938	.971
4 10	.912	.910	19 28	.924	.970
5 26	.924	.920	21 00	.918	.974
7 52	.929	.931	23 00	.913	.974
8 58	.934	.942	29 58	.896	.975
10 20	.938	.950	40 20	.869	.980
12 40	.942	.955	48 50	.847	.984

The above is Castel's table derived from experiments made with conical adjutages or tubes, whose length was 2.6 times the diameter at the extremity or outlet. In the annexed figure A C D B represents Castel's tube where $m n$ is 2.6 times C D and angle A O B = $\angle$ of convergence.

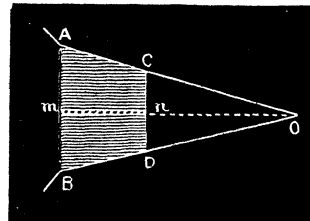
NOTE. It appears that when the angle at O is $13\frac{1}{2}$ degrees the coefficient of discharge will be the greatest.

The discharge may be increased by making $m n$ equal to C D, A B = 1.2 times C D, and rounding or chamfering the sides at A and B.

In the next two tables, we have reduced Blackwell's coefficient from minutes to seconds, and call $C = m$. $Q = 8.03 m A \sqrt{h}$ or $Q = C A \sqrt{h}$, where C is the value of 8.03 m in the last column. h is always taken back from the overfall at a point where the water appears to be still.

Experiments 1 to 12, by Blackwell, on the Kennet and Avon Canal.

Experiment 13, by Blackwell and Simpson, at Chew Magna, England.



310E*. OVERFALL WEIRS, COEFFICIENT OF DISCHARGE.

No.	Description of Overfall.	Head in inches.	Value of m	Value of $8.08 m = C$.
1	Thin plate 3 feet long.	1 to 3	.440	3.533
	" " "	3 to 6	.402	3.228
2	" 10 feet long.	1 to 3	.501	4.023
	" " "	3 to 6	.435	3.493
	" " "	6 to 9	.370	2.971
3	Plank 2 inches thick with a notch 3 feet long.	1 to 3	.342	2.746
	" " "	3 to 6	.384	3.083
	" " "	6 to 10	.406	3.260
4	Plank 2 in. thick, notch 6 ft	1 to 3	.359	2.883
	" " "	3 to 6	.396	3.179
	" " "	6 to 9	.392	3.148
	" " "	9 to 14	.358	2.878
5	Pl'k 2 in. thick, notch 10 ft.	1 to 3	.346	2.778
	" " "	3 to 6	.397	3.191
	" " "	6 to 9	.374	3.003
	" " "	9 to 14	.336	2.698
6	Same as 5, with wing walls	1 to 2	.476	3.822
	" " "	4 to 5	.442	3.549
7	Overfall with crest 3 feet. Wide sloping 1 in 12—3 ft. Long like a weir.	1 to 3	.342	2.746
	" " "	3 to 6	.328	2.634
	" " "	6 to 9	.311	2.497
8	Same as 7, but slopes 1 in 18	1 to 3	.362	2.907
	" " "	3 to 6	.345	2.737
	" " "	6 to 9	.332	2.666
9	Same as 7 & 8 but 10 ft long	1 to 4	.328	2.634
	" " "	4 to 8	.350	2.810
10	Level crest 3 ft w. & 6 long	1 to 3	.305	2.449
	" " "	3 to 6	.311	2.497
	" " "	6 to 9	.318	2.553
11	" " "	3 to 7	.330	2.649
	" " "	7 to 12	.310	2.489
12	Same as 11 but 10 ft. long.	1 to 5	.306	2.457
	" " "	5 to 8	.327	2.626
	" " "	8 to 10	.313	2.513
13	Overfall bar 10 feet long	1 to 3	.437	3.509
	And 2 inches thick.	3 to 6	.499	4.007
	" " "	6 to 9	.505	4.055

BLACKWELL'S SECOND EXPERIMENTS.

Overfall of cast iron, 2 inches thick, 10 ft. long, square top. Canal had wing walls, making an angle of 45 degrees.

Head in feet.	Coeff. m	Head in ft.	Coeff. m	Head in ft.	Coeff. m
.083 to .073	.591	.344	.743	.500	.749
.083 to .088	.626	.359	.760	.516	.748
.182 to .187	.682	.365	.741	.521	.747
.229	.665	.361	.750	.578	.772
.244	.670	.375	.725	.639	.717
.240	.655	.416	.780	.667	.802
.242	.653	.423	.781	.734	.737
.245	.654	.451	.749	.745	.750
.250 to .252	.725	.453	.751	.750	.781
.333	.745	.495	.728		

From the above we have a mean value of $m = 0.723$.

The reservoir used on the Avon and Kennet canal, in England, contained 106,200 square feet, and was not kept at the same level, but the quantity discharged for the experiment was not more than 444 cubic feet, which would reduce the head but .05 inch. In the Chew Magna we have an area of 5717 square feet kept constantly full by a pipe 2 inches in diameter from a head of 19 feet. The inlet of the pipe to the overfall being 100 feet, consequently the water approaches the fall with a certain degree of velocity, which partially accounts for the difference in value of m , in experiments 13 and 5.

Poncelet and Lebros' experiments on notches, 8 inches long, open at top:

Size of Notches.	Coefficient m .	Size of Notches.	Coefficient m .
8 × 0.4	.636	8 × 3.2	.595
8 × 0.8	.625	8 × 4.	.592
8 × 1.2	.618	8 × 6.	.590
8 × 1.6	.611	8 × 8.	.585
8 × 2.4	.601	8 × 9.	.577

From these small notches we have a mean value of $m = .603$.

Du Buat's experiments on notches 18.4 long, give a mean coefficient $m = .632$.

Smeaton and Brindley, for notches 6 inches wide and 1 to 6½ high, give $m = .637$.

Rennie, for small rectangular orifices, gives as follows:

Head 1 to 4 feet, orifice 1 inch square, mean value of $m = .613$.

“ “ “ 2 inches long and ½ high, $m = .613$.

“ “ “ 2 inches long and ⅙ deep, $m = .632$.

The following table is from *Poncelet and Lebros' experiments on covered orifices in thin plates*. Width of the orifice .20 metre (about 8 inches) $l =$ length, and $h =$ height of the orifice.

310r*.

HEIGHT OF THE ORIFICES.

Head on centre of orifice.	0.20 m $l = h$.	0.01 m $l = 2h$	0.05 m $l = 4h$	0.08 m $l = 6.7h$	0.02 m $l = 10h$	0.01 m $l = 20h$
m	m	m	m	m	m	m
0.02					.660	.698
.03				.688	.660	.691
.04			.612	.640	.659	.685
.05			.617	.640	.659	.682
.06		.590	.622	.644	.658	.678
.08		.600	.626	.639	.657	.671
.10		.605	.628	.638	.655	.667
.12	.572	.609	.630	.637	.654	.664
.15	.585	.611	.631	.635	.653	.660
.20	.592	.613	.634	.634	.650	.655
.30	.598	.616	.632	.632	.645	.650
.40	.600	.617	.631	.631	.642	.647
.50	.602	.617	.631	.630	.640	.643
.70	.604	.616	.629	.629	.637	.638
1.00	.605	.615	.627	.627	.632	.627
1.30	.604	.613	.623	.623	.625	.621
1.60	.602	.611	.619	.619	.618	.616
2.00	.601	.607	.613	.613	.613	.613
3.00	.601	.603	.606	.607	.608	.609

Here the water takes the form of the hydraulic curve, nearly that of a parabolic, and its sectional area $= \frac{2}{3} l h$. The co-efficient increases as the orifice approaches the sides or bottom.

Let C = coëff. of perfect contraction, and C' = coëff. of partial contraction, then $C' = C + .04$ n.—*Neville*.

The presence of a *courseir*, mill-race, or channel, has no sensible effect on the discharge, when the head on its centre is not below .50 to .60 metres, for orifice of .20 to .15 metres high, .30 to .40 for .10 metres high, and .20 for .05 metres high.

The charge on the centre is seldom below the above.—*Morin's Aide Memoire*, p. 27.

310f. Example 10: From *Neville's Hydraulics*, p. 7.—What is the discharge in cubic feet per minute from an orifice 2 ft. 6 in. long, and 7 in. deep; the upper edge being 3 in. under the surface of apparent still water in the reservoir.

$l h = 2.5 \text{ ft.} \times 7'' =$ area, S of orifice $= 1.458$ square feet.

$H =$ half of $7'' + 3 = 6.5 \text{ in.} = 0.541666 \text{ ft.} =$ surface of the water in the reservoir above the centre of the orifice. The square root of

$0.541666 = \sqrt{H} = 0.736$. Head on centre of orifice $= 6.5 \text{ in.} = 165$ metres. Ratio of length of orifice to its height $= 4$. Then opposite, 165 metres, and under $l = 4 h$, find $m = 0.616$

$Q = 8.03 \times 0.616 \times 1.458 \times 0.736 =$ cubic ft. per second.

$Q = 481.8 \times 0.616 \times 1.458 \times 0.736 =$ cubic ft. per minute.

Neville makes $m = 0.628$, and $Q = 320.4$ cubic feet.

M. Boileau, in his *Traite de la Mesure des eaux courantes*, (Paris, 1854,) recommends Poncelet and Lebros' value of m in the general formula.

$Q = m A \sqrt{2 g h}$ or $Q = m l h \sqrt{2 g h}$

Complete contraction is when the orifice is removed 1.5 in. to twice its lesser diameter of the fluid vein.

The French make $m = .625$ for sluices near the bottom, discharges either above or under the water.

Castel has found that 3 sluices in a gate did not vary the value of m .

310g. Let $R = h y d$, mean depth; $V =$ surface velocity, by Sec. 312; $D =$ diam.; $r =$ radius of circular orifices; $v =$ mean, and $w =$ bottom velocities; $Q =$ discharge in cubic feet per second; $T =$ time in seconds; $A =$ area of section of conduit; $I =$ the head; per unit $=$ height divided by the horizontal distance between the reservoir and out-let.

$v = 0.90 V$ for rectangular canals, and $v = 0.903 V$ for those with earthen slopes.—*Boileau*.

$v = .80 V$ for large channels, by *Prony*.

$v = 0.835 V$ for large channels, by *Ximes, Funk, and Bruning*.

$V =$ surface, $W =$ bottom velocities.

$v = 0.80 V$, and $W = .60 V$, by Conference on Drainage and Irrigation at Paris in 1849 and 1850.

$Q = 8.025 m A \sqrt{h}$ is the general formula where $A =$ sectional area.

$Q =$ quantity in cubic feet; $h =$ height of reservoir; $m =$ co-eff. of efflux.

$Q = 8.025 m A T \sqrt{h}$ in time T .

$R I = 0.00002427 V + 0.000111416 V^2$ all in feet, *Eytkewin*; from which he gives $V = \frac{10}{11} \sqrt{R I}$ in which formulas he puts $R = h y d$, mean depth, $f =$ twice the fall in feet per mile, and $I =$ inclination, $=$ head divided by the length.

$V = \frac{10}{11} \sqrt{Rf}$ is used by Beardmore and many Engineers.

310g. For clear, straight rivers, with average velocities of 1.5, *Neville* gives $V = 92.3 \sqrt{R \text{ I}}$, and for large velocities $V = 93.3 \sqrt{R \text{ I}}$. He says that co-effts. decrease rapidly when velocities are below 1.5 ft. per second. In his second edition of his valuable treatise on hydraulics, he states that the best formula proved by experiments for discharges over weirs is,

$$Q = 1.06 (3h + V a^2)^{2/3} - V a^3. \quad \text{Here } V a = \text{velocity of approach.}$$

310h. *M. Boileau*, in his *Traites de la Mesure des eaux courantes*, p. 345: For discharge through orifices,

O = sectional area of reservoir at still water, h = diff. of level between the summit of the section O and that of the section (*remous d' aval*), where the ripple begins.

$$Q = A \sqrt{2g \frac{h}{1 - \frac{A^2}{O^2}}} = 8.025 A \sqrt{\frac{h O^2}{O^2 - A^2}}$$

In his tables he makes the value of m , coëff. of contraction for short *remous*, or eddy, = 0.622, 0.600 when it attains the summit, and 0.688 when the orifice is surrounded by the *remous*.

310i. Let Q = the quantity in feet per second.

$Q = 8.025 m \sqrt{h}$ = effective discharge in cubic ft. per second, m = variable.

$Q = 4.879 A \sqrt{h}$ orifice surrounded on all sides, $m = 0.608$

$Q = 5.048 A \sqrt{h}$ orifice surrounded on three sides, $m = 0.629$

$Q = 5.489 A \sqrt{h}$ orifice coincides with sides and bottom, $m = 0.684$

$Q = 5.939 A \sqrt{h}$ as last sluice makes angle 60° against stream, $m = 0.740$

$Q = 6.420 A \sqrt{h}$ as last but. sluice makes the angle 45° , $m = 0.800$

$Q = 5.016 A \sqrt{h}$ sluice vertical, orifice near the bottom, $m = 0.625$

$Q = 4.253 A \sqrt{h}$ 2 sluices, or orifices, within 10 ft. of each other, $m = 0.530$

$Q = 6.019 A \sqrt{h}$ the flood gates make 160° with the current, and $m = 0.750$ that there are 3 sluices guarded to conduct the water into the buckets of a water wheel = sum of the areas.

$T v = 5.35 \sqrt{h}$ = mean vel. for regular orifices, open at top, and is the time required to empty a given vessel when there is no efflux, and is double the time required to empty the same when the vessel or reservoir is kept full.

$$T = \frac{A \sqrt{h}}{4.013 m S} \quad \text{Where } S = \text{sectional area of orifice, and } A = \text{that of the reservoir.}$$

$$t = A \left\{ \frac{\sqrt{H} - \sqrt{h}}{4.013 m S} \right\} = \text{time required to fall a given depth, } H - h$$

$$Q = 8.025 t m S \left\{ \frac{8.025 t m S}{4 A} + \sqrt{h} \right\} = \text{discharge in time } t.$$

$Q = 8.025 m S \sqrt{H - h}$ when reservoir A discharges into A' under water.

$$T = \frac{A \sqrt{H}}{4.013 m S} = \text{time required to fill the inferior A'.$$

$$t = \frac{A \cdot A' \sqrt{H - h}}{4.013 m S \sqrt{A - A'}} \quad \text{time to bring both to the same level in canal locks.}$$

$V = 5.35 \sqrt{(h + 0.0349410 w^2)}$. Here the water comes to the reservoir with a given velocity, w .

310i. For *D'Arcy's Formula*, see p. 264.

He has given for $\frac{1}{2}$ inch, pipes $m = 65.5$ and $v = 65.5 \sqrt{r s}$

For 1" diameter $v = 80.3 \sqrt{r s} = m \sqrt{r s}$

2", $m = 94.8$, 4" $m = 101.7$, 6" $m = 105.3$

for 9", $m = 107.8$, 12" $m = 109.3$, 18" $m = 110.7$

24" diam. $v = 111.5 \sqrt{r s} = m \sqrt{r s}$

for large pipes $v = \left(\frac{r s}{0.00007726} \right)^{\frac{1}{2}} = 118 \sqrt{r s}$

310i. Neville's general formula for pipes and rivers:

$v = 140 (r i)^{\frac{1}{2}} - (r i)^{\frac{1}{3}}$ here $r = h y d$, mean depth, and $i =$ inclination.

Fitches, in Lowell, Mass., has found for over falls, $m = .623$. (See his valuable experiments made in Lowell.)

Thompson, of Belfast College, Ireland, has found from actual experiments that for triangular notches, $m = 0.618$, and $Q = 0.317 h^{5/3}$ = cubic feet per minute, and h = head in inches.

M. Girard says it is indispensable to introduce 1.7 as a co-efficient, due aquatic plants and irregularities in the bottom and sides of rivers. Then the hydraulic mean depth (see Sec. 77,) is found by multiplying the wetted perimeter by 1.7 and dividing the product into the sectional area.

A velocity of $2\frac{1}{2}$ feet per second in sewers prevents deposits.—*London Sewerage*.

310j. *Spouting Fluids*.—Let T = top of edge of vessel, and B = bottom, O = orifice in the side, and B S = horizontal distance of the point where the water is thrown. (See fig. 60.)

$B S = 2 \sqrt{T O}$, $O B = 2 O E$, by putting O E for the ordinate through O, making a semi-circle described on F B.

310k. On the application of water as a motive power: Q = cubic ft. per minute, h = height of reservoir above where the water falls on the wheel, P = theoretical horse-power.

$$P = 0.00189 Q h, \text{ and } Q = \frac{528 P}{h}$$

Available horse-power = 12 cubic ft., falling 1 ft. per second, and is generally found = to 66 to 73 per cent. of the power of water expended. Assume the theoretical horse-power as 1, the effective power will be as follows:

Over-shot wheels = .68	For turbine wheels, .70
Under-shot wheels, .35	For hydraulic rams in raising water, .80
Breast wheels, .55	Water pressure engines, .80
Poncelet's under-shot .60	High breast wheels, .60

Let P = pressure, in lbs., per square inch.

$$P = Q, 4333 h \text{ and } h = 2.31 p$$

$$P = .00123 Q h \text{ for over-shot wheels, and } Q = 777 P \text{ divided by } h$$

$$P = .00113 Q h \text{ for high-breast wheels, and } Q = 882 P \text{ divided by } h$$

$$P = .00101 Q h \text{ for low-breast wheels, and } Q = 962 P \text{ divided by } h$$

$$P = .00066 Q h \text{ for under-shot wheels, and } Q = 1511 \text{ divided by } h$$

$$P = .00113 Q h \text{ for Poncelet's undershot wheels, and } Q = 822 \text{ divided by } h$$

For under-shot wheels, velocity due to the head $\times 0.57$ will be equal to the velocity of the periphery, and for Poncelet's, 0.57 will be the multiplier.

310j. <i>High-pressure turbines for every 10-horse power.</i>							
$h =$	30	40	50	60	70	80	90
$Q =$	4.2	3.1	2.5	2.1	1.8	1.6	1.4
$V =$	36	42	47	51	55	59	63
							66

We have seen, S.-E. of Dedham, in Essex, England, a small stream collected for a few days, in a reservoir, thence passed on an over-shot wheel, and again on an undershot wheel. If possible, let the reservoirs be surrounded by shade trees, to prevent evaporation.

310k. *Artesian Wells* may be sunk and the water raised into tanks to be used for household purposes, irrigating lands, driving small machinery, and extinguishing fires.

310l. *Reservoirs* are collected from springs, rivers, wells, and rain-falls, impounded on the highest available ground, from whence it may be forced to a higher reservoir, from which, by gravitation, to supply inhabitants with water.

310p. *Land and City Drainage.*

In draining a hilly district.—A main drain, not less than 5 ft. deep, is made along the base of the hill to receive the water coming from it and the adjacent land; secondary drains are made to enter obliquely into the main, these ought to be 4 to 5 ft. deep, filled with broken stones to a certain height; tiles and soles, or pipes. The first form is termed French draining; the last two mentioned are now generally used. In 1838 to 1842 we have seen, near Ipswich, England, drains made by digging 4 feet deep, the bottom scooped 2 to 3 inches and filled with straw made in a rope form, over this was laid some brushwood, then the sod, and then carefully filled.

The French drains were sometimes 15 inches deep, 5 inches at bottom and 8 inches at top, all filled with stone, then covered with straw and filled to the top with earth.

In tile draining the sole is about 7 inches wide, always $\frac{3}{4}$ in. on each side of the tile, and is about 12 to 15 inches long, its height is to be one-fourth its diameter. The egg shape is preferable. Never omit to use the tile, let the ground be ever so hard.

Pipe Drains.—Pipes of the egg shape are the best; pipes 2 to 4 in. diameter have a 4 in. collar. In retentive land put 4 feet deep and 27 feet apart; when $3\frac{1}{2}$ feet deep, put 33 feet apart.

From the best English sources we find the comparative cost. $2\frac{1}{2}$ ft. deep cost $3\frac{1}{2}$ pence, add $1\frac{1}{2}$ pence for every additional 6 inches in depth. Profit by thorough drainage is 15 to 20 per cent. See Parliamentary Report.

310q. *In draining Cities and Towns* our first care is to find an outlet where the sewage can be used for manure, and to avoid discharging it into sluggish streams. The result of draining into the river Thames, and the Chicago river with its far-famed Healy slough ought to be sufficient warning to Engineers to beware of like results. (See Sec. 310j.) Where the city or town authorities are not prepared to use the sewage as a fertilizer, and that there is a river near, or through it, let there be intercepting sewers, egg-shaped, with sufficient fall to insure $2\frac{1}{2}$ feet per second, which in London is found sufficient to prevent deposit; should not exceed $4\frac{1}{2}$ feet per second. When these main sewers get to a considerable depth, the sewage is lifted from these into small, covered res-

ervoirs, thence to be conveyed to another deep level, and so on until brought far enough to be discharged into the river, or some outlet from which it cannot return. But we hope it will not be wasted; the supply of Guano will fail in a few years, then the people will have to depend on the home supply.

Sewers under 15 inches diameter are made of earthenware pipes, with collars, laid in cement; 2 foot diameter are 4 inches, or half a brick, thick; 3 to 5 feet, 8 inches thick; 6 to 8 feet, 12 inches thick, according to the nature of the earth. Where the soil is quick-sand, the bottom ought to be sheeted, to prevent the sinking of the sewer.

As the sewers are made, connecting pipes are laid for house drainage at about every 20 feet, and man-holes at proper intervals to allow cleansing, flushing, and repairing. A plat is on record, showing the location of each sewer, with its connections, man-holes, and grade of bottom, to guide house and yard drains or pipes, whose fall is one-quarter inch per foot, in Chicago.

310Q. *Irrigation of Land.*

In wet districts the land is cut up in about 10-acre tracts; the ditches deep; ponds made at some points to collect some of the water, these ponds to be surrounded by a fence and shade trees, such as willow and poplar, a place on the North side of it may be sloped, and its entrance well guarded with rails, so that cattle may drink from, but not wade in, the pond, which may be of value in raising fish.

$v = 55 \sqrt{2af}$ and $Q = va$. Here v = vel. in feet, a = area, and f = fall in feet per mile.

In irrigating, the land is laid off and levelled so that the water may pass from one field to another, and may be overflowed from sluices in canals fed from a reservoir or river. The water from a higher level, as reservoir, may be brought in pipes to a hydrant, where the pressure will be great enough to discharge, through a hose and pipe, the required quantity in a given time. Water or sewage can be thus applied to 10 acres in 12 hours by one man and two boys.

The profit by irrigation is very great,—witness the barren lands near Edinburgh, in Scotland, and elsewhere.

In England, on irrigated land, they grow 50 to 70 tons of Italian rye grass per acre. Allowing 25 gallons of water to each individual will not leave the sewage too much diluted, and 60 to 70 persons will be *sufficient for one acre*, applied 8 times a year. At the meeting of the Social Science Association in England, in 1870, it was decided that the sewage must be taken from the fountain head, as they found it too much diluted, and that alum and lime had been used to precipitate the fertilizing matter, but had failed. They estimated the value due to each person at $8\frac{1}{3}$ shillings, but in practice realized but 4 to 5 shillings.

Mr. Rawlinson recommended its application diluted; others advocated the dry earth closet system, which in small towns is very applicable, owing to the facility of getting the dry earth and a market for the soil.

310R. The supply of guano will, in a few years, be exhausted, then necessity will oblige nations to collect the valuable matter that now is wasted. See Sec. 310I.

310s. *On the Steam Engine.*

H = horse-power capable of raising 33000 pounds 1 ft. high in 1 minute.

P = pressure in pounds per square inch.

D = diameter of cylinder piston in inches.

A = area of cylinder or its piston.

S = length of stroke, and 2 S = total length travelled.

R = number of revolutions per minute.

V = mean vel. of piston in feet per minute.

Q = total gallons (Imperial) raised in 24 hours.

q = quantity raised by each stroke of the piston.

C = pounds of coal required by each indicated horse-power.

$$H = \frac{2 S A P R}{33000} = \text{indicated horse-power.}$$

$$H = \frac{D^2 \sqrt[3]{S}}{15.6} \text{ indicated horse-power for high-pressure engines.}$$

$$D = \frac{15.6 H}{\sqrt[3]{S}} \text{ and } V = 12S \sqrt[3]{S}$$

$$H = \frac{D^2 \sqrt[3]{S}}{47} \text{ for condensing engines, from which we have}$$

$$D = \frac{\sqrt[3]{47 H}}{\sqrt[3]{S}} \text{ and } V = 12S \sqrt[3]{S}$$

$$\text{Admiralty Rule. } H = \frac{D^2 V}{6000} = \text{nominal horse-power.}$$

The American Engineers add one-third for friction and leakage.

Example. The required gallons in 12 hours = 3,000,000; Stroke, 10 feet; number of strokes per minute = 12; time in minutes = 1440. From the above, we find $q = 173.6$ Imperial gallons; $d = 22.6$ inches—the diameter of the pump, as taken by the American engineers; $d = 22$, as taken by the English.

For much valuable information on the steam engine, see Appleton's (Byrne's) Dictionary of Mechanics, and Haswells' tables.

Average duty of a Cornish engine is 70 million lbs., raised one foot high, with 112 lbs. of bituminous coal.

Example. From Pole on the Cornish Engine, as quoted by Hann on the Steam Engine.

Cylinder, 70 inches diameter; stroke, 10 feet; pressure per square inch, 45 lbs. during one-sixth the stroke, and during the remainder the steam is allowed to expand.

$$70 \times 70 \times 0.7854 = \text{area of piston} = 3858 \text{ square inches.}$$

$$\frac{3858 \times 45}{6} = \text{pounds raised one foot high} = 288,600.$$

This is the work performed before the steam is cut off.

To find the work done by expansion.—Find from a table of Hyperbolic Logarithms for $C = 1.7916$, which, multiplied by the work done before the steam is cut off, will give the work required, that is, 1.7916×288600

$$\text{Work done after the steam is cut off, } 517102$$

310T. *Pressure of Fluids and Retaining Walls.*

(DEF.—Retaining Wall is that which sustains a fluid, or that which is liable to slide.)

310. *The Centre of Pressure* is that point in the surface pressed by any fluid, to which, if the whole pressure could be applied, the pressure would be the same as if diffused over the whole surface.

If to this centre a force equal to the whole pressure be applied, it will keep it in equilibrium.

Against a rectangular wall the centre of pressure is at two-thirds of the height from the top, and the

Pressure $P = \frac{h^2}{2} \cdot l w$. Here w = specific gravity of the fluid, and l the length pressed.

In a cylindrical vessel or reservoir the same formula will hold good, by substituting the circumference for the length, l , of the plane.

Example.—For a lock-gate 10 ft. long, 8 ft. deep, the pressure
 $P = \frac{64}{2} \times 10 \times 62.5 = 20,000$ pounds.

Example.—For a circular reservoir, diameter 20 ft., depth 10 ft., filled with water, we have

$P = \frac{10 \times 10 \times 20 \times 3.1416 \times 62.5}{2} = 196,350$ lbs., the pressure on the sides of the reservoir.

The pressure on the bottom = $20 \times 20 \times .7854 \times 62.5 = 19,635$ lbs.

Total pressure, 215,985 lbs.

Dams are built at right angles to the stream entering the reservoir. All places of a porous nature are made impervious to water by clay or masonry laid in cement; *top* to be 4 ft. above the water; *width*, in ordinary cases, equal to one-third the height; the *inner* slope, next the water, to be 3 to 1; the outer slope 2 to 1. *In low Dams*, width at top equal to the height.

Dams, in Masonry, by the French Engineers, *Morin* and *Rondelet*, at bottom $0.7 h$, at middle $0.5 h$, and top $0.3 h$.

310L. Thickness of rectangular walls is found from

$t = 0.865 (H - h) \cdot \sqrt{\frac{1000}{w}}$ Here 1000 = weight of a cubic ft. of water.

w = weight of 1 cubic foot of masonry, and t = required thickness,

H = total height, and h = height from top of dam to water.

Foundations of Basins and Dams are to rest on solid clay, sometimes on concrete, laid with puddled clay. The side next the water is laid with stones 12 inches deep, laid edgewise; sometimes they are laid with brick in cement, the outer face covered with sod. A puddled wall is brought up the middle whose base = one-third the height, and top = one-sixth the height; the top is made to curve, to carry off the rain water.

Waste-weir is regulated with a waste-gate, and made so as to carry off the surplus water; the sluice or gate may be made self-acting. *Bywash* receives the surface water from the waste-weir, and from the supply streams when not required to enter the reservoir in times of heavy rains and when the water becomes muddy.

310M. *Cascade.* Let f = fall from crest of weir, h , as usual, the height of still water above the crest of the weir, $v = 5.35 \sqrt{h}$ and $x = \frac{4}{3} \sqrt{h f}$ = distance to which the water will leap; this distance is to be covered with large stones, to break the fall of the water.

310t. *Retaining Walls* are sometimes built along the base of the dam.

St. Ferrel Reservoir, destined to feed the Languedoc Canal, in France, contains 1541 million gallons of water; the dam at its highest part is 106,2 feet.

One reservoir in *Ancient Egypt* contains 35,200 million cubic feet of water. Some are in Spain holding 35 to 40 million cubic feet—similar ones are found in France. The *Chinese* collect water into large reservoirs for the supply of towns and cities, and the irrigation of their lands.

The Hindoos have built immense reservoirs to meet the periodical scarcity of rain, which happens once in about five years. One of their reservoirs, the Veranum, contains an area of 35 square miles, made by a dam 12 miles long. The evaporation in India for 8 months is $\frac{1}{2}$ inch in depth per day. One-fourth of an inch may be a safe calculation in milder or colder climates.

In *Dams of Masonry*, buttresses are made at every 18 to 20 feet. Depth = the thickness of the wall, and length = double the thickness. Mahan and Barlow, in their Treatise on Engineering, say, "It is better to put the material uniformly into the wall."

310U. To find the thickness of a rectangular wall, *A B*, to resist its being turned over on the point *D*. (See Fig. 70.) Let the perpendicular, *E F*, pass through the centre of the rectangle; by Sec. 313 it passes through the centre of gravity *G*, makes *C P* = one-third of *B C*. We have the vertical pressure = weight of the wall, and the lateral pressure equal to that of the pressing fluid or mass. Let *w* = specific gravity of the water, and *W* that of the wall. We have the pressure of the fluid represented by *H D* = *C P*, and that of the wall by *D F*, and *T D H* is a bent lever of the first order.

$$\text{By Section 319c, } P : W :: \frac{D C}{2} : \frac{B C}{3}$$

$$\text{and } \frac{P \times B C}{3} = \frac{D C \times W}{2} \text{ clear of fractions.}$$

$$\text{then } P = \frac{3 D C \times W}{2 B C}$$

$$\text{and } D C = A B = \frac{P \times 2 B C}{3 W}$$

We have the value of $P \times 2 B C$ per lineal foot, and find the value of $3 W$ for height, *B C*, and one foot thick, which, divided into $P \times 2 B C$, will give the value of *A B* or *D C* when on the point of turning over.

Let *w* = weight of material, and *S* = weight of water; *h* = height of wall = that of the water, and *b* = breadth of wall required, then we have

$$P = \frac{h^2}{2} \cdot 62\frac{1}{2} \text{ lbs.} = \text{pressure of water against the wall, and}$$

$$P = \frac{3 b \times h b w}{2 h} = \frac{3 b^2 w}{2}$$

$$62 \cdot 5 \frac{h}{2} = \frac{3 b^2 w}{2}$$

$$62.5 h^2 = 3 b^2 W$$

$$b = \left\{ \frac{62.5 h^2}{3 W} \right\}^{\frac{1}{2}} = h \sqrt{\frac{62.5}{3 W}}$$

$$h = b \sqrt{\frac{3}{62.5}}$$

Example.—Height of dam and water = 20 ft.; specific gravity of water = $62\frac{1}{2}$ lbs., and that of the masonry 120 lbs.—to find thickness b .

$$b = \left\{ \frac{62.5 \times 20 \times 20}{3 \times 120} \right\}^{\frac{1}{2}} = 8.33 \text{ feet.}$$

As this formula gives but the thickness, to form an equilibrium, add one foot to the thickness, for safety.

Rondelet recommends, to find the required thickness of 1.8 times the calculated pressure, which in this case would be 28800, which divided by 263, gives $b = 79.33088$, whose square root = 8.91 feet. We prefer to use *Rondelet's* formula for safety.

310U*.

REVETMENT WALLS.

In retaining walls we have to support water, but in revetment walls we have to support moveable matter, such as sand, earth, etc. (See fig. 71) Let C = tangent squared of half the angle of repose, which may be taken at $22\frac{1}{2}$ deg., which angle is called the angle of rupture, as shown by *Coulomb* and others. The angle of $V D W$ is the angle of repose, and the angle $W D S$ being half the angle, $w D S$ is the angle of rupture, and the line $D S$ = line of rupture. Assume the angle $W D S = 22\frac{1}{2}^\circ$ whose tangent squared equals $.41421 \times .41421 = 0.1715699$, nearly 0.1716, which we take for the coefficient of c in the following formula: b = width at top

$$b = h x \left\{ \frac{c w}{3 W} \right\}^{\frac{1}{2}} \quad \text{And } h = b \left\{ \frac{3 W}{c w} \right\}^{\frac{1}{2}} \quad \text{And } P = \frac{h^2}{2} \times \frac{W c}{2}$$

$$b = h \left\{ \frac{0.1716 w}{3 W} \right\}^{\frac{1}{2}} \quad \text{And } h = b \left\{ \frac{3 W}{0.1716 c} \right\}^{\frac{1}{2}} \quad \text{And } P = \frac{0.1716 h^2 w}{2}$$

Here w = specific gravity of the material to be sustained, and W = that of the wall $C = 0.625$ for water. 0.410 for fine dry sand. 0.350 earth in its natural state; and for earth and water mixed, 0.40 to 0.65. To the value of b thus found the English engineers add for safety about one-sixth of it.

310u1. When the wall has an external batter. Let t equal the mean thickness; then we have:

$$t = ch \sqrt{\frac{w}{W}} = ch \sqrt{\frac{w}{W}} \text{ for a vertical wall.}$$

$$t = 0.95 ch \sqrt{\frac{w}{W}} = ch \sqrt{\frac{w}{W}} \text{ batter 1 in 16.}$$

$$t = 0.90 ch \sqrt{\frac{w}{W}} \text{ " 1 in 14.}$$

$$t = 0.86 ch \sqrt{\frac{w}{W}} \text{ " 1 in 12.}$$

$$t = 0.83 \sqrt{\frac{w}{W}} \quad " \quad 1 \text{ in } 10.$$

$$t = 0.80 \sqrt{\frac{w}{W}} \quad " \quad 1 \text{ in } 8.$$

$$t = 0.76 \text{ ch } \sqrt{\frac{w}{W}} \quad " \quad 1 \text{ in } 6.$$

From the mean thickness t , take half the total batter, and it will give the thickness at top; and to t add the half batter it will give the thickness at the base.

310u2. Where there is a surcharge running back from the walls at a slope of $1\frac{1}{2}$ to 1. Column A for hewn stone or rubble laid in mortar. B for well scabbled rubble in mortar, or brick. Col. C, well scabbled dry rubble. Col. D the same as A. Col. E the same as B. Columns A, B, and C are from the English. Cols. D and E are from *Poncelet*. H = total height of the walls and surcharge. h = that of a rectangular wall above the water. *Poncelet* has the surcharge:—

WHEN.	A	B	C	D	E
$H = h$	.35 <i>h</i>	.40 <i>h</i>	.50 <i>h</i>	.35 <i>h</i>	.45 <i>h</i>
$H = 1.2h$	.46 <i>h</i>	.51 <i>h</i>	.61 <i>h</i>	.44 <i>h</i>	.55 <i>h</i>
$H = 1.4h$	.51 <i>h</i>	.56 <i>h</i>	.66 <i>h</i>	.53 <i>h</i>	.67 <i>h</i>
$H = 1.6h$	.54 <i>h</i>	.59 <i>h</i>	.69 <i>h</i>	.62 <i>h</i>	.78 <i>h</i>
$H = 1.8h$	.56 <i>h</i>	.61 <i>h</i>	.71 <i>h</i>	.67 <i>h</i>	.85 <i>h</i>
$H = 2.h$	.58 <i>h</i>	.63 <i>h</i>	.73 <i>h</i>	.71 <i>h</i>	.93 <i>h</i>

WALLS OF DAMS.

310u3. *Morin* in his *Aide Memoire*, gives for thickness at base

$$t = 0.865 (H-h) \cdot \sqrt{\frac{1000}{p}} \quad \text{Here } H = \text{height of the wall and } h = \text{height}$$

from the surface of the water to the top of the wall. 1000 = specific weight of one kilogramme of water, and p = specific weight of one kilogramme of the masonry.

Example wall four metres high. $h = 0.50$ m. $p = 2000$,

$$t = 0.865 \cdot (4.0 \text{ met} - 0.50) \cdot \sqrt{\frac{1000}{2000}} = 2.04 \text{ metres.}$$

310u4. DRY WALLS are made one-fourth greater than those laid in mortar.

310u5. *Line of resistance in a wall or pier.* (See fig. 71.)

Let PQ = the direction of the pressure P , which is supported by the wall. The line EF passing through the centre of gravity meet PQ at G . Make GL = the pressure P , and GH = pressure by the weight of the wall $ABCD$. Complete the parallelogram $GHLK$. Join GK and produce it to meet the base CD at M . Then M is a point in the line of resistance.

310u6. *The celebrated Vauban* in his walls of fortifications, makes $MF = \frac{1}{9}$ of CF . F being where the line through the vertical of the centre of gravity of the wall intersects the base.

Let w = weight of the wall. $h = BD$. $b = AB$. $a = \text{angle } PGE$. $d = SE$ and $x = MF$.

$$x = P \cdot \frac{h \sin a - d \cos a}{wbh + P \cos a} -$$

310u6a. *The greatest height to which a pier can be built*, is when the line of resistance intersects the base at C, that is, when H is a maximum, $x = \frac{1}{2}b$ MF must not exceed from 0.3 to 0.375 the thickness of CD.

Vauban in his walls of fortifications makes the base 0.7h. At the middle 0.5h, and at the top 0.3h.

310u6b. In fig. 72. Let CE = nat. slope. G = centre of gravity of the triangular piece to be supported. Draw FGR parallel to CE, then the triangular wall BCR will be a *maximum* in strength. And by making BA = 1.5 to 2 ft. and producing EB to O, making AO = OR and describing the curve AKR the figure ABCRK will be a strong and graceful wall.

310u7. (See fig. 72.) *Rondelet's Rules*.—Assume the nat. slope to be 45 degrees. In the parallelogram BCDE draw the diagonal CE. When the wall is rectangular, then BA = CR = one-sixth of CE.

When the wall batters 2 inches per foot AB = one-ninth do.

do do do 1 1-2 inches per foot AB = one-eighth do.

The English Engineers, make their walls less than the French. They put 1-15 1-10 respectively where Rondelet has 1-8 and 1-9. When the batter is one inch per foot, the English make AB = one-eleventh of CE.

For dry walls, make AB = 2-3 of CE, never less than one-half; and in order to insure good drainage, ought to be built of large stones, and batter three inches per foot.

310u8. *Colonel Wurmb* in his *Military Architecture*, gives

$$T = 0.845 h \tan. \frac{a}{2} \sqrt{\frac{w}{W}}, \text{ and } t = T + \frac{nh}{10}.$$

Here T = thickness of a rectangular wall and t = that of a sloping one at the base, n = ratio of batter to h and $\frac{a}{2}$ = half the complement of the angle of repose = WDS. (fig. 71.)

310u9. *Safety pressure per square foot*. White marble 83,000 lbs.; variegated do. 129,000 lbs.; veined white do. 17,400 lbs.; Portland stone 30,000 lbs.; Bath stone 17,000 lbs.

Pressure on—The Key of the Bridge of Neuilly, Paris, 18,000 lbs. Pillars of the dome of the Invalides, Paris, 39,000 lbs. Piers of the dome of St. Paul, London, 39,000 lbs. Do. of St. Peter's, in Rome, 33,000 lbs.; of the Pantheon, in Paris, 60,000 lbs. All Saints, Angiers, 80,000 lbs.

Rankine gives on firm earth 25,000 to 35,000.

do on rock a pressure equal to one-eighth of the weight that would crush the rock.

Fox on the Victoria R. R., London, clay under the Thames 11,200 lbs., and for cast iron cylinders filled with concrete and brickwork 8,960 lbs.

Brunlee on the Leven and Kent viaduct, gravel under cast iron 11,200 lbs.

Blyth—On Loch Kent viaduct, gravel under the lake 14,000 lbs.

Hawkshaw.—Charing Cross R. R., London, clay 17,920 lbs.

Built on cast iron cylinders 14 ft. diameter below the ground and 10 ft. dia. above it, sunk 50 to 70 ft. below high water mark, filled with Portland cement, concrete, and brickwork.

General Morin, of France, recommends for Ashlar one-twentieth of the crushing weight, for a permanent safe weight.

Vicat says that *sometimes* we may load a column equal to one-tenth of the crushing weight, but it is safer to follow *Morin*.

OUTLINES OF SOME IMPORTANT WALLS.

310u11. (Fig. 72 a.) Wall built at the India Docks, London. Radius 72 ft. = DB = DE. Wall is 6 ft. uniform thickness. Counterforts 3' x 3', 18 ft. apart. AE = h = 29 ft.

The wall at East India Dock, built by Walker, is 22 ft. high, 7 1-2 ft. thick at base and 3 1-2 ft. at top. Radius 28 ft. Counterforts 2 1/4 ft. wide, 7 1-2 ft. at bottom and 1 1-2 at top. Lines of the two walls are on the same line with the top. Their backs vertical.

Fig. 73. *Liverpool Sea Wall*, built in 1806, base 15', top 7 1-2. Front slope 1 in 12. Counterforts 15 wide and 36' from centre to centre. Height 30 ft.

Fig. 73 a. *Dam at Poona*, near Bombay, in the East Indies. Top of dam is 3 ft. above water. 60 1-2 ft. thick at base and 13 1-2 at top. 100 ft high.

(Fig. 74.) *The Toolsee Dam*, near Bombay, is built of Basalt, rubble masonry. Mortar of lime and Roman cement. Height 80 ft., thickness at base 50 ft., at top 19 ft.

(Fig. 75.) *Dublin Quay Wall*, 30 ft. high. Counterforts 7 ft. long and 4 1-2 ft. deep, and 17 1-2 ft. from side to side. A puddle wall at the back, built on piles. Sheeted on top to receive the masonry.

(Fig. 76.) Wall of *Sunderland Docks*, England.

(Fig. 77.) *Bristol Docks*.

(Fig. 78.) Revetment wall on the *Dublin and Kingston R. R.* This is in face of a cut and is surcharged.

(Fig. 79.) *Chicago street revetment walls*.

Blue Island Avenue viaduct in Chicago.

Steepest grade on the streets crossing is 1 in 30, rather too steep for traffic. On the avenue it is but 1 in 40.

310u12. *Blue Island dam* on the Calumet feeder taken away in 1874. Timber of Oak and Elm. Built in compartments, well connected and the spaces filled with stones. It was down 27 years and did not show the slightest decay in the timber used.

Jones' Falls dam, on the Rideau canal, is 61 feet high, built of sand stone, with puddle embankments behind it. Several other dams made similar to that at Blue Island, are between Kingston and Ottawa (formerly Bytown), in Canada.

PILE-DRIVING, COFFER-DAMS, AND FOUNDATIONS.

Pile driving machines are of various powers and forms. A simple portable machine may be 12 to 16 feet high, hammer 350 to 400 pounds weight, without nippers or claws, and worked by about 10 men.

A *Crab* may be placed and worked, but where a small engine can be placed it is preferable. The locality and ground will control which to use. The site is bored to find the under lining stratas, both sides of the banks, (if for a bridge,) to be brought to the same level.

It is an old rule that a pile that will not yield to an *impact* of a ton, will bear a constant pressure of 1 1/2 tons.

The *power of a pile driver* may be determined from the following formulas:

310v1. *Screw Piles* 6 1-2 ft. in dia. have been driven in India and elsewhere. 4 levers are attached to a capstan, each lever moved by oxen,

Hollow Cast Iron Piles.—When these are driven, a wooden punch is put on top to receive the blows and protect the piles from breaking.

m = velocity in feet acquired at the time of impact.

h = height fallen through in time s , in seconds.

s = time of descent in seconds, w = weight of hammer.

$$s = \sqrt{\frac{h}{16.083}} = \sqrt{\frac{h}{4.01}} \quad v = 8.82 \sqrt{h}$$

$$m = 2 w \sqrt{\frac{16.083 h}{w}} \quad \text{Let } h = 10 \text{ feet, } w = 2 \text{ tons;}$$

$$\text{Then } m = 4 \sqrt{160.83} = 30.4 \text{ tons.}$$

$$v = 25.2 \text{ feet.}$$

Otherwise We determine the safe load to be borne by each pile, and in driving find the depth driven by the last blow = d . W = weight of the hammer in cwts., H = height fallen, and L = safe load in cwts. of 112 lbs.

$$L = \frac{W H}{8 D} \quad \text{and} \quad D = \frac{W H}{8 L}$$

Example.—Hammer 2000 lbs., fall 35 feet. Safe load $L = 44,000$ lbs.,

$$\text{then } D = \frac{2000 \times 35}{8 \times 40,000} = 0.22 \text{ inches, nearly the length to be driven by the last blow.}$$

Let w = safe weight that a pile will bear where there is no scouring or vibration caused by rolling pressure on the superstructure.

R = weight of ram in pounds. f = fall in feet and d equal depth driven by the last blow.

$$w = \frac{R h}{8 d} \quad \text{this is the same as Major Sander's, U.S. Engineers.}$$

$$w = \frac{80}{3} \cdot (R + 0.228 \sqrt{h} - 1) \quad \text{The same as Mr. McAlpine's formula}$$

assuming w = one-third of the extreme weight supported.

$w = 1,500$ lbs. $\times$ by the number of square inches in the head of the pile. This agrees with the late Mahan and Rankine's formulas for piles driven to the firm ground.

$w = 460$ lbs. (mean safe working load) per inch, by *Rondelet*.

$w = 990$ lbs. per square inch for piles 12 in. dia., by *Perronet*.

$w = 880$ lbs. do. do. do. 9 do. do.

$w = 0.45$ tons in firm ground. According to English Engineers.

$w = 0.09$ tons in soft ground. do. do. do.

Piles near, or in, salt water deteriorate rapidly and must be filled with masonry or concrete.

Lime stone exposed to sea air also suffers, and ought not to be used, as granite laid in cement can alone remain permanent.

Piles are driven, according to the French standard, until 120,000 lbs. pressure equal to 800 lbs. falling 5 ft. 30 times will penetrate but one-fifth of an inch. The most useful fall is 30 feet—should not exceed 40 ft.

Where there is no vibration of the pile the friction of the sand and clay in contact with it increases its strength, and is greater under water where there is no scouring, than in dry land.

The Nasmith Steam Hammer strikes in rapid succession, so as to prevent the material being displaced at each blow to settle about the pile. The blows are given about every second.

When men are used as a force, there is one man to every 60 lbs. of the weight. Piles driven in hard material are shod with iron and an iron hoop put on top, to prevent splitting.

For much valuable information, see a paper by Mr. McAlpine, in the *Franklin Journal*, vol. 55, pp. 98 and 170.

It sometimes happens that below a hard strata there is one in which the pile could be driven easier, therefore boring must be first used to find the stratas, and observations made on the last three or four blows.

310v. Mr. McAlpine's formula, from observations made at the Brooklyn Navy Yard, gives as follows:

$x = W + .0228 \sqrt{F-1}$. Here x = supporting weight of the pile. W = weight of the ram in tons. F = fall in feet.

He says that only 1-3 of the value of x should be used for safety weights.

These piles were driven until a ram 2,200 lbs. falling 30 ft. would not drive the piles but 1-2 an inch. They were made to bear 100 tons per square foot.

Piles in firm ground will bear 0.45 tons per square inch, and in wet ground 0.09 tons. The greatest load ranges from .9 to 1.35, tons per square inch.

310v1. *Cast iron cylinders* were first used in building the railway bridge across the Shannon, in Athlone, Ireland; next at Theis, in Austria, and now generally used. Those used in the bridge of Omaha, United States, are in cylinders 10 ft. long, 8' inner diameter; thickness $1\frac{5}{8}$ inches. Flanges on the inside 2". These when down are filled with concrete. The lower ends of those sunk in Athlone were bevelled, and sunk by Potts' method of using atmospheric pressure—that is, by exhausting the air in the cylinder, which caused the semifluid to rise and pass off. The pipe of the air pump was attached to the cap of the cylinder.

310v2. *Foundations of Timber*.—Where timber can be always in water, several layers of oak or elm planks are pinned together. We have seen the Calumet dam, on the Illinois and Michigan Canal, removed, in 1874, after being built 27 years. The foundation was of oak logs, pinned together, and in compartments filled with stones. The lumber did not show the least sign of decay.

Timbers 10 to 12 in. square are laid $2\frac{1}{2}$ to 3 feet apart, and another layer is laid across these, and the spaces between them filled with concrete, the whole floored with 3-inch plank.

Pile Foundations.—Piles ought to have a diameter of not less than one-twentieth of their length, to be $2\frac{1}{2}$ to 3 feet apart, and the load for them to bear, in soft ground, 200 lbs. and in hard, firm ground, 1000 lbs. per square inch of area of head. Piles ought to be driven as they grew—with butt end downwards—all deprived of their bark; a ring is sometimes put on top, to prevent their splitting and riving.

Pile-Driving Engine.—When worked by men, there is one man to every 40 lbs. weight of the ram or hammer used. A pile is generally said to be deep enough when 120,000 foot lbs. will not drive it more than one-fifth of an inch. 120,000 foot lbs. pressure is a hammer of 1000 lbs. weight falling 6 feet 20 times.

Let W = weight of ram, h = height of fall, x = depth driven by the last blow, P = greatest load to be supported, S = sectional area of the pile, l = its length, E = its modulus of elasticity.

$$P = \sqrt{\left\{ \frac{4 E S p^2 l}{4 E S l} + \frac{4 E^2 S^2 x^2}{l^2} \right\} - \frac{2 E S x}{d}}$$

By this formula P is to be 2000 to 3000 lbs. per square inch of S , and the working load is taken at 200 to 1000 lbs.

COFFER-DAMS.

310v3. In building the Victoria bridge, in Montreal, the coffer-dam was 188 ft. long, width 90, pointed against the stream, and flat at the other end. Double sides made to be removable. Depth of rapid water 5 to 15 ft. On the outside at intervals of 20 ft., strong piles were driven, in which steel pointed bars, 2 in. dia. were made to drill to a depth of two feet in the rock, to keep the dam in position. When the pier was built these bars, etc., were removed as required. In floating it to its required place the dam drew 18" of water.

For building cofferdams in deep water, see Mr. Chanute's treatise on the Kansas City bridge, on the Missouri.

Cofferdam of earth, where it is feasible, is the cheapest. It has to be built slowly. There are two rows of piles driven, then braced and sheeted, and filled with clay of a superior quality.

The Thames embankment reclaimed a strip of land 110 to 270 ft. wide. Depth of water in front 2 ft. Rise of tide $18\frac{1}{2}'$. *Strata*, gravel and sand resting on London clay at a depth of 21 to 27 ft. Depth of wall 14 ft. below low water mark. Dams were $11\frac{1}{2}$ ft. long and 25 broad inside, made of two rows of piles 40 to 48 ft. long, 13 in. square, shod with cast iron shoes 70 lbs. each, and driven 6 ft. apart. The sheeting driven 6 ft. in the clay. At intervals of 20 ft., other piles were driven as buttresses and supported by walling at every $6\frac{1}{2}$ ft. horizontally, and connected with two other piles bolted with iron bolts $2\frac{1}{2}$ in. dia., with washers 9" dia. and $2\frac{3}{4}"$ thick. An iron cylinder 8 ft. dia. sunk in each dam as pump wells.

WOOD PRESERVING.

310v4. Trees ought to be cut down when they arrive at maturity, which, for oak, is about 100 years, fir, 80 to 90, elm, ash, and larch, 75. Should be cut when the sap is not circulating, which, in temperate climates, is in winter, and in tropical climates in the dry season—the bark taken off the previous spring. When cut, make into square timber, which, if too large, ought to be sawed into smaller timbers.

310v4a. *Natural Seasoning*.—By having it in a dry place, sheltered from the sun, rain, and high winds, supported on cast-iron bearers, in a yard thoroughly drained and paved, this requires two years to fit it for the carpenter's shop, and for joiners, four years. Timber steeped in water about two weeks after felling, takes part of the sap away. Thus, the American timber, rafted down stream to the sea-board, affords a good opportunity for this natural process.

310v4b. *Artificial Seasoning*, is exposing it to a current of hot air, produced by a fan blowing 100 feet per second. The fan air-passages and chambers are so arranged that one-third the air in the chamber is expelled per minute. The best temperature is, for oak, 105° Fahr., pine in thick pieces, 120° , pine in boards, 180° to 200° , bay mahogany, 280° to 300° .

Thickness in inches,	1	2	3	4	6	8
Time required in days,	1	2	3	4	7	10

each day, only twelve hours at a time.

310v4c. *Robert Napier's Process* is by a current of hot air through the chamber, and thence into a chimney, is found very successful. The air admitted at 240° , requires 1 lb. of coke to every 3 lbs. moisture evaporated.

The short duration of wooden bridges, ties, etc., calls for a method for preventing the dry rot in timber. The following brief account will be sufficient to inform our readers of the means used to this time:

Tanks are made to hold the required cubic feet, and sunk in the ground level with the surface.—*Kyan's Process*, patented March, 1832.

On the Great Western Railway, England, the tank was 84 feet long, 19 feet wide at top, 60 feet long and 12 feet 8 in. wide at bottom, and 9 feet deep.

Corrosive sublimate (bichlorate of mercury) was used at the rate of 1 lb. to 5 gallons of water. Cost per load of 50 cubic feet, 20 shillings, sterling; of this sum, one-fourth was for the mercury, one-fourth for labor, and one-half for license, risk, and profit. The solution is generally made of 1 lb. of the mercury to 9 to 15 lbs. of water. Time of immersion, eight days; timber to be stacked three weeks before using. Experiments are reported against Kyan's method.

Sir William Burnet's Method—Patented in England, March, 1840. He uses *chloride of zinc* (muriate of zinc). Timber prepared with this was kept in the fungus-pit at Woolwich dock-yard for five years, and was found perfectly sound. The specimens experimented on were English oak, English elm, and Dantzic fir. Cost—one pound at one shilling is sufficient for ten gallons of water, a load of 50 cubic feet thus prepared in tanks costs, for landing, 2 shillings, preparation, labor, etc., 14 shillings, total, 16 shillings.

Bethell's Method.—Close iron tanks are provided, into which the wood is put, also coal-tar, free from ammonia and other bituminous substances. The air is exhausted by air-pumps under a maximum pressure of 200 lbs. per square inch during 6 or 7 hours, during which time the wood becomes thoroughly impregnated with the tar oil, and will be found to weigh from 8 to 12 lbs. per cubic foot heavier than before. The ammonia must be taken away from the tar oil by distillation.

Payne's Method—Patented 1841.—The timber is enclosed in an iron tank, in which a vacuum is formed by the condensation of steam, and air-pumps. A solution of sulphate of iron is then let into the tank, which immediately impregnates all the pores of the wood. The iron solution is now withdrawn, and replaced with a solution of chloride of lime, which enters the wood. There are then two ingredients in the wood—sulphate of iron and muriate (chloride) of lime. The timber thus prepared has the additional quality of being incombustible.

Bouchéri's Method.—Use a solution of 1 lb. of sulphate of copper to 12½ gallons of water. Into this solution the timber is put endwise, and a pressure of 15 lbs. per square inch applied.

W. H. Hyett, in Scotland, impregnated timber standing,—found the month of May to be the best season. From his experiments on beech, larch, elm, and lime, we find that prussiate of potash is the best for beech—½ lb. per gallon—chloride of calcium the best for larch. Time applied, 17 to 19 days. For further information, see Parnell's Applied Chemistry.

A. Lege and Fleury Peronnet, in France, in 1859, used sulphate of copper, which they found to be better and cheaper than Bouchéri's method.

310v5. *By exhausted steam*.—In Chicago, at Harvey's extensive lumber yard and planing mill, the following process is found very cheap and effective:—

The machinery is driven by a 100-horse power engine, the fuel used is exclusively shavings; the exhausted steam is conducted from the engine house to the kiln, where it is conveyed along its east side in a live steam

coil of 20 pipes, 2 inches in diameter. The heat thereof passes up and through the timber, separated by inch strips and loaded on cars. The heat passes to the west through the lumber cars, and thence to the north-west corner of the kiln, where it escapes. Connected with the last main pipe (8 inches in diameter,) are condensing pipes, 2 inches in diameter, laid within 4 inches of one another, and connected with a main exhaust pipe 4 inches into a chimney—one of which is over each car.

There are five tracks, or places for ten cars in each, about 80 by 60 feet; each car is 16 feet long, 6 feet wide, and 7 feet high, and is moved in and out on a railway; the whole, when filled, contains 200,000 feet of lumber. The temperature is kept, day and night, at 160° Fahr., and the whole dried in 7 days, losing about half its weight, and selling at about one dollar more per thousand. This makes a great saving in the transportation of lumber from the yard to various places in the west, as the freight is charged per ton.

MORTAR, CONCRETE, AND CEMENT.

From experiments made by the Royal Engineers, they find that 1120 bu. gravel, 160 bu. lime, and 9 of coals, made 1440 cubic feet in foundation; 4522 bu. gravel, 296 lime, and 30½ coal, made 2325 feet in abutments; 3591 bu. gravel, 354 lime, and 30 bu. coal, made 2180 cubic feet in arches. Cost per cubic foot—in foundations, 3½d, abutments, 4½d, arches, 5¼d; specific gravity, 2,2035; 16 cubic feet = 1 ton = 2240 lbs. Breaking weight of concrete to that of brick-work, as 1 to 13.

At Woolwich that concrete in foundations cost one-third, and in arches one-half that of brickwork.

Stoney, in his *Theory of Strains*, p. 234, edition of 1873, says Rondelet states that plaster of Paris adheres to brick or stone about two-thirds of its tensile strength; is greater for mill-stones and brick than for limestone, and diminishes with age; lime mortar, its adhesion to stone or brick exceeds its tensile strength, and increases with time.

On the Croton Water Works. Stone backing. 1 cement to 3 of sand. Brick work, inside lining 1 c to 2 s.

At Fort Warren, Boston Harbor, the proportions for the stone masonry were stiff lime paste 1 part, hydraulic cement 0.9, loose damp sand 4.8.

At Fort Richmond, hyd. cement 1.00, loose damp sand 3.2.

Vicat, a well-known French Engineer, recommends pure limepaste 1, sand 2.4, and hyd. lime paste 1, sand 1.8.

Cement for water work. Friessart recommends hyd. lime 30 parts, Terras of Andrenach 30 parts, sand 20, and broken stones 40.

Grouting. *Smeaton*, who built the Eddystone light house, recommends 4 parts of sand, one of lime made liquid. For Terras mortar he substitutes iron scales 2 parts, lime 2 and sand 1 part. This makes a good cement.

Iron cement. Gravel 17 parts by weight, iron filings or turnings 1 part, spread in alternate layers. Used in sea work, forms a hard cement in two months.

310v6. *Stoney* at Sec. 304, edit. 1873, gives the crushing weight per square inch at 3, 6, and 9 months, as follows:

Specimens acted on were made into bricks 9 × 4½ × 2¾ inches. They began to fail at five-eighths of the ultimate load.

At Sec. 688 of *Stoney* on strains, the working load is taken at one-sixth of the crushing weight.

Vicat gives tenacity (one year after mixture) of hydraulic cement 190 lbs. to 160, and common mortar 50 to 20.

Cement for moist climates. Lime one bushel, $\frac{1}{2}$ bu. fine gravel sand, $2\frac{1}{2}$ lbs. copperas, 15 gallons of hot water. Kept stirred while incorporating.

CONCRETE.

310z7. In London, architects use one part of ground lime and 6 parts of good gravel and sand together. Broken bricks or stones are often added. Strong hydraulic concrete, is made of 2 parts of stone and 1 of cement.

In the United States, 1 of cement to 3 of broken stone and sand is frequently the proportions.

The stones and sand are spread in a box to a depth of 8 inches, the proportion of cement is then spread on the whole and sufficiently wetted. Four men with shovels and hoes mix up the ingredients from the sides to the centre, and mix one time in one direction and again in the opposite one. It is then taken on wheel-barrows and thrown from a height where it is spread and well rammed. One part of the materials before made makes $\frac{5}{8}$ in foundation. Lime must not be mixed when used in sea-walls.

Concrete is made into domes and arches.

The central arch of Ponte d'Alma, 161 ft. span and 28 ft. rise is made of concrete. Also the dome of the Pantheon at Rome, 142 ft. diameter.

Beton is concrete where cement takes the place of lime. In building the harbor at Cherbourg, in France, Beton blocks 52 tons weight, dimensions $12 \times 9 \times 6$ 1-2ft., 712 cubic feet, built of stone and cement, mortar made of sand 3 and cement $\frac{1}{4}$. These blocks at nine months old bore a compressive strength of 113 tons, nearly equal to that of Portland stone.

The Mole, at Algiers, Africa, built by French Engineers, is made of blocks of Beton, not less than 353 cubic feet each. All the blocks are of the same form, 11' long, $6\frac{1}{2}$ ft. wide and 4 ft. 11" high. *Composition of Beton Mortar* is made of lime 1, Pozzuolana 2, makes two parts of mortar. Beton is composed of mortar 1, stone 2. The stones are broken into pieces of about $1\frac{1}{2}$ cubic ft. each. Weight per cubic foot of this Beton = 137 lbs.

An adjustable frame is made so as to be removable when the block is dry, the bottom is covered with two inches of sand and the sides of the frame lined with canvass to prevent their being washed. They are cast in making a slope on the outside 1 to 1, and on the land side $\frac{1}{2}$ to 1. The blocks are put on small wheeled trucks and moved on a tramway to an inclined float, where it is lowered to a depth in water of 3 ft. 3 inches, and placed by chains between two pontoons and floated to the required place in the Mole.

PRESERVATION OF IRON.

310z8. The iron is heated to the temperature of melting lead (630° Fahr.), then boiled in coal tar.

Where the iron is to be painted with other parts of the structure, the iron is heated as above, and brushed over with linseed oil—this forms a good priming coat for future coats of paint. Galvanizing with zinc is not successful, being acted on by the acid impurities found in cities, towns, and places exposed to the sea, or sea air.

Steel hardened in oil is increased in strength.—*Kirkaldy*.

VICTORIA ARTIFICIAL STONE.

310z9. Rev. H. Heighton, England, uses at his works, Mount Sorrel and Guernsey granite, refuse of quarries, broken into small fragments and mixed with one-fourth its bulk of granite and water, to make the whole into a thick paste, which is put into well-oiled moulds, where it is allowed to stand for four or five days, or until the mass is solidified. After this, it is placed in a solution of silicate of soda for two days, after which it is ready for use. He keeps the silicate of soda in tanks which are to receive the concrete materials, the silica is ground up and mixed with the bath. The lime removes the silica, forming silicate of lime. The caustic soda is set free, which again dissolves fresh silica from the materials containing it. This, in flags of 2 inches thick, serves for flagging. It is made into blocks for paving, is impervious to rain and frost. Mr. Kirkaldy has found the crushing weight to be 6441 lbs. per square inch—Aberdeen granite being 7770, Bath stone, 1244, Portland stone, 2426.

310z10. *Ransom's Method* to prevent the decay of stone, and when dried then apply a solution of phosphate of lime, then a solution of baryta, and lastly, a solution of silicate of potash, rendered neutral by Graham's system of dialysis—this is Frederick Ransom's process. With Mr. Ransom, of Ipswich, England, in 1840 and 1841, we have spent many happy hours in constructing equations, etc. The above process, by Mr. Ransom sets the opposing elements at defiance. Ransom dissolves flint in caustic soda, adds dry silicious sand and lime-stone in powder, forms the paste into the desired forms, and hardens it in a bath of a solution of chloride of calcium, or wash it by means of a hose.

Make blocks of concrete with hydraulic cement. When well dried, immerse in a bath of silicate of potash or soda, in which bath let there be silica free or in excess. Here the lime in the block takes the alkali, leaving the latter free to act again on the excess of silica, and so proceed till the block is an insoluble silicate of lime, known as the silicated concrete, or Victoria stone, of which pavements have been made and laid in the busiest part of London; also, as above stated, enormous buildings, such as the new *warehouses*, 27 South Mary Ave., London.

Silicate of Potash is composed of 45 lbs. quartz, 30 lbs. potash, and 3 lbs. of charcoal in powder.

Silicate of Soda—Quartz 45, soda 23, charcoal 3. These are fused, pulverized, and dissolved in water.

This silica absorbs carbonic acid, therefore it must be kept closely stopped from air. The strength is estimated by the quantity of dry powder—40 degrees means 40 of dry powder and 60 of water.

In applying this, begin with a weak solution, make the second stronger. One pound of the silica to five pounds of water will answer well. It is not to be applied to newly-painted surfaces.

Mortar and lime stones ultimately produce silicate of lime.

If the surface is coated with a solution of chloride of calcium, the chlorine will combine with the soda, making the soluble salt, chloride of sodium, and there remains on the surface silicate of lime, which is highly insoluble. The surface is washed with cold water, to remove the chloride of sodium.

When applied to stone or brick, add 3 parts of rain-water to a silicate of 33 degrees. A final coating of paint, rubbed up with silicate of soda, will render the surface so as to be easily cleaned with soap and water.

This silicate adheres to iron, brass, zinc, sodium, etc. Enormous buildings have been built and repaired by this means. The best colors to be used with it are Prussian blue, chromate of lead and of zinc, and blue-green sulphide of cadmium.

BEAMS AND PILLARS.

310v11. The strongest rectangular beam that can be cut out of a log is that whose breadth = d divided by 1,732, where d = diameter of the log. (See Fig. 80.)

In the figure, $a c$ = diameter, make $a f$ = one-third of d , erect the perpendicular $f b$, join $b c$ and $a b$, make $c d$ parallel to $a b$, join $a d$, then the rectangle, $a b c d$, is the required beam. See Sections 21, 22.

A beam supported at one end and loaded at the other will bear a given load, = w , at the other end.

When the load is uniformly distributed, it will bear 2 w .

Beam supported at both ends and loaded at the middle = 4 w .

Beam supported at both ends and the weight distributed = 8 w .

When both ends are firmly fixed in the walls, the beam will support fifty per cent. more.

The following table are the breaking weights for different timbers and iron—the safe load is to be taken at *one-fourth to one-sixth of these*:—one-sixth is safer.

310v12.

TABLE.

SPECIFIC GRAVITIES, BREAKING WEIGHTS, AND TRANSVERSE STRAINS OF BEAMS SUPPORTED AT BOTH ENDS AND LOADED IN THE MIDDLE.

KIND OF WOOD.	Sp'cific Gr vity	Brking Weight W	Transv Strain. S	AUTHORITY.
Ash, English, - - -	760		2022	Barlow.
" African, - - -	985	1701	2484	Nelson.
" American, - - -	611	274	1550	"
" White, " seasoned,	642		2041	Lieut. Denison.
" Black, " - - -	533		8861	Moore.
Elm, English, - - -	605		551	Nelson.
" Canada, - - -	703	1377	1966	"
" " - - -	685	1265	1819	Denison.
" Rock, seasoned, -	752		2312	"
" " green, - - -	746		2049	Nelson.
Hickory, American, -	838	1857	1332	"
Iron-wood, American, -	879		1800	"
Butternut, green, -	772		1387	"
Oak, American, mean of 11,	1034	1000+	1806	"
" Live, - - -	1120	1041	1513	"
Pine, White, mean of 6, -	453	966	1456	"
" North of Europe, -	587		1387	Moore.
" Red, West Indies, -			1799	Young.
" " American, mean 3,	621	1292	1944	Nelson.
Hemlock, - - -	911		1142	Chatham, England.
Larch, Scotch, - - -	480		1193	" "
Coudie, New Zealand, -	550		1873	" "
Bullet-tree, West Indies, -	1075		2733	Young.
Green-heart, " - - -	1006		2471	"
Kakarally, " - - -	1223		2379	"
Yellow-wood, mean of 3,	926	1364	2103	"
Wallabia, - - -	1147		1643	"
Lancewood, South African,				
mean of 4, - - -	1066	1167	2305	Nelson.
Teak, mean of 9, - - -	719	1292	1898	"

Let l = length, b = breadth, d = depth, W = breaking weight, loaded at the centre, S = transverse strain acting perpendicularly to the fibres. l , b , and d in inches— W and S in pounds.

$$S = \frac{l W}{4 b d^2} \quad W = \frac{4 b d^2 S}{l}$$

$$b = \frac{W l}{4 d^2 S} \quad d = \frac{W l}{4 b S}$$

TIMBER PILLARS. BY RONDELET.

310v13. Let w = the weight which would crush a cube of fir or oak. When height = 12 times the thickness of the shorter side, the face = $0.833w$

"	24	"	"	"	"	"	$0.500w$
"	36	"	"	"	"	"	$0.334w$
"	48	"	"	"	"	"	$0.166w$
"	60	"	"	"	"	"	$0.083w$
"	72	"	"	"	"	"	$0.042w$

1. Example. A white pine pillar 24 ft. long, 12 inches wide and 6 inches thick. Required the breaking weight.

From Sec. 3107. The crushing weight of white deal = 7293

$$\frac{72 = 12 \times 6}{525096}$$

Length = 48 times the shorter side.

$$.166 = \frac{1}{6}$$

$$\frac{87,516 \text{ lbs.}}{Rondelet = 39.07 \text{ tons.}}$$

310v14. *Hodgkinson's formula for long square pillars more than thirty times the side—*

W = breaking weight in tons, l = length in feet, d = breadth in inches.

Note. With the same materials a square column is the strongest, the timber in all cases being dry.

$$W = 10.95 \frac{d^4}{l^2} \text{ for Dantzic oak.}$$

$$W = 6.2 \frac{d^4}{l^2} \text{ for American red oak.}^*$$

$$W = 6.8 \frac{d^4}{l^2} \text{ for red pine.}$$

$$W = 6.9 \frac{d^4}{l^2} \text{ French oak.}$$

$$W = 12.4 \frac{d^4}{l^2} \text{ for Teak.}^*$$

Note. These marked * are put in from the values of C. Sec. 319v6.

310v15. *Brereton's experiments on pine timber.* For pieces 12 inches square and 20 feet long, he finds the breaking weight in tons 120, for 20, 30 and 40 ft., he finds 115, 90, and 80 tons respectively. *Stoney* says "this is the most useful rule published," and gives a table calculated from Brereton's curve to every five feet.

Ratio of length to the least breadth, 10, 15, 20, 25, 30, 35, 40, 45, 50.

Corresponding breaking wt. in tons per sq. ft. of section, 120, 118, 115, 120, 90, 89, 80, 77, 75.

2. Example. White pine pillar 24' ft. by 12" × 16".

Ratio 24 ft. to 6 in. = 1.48 tabular number for 50 = 75 and for 65 = 77 ∴ or therefore for 48 = 75.8.

$$\frac{12'' \times 6'' \times 75.8}{12 \times 12} = 37.9 \text{ tons. Brereton.}$$

By Hodgkinson least side 6" in the fourth power 1296
which multiply by the coefft for red deal 7.8

$$10108.8$$

Divide by the square of the length in feet 576 and the quotient will be for red pine and 6 inches square 17.55 tons.

As 6":17.55:12" = for 12" x 6" = 35.10 tons.

The crushing weight of white deal = 7293 lbs. and of red deal 6586, that is white deal is 1.11 times that of red = $35.1 \times 1.11 = 38.96$ tons. Hodgkinson's.

Safe load in structures, includes weight of structure.

Stone and brick one-eighth the crushing weight.

Wood one-tenth. Cast iron columns, wrought iron structures and cast iron girders for tanks each one-fourth, and for bridges and floors one-sixth. A dense crowd, 120 lbs. per sq. ft. For flooring $1\frac{1}{2}$ to 2 cwt. per sq. ft., exclusive of the weight of the floor.

310v16. The strength of cast iron beams are to one another as the areas of their bottom flanges, and nearly in proportion to their depths.

$W = \frac{cad}{l}$ = theoretical weight, which is from 4 to 6 times the weight to be sustained. Here W = breaking weight in tons placed on the middle of the beam. c and a constant multipliers derived from experiments. One-sixth the breaking weight where there is rolling or vibration and one-fourth where stationary and quiet, generally taken at 26. a = sectional area of the bottom flange, taken in the middle. d = depth of beam = g a (fig. 81) b = length between the supports.

The strongest form, according to Hodgkinson, is where the area of the lower flange is six times that of the upper flange.

Fairbairn's form is shown in fig. 81, where $e d = 1$, $a b = 2.5$, $ag = 4$, $a h = 0.42$, $ef = 0.20$ and $ik = 0.25$.

Area of bottom flange = 1.05 and of top one = 0.20. Here we have the bottom flange area = $5\frac{1}{4}$ times that of the top.

Mr. Fairbairn says, at page 32 of his treatise, that "a beam made in the above form, will be safer without truss, bars, or rods than with them."

At page 65, he shows that the advantage of a truss beam is but two-thirds of that of the simple beam as determined by experiments.

310v17. To calculate the strength of a truss beam, dimensions in inches.

$$W = \frac{(26a + 3a_1) \cdot d}{3l} \text{ tons. Here } w = \text{safe weight, } a = \text{area of bottom}$$

flange, and b = area of the truss rods, l = the distance between the points of support, and d = depth of the cast metal beam. At p. 51, he states that when the broad flange is uppermost its strength is 100, and when undermost its strength is 173.

Note A. There are various causes which render cast iron beams unsafe for bridges, ware-houses, and factories. The wrought iron beams are lighter, easier handled in building, stronger, and cheaper than cast iron, and are only about two-fifths the weight of cast iron beams of the same strength.

Note B. By comparing thirty principal American trussed bridges, we find that their depth is about one-eighth their span, ranging from one-fifth to one-tenth.

310e18. *Wrought iron beams.*

Note C. *The box-beam* (fig. 82) is the strongest form, weight for weight, best beam (fig. 83) on account of its simple construction, facility of painting; it is recommended by Fairbarn, who says that "taking the strength of a box beam (fig. 82) at 1, that in the form of Fig. 83 would be 0.93, each of equal weight. Beams like Fig. 83 can be made for buildings 60 ft. wide without columns, and with one row of columns they may be 22 inches deep and 5-16 inches thick, with angle iron rivetted.

Let W = breaking weight in tons, $d = 22''$ = depth of beam, a area of the bottom flange, l distances between the supports in inches = 360

$$W = \frac{ad^3}{l} \quad \text{Here} = \text{constant} = 75 \text{ and } a = 6''$$

that is $W = \frac{6 \times 22 \times 75}{360} = 27.5$ tons in the middle, or 55 tons distributed. Fairbarn gives the weight of this beam equal to 40 cwt. and that of wrought iron, having the same strength, equal to 16 cwt. 1 qr. and 14 lbs.

CAST IRON PILLARS.

310e19. $W = m \frac{D^{3.5}}{l^{1.63}}$ tons. W = breaking weight in tons. D = external and d = internal diameters in inches, and b = length in feet. Hodgkinson gives a mean value of 13 irons = 4.6.

To find D in the power $3\frac{1}{2}$. Find the logarithm of D . Multiply it by $3\frac{1}{2}$ and find the natural number corresponding to it.

$W = 42.6 \frac{D^{3.5}}{l^{1.63}}$ tons. The thickness of metal in a hollow pillar is usually taken at one-twelfth its diameter. Assuming the strength of a round pillar at 100, then a square pillar with the same amount of material = 93, a triangular pillar with the same amount of material = 110.

310e20. *Gordon's rule* is considered the best formula.

$P = \frac{fS}{1 + a \frac{l^2}{h^2}}$ Here P = breaking weight in lbs., S = sectional area, l = length, and h = the least external diameter on the least side of a rectangular pillar, f and a = constants. (All in inches.)

For Wrought iron, $f = 36,000$ and $a = .00033$.

" Cast iron, $f = 80,000$ and $a = .0025$

" Timber, $f = 7,200$ and $a = .004$.

Example 1. Let length $l = 14$. Diameter $= h = 8$ inches of a timber pillar or column.

Sectional area = 50,265 multiplied by the value of $f = 72,000$ gives 361908 = fS .

$\frac{14 \times 12 \times 14 \times 12}{8 \times 8} = 336 = \frac{l^2}{h^2}$. This multiplied by .004 = 1,344 and $1 + 1.344 = 2.344$ = the denominator in the formula, which divided into 361908, gives the value of $P = 154,397$ lbs.

The safe weight to be taken at one-sixth to one-eighth for permanent loads and one-third to one-fourth for temporary loads.

310w. We are to find the weight of the proposed wall with the pressure of the roof thereon, and prepare a foundation to support eight times this weight on rock foundation, and in hard clay the safe load may be taken from 17 to 23 lbs. per square inch. In Chicago, on blue clay the weight is

taken at 20 lbs. per square inch. The foundation must be beyond the influence of frost at its greatest known depth.

310w1. *Depth of foundation.* Let P = pressure per lineal foot of the wall, w = weight of one cubic foot of the load to be supported. W = weight of one cubic foot of masonry. f = friction of masonry on argillaceous soil. d = the required depth of the foundation. a = the complement of the angle of repose.

Let us take $f = 0.30$ which is the friction of a wall on argillaceous soil.

$$d = 1.4 \tan \frac{a}{2} \cdot \left\{ \frac{2(P-f)}{W} \right\}^{\frac{1}{4}} \quad (\text{See Fig. 71.})$$

Example. A dam has to sustain water 4 metres high. The specific weight of masonry = 2000 and that of water is = 1000. Let t = thickness at top of wall and T = thickness at the bottom.

$$t = 0.865 \times 4 \sqrt{\frac{1000}{2000}} = 2.44 \text{ metres.}$$

Weight of one lineal metre = $4 \times 2.44 \times 2000 = 19520$ kilogrammes.

$$\text{Friction } -f = 19520 \times 0.30 = 5856$$

$$\text{Pressure } P = 1000 \times \frac{h^2}{2} = 1000 \times 8 = 7000$$

$$\text{and } 8000 - 5856 = P - f = 2144.$$

Taking the complement of the angle of repose = $60^\circ = a$

$f = \tan$ of half a $\tan 30^\circ = 0.578$, then from the above formula

$$d = 1.4 \times 0.578 \sqrt{\frac{288}{2000}} = 1.185 \text{ metres, the required depth of foundation.}$$

The footing is to be equal to the thickness of the wall at base; that is the base of footing will be twice as wide as the wall, and diminish in regular offsets.

The foundation of St. Peter's, in Rome, are built on frustums of pyramids connected by inverted arches.

310w2. The area of the base of footing must be in proportion to the weight to be carried. It is usual to have one square foot of base for every two tons weight. In Chicago, where clay rests on sand, the bearing weight is taken at 20 lbs. per square inch, but there are buildings where the weight is greater, in some cases as high as 34 lbs.

Mr. Bauman, in a small practical treatise on Isolated Piers, makes the offsets for Rubble masonry 4 inches per foot in height. For concrete 3 inches. For dimension stone about the thickness of the stone, but his plan shows the offsets for dimension stone to be four-fifths of the height, and the height = to 1.2 the width at the lowest course of dimension stone.

WALLS OF BUILDINGS.

310w3. Let l , h and t represent the width, height and thickness respectively in French metres.

$$t = \frac{2l+h}{48} = \text{minimum thickness for outer walls.}$$

$$t = \frac{l+h}{48} \text{ for walls of double buildings or of two stories.}$$

$$t = \frac{l+h_1}{36} \text{ for partition walls.}$$

Example. A building having a basement story 5 metres high, 1st story = 2.50 met. high, and the 2d story = 2.50 met. high.

l = width = 11 metres.

$$t = \frac{11+10}{48} = 0.44 \text{ for basement.}$$

$$t = \frac{11+5}{48} = 0.33 \text{ for 1st story.}$$

$$t = \frac{11+2.5}{48} = 0.28 \text{ for 2nd story. These are from } \textit{Guide de Me-}$$

chanique Pratique, by Armegaud.

310w4. *Rondelet* says the thickness of isolated walls ought to be from one-eleventh to one-sixteenth of their height, and walls of buildings not less than one-twenty-fourth the distance of their extreme length. He gives the following table:

Kind of Building.	Outer Walls.		Middle Walls.		Partitions.	
	met.	met.	met.	met.	met.	met.
Odd houses,	0.41 to 0.65		0.43 to 0.54		0.32 to 0.48	
Large buildings,	0.65 to 0.95		0.54 to 0.65		0.41 to 0.54	
Great edifices,	1.30 to 2.30		0.65 to 1.90		0.65 to 1.95	

Rondelet examined 280 buildings, with plain tiled roofs, in France; finds $t = 1.24$ of the width in the clear.

310w5. *Thickness of walls by Gwilt.* To the depth add half the height and divide the sum by 24. The quotient is the thickness of the wall, to which he adds one or two inches.

For Partitions, he says:—To their distance apart add one-half the height of the story and divide by 36 will give t . To this add $\frac{1}{2}$ inch for each story above the ground.

310w6. *To connect Stones.* Iron clamps are put in red hot and filled up with asphalt. This protects the iron forever. Where the clamps are fastened with lead, the iron and lead in the course of time, decompose one another.

Duals of wood dove-tailed 2 inches square, have been found perfect, imbedded in stones as clamps, after being 4000 years in use. In large, heavy buildings, pieces of sheet lead are put in the corners and middle of the stones to prevent their *fleshing*.

310w7. Molesworth & Hurst, of England, in their excellent hand-books, have given valuable tables on walls of buildings. From these and other reliable English sources we find—

First-class houses, 85 ft. high, six stories. The ground and first story are each one-forty-seventh of the total height.

The 2d, 3d, and 4th stories are each 6 inches less; the 5th and 6th stories are each $4\frac{1}{2}$ inches less than the latter.

Second-class, 70 ft. high. The ground, 1st and 2d stories are each one-fifty-fourth of the total height, and 4th and 5th stories, each $6\frac{1}{2}$ inches less than these.

Third-class, 52 ft. high. The ground floor is 1.40 of the total height, and the 1st, 2d, 3d, and 4th stories are $6\frac{1}{2}$ inches less than these.

Fourth-class, 38 ft. high. The ground and first stories are one-thirty-fifth of the total height, and 2d and 3d stories are $4\frac{1}{2}$ in. less than these.

When the wall is more than 70 ft. long, add one-half brick ($6\frac{1}{2}$ inches) to the lower stories.

The footing is double the thickness of the wall, and also double the height of the footing, laid off in regular offsets. The bases must be level.

310w8. In *Chicago*, there is the following ordinance, strictly enforced since the great and disastrous fire of Oct. 9, A., D. 1871. Outside walls

for two stories, 12 inches thick. If more than two stories above the basement, the basement and first story shall not be less than 16 inches, and the walls above the second story not less than 12 inches, and with the exception of the front walls the others will extend at least 12 inches above the roof. For dwelling-houses the above may be reduced 4 inches.

In all buildings over 25 ft. in width, and having no partitions or girders supported by columns or piers, 4 inches must be added for every additional 10 ft. in width of said building.

CHIMNEYS.

310w9. *Chimneys.* That of the *Refining Company*, in Chicago, is 151 ft. high, 12 ft. square at base. Two courses of heavy dimension stones. Base 16 ft. square. Mortar made of cement and roofing gravel. Pressure on foundation 34 lbs. per square inch.

The McCormick Reaper, 160 ft. high, 14 ft. square at base of column, which is circular. It is 25 ft. square at the ground. Flue 6' 8" diameter. Covers an area of 625 square ft. Weighing 1100 tons. Pressure $24\frac{1}{2}$ lbs. per square inch. Built on hard, dry clay.

The Tobacco Chimney, in Paris, is 29 metres high. Outer diameter at the bottom 3.45 metres. Inner dia. at the same = 2.15 metres. At the top the outer dia. = 1.30 and the inner = 1.03 met. This burns 700 kilogrammes of coal, which gives about 150 horse power. Chimneys in France are built from 20 to 30 metres high, rarely 40 metres.

M. Darct recommends one square decimeter to every 3 kilogrammes of coal consumed per hour = 0.64 horse power.

The surface for the grating three times greater than that for the chimney.

Best forms in order are, circular, octagonal, or many sided.

Thickness at top usually one brick in length.

Thickness at top 0.11 to 0.22 metres, and putting d = inner diameter and d_i = outer diameter at top, we have $d = d_i + 0.22$ or $d + 0.44$. And putting D = bottom diameter and D_i = outer dia. at do., we have H = height and m = a tabular constant for slant or inclination. Then $D_i = D + 2 Hm$, where m for the interior is taken bet. 0.012 and 0.018 and for the outer dia. m is taken at 0.025 to 0.035.

Example. $H = 60$ metres. $d = 0.60$ = interior diam.

Then $D = d + 2Hm = 0.60 + 120 \times 0.015 = 2.40$ metres mean

$I' = 0.60 + 0.33 + 120 \times 0.30 = 4.53$ took mean thick-

ness and constants.

Chimney at St. Rollox, Glasgow, 455½ ft. high, and 435½ ft. above bottom of the base, which is of concrete, 6 ft. thick. Load per square ft. 6720 lbs. or 3 tons on concrete.

Chimney at Muspratt's Chemical Works, St. Helen's, near Liverpool, is 406 ft. high.

Chimney at West Cumberland, Harmetide Iron Works. Base of concrete 3 ft. thick, made with hydraulic lime. Height above ground, 250; total height, 267 ft. Pressure at a section 2 ft. above the ground. Working load on concrete 3 tons, and on brickwork 8 tons.

Chimney at Atkins' Soap Works, near Birmingham, England. Total height, 312 ft. Brickwork. Pressure per square ft. at base, 6 tons.

Shot-Tower, Baltimore. Brickwork, 244 ft. high. Working load on base, 6½ tons. The crushing weight = 4.8 times the working load.

United States Light House, 160 ft. high. Brick. Pressure, 3.7 tons. Crushing weight at base = 8½ times the working or safety load.

Pressure on roof for weight of roof, snow, and pressure of the wind, 40 lbs. per square, weather side, and 20 lbs. on the other, under 150 foot span; add 1 lb. for every additional 10 feet, stoney, 524.

Greatest pressure of wind observed in Great Britain was 55 lbs. per square foot = 0.382 lbs. per square inch.

TUNNELS.

310w1. *Tunnels*.—Within brick work, for single track, 20 feet high, 15 feet wide. Double track, 24 feet high, and 24 to 30 feet wide. For canals, 14 to 30 feet high and 14 to 30 feet wide.

Minimum tunnels, $4\frac{1}{2}$ by 3 feet. The form of a horse-shoe, or elliptical, is generally adopted, resting on an inverted arch, excepting that directly under the side walls, which is horizontal. Shafts are sunk to facilitate the lifting of materials and supplying fresh air; they are about 8 feet in diameter. The Mount Cenis tunnel is 7.59 miles long, has no shaft, owing to its great depth below the top of the surface. At each end there is a sump or well, to collect the water of the tunnel, and from which it is lifted.

Some are made by sinking cast-iron cylinders, others steined with brick, those of iron ought to be preserved from oxidation. See Sec. 310w8.

In sinking a shaft lined with brickwork, a drum curb, made of oak timber, mounted with bevelled iron to enable it to sink. The outer diameter is the same as that of the brickwork. The excavation is made greater than the intended shaft. The drum is lowered horizontally as low as the excavation will permit, and the brickwork laid in single courses or rings. There are temporary supports to keep the drum in position until another length is excavated, then these are removed. The sharp edge of the drum penetrates deeper, carrying its already built load of masonry, and giving an additional length on top to be built, etc. At the London Thames tunnel, the Wapping-Street shaft was sunk on a drum to 72 feet deep.

Tunnels in soft ground must be timbered in length. The cost of brickwork in tunnels is about double of what it is over ground.

The Mount Cenis tunnel was made by driving jumpers by atmospheric pressure, using the pressure of five atmospheres, each jumper driven by its own air-cylinders, holes 1 3-16 in. diameter, 80 holes driven in the surface of the tunnel, making a progress of $1\frac{1}{2}$ yards per day.

Thames tunnel, built in soft earth by Brunel, $37\frac{1}{2}$ feet wide, 22 feet high, being in two parallel elliptical archways, each 14 feet wide and 17 feet high; thickness at invert, $2\frac{1}{2}$ feet, at the crown $2\frac{1}{2}$ feet, sides 3 feet, central wall $3\frac{1}{2}$ feet, the whole laid on a flooring of elm planks 3 inches thick. In constructing this, Brunel used a *shield*—a machine resembling a man, moving forward step by step and arm after arm. Cost £12 5s per cubic foot of its bulk, and £1137 per lineal foot. For a full account of this great feat of engineering, see Weal's Quarterly Papers on Engineering.

We believe the mortar made by the ancients was mixed with blood and iron filings.

From the Professional Papers of the Royal English Engineers.

310w2. *Concrete of lime and stones* will not concrete.

Godwin gives stones, 2, sand, 1, lime sufficient to mix well, then add boiling water; use while hot, and ram; the stones not larger than a hen's

egg. *Gravel* means coarse gravel 5, sand, 3. $3\frac{1}{2}$ buckets of *gravel*, $\frac{1}{2}$ bucket of lime, and — bucket of boiling water—ready for use in $2\frac{1}{2}$ minutes.

An arch of concrete, 4 feet thick, was found to be bomb-proof, at Woolwich, England.

TUNNELS.

310w3. *Hoosac Tunnel*, (fig. 83c), has shafts, the central one of which is 1030 ft. deep, of an elliptical form. The conjugate diameter across the roadway is 15 ft., and the transverse along the road 27 ft. There are other shafts, some $6' \times 6'$, $10' \times 8'$, and $13' \times 8'$. Where the shaft is not in rock, it is lined on one side $2' 8''$ to $2' 2''$, and on the other side, $2' 4''$ to $1' 8''$. The work was carried on the same as Mount Cenis, using the Burleigh rock drill, mounted on two carriages; each carrying five drills, standing on the same cross section, 6 ft. asunder. The explosives used, were nitroglycerine in hard rock, and powder in other places. The compressed air, at the time of the application, was 63 lbs. per square inch, which was 2 lb. less, due to its passage through two cast-iron pipes, each 8 inch. in diameter, through which fresh air was supplied to the workmen. Three gangs of men worked each eight hours per day, excepting Sundays.

Average shafts, 26 ft. high and 26 ft. at widest part, sunk 25 feet per month, and in rock, about 9 ft. per month.

Tunnel for one track is 19 ft. from the top of the rail to the intrados of the crown, and widest width = $18\frac{1}{2}$ ft. Thickness of the arch = $1' 10''$, horse shoe form.

310w4. *The Box Tunnel*, (Fig. 83a), on the Great Western Railroad, England, (horse shoe form), is 28 ft. wide at the top of the rail and $24\frac{1}{2}$ ft. high. Thickness of arch $2' 3''$.

At 13 ft. above the rail, width is 30 ft. At 20 ft. above the rail, width is 20 ft. At $24\frac{1}{2}$ ft., width is 0. Length 9600 ft. in clay and lime stone. Shafts at about every 1200 ft.

310w5. *The Sydenham Tunnel*, (Fig. 83b). On the London and Chatham Railroad, England. Length 6300 ft. Five shafts, each 9 ft. diameter. Thickness of arch 3 ft. Width at level of rail $22\frac{1}{2}$ ft. At 5 ft. above rail 24 ft. At 10 = 23 ft. At 16 = 18 ft. At $20\frac{1}{2}$ ft. met under part of the crown.

310w6. *Tunnel for one track*. (Fig. 83c.)

310w7.

BLASTING ROCK.

Let P = lbs. of powder required when l = the length of line of least resistance, that is, to the nearest distance to the surface of the rock in feet, which should not exceed half the depth of the hole.

$P = \frac{l^3}{32}$. One pound of powder will loosen about 10,000 lbs. of rock.

Nitroglycerine is ten times as powerful as powder, but extremely dangerous. *Dualine* is ten times as powerful as powder. *Gun-cotton* is about five times that of powder. Giant, Rendrock, Herculan, and Neptune, about the same as nitroglycerine. Giant powder is preferable, but is more expensive.

In small blasts, 1 pound of powder loosens $4\frac{1}{2}$ tons of rock; and in large blasts, it loosens 2 3-5ths. tons.

It is usual to use $\frac{1}{2}$ to $\frac{3}{4}$ lb. of powder for ton weight of stone to be removed, taking advantage of the veins and fissures of the rock in sinking.

A man in one day will drill in granite, by hammering,	100 to 200 in.
" " " " " churning,	200 "
" " " " " lime stone,	500 to 700 "

310æ8. The bottom of the hole may be widened by the action. Of one part nitric acid added to three parts of water. See Fig. 85, which represents a copper funnel of the same size as the hole. Inside of this is a lead pipe an inch in diameter, reaching to within one inch of the bottom. About the outside of the funnel is made air-tight at the surface with clay around it. At g, above the neck, is a filling of hemp. The acid acting on the limestone in a bore of $2\frac{1}{2}$ inches, will remove 55 lbs. of stone in four hours. The frothy substance of the dissolved rock will pass through the copper tube. And after a few hours, the hole is cleaned and dried, and made ready to receive the powder.

One lb. of powder occupies 30 cubic inches of space, fills a hole 1 inch in diameter and 38 inches deep.

As the square of 1 inch diameter filled with 1 lb. of powder is to 38 inches in depth, so is the square of any other diameter to the depth filled with 1 lb. of powder. See *Sir John Burgoyne's Treatise on Blasting*.

When the several holes are charged they are connected by copper wires with a battery and then discharged.

The blowing up of Hell Gate, by Mr. Newton, is the greatest case of blasting on record.

At the Chalk Cliff, near Dover, England, 400,000 cubic yards were removed by one blast. Length of face removed, 300 feet. Total pounds of powder, 18,500.

ARCHES, PIERS, AND ABUTMENTS.

310æ9. Next page is a table showing several bridges built by eminent engineers, giving their thickness at the crown or key of each, as actually existing, and the calculated thickness, by Levell's formulas. We also give Trautwine. Rankine & Hurst's formulas. M. Levell, in 1855, and since, has been chief engineer of Roads and Bridges in France. We believe that all surveyors and engineers are familiar with the names and works of Trautwine. Rankine & Hurst.

C = thickness of the crown. r = radius of the intrados. h = height of the arch. s = half span. v = height of the arch to the intrados, and r = the radius of the circle. Then,

$$r = \frac{s^2 - v^2}{2v}$$

See Euclid, Book III, prop. 35.*

$$\text{By Levell. } C = \frac{S+10}{30} \text{ for French meters, } = \frac{S+32.809}{30} \text{ for English ft.}$$

By Prof. Rankine. $C = \sqrt{0.12r}$ for a single arch and $\sqrt{0.17r}$ for a series of arches.

$$\text{By Trautwine. } C = \sqrt{\frac{r + \frac{1}{2}S}{4}} + 0.2 \text{ feet for first-class work.}$$

To this add one-eighth for second-class work, and one-fourth for brick or fair rubble work.

$$\text{By Hurst. } C = 0.3 \sqrt{r} \text{ for block stone work.}$$

$$\text{" " } C = 0.4 \sqrt{r} \text{ for brickwork and } 0.45 \sqrt{r} \text{ for rubble work.}$$

$$\text{" " } C = 0.45 \sqrt{S} + \frac{S}{12} \text{ for straight arch of brick, with radiating joints.}$$

Mr. Levell finds his formula to agree with a large number of arches now built from spans of 5 to 43 meters, including circular, segments of circles, semicircular, and elliptical.

* If two lines intersect one another in a circle, the product of the segments of one = the product or rectangle of the others.

BRIDGES, WITH THEIR ACTUAL AND CALCULATED DIMENSIONS.

310w10. THE CALCULATED ARE BY LEVELLE'S FORMULA.

NAMES OF BRIDGES.	Span.	Rise to intrados.	Actual thickness of the key.	Calculated thickness of the key.	Height of the Abutment.	Actual thickness of abutment.	Calculated thickness of abutment.	Function of $\frac{1}{D}$ formula.
SEGMENTS OF CIRCLES.								
Pont de la Concorde, Paris.....	23.40	1.93	0.97	1.11				
" de Pasia, ".....	5.00	.80	.52	.50	2.0	1.70	1.95	.160
" de Courcelles du Nord.....	9.80	.90	.65	.66				
" des Abbatoirs, Paris.....	16.05	1.55	.90	.87	3.93	10.	7.24	.097
" de Ecole Militaire, ".....	28.	2.99	1.14	1.29				
" de Melisey.....	11.40	1.50	.60	.71	3.55	5.20	4.63	.132
" sur le salat.....	14.	1.90	1.10	.80	6.21	5.80	6.06	.136
" de Marbre, Florence, Italy.....	42.23	9.10	1.62	1.74				
" on the Forth, at Stirling, Scotl'd.....	16.30	3.12	.84	.88	6.32	4.88	5.15	.192
" de Bourdeaux, France.....	26.49	8.83	1.20	1.23				
" Saint Maxence Sur la Oise, ".....	23.40	1.95	1.46	1.11	8.45	11.8	12.2	.083
" de la Boucherie, Nuremburg.....	29.60	3.90	1.22	1.32				
" de Dorlaston.....	26.37	4.11	1.07	1.21	5.03	9.76	9.00	.156
" du Rempart, R. R. Orleans to Tours.....	1.20		.45	.37	1.20	.55	.74	1.70
" de Saint Hylarion, R. R. Paris to Chartres.....	2.0		.40	.40	3.80	1.20	1.09	4.40
" de la Tuilerie, ".....	4.0		.50	.47	3.40	1.40	1.58	4.10
" des Voisins, ".....	5.0		.55	.50	2.50	1.50	1.73	5.15
" y Prydd, Wales.....	140	35.	1.5	5.76				
Cabin John, Washington Aqueduct.....	220	57.	4.16	8.42				
Ballochmyle, Ayr, Scotland.....	181	90.5	4.5	7.16				
Dean, Edinburgh, ".....	90	30.	3.0	4.09				
Ordinary over a double R. R. track.....	30	7.5	1.83	2.09				
Grovenor, on the Dee.....	200	42.	4.	7.76				
Turin, Italy.....	147.6	18.	4.90	6.01				
Mersey Grand Junction.....	75.	14.5	3.	3.59				
Philadelphia & Reading R. R.....	44	8.	2.50	2.56				
SEMICIRCLES.								
Pont des Tetes, on the Durance.....	38.0	19.	1.62	1.60				
" de Sucres.....	18	9.	1.	0.93				
" de Corbeil.....	16.82	8.41	0.75	0.89				
" de Franconville.....	7.40	3.70	.60	.58				
" du Crochet.....	4.	2.	.50	.47				
" des Chevres.....	1.50	.75	.35	.38				
" de Orleans A'Tours.....	20.	10.	1.	1.	1.	4.50	4.49	
ELLIPTICAL.								
Pont de Neuilly, Paris.....	38.98	9.74	1.62	1.63	2.30	1080	1080	.250
" de Vissile Sur le Romanche B.....	41.90	11.69	1.95	1.73				
" du Canal Saint Denis.....	12.	4.50	.90	.73	3.10	3.75	3.40	.375
" de Moielins A' Nojent.....	18.	5.13	1.	.93				
" du Saint du-Rhone.....	34.	9.74	1.30	1.47				
" de Wellesly a' Limerick, Ireland.....	21.34	5.33	.61	1.04	3.66	5.03	6.47	.250
" Sur le Loir.....	24.23	8.	1.20	1.14				
" de Trilport.....	25.61	8.77	1.95	1.19				
" ".....	24.50	8.44	1.36	1.15	1.95	5.85	6.21	.344
" Royal, Paris.....	23.52	9.30	1.10	1.12				
" Gignac sur le Herault.....	48.72	13.30	1.95	1.96				
" Alma sur le Seinne.....	43	8.60	1.50	1.76				
" de Vieille Brioude sur le Allier.....	54.20	21.	1.30	2.14				
" Aauss, on the Vienna R. R.....	20.	6.67	1.10	1.00				

The Line of Rupture in a semicircle arch, with a horizontal extrados, is where the line of 60 degrees from the vertical line through the crown meets the arch.

This has been established by Mr. Mery, and Mr. Petit, of France, the latter a Captain of Engineers.

Mr. Lavelle, from Petit, gives for semicircular arches, where d = diameter, t = thickness of the arch or key at the crown.

When the diameter = 2,000, 5,000, 10,000, 20,000, then

$t = \left(\frac{1 + 0.1d}{3} \right) = 0.40, 0.50, 0.67, 1.00$, whose corresponding angles of rupture are 59°. 63°. 64°. and 65°, from the vertical line CD.

Lavelle adopts 60°.

310x. TELFORD'S TABLE.—HIGHLAND BRIDGES.

Span, in feet.	Versed Sine.	Depth of key-stone.	Height of Abutment to Springing.	Mean Thickness of Abutment.	Thickness of Spandries.	Thickness of necessary inverted arches.
6	2'.0"	1'.0"	2'.6"	2'.0"	1'.6"	1'.0"
8	1.6	1.2	2.6	2.0	2.0	1.0
10	3	1.3	3.0	2.6	2.0	1.0
12	3.6	1.4	3.0	3.0	2.6	1.0
18	4	1.6	3.0	4.6	2.9	1.4
24	6	1.9	4.0	5.0	2.9	1.4
30	8	2.0	4.0	5.6	3.0	1.6
50	..	2.6	6.0	6.0	3.6	1.6

310x0. SEGMENT ARCHES.

BATTER OF PIERS $\frac{1}{4}$ -INCH IN ONE FOOT.

Span.	Depth of Crown.	Height of Piers.	Thickness of Piers.	Thickness of Pier at Top, Battered.	Thickness of Piers for Flat Girder Trusses.	Thickness of Piers for Flat Girders with Batter.	Rise of Arch to the Intrados.
10ft	1'.2"	5' to 20'	3' to 3'. 9"	3'. 0'	1'. 3' to 2'. 7 $\frac{1}{2}$ '	2. 3	3'.0'
15	1.6	5 " 20 "	" " "	3. 0	2. 7 $\frac{1}{2}$ " 3. "	2. 7 $\frac{1}{2}$	4.6
20	1.6	5 " 40 "	3 " 4. 6	3. 0	2. 7 $\frac{1}{2}$ " 3. 4 $\frac{1}{2}$	" "	6.0
25	1.6	5 " 40 "	3 " 4.10 $\frac{1}{2}$	3. 9	3. 0 " 4. 1 $\frac{1}{2}$	3. 4 $\frac{1}{2}$	7.3
30	1.10 $\frac{1}{2}$	5 " 40 "	4. 1 $\frac{1}{2}$ " 6. "	4. 1	4. 1 $\frac{1}{2}$ " 6. 0	4. 1 $\frac{1}{2}$	9.0
35	2.3	10 " 40 "	4.10 $\frac{1}{2}$ " 6. 4 $\frac{1}{2}$	4.10	5. 3 " 6. 4 $\frac{1}{2}$	4. 6	10.6
40	2.3	" " "	5.77 " 7. 1 $\frac{1}{2}$	5. 3	4.10 $\frac{1}{2}$ " 6. "	4.10 $\frac{1}{2}$	11.3
45	2.7	" " "	6.47 " 7. 6	6. 0	5. 7 $\frac{1}{2}$ "	" "	13.0
50	3.0	" " "	7. 1 " 8. 3	7. 1	6. 9	" "	14.6
55	3.0	" " "	7.10 " 9. 4	7.10	7. 1 $\frac{1}{2}$	" "	16.0
60	3.0	" " "	8. 7 " 9. 8	8. 3	7.10 $\frac{1}{2}$	" "	17.3

310x1. RADIUS OF CURVATURE. Fig. 86—Let ABCD be a curve of hard substance. Wind a cord on it from D to A. Take hold of the cord at A and unwind it, describing the oscillatory curve a, b, c, d. When the cord is unwound as far as B and C, etc., the point or end A will arrive at B, C, etc., and the line BC will be the radius of curvature to the point B, and the line Cc will be the radius of curvature to the point C.

The curve ABCD may be made on thick pasteboard, and drawn on a large scale, by which mechanical means the radius of curvature can be found sufficiently near.

The radius of curvature of a circle is constant at every point.

310x2. *Tension* is the radius of curvature at the crown.

310x3. *Piers*. L. B. Alberti says piers ought not to be more than one-fourth or less than one-fifth the span.

The pier of Blackfriar's Bridge, London, is about one-fifth the span. The pier of Westminster Bridge, London, is about one-fourth the span. The pier at Vicenza, over the Bacchilione, Palladio, makes one-fifth the span.

Piers generally are found from one-fourth to one-seventh of the span. The end of the pier against the current is pointed and sloped on top, to

break the current and floating ice, if any. When the angle against the current is ninety degrees, the action of the water is the least possible, and half the force is taken off.

310x4. *The horizontal thrust of any semi-arc.* Fig. 87, AEKD. By section 313, find G, the centre of gravity of said arc, or by having the plan drawn on a large scale—about four feet to one inch—the point G can be found sufficiently near.

Draw OGM at right angles to AQ, and draw DO parallel to AQ. We find the area A, of AEKD. We have AM from construction, and OM = QD = rise at the arch, and AQ = one-half the span, and the height of the pier, XY, to find the thickness of FE = BL. We have OM : AM :: A : T, equal to its thrust in direction of AH on the pier. We have taken the area A to be in proportion to the weight, and make the pier to resist three times the thrust, T. This fourth term F, will be the surface of the pier BEFL, whose height, XY, is given. Therefore,

$$\text{Thickness of the pier out of water.} = \frac{3T}{XY}$$

Let AQ = 28, MO = 18, AM = 9, A = 270, and XY = 30.

18 : 9 :: 270 : 135 = T = thrust on the pier at B.

The pier 30 feet high is to sustain for safety three times 135 = 405
 $\frac{405}{30} = 13.5 \text{ ft.} = \text{BL, the required thickness.}$

310x5. The thrust to overturn the pier about the point L,

$$= \frac{AM \times A \times CB}{OM} \text{ which must be } = EB \times BL.$$

$$\therefore BL = \left(\frac{2AM \times A \times CB}{OM \times EB} \right)^{1/2} = \text{thickness of a dry pier.}$$

$BL = \left(\frac{7AM \times A \times CB}{OM(3.5EB - AB)} \right)^{1/2}$ thickness *in* water. Here we take A, as before, three times the area of AEKD.

In circular and elliptical arches, we take AB = diameter for circular, and transverse axis for elliptical; CD for rise or versed sine in the circular, and the conjugate diameter in elliptical, and DQ for the generating circle of the cycloid. DP = abscissa, and PC its corresponding ordinate to any point, C, in the curves.

Having determined on the span and rise of the arch, and the thickness, DK, at the crown, we find the height, CI, at the point C, corresponding to the horizontal line, PC, an ordinate to the abscissa DP. See the above figure.

$$CI = \frac{DK \times DQ^3}{PQ^3} \text{ For the circle.}$$

$$CI = \frac{DK \times DQ^3}{PQ^3} \text{ For the ellipse - same as for the circle.}$$

$$CI = \frac{DK \times DQ^2}{(DQ - DP)^2} \text{ For the cycloid.}$$

$$CI = \frac{DK \times (C + DP)}{C} \text{ For the catenary.}$$

Here C is the tension or radius of curvature at D.

The above three forms are practicable. Sometimes for single arches the parabolic arch is used.

CI = DK for every point, C, in a parabola.

In all cases, CI is at right angles to the line AB.

Gwilt, in his work on the equilibrium of arches, says: "The parabola may be used with advantage where great weights are required to be discharged from the weakest part of an edifice, as in warehouses, but the scantiness of the haunches renders them unfit for bridges."

310x6. THE CATENARIAN is correctly represented by driving two nails in the side of a wall or upright scantlings, at a distance equal to the required span BA. From the centre, drop a line marking the distance DQ equal to the rise of the arch, and let a light chain pass through the point to ADB, and we have the required curve. Let DP and CP be any abscissa and corresponding ordinate, to find CI from the intrados to the extrados.

TO FIND THE TENSION AT D.

310x7. Let c = tension constantly at the vertex.

KD = thickness of the arch at crown = a .

DP = any abscissa x , and PC = y , its corresponding ordinate.

$$C = \frac{x}{2} \times \left(\frac{y^2}{x^2} + 0.3333 - \frac{8x^2}{45y^2} + \frac{691x^4}{3780y^4} - \frac{23851x^6}{453600y^6} \&c. \right) \text{ This is}$$

Dr. Hutton's formula, excepting that the parenthesis, is erroneously omitted.

$$C = \frac{x}{2} \times \left(\frac{y^2}{x^2} + 0.3333 - 0.1778 \frac{x^2}{y^2} + 0.1828 \frac{x^4}{y^4} - 0.0526 \frac{x^6}{y^6} \&c. \right)$$

Example given by Hutton. Let DQ = 40 = x , and one-half the span AQ = 50 = y .

Here the tension $C = 20 \times (1.5625 + 0.3333 - 0.1137 + 0.0749 - 0.0138, \&c.)$ That is $C = 20 \times 1.8432 = 36.864$, as given by Hutton.

TO FIND THE RADIUS OF CURVATURE AND TANGENT TO ANY POINT C OF THE CATENARIAN. Fig. 90.

310x8. Produce QD to P making $OP = CO \times \sqrt{2c + DO + DO^2}$.

Join PC, which will be the tangent to the point C. From the point C, draw CW at right angles to AP. And make As $c : c + DO :: c + DO : CR$ = Rad. of curvature.

When the abscissa $DO = o : C : c :: c : CR = c$. Hence the tension at the lowest point D is equal to the radius of curvature.

Let the span = 100 and rise = 40 feet, then radius of curvature for a segment of a circle = 51.25 = radius of curvature.

" Parabola,	= 30.125	"
" Ellipsis,	= 62.5	"
" Catenary,	= 36.864	"

The strength of the Parabola at the crown is to the above figures as the rad. of curvature of the other figures, to that of the parabola; hence the strength of the parabola is 2.1 times that of the ellipsis, and P : C :: 36.864 : 30.129.

Parabola is 1.22 as strong as the Catenarian.

To find the extrados to the point C. Whose abscissa DO = x and ordinate CO = y are given. Fig. 90.

Let KD = a and DO = x and CO = y as above. Then from Hutton:

$$CI = \frac{ac + ax}{c} = a + \frac{ax}{c}$$

$$KV = \frac{c - a}{c} \times x = x - \frac{ax}{c}$$

DO : KV :: always as $c : c - a$.

The extrados will be a straight line when $a = c$, the tension at K.

In the above example, where we have found $c = 36,864$ feet to have the extrados a straight line, would require $a = KD$, to be nearly 37 feet.

Assuming the same span 100, rise = 40, and putting $DK = 6$ feet, the extrados and the arch will be as figure 91. This arch is only proper for a single arch, where the extrados rises considerably from the springing to the top.

$AC = CB$ is given = $a = \frac{1}{2}$ -span. $CD = h =$ height. Figure 92. $DE =$ distance of chain to the lower part of the roadway parameter. K and M any points in the curve, from which we are to find the suspension rods KD and MP , etc.

$$DK = \frac{CD - DE}{AC} \times HK^2 + \text{and } \frac{CD - DE}{AC} \times DM^2 + DE = MP$$

We have $\frac{CD - DE}{AC}$, a constant quantity; Let it = c , and divide EG into any parts as Q, P, D, R , etc. Then the length of the rod at $R = RS = c \times ER^2$ and rod $QT = c \times EQ^2$.

310x9. To find the sectional area in inches of any rod, as DK , and the strain in pounds on it, at K .

Let $W =$ weight of one lineal foot of the roadway when loaded with the maximum weight.

Strain on K .—Let $2 \frac{h - t}{a^2} - 0.0003$ be divided into W , it will give the strain in pounds on K . Let this strain be represented by S .

Sectional area of the rod $DK = S + 0.0000893$ lbs.

$$DK = \frac{CD - DE}{AC^2} \times HK^2 + DE = \text{length of the rod } DK.$$

Let $W =$ weight of every lineal foot of roadway and its maximum load thereon. Strain = $2 \frac{CD - DE}{AC^2} - 0.0003$, this divided into W , gives the strain on the lowest point D of the chain.

Sectional area of chain at D is found by multiplying the last, by .0000893.

Example. Half span $AC = 200$. $DE = 2$ feet, wt. of one lineal foot of road = 500. Horizontal distance $HK = 100$ ft. $CD = 40$ ft.

$$40 - 2 = \frac{38 \times 100^2}{200 \times 200} = \frac{380000}{200 \times 200} = 9.5 = HD. \text{ And } 9.5 + 2 = 11.5 = \text{rod } KD.$$

$$2 \frac{(40 - 2)}{200 \times 200} = \frac{38 \times 2}{40000} = \frac{76}{40000} = 0.0019, \text{ and } .0019 - 0.0003 = 0.0016.$$

$$\text{And } \frac{500}{.0016} = 3125.000 \text{ lb. lb.} = \text{strain on the lowest point of chain at } B.$$

$$\text{And } 31250000 \times 0.0000893 = 279 \text{ square inches} = \text{sec. area at } B.$$

$$\frac{2 \times 9\frac{1}{2}}{100} = \frac{19}{100} = .190, \text{ this squared} + .0261 + 1 = 1.0262, \text{ whose}$$

square root = 1.013, which $\times$ by 3125000 = 3165625 lbs. strain on the point K , which $\times$ by 0.0000893 = sectional area = 283 square inches of chain at K .

Basis here. Took one-sixth the load for coefficient of safety.

A bar of iron 12 feet long and 1 inch square weighs 3.3 lb.

The tensile strain to break a square inch of wrought-iron is taken at 6720 lb., the iron loaded with one-sixth its breaking weight.

On bridges, the load should not exceed one-twentieth of the weight which would crush the materials in the arch stones; and where there is a heavy travel, should not exceed one-thirtieth.

PIERS AND ABUTMENTS.

310x10. When the angle at the point of an abutment against the stream is 90 degrees, then the pressure on the pier is but one-half what it would be on the square end. The longer the side of the triangular end of the pier is made the less will be the pressure. Let ABC represent the triangular end against the stream, and C the furthest point or vertex. *Gwill* says "that the pressure on the pier is inversely proportional to the square of the side AC, or BC, and that the angle at C ought not to be made too acute, lest it should injure navigation, or form an eddy toward the pier. *Abutments.* In a list of the best bridges, we find the abutment at the top from one-third to one-fifth the radius of curvature at the crown of the arch.

Molesworth gives the following concise formula :

$$T = \text{thickness of abutments} = \left(6R + \left(\frac{3R}{2H} \right)^2 \right)^{1/2} - \frac{3R}{2H}$$

Here R = rad. at crown in feet, H height of the abutment to springing in feet, for arches whose key does not exceed three feet in depth.

Example. R = 20 +. H = 10.

$(120 + 9)^{1/2} = 11.36$ from which take 3, will give the abutments without wing walls or counterforts.

Abutments.—To counteract the tendency to overturn an abutment, let the arch be continued through the abutment to the solid foundation, or by building, so as to form a horizontal arch, the thrust being thrown on the wing walls, which act as buttresses.

2d.—By joggling the courses together with bed dowel joggles so as to render the whole abutment one solid mass.

310x0. The depth of the voussoirs must be sufficient to include the curve of equilibrium between the intrados and extrados.

The voussoirs to increase in depth from the key to the spangling, their joints to be at right angles to the tangents of their respective intersections and curve of equilibrium.

The curve of equilibrium varies with the span and height of the arch stones, the load and depth of voussoirs, and has the horizontal thrust the same at any point in it.

The pressure on the arch stones increase from the crown to the haunches.

310x1.

SKEW ARCHES.

In an ordinary rectangular arch, each course is parallel to the abutments, and the inclination of any bed-joint with the horizon will be the same at every part of it. In a skew arch this is not possible. The courses must be laid as nearly as possible at right angles to the front of the arch and at an angle with the abutments. The two ends of any course will then be at different heights, and the inclination of each bed-joint with the horizon will increase from the springing to the crown, causing the beds to be winding surfaces instead of a series of planes, as in the rectangular arch. The variation in the inclination of the bed-joints is called the thrust of the beds, and leads to many different problems in the cutting. See *Buck on Skew Bridges*.

EAST RIVER BRIDGE, NEW YORK.

310x2. Brooklyn tower, 316 feet high, base of caisson, 102 × 168 feet.

New York tower, 349 feet high, base of caisson, 102 × 178 feet.

The Victoria Bridge, at Montreal, 7000 feet long, one span, 330 feet and fourteen of 242 feet, built in six years. Cost, \$6,300,000. Built by Sir Robert Stephenson.

Concrete Bridges.—One of these built by Mr. Jackson in the County of Cork, Ireland, is of cement, one part sand. Clear sharp gravel, six to eight parts. Rammed stones in the piers. He also built skew bridges of the same materials.

Mr. McClure built one 18 feet span, $3\frac{1}{2}$ feet rise, and $1\frac{1}{2}$ feet thick at the key, and $2\frac{1}{2}$ feet at the springing. Built in ten hours, with fifteen laborers and one carpenter. Piers are of stone, centre not removed for two months. Proportions of materials used: Portland cement, 1, sand, 7 to 8, 40 per cent of split stone can be safely used in buildings, and 25 per cent in bridges. Stones used in practice, 4 to 6 inches apart. Cottage walls, 9 inches thick. Chimney walls, 18 inches. Partitions, 4 inches. Walls, sometimes 18 feet high and 12 inches thick. Garden walls, $\frac{1}{4}$ mile long, 11 feet high, and 9 inches thick. Cisterns, 5 feet deep and 6×5 feet 9 inches thick.

Cost of one cubic yard of this concrete wall, 12 to 15 shillings, at 3 to 4 dollars.

310x3. These kind of buildings are common in Sweden, since 1828, and built in many towns of Pomerania, where its durability has been tested. It is applicable to moist climates. Where sand can be had on or near the premises, walls can be built for one-fourth the cost of brickwork. In Sweden, they use as high as 10 parts of sand to 1 of hydraulic lime. The lime is made into a milk of lime, then 3 parts of the sand is added, and mixed in a pug-mill made for that purpose. After thus being thoroughly mixed the remainder of the sand is added. These walls resist the cold of winter, as well as the heat of summer.

The pug-mill is made cylindrical, in which on an axis are stirrers, moved by manual labor, or horse power, as in a threshing machine. One of these, in ordinary cases, will mix 729 cubic feet in one day. Let us suppose a house, 40 feet long, 20 feet wide, and 1 foot thick. This *caisse* will mix 1 to $1\frac{1}{2}$ *toise*, cubic, per day, which will be made into the wall by three men, making the wall all round, 6 feet high, moved upwards between upright scaffolding poles. There is a moveable frame, stayed at proper distances, laid on the wall to receive the beton where two men are employed in spreading it.

310x4.

TO TEST BUILDING STONES.

Take a cubic 2 inches each way, boil it in a solution of sulphate of soda (Glauber salts) for half an hour, suspend it in a cold cellar over a pan of clear sulphate of soda. The deposit will be the comparative impurities.

Rubble work.—The stones not squared.

Coursed work.—Stones hammered and made in courses.

Ashlar.—Each stone dressed and squared to given dimensions.

To prevent sliding.—Bed dowels are sunk one-half inch in each, made of hardwood.

Walls faced with stone and lined with brick are liable to settle on the inside, therefore set the brickwork in cement, or some hard and quick setting mortar. The stones should be sizes that will bond with the brickwork.

Bond in masonry is placing the stones so that no two adjoining joints are above or below a given point will be in the same line. The joints must be broken.

Stones laid lengthways are called stretchers, and those laid crossways, headers.

Brick work.—English bond is where one course is all stretchers and the next all headers.

Flemish bond is where one brick is laid stretcher, the next a header and in every course a header and stretcher alternately.

Tarred hoop-iron is laid in the mortar joints as bonds.

310x5. ANGLES OF ROOFS, WITH THE HORIZONTAL.

CITY.	COUNTRY.	N. Latitude.	Plain tiles.	Hollow tiles.	Roman tiles.	Slates.
Carthagea, ..	Spain,	37° 32'	24° 12'	16° 12'	19° 12'	22° 12'
Naples,	Italy,	40 52	26 12	18 12	21 12	24 12
Rome,	do	41 53	27 0	19 0	22 0	25 0
Lyons,	France,	45 48	30 0	22 0	25 0	28 0
Munich,	Germany,	48 7	31 48	23 48	26 48	29 48
Viena,	Austria,	48 5	32 0	24 0	27 0	30 0
Paris,	France,	48 52	32 36	24 36	27 36	30 36
Frankfort,	On the Main,	50 8	33 48	25 48	28 48	33 48
Brussels,	Belgium,	50 52	34 36	26 39	29 36	32 36
London,	England,	51 31	35 24	27 24	30 24	33 24
Berlin,	Germany,	52 33	36 36	28 36	31 36	34 36
Dublin,	Ireland,	53 21'	37 48	29 48	32 48	35 48
Copenhagen,	Denmark,	55 42	40 48	32 48	35 48	38 48
St Petersburg,	Russia,	59 40	48 24	40 24	43 24	46 24
Edinburgh,	Scotland,	55 57	41 12	33 12	36 12	39 12
Bergen,	Norway,	60 5	48 24	40 24	43 24	46 24

From the above table, we see that the elevation of the roof increases one degree for every $\frac{3}{4}$ ths degree of latitude, from Carthagea to Bergen.

Pressure on Roof. For weight of roof, snow, and pressure of the wind, 40 lbs. per square foot, on the weather side, and 20 lbs. on the other, under 150 feet span. Add 1 lb. for every additional 10 feet.—*Stoney*, p. 524.

Greatest pressure of wind observed in Great Britain has been 55 lbs. per square foot = 0.382 lbs. per square inch.

310x6. TRUSSED BEAMS AND ROOFS.

Let AD = b = half the span. AB = tie-beam resting on the wall-plates
CD = h = height = king-post. AC = b = length of principal rafters, 10 to 12 feet asunder.

Q = angle BAC = angle of *minimum* pressure on the foot of the rafter.
Secant of the angle Q = l . See fig. 83 A.

When Q = 35° 16', the pressure P is a minimum. *Moseley's Mechanics*, Sec. 302, Eq. 395.

Then $h = 0.7072b$
 $l = 1.2248b$
 $W = 1.2248b s w$ } Here s = distance between each pair of rafters.
" w = weight of each square foot of roof,
including pressure of the wind and snow, as determined in the locality. W = weight on each rafter.

310x7. To calculate the parts of a common Roof. Let a = sectional area of a piece of timber. d = its breadth, and l = its length. s = span of the roof in feet. p = length in feet of that part of the tie-beam supported by the queen-post.

King-post. $a = ls \times 0.12$ for fir, and $a = ls \times 0.13$ for oak.

Queen-post. $a = lp \times 0.27$ for fir, and $a = lp \times 0.32$ for oak.

The Beam. $d = \sqrt[3]{\frac{l}{b}} \times 1.47$ for fir.

Principal rafters with a king-post, $d = \frac{l^2 s}{b^3} \times 0.96$ for fir.

" with two queen-posts, $d = \frac{l^2 s}{b^3}$

Straining Beam. Its depth ought to be to its thickness as 10 to 7.

$$d = \sqrt[4]{l s^{\frac{1}{2}}} \times 0.9 \text{ for fir.}$$

Struts and Braces. $d = \sqrt[4]{l p^{\frac{1}{2}}} \times 0.8$ for fir, and $b = 0.7 d$.

Purlins.— $d = \sqrt[4]{b^{\frac{1}{2}} c}$ for fir, or multiply by 1.04 for oak, and $b = 0.6 d$.

Common Rafters. $d = \sqrt[3]{\frac{l}{b}} \times 0.72$ for fir, or 0.74 for oak.

Two inches is the least thickness for common rafters, therefore, in this case, $d = 0.571 l$ for fir.

310x8. *Laminated arched beams* formed of plank bent round a mould to the required curve and bolted are good for heavy travel and great speed.

JETTIES.

310x9. In rivers, at and near their outlets, sand bars are formed where the velocity is less than that of the deep water on either side. The desired channel is marked out, and two rows of piles are driven on the out-sides, to which the mattresses are tied. The space or jetties thus piled are filled with mattresses made of fascines of brushwood, bolted by wooden bolts and boards on the top and bottom of each, sloping from the outside towards the channel.

One in New Orleans, now in progress of construction by Capt. Eads, C. E., is from 35 to 50 at bottom, and 22 to 25 at the top, mattresses 3 feet thick. From 3 to 6 layers are laid on one another. Mud and sand assist to fill the interstices. They are loaded with loose stones, and the top covered with stone. The water thus confined causes a current, which removes the bars. Drags may be attached to a boat and dragged on the bars, which will assist in loosening the sand.

The mattresses are built on frames on launchways on the shore, and then floated and tied to the piles.

Jetties may be from 10 feet upwards, according to the location. Those of the Delta, at the mouth of the Danube, are filled with stones.

See Hartley on the Delta of the Danube, for 1873.

General Gilmore, U. S. Engineer's report on the Jetty System, for 1876.

General Comstock's, U. S. A., report on the New Orleans South Pass.

310x9. *Excavations for Foundation*, measured in cubic yards, pit measurement. Allow 6 inches on each side for stone and brickwork, and no allowance is made where concrete is used. Where excavation is made for water or gas pipes, slopes of 1 to 4 is allowed. State for moving away the earth not required for backfilling, the distance to which it is to be moved, and inclination, and how disposed of, whether used as a filling or put in a water embankment. This done for first proposed estimate.

Filling is measured as embankment measurement, for the allowances for shrinkage add 8 per cent for earth and clay when laid dry. When put in water, add one-third. Bog stuff will shrink one-fourth. See p. 210.

100 cubic feet of stone, broken so as to pass through a ring $1\frac{1}{2}$ inch in diameter, will increase in bulk to..... 190 cubic feet.

Do do to pass through a 2 inch ring, 182 " "

Do do " " $2\frac{1}{2}$ " 170 " "

Rubble Masonry.—One cubic yard requires 1 1-5 cubic yards of stone and 1-4 cubic yard of mortar. Ashlar masonry requires 1-8 its bulk of mortar.

All contractors ought to be informed that when they haul 100 yds from the pit, that it will not measure the same in the "fill" or embankment.

Isolated Peirs are measured solid, to which add 50 per cent.

Brick Walls are measured solid, from which deduct one-half the openings; then reduce to the *standard measurement*, for example: multiply the cubic feet by $2\frac{1}{2}$, and divide by 1000, to find the number of thousands of bricks, as calculated in Chicago, where the brick is 8 by 4 by 2 inches.

NOTE.—One must observe the local customs.

The English standard rod is $16\frac{1}{2} \times 16\frac{1}{2} \times 13\frac{1}{2}'' = 272$ superficial feet of the standard thickness of $1\frac{1}{2}$ bricks or $1\frac{1}{2}'' = 306$ cubic feet. 100 cubic feet brickwork requires 41 imperial gallons of water, or 49 United States to slake the lime and mix the mortar. When the wall is circular and under 25 feet radius, take the outside for the width. Include sills under 6 inch.

Cornices. The English multiply the height by the extreme projection for a rectangular wall.

In various places in America, the height of the cornice is added.

Chimneys, flues, coppers, ovens, and such like, are measured solid, deducting half the opening for ash-pits and fireplaces.

Three inches of the wall-plate is added to the height for the wall; this compensates for the trouble of embedding the wall-plates.

Stone Walls. Measured as above, and take 100 cubic feet per cord of stone mason's measurement. The cord is 8x4 feet by 4 feet, or 12½ cubic feet, or it is measured in cubic feet. The surface is measured by the superficial foot, as ashler hammered cut stone, and entered separate.

Chimneys are measured solid, only the fireplaces deducted in England.

Slater's Work. Measured by the square of 100 feet. Measure from the extreme ends. Allow the length by the gauge for the bottom course or eve. Deduct openings; but add 6 inches around them; also 6 inches for valley hips, raking, and irregular angles.

Filling. Measured as above. Add for valleys, 12", eaves, 4". All cutting hip, etc., 3 inches.

A Pantile is $1\frac{1}{2}'' \times 1\frac{1}{2}'' \times \frac{1}{2}''$ inch, weighs $5\frac{1}{2}$ lb, more or less, 1 sq. = 897 lb.

A Pantile $10\frac{1}{2}'' \times 6\frac{1}{2}'' \times \frac{5}{8}''$ inch, weighs $2\frac{1}{2}$ lb, " " " = 1680 lb.

Pantile laths, are 1 inch thick and $1\frac{1}{2}$ inch wide and 10 feet long.

Plastering. Render two coats and set. Lime, 0·6 cubic feet; sand, 68; hair, 19 lb; water, 2·7 imperial gallons.

Measure from top of baseboard to one-half the height of the cornice; deduct one-half for openings, or as the custom may be.

Gutters should have a fall of 1 inch in 10 feet.

Painting. 1 lb. of paint will cover 4 superficial yards, the first coat, and about 6 yards each additional coat. About 1 lb of putty for stopping every 20 yards.

1 gallon of tar and 1 lb of pitch cover 12 yards first coat, and about 17 yards the second coat.

1 gallon of priming color will cover 50 superficial yards.

" white paint " 44 " "

Other paints range from 44 to 50 " "

Take where the brush touched. Keep difficult and ornamental work separate. Also, the cleaning and stopping of holes, and other extras.

Joiner's Work. Measured as solid feet or squares of 100 feet superficial.

Flooring by the square of 100 feet superficial.

Skirting, per lineal foot, allowing for passages at the angles.

Sashes and frames. Take out side dimensions, add 1 inch for any middle bar in double sashes.

Engineers and architects ought to discountenance draining and wasting sewage into rivers. The paving of streets with wooden blocks, which is certainly unhealthy, causing malarial fevers. MacAdam stones, heavily rolled, etc., or stone blocks, are better. The French pavement, now used in London, is the best, which is made by putting a coat of asphalt $2\frac{1}{2}$ to 3 inches thick, on a bed of concrete 8 to 10 inches thick.

CHICAGO, Oct. 15, 1878.

M. McDERMOTT.

SANITARY HINTS.

310x10. The surveyors and engineers are frequently obliged to encamp where they encounter mosquitoes and diseases of the bowels.

Oil of pennyroyal around the neck, face, and wrists.

Apply around the neck and face, at the line of hair, and around the wrists, two or three times during the day and once or twice at night. This is a pleasant application to use, but disagreeable to the mosquitoes. We used to use a mixture of turpentine and hog's fat or grease, and at other times, wear a veil; both were but of temporary benefit; the first, was a nuisance, and the latter, by causing too much perspiration, was unhealthy.

Drinking too much water can be avoided by using it with finely ground oatmeal; by using this, the surveyor and engineer, and all their men using it, will not drink one-fifth as much water as if they did not use it.

DIARRHŒA.

The best known remedy is tincture of opium; tinct of camphor; tinct of rhubarb; tinct of capsicum (Cayenne pepper); of each one ounce. Add, for severe griping pains, 5 drops of oil of Anisee to each dose.

Dose.—25 drops in a little sweetened water, every hour or two, till relieved. Sometimes we put a little tannic acid, which is a powerful astringent. Avoid fresh meat, and use soda crackers.

To escape Chills and Fever, use quassi, by pouring some warm water on quassi chips, and letting it stand for the night. Take a cupful every morning. Never allow wet clothes to dry on you, if it can possibly be avoided. Tannic acid and glycerine will heal sore or scald feet.

Wafers applied to your corns, after being well soaked in lye water, will cure them. Apply the wafer after being moistened on the tongue; then apply a piece of linen or lint. Repeat this again when it falls off, in two or three days, and it will remove the corn and the pain together.

To Disinfect Gutters, Sewers, etc. Take one barrel of coarse salt and two of lime; mix them thoroughly, and sprinkle sparingly where required. This acts as chloride of lime.

To Disinfect Rooms in Buildings. Take, for an ordinary room, half an ounce of saltpetre; put on a plate previously heated, on this pour half an ounce of sulphuric acid (oil of vitriol); put the plate and contents on a heated shovel, and walk into the room and set the plate on some bricks previously heated. This destroys instantaneously every smell, enables the nurse to go to the bedside of any putrid body and remove it. Where there is sickness, as now in Memphis, etc., it causes great relief to the sick and protection to those in attendance. This is Dr. Smith's disinfectant, used at Gross Isle, Quarantine Station, below Quebec, Canada, in 1848. We have used it on many occasions, with satisfactory results, since then. Clothes hung in a well-closed room for two days, and subjected to this on three plates, would be rendered harmless.

CHICAGO, 23d Sept., 1878.

M. McDERMOTT.

FORCE AND MOTION.

311. *Matter* is any substance known to our senses.

Inertia of Matter is that which renders a body incapable of motion.

Motion is the constant change of the place of a body.

Force is a power that gives or destroys motion.

Power is the body that moves to produce an effect.

Weight is the body acted upon.

Momentum of a body is the product of its velocity by the quantity of matter in it.

Gravity is the force by which bodies descend to the centre of the earth.

Centrifugal Force is that which causes a body, moving around a centre, to go off in a straight line.

Centripetal Force is that which tends to keep the body moving around the centre.

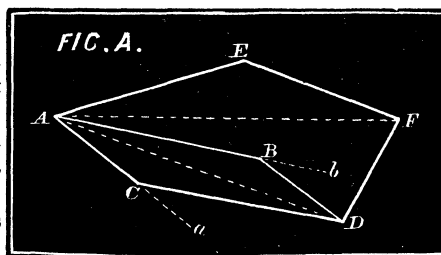
Let D B represent a straight line; D C A B
•-----•-----•-----•
D, C, A and B, given forces.

If D and C in the same direction act on A, their force = their sum.

If D and B in the same line act on A, but in different directions, the effect of their force will equal their difference, as D - B, where D is supposed the greater.

If D and C act on A in one direction, and B in the other, then the effect = D + C - B.

When the forces C and B act on A, making a given $\angle BAC$, the single force equal to both is called the resultant.



Resultant of the forces B and C acting on A is = D; or by representing forces B, C and D by the lines AB, AC, AD, then the resultant in the above will be the diagonal AD, and AB and AC are its components.

Component or Component Forces are those producing the resultant, as AB and AC.

Rectangular Ordinates are those in which the $\angle BAC$ is right angled, or when a force acts perpendicularly to the plane AC or AB.

In the last figure, the force AC forces A in a direct line towards a, and the force AB forces A towards b in the same line; but when both forces act at the same time, A is made to move in AD, the diagonal of the parallelogram made by the forces AC and AB, by making CD = AB, and AC = BD.

Parallelogram of Forces is that in which AB and AC, the magnitudes of forces applied to the body A, gives the diagonal AD in position and magnitude. The diagonal AD is called the *resultant*, or resulting force.

Example. The force AB = 300 lbs., the force AC = 100 lbs., the angle BAC = right angle. Here we have AC and AB = BD and CD; $\therefore \sqrt{(AC^2 + CD^2)} = AD$; i. e., $\sqrt{(10000 + 90000)} = \sqrt{(100000)} = AD = 316.23$ lbs.

Otherwise, $AD = (AB^2 + AC^2 + 2AB \times AC \times \cos. \angle BAC)^{\frac{1}{2}}$
m

Let the $\angle BAC = 60^\circ$; $\therefore$ its cosine $= .5$; then

$$300^2 = 90000$$

$$100^2 = 10000$$

$$2 \times 300 \times 100 \times 0.5 = 30000$$

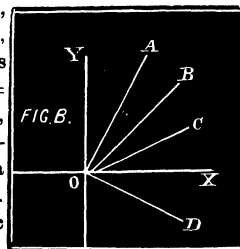
$$AD^2 = 130000 \therefore AD = 360.55 \text{ lbs.}$$

Having the $\angle BAC$, to find the $\angle CAD$. $AD : AB :: \sin \angle BAC : \sin \angle DAC$.

$$\therefore \sin \angle DAC = \frac{AB \cdot \sin \angle BAC}{AD}.$$

Let CA , BA and EA be three forces in magnitude. We find the resultant AD of the forces CA and BA ; then between this resultant and the force EA find the line EF , the required resultant of the three forces; and so on for any other number of forces. By drawing a plan on a scale of 100 lbs. to the inch, we will find the required forces.

Or, let OX and OY be two rectangular axis, and AO , BO , CO and DO represent forces, and a , b , c , d = the angles made by the forces A , B , C and D , with the axis OX . Let S = sum of the forces acting in direction of axis OX , and s the sum of the forces acting in the direction of OY ; then we have $S = AO \cdot \cos. a + BO \cdot \cos. b + CO \cdot \cos. c + DO \cdot \cos. d$.
 $s = AO \cdot \sin a + BO \cdot \sin b + CO \cdot \sin c - DO \cdot \sin d$. Resultant $= \sqrt{(S^2 + s^2)}$.



In this case, the forces are supposed to move inclined to the axis OX , as well as to OY .

Note. In the first quadrant XOY , the sines and cosines are positive; but in the fourth quadrant XOW , the sines must be negative.

The effect of any force acting on a body is in proportion to the cosine of its inclination.

If three forces, B , C and D , act on a point A , so as to keep it in equilibrium, each of these is proportional to the sine of the $\angle$ made by the other two. (See fig. B.)

Let B and C be the components of the resultant D , then

$$D : C :: \sin \angle BAC : \sin \angle BAD.$$

$$D : B :: \sin \angle BAC : \sin \angle CAD.$$

If we represent the three forces meeting in A , by the three contiguous edges of a parallelopiped, their resultant will be represented in magnitude and direction, by the diagonal drawn from their point of meeting to the opposite angle of the parallelopiped.

If four forces in different planes act upon a point and keep it in equilibrium, these four forces will be proportional to the three edges and diagonal of a parallelopiped formed on lines respectively parallel to the directions of the forces.

Polygon of Forces. Let OA , OB , OC and OD in fig. B. represent forces in position and magnitude. From A draw AE = and parallel to OB , EF = and parallel to OC , FG = and parallel to OD ; then the line OG = resultant in magnitude and direction.

The sum of the moments, of any number of forces acting on a body, must be equal to sum of the moments of any number of forces acting in opposite directions, so as to keep the body from being overthrown.

FALLING BODIES.

312. All bodies are attracted to the centre of the earth, fall in vertical lines, and with the same velocity.

Velocity acquired by a body in falling increases with the time.

Uniformly accelerated motion is that which augments in proportion to the time from its commencement.

If a body falls through a given space in a given time, it acquires a speed or velocity which would carry it over twice that space in the same time.

ANALYSIS OF THE MOTION OF A FALLING BODY.

Time in seconds from a state of rest.	Comparative spaces fallen through in each successive second.	Constant difference. Velocities acquired at the end of times in second col. = V.	Comparative heights fallen through from a state of rest = H.
1	1 h	2 h	1 h
2	3 h	4 h	4 h
3	5 h	6 h	9 h
4	7 h	8 h	16 h
5	9 h	10 h	25 h
6	11 h	12 h	36 h
etc.	etc.	etc.	etc.
n	(2 n - 1) h	2 n h	n ² h

Here h = half the initial of gravity, being half the velocity acquired by a body falling in *vacuo* at the end of the first second. As g, the initial of gravity, is = 32.2, $\therefore$ h = 16.1. The value of g varies with the latitude, but the above is near enough.

From the above, we find that by putting H = total height, and V = the acquired velocity, $V = 12 h = \sqrt{4 h \times 36 h} = \sqrt{2 g H}$. Here $2 g = 4 h$.

Let $V = 10 h = \sqrt{4 h \times 25 h} = \sqrt{2 g H} = 8.02 \sqrt{H}$, etc.
 $V = 2 n h = \sqrt{4 h \times n^2 h} = \text{do.} = \text{do.}$

This is the general equation for the velocities of bodies moving in *vacuo*, from which it appears that

Velocities are to one another as the square roots of their heights.

Heights are to one another as the squares of their velocities. But as bodies do not move in *vacuo*, the velocities are less by a constant quantity of resistance, which we put = m.

Theoretical Velocity = $8.02 \sqrt{H}$, or as now used = $8.03 \sqrt{H}$.

Actual Velocity = $8.03 m \sqrt{H}$, in which m is to be determined by experiments.

To find the velocity of a stream of water. Take a ball of wax, two inches in diameter, or a tin sphere partly filled with water, and then sealed, so that two-thirds of it will be in the water. Find the elapsed time from the ball passing from one given point to another. Repeat the measurements until two of them agree.

Mean velocity is in the middle of the stream and at half its depth.

Let V = surface, and v = mean velocity; then, according to Prony, $v = 0.816 V$ for velocities less than 10 feet per second. (*See Sequel for Water Works.*)

Composition of Motions is like the composition of forces, and the same operations may be performed. If, in fig. A, last page, a body acting on

A drives it to B in 300 seconds, in the direction A B, and in the direction A C drives it to C in 100 seconds, $\therefore$ it is driven by the united forces to D in 360.55 seconds.

$V = t g$. Here t = time in seconds, and $g = 32.2$.

$$H = \frac{v t}{2} = \frac{t^2 g}{2} = \frac{v^2}{2g}, \text{ because } t = \frac{v}{g}. \text{ Here } H = \text{space fallen through.}$$

Example. Let a body fall during 10 seconds; then we have,

$$V = 10 \times 32.2 = 322 = \text{velocity at the end of 10 seconds.}$$

$$H = \frac{322}{2} \times 10 = 1610 = \text{space passed through in 10 seconds.}$$

$$\text{Or, } H = \frac{100 \times 32.2}{2} = 1610; \text{ or, by the third equation for } S,$$

$$H = \frac{(322)^2}{2 \times 32.2} = \frac{103684}{64.4} = 1610.$$

When the velocity begins with a given acquired velocity = c ,

$V = c + t g$. Here c is constant for all intervals.

$$H = c t + \frac{t^2 g}{2} = \left(\frac{c + V}{2}\right) t = \frac{V^2 - c^2}{2g} \text{ for accelerated motion.}$$

When the motion is retarded, and begins with velocity c ,

then $V = c - t g$.

$$H = c t - \frac{t^2 g}{2} = \left(\frac{c - V}{2}\right) \cdot t = \frac{c^2 - V^2}{2g}$$

$$\text{From above we have } V = t g; \therefore t = \frac{V}{g}$$

$$\text{Also, } H = c t - \frac{t^2 g}{2}. \text{ Substituting the value of } t, \text{ we have,}$$

$$H = \frac{V^2 g}{2g^2} = \frac{V^2}{2g}$$

$$V^2 = 2gH; \text{ but } H = \text{the total height} = H;$$

$$\therefore V = \sqrt{2gH} = 8.02 \sqrt{H} = \text{formula for free descent.}$$

$$H = \frac{V^2}{64.4}, \text{ and by putting } m = \text{coefficient or constant of resistance, we}$$

$$\text{find } V = m \sqrt{2gH}, \text{ and } H = \frac{V^2}{m^2 \times 2g}$$

$$\text{Actual velocity } V = (8.02 m \sqrt{H}) \text{ and } H = \left(\frac{V^2}{64.4 \times m^2}\right) \text{ all in feet.}$$

CENTRE OF GRAVITY.

313. *Centre of Gravity* is that point in a mass which, if applied to a vertical line, would keep the whole body or mass in equilibrium.

In a Circle, the centre of gravity is equal to the centre of the circle.

In a Square or Parallelogram—where the diagonals intersect one another.

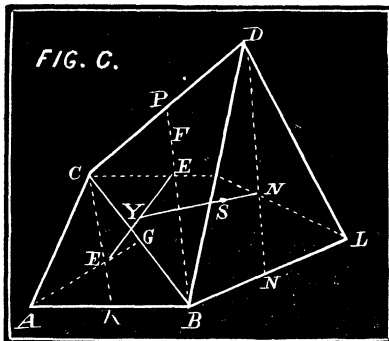
In a Triangle—where lines from the angles to the middle of the opposite line cut one another (see annexed figure). Where C H, D G and B P cut one another in the same point F, then G F = one-third of G D, and H F = one-third of C H. Hence, the centre of gravity of a triangle is at one-third of its altitude.

In a Trapezoid, A B C D, let E F be perpendicular to A B and C D.

$$\text{When } E G = \frac{E F}{3} \times \frac{C D + 2 A B}{C D + A B}, \text{ let } E F = h, A B = b, \text{ and } C D = c;$$

$$\text{then } EG = \frac{h}{3} \times \left(\frac{c+2b}{c+b} \right).$$

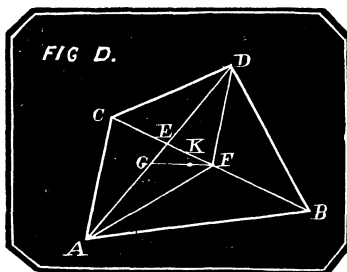
Trapezium. Let $ABCD$ be the given trapezium; join B and C ; find the centre of gravity E of the $\triangle ACB$, and also the centre of gravity F of the $\triangle CBD$; join E and F ; let $EF = 36$; let the area of $\triangle ACB = 1200$, and that of $CBD = 1500$; then, as $1200 + 1500 : 1200 :: 36 : FG = 16$; and in general figure, $ABDC : ACB :: FE : FY$.



In the annexed figure, $AK = KB$, $CG = GB$, $BH = HD$, and Y is the required centre of gravity of ABD .

Let the figure have three triangles, as $ABLD$. Find the centre of gravity N of the $\triangle BLD$; join Y and N ; then, $ABLD : \triangle BLD :: YN : YS$. Hence, S = required centre of gravity of $ABLD$. Points E, F, N , are the centres of the inscribed circles. By laying down a plan of the given figure on a large scale, we can find the areas and lines EY and YS , etc., sufficiently near.

Otherwise, by Construction. Let $ABCD$ be the required figure. Draw the diagonals AD and CB ; bisect BC in F ; make $DE = AG$; join FG , and make $FK =$ one-third of FG ; then the point K will be the required centre of gravity.



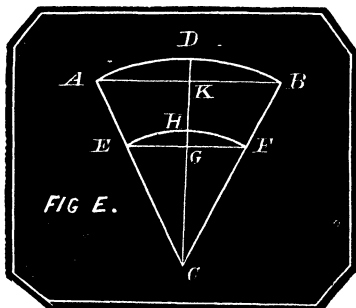
Cone or pyramid has its centre of gravity at one-fourth its height.

Frustum of a Cone has its centre of gravity on the axis, measured from the centre of the lesser end, at the distance $\frac{h}{4} \left(\frac{3R^2 + 2Rr + r^2}{R^2 + Rr + r^2} \right)$. Here R = radius of the greater end, and r = that of the lesser; h = height of the frustum.

Frustum of a Pyramid, the same as above, putting S = greater side, instead of R , and s = lesser side, instead of r .

In a Circular Segment, having the chord b , height h , and area A , given. Distance from the centre of the circle to the centre of gravity on $h = \frac{1}{12} \left(\frac{b^3}{A} \right)$

In a Circular Sector CAB , there are given the arc ADB , the angle ACB , AB and the arc ADB can be found by tab. 1 and 5, the radius CD bisecting the arc ADB , and putting G = centre of gravity, then its distance from the centre

$$= CG = \frac{\text{chord}}{\text{arc}} \times \frac{C}{D}.$$


Example. Let $\angle A C B = 40^\circ$, and $C D = 50$ feet, to find $C G$. Here the $\angle A C D = 20^\circ$, and $C A = 50$, $\therefore$ by table 1, its departure $A K = 17.10$; this multiplied by 2, gives the chord $A B = 34.20$. By table 5, $40^\circ = .698132$; this multiplied by 50, gives arc $A D B = 34.91$.

$$\text{Now, } C G = \frac{34.907}{34.2} \times \frac{2}{3} \times 50 = \frac{3490.7}{102.6} = 34.02.$$

In a Semicircle, the centre of gravity is at the distance of $0.4244 r$ from the point C .

In a Quadrant, the point G is at the distance $C G = 0.60026$.

In a Circular Ring, $E H F B D A$, there are given the chords $A B$, $E F = a$ and b , and the radius $C A = R$, and radius $C E = r$, and $C G = \frac{4}{3} \cdot \frac{\sin. \frac{1}{2} c}{c} \cdot \left(\frac{R^3 - r^3}{R^2 - r^2} \right)$. Here $c = \text{angle } A C B$.

Centre of Gravity of Solids.

314. *Triangular Pyramid or Cone.* The point G , or centre of gravity, is at three-fourths of its height measured from the vertex.

Wedge or Prism. The point G is in the middle of the line joining the centres of gravity of both ends.

In a Conic Frustrum, the distance of G from the lesser end is equal to $\frac{h}{4} \left(\frac{3 R^2 + 2 R r + r^2}{R^2 + R r + r^2} \right)$. Here $R = \text{radius of greater base}$, and $r = \text{that of the lesser}$.

In a Frustrum of a Pyramid, the above formula will answer, by putting R for the greater side and r for the lesser side of the triangular bases. The value will be the length from lesser end.

In any Polyhedron, the centre of gravity is the same as that of its inscribed or circumscribed sphere.

In a Paraboloid, the point G is at $\frac{2}{3}$ height from the vertex.

In a Frustrum of do. The distance of G from lesser end $= \frac{h}{3} \left(\frac{2 R^2 + r^2}{R^2 + r^2} \right)$.

In a Prismoid or Ungula, the point G is at the same distance from the base as the trapezoid or triangle, which is a right section of them.

In a Hemisphere, the distance of the centre of gravity is three-eighths of the radius from the centre.

In a Spherical Segment, the point G , from the centre of the sphere $= \frac{3.1416 h^2}{S} \left(r - \frac{h}{2} \right)$. Here $h = \text{height}$, and $S = \text{solidity}$.

SPECIFIC GRAVITY AND DENSITY.

315. *Specific Gravity* denotes the weight of a body as compared with an equal bulk of another body, taken as a standard.

Standard weight of solids and liquids is distilled water, at 60° Fahrenheit or 15° Centragrade. At this temperature, one cubic foot of distilled water weighs 1000 ounces avoirdupois.

When 1 cubic foot of water, as above, weighs	1000 ounces,
1 cubic foot of platinum weighs	21500 “
That is, when the specific weight of water =	1,
then the specific weight of platinum =	21.5.
One cubic foot of potassium weighs	865 “
$\therefore$ its specific gravity, compared with water, =	0.865.

316. *To find the Specific Gravity of a liquid.* The annexed is a small bottle called specific gravity bottle, which, when filled to the cut or mark a b on the neck, contains, at the temperature of 60° Fahrenheit, 1000 grains of distilled water. Some bottles have thermometers attached to them; but it will be sufficiently accurate to have the bottle and thermometer on the same table, and raise the heat of the surrounding atmosphere and liquid to 60°. Some bottles contain 500 grains. Some have a small hole through the stopper. The bottle is filled, and the surplus water allowed to pass through the stopper.

C is a Counterpoise, that is, a weight = to the empty bottle and stopper.

To find the Specific Gravity. Fill the bottle with the liquid up to the mark a b (which appear curved), and put in the stopper. Put the bottle now filled into one scale, and the counterpoise and necessary weight in the other. When the scales are fairly balanced, remove the counterpoise. Let the remaining weight be 1269 grains; then the specific gravity = 1.269, which is that of hydrochloric or muriatic acid.

Density of a body is the mass or quantity compared with a given standard. Thus, platinum is $21\frac{1}{2}$ times more dense than water, and water is more dense than alcohol or wood.

Hydrometer is a simple instrument, invented by Archimedes, of great antiquity (300 B. C.), for finding the specific gravity of liquids. It can be seen in every drug store. See the annexed figure, where A is a long, narrow jar, to contain the liquid; B, a vessel of glass, having a weight in the bulb and the stem graduated from top downward to 100. The weight is such that when the instrument is immersed in distilled water at 60° Fahrenheit, it will sink to the mark or degree 100.

Example. In liquid L the instrument reads 70°. This shows that 70 volumes of the liquid L is = to 100 volumes of the standard, distilled water; $\therefore 70 : 100 :: 1 : 1.428 =$ specific gravity of L.

The property of this instrument is, that it sustains a pressure from below upwards = to the weight of the volume of the liquid displaced by such body. Those generally used have a weight in the bulb and the stem graduated, and are named after their makers, as Baume, Carties, Gay Lussac, Twaddle, etc. Syke's and Dica's have moveable weights and graduated scales.

To find the Specific Gravity by Twaddle's Hydrometer. Multiply the degrees of Twaddle by 5; to the product add 1000; from the sum cut off three figures to the right. The result will be the specific gravity.

Example. Let $10^\circ =$ Twaddle; then $10 \times 5 + 1000 = 1.050 =$ specific gravity.

317. *To find the Specific Gravity of a solid, S.* Let S be weighed in air, and its weight = W. Let it be weighed in water, and its weight = w. Then $\frac{W - w}{w} =$ weight of distilled water displaced by the solid S. Then $\frac{W}{W - w} =$ specific gravity.

Rule. Divide the weight in the air by the difference between the weight in air and in water. The quotient will be the specific gravity.

Let a piece of lead weigh in air =	398 grains,
and suspended by a hair in distilled water =	362.4 "
Difference =	35.6

This difference divided into 398, gives specific gravity = 11.176, because $35.6 : 1 :: 398 : 11.176 =$ specific gravity of the lead.

318. *To find the Specific Gravity of a body lighter than water.*

<i>Example.</i> A piece of wax weighs in air = B =	133.7 grains;
Attached to a piece of brass, the whole weight in air = C =	183.7
Immersed in water, the compound weighs = c =	38.8
Weight of water = in bulk to brass and wax = C - c =	144.9
Weight of brass in air = W =	50
“ “ in water = w =	44.4
Weight of equal bulk of water = W - w =	5.6
Bulk of water = to wax and brass = C - c =	144.9
“ “ = to brass alone = W - w =	5.6
“ “ = to wax alone = C - c - (W - w) =	139.3

That is, $C - c + w - W = 139.3$.

B : C - c + w - W :: specific gravity of body : specific gravity of water. That is,

W : C - c + w - W ; specific gravity of body : 1.

$$\text{Specific gravity of body} = \frac{B}{C - c + w - W} = \frac{133.7}{139.3} = 0.9598.$$

The above example is from Fowne's Chemistry; the formula is ours.

319. *To determine the Specific Gravity of a powder or particles insoluble in water.* Put 100 grains of it into a specific gravity bottle which holds 1000 grains of distilled water; then fill the bottle with water to the established mark, and weigh it; from which weight deduct 100, the weight of the powder. The remainder = weight of water in the bottle. This taken from 1000, leaves a difference = to a volume of water equal to the powder introduced.

<i>Example.</i> In specific gravity bottle put B =	100 grains
Filled with water, the contents = C =	1060 “
Deduct 100 from 1060, leaves weight of water = C - B =	960 “
This last sum taken from 1000, leaves 1000 - C + B =	40 “

Which is = to a volume of water = to the powder.

$$40 : 1. :: 100 : 2.5 = \text{required specific gravity} = \frac{B}{1000 + B - C}$$

To find the Specific Gravity of a powder soluble in water. Into the specific gravity bottle introduce 100 grains of the substance soluble in water; then fill the bottle with oil of turpentine, olive oil, or spirits of wine, or any other liquid which will not dissolve the powder, and whose specific gravity is given; weigh the contents, from which deduct 100 grains. The remainder = the weight of liquid in the bottle, which taken from 1000 leaves the weight of the liquid = to the bulk of the powder introduced.

<i>Example.</i> In specific gravity bottle put of the powder =	100 grains
Fill with oil of turpentine, whose specific gravity =	0.874
Found the weight of the contents	890 “
890 - 100 = weight of oil of turpentine in bottle =	790 “

which has not been displaced by the powder.

But the bottle holds 874 grains, $\therefore 874 - 790 =$ 84

That is, 84 is the weight of a volume of the oil, which is equal to the volume of powder introduced. Consequently,

$874 : 1000 :: 84 : 96.1 = \text{weight of water} = \text{to the volume of powder introduced. And again, as } 96.1 : 100 :: 1 : 1.04 = \text{required specific gravity.}$

319a. SPECIFIC GRAVITIES OF BODIES.

SUBSTANCES.	Specific Gravity, one cubic ounces.	Weight of foot in lb.	SUBSTANCES.	Specific Gravity, one cubic ounces.	Weight of foot in lb.
<i>Metals.</i>			Mansfield sandstone.	2338	146.1
Brass, common.....	7820	489.8	Unhewn stones.....		135
Copper wire.....	8878	554.8	Hewn freestone.....		170
“ cast.....	8788	549.2	Coal, bituminous....	1270	79.3
Iron, cast.....	7207	450.1	Coal, Newcastle.....	1270	79.3
“ bars.....	7788	486.7	“ Scotch.....	1300	81.2
Lead, cast.....	11352	709.5	“ Maryland.....	1355	84.6
Steel, soft.....	7833	489.5	“ Anthracite.....	1436	89.7
Zinc, cast.....	6861	428.8	<i>Granites.</i>		
Silver, not hammer'd	10474	654.6	Granite, mean of 14.		169
“ hammered....	10511	656.9	Granite, Aberdeen...	2625	164
<i>Woods.</i>			“ Cornwall.....	2662	166.4
Ash, English.....	845	52.8	“ Susquehanna.	2704	169
Beech.....	700	43.8	“ Quincy.....	2652	165.8
Ebony, American...	1331	83.1	“ Patapsco.....	2640	165.7
Elm.....	671	41.9	Grindstone.....	2143	133.9
Fir, yellow.....	657	41.1	<i>Limestones.</i>		
“ white.....	569	35.5	Limestone, green....	3180	193.7
Larch, Scotch.....	540	33.8	“ white....	3156	197.2
Locust.....	950	59.4	Lime, quick.....	804	50.3
Norway spars.....	580	36.3	Marble, common.....	2686	167.9
Lignumvitæ.....	1333	83.3	“ French.....	2649	165.6
Mahogany.....	1063	66.4	“ Italian white..	2708	169.3
Maple.....	750	46.8	Mill-stone.....	2484	155.3
Oak, live.....	1120	70	Paving do.....	2416	151
“ English.....	932	58.2	Portland do.....	2428	151.7
“ Canadian.....	872	54.5	Sand.....	1800	112.5
“ African.....	980	61.3	Shale.....	2600	162.5
“ Adriatic.....	990	61.9	Slate.....	2672	167
“ Dantzic.....	760	47.5	Bristol stone.....	2510	156.9
Pine, yellow.....	660	41.2	Common do.....	2033	127
“ white.....	554	34.6	<i>Grains and Liquids.</i>		
Walnut.....	671	41.9	Water, distilled.....	1000	62.5
Teak.....	750	46.9	“ Sea.....	1026	64.1
<i>Stones, Earth, etc.</i>			Wheat.....		46.08
Brick.....	1900	118.7	Oats.....		24.58
Chalk.....	2784	174	Barley.....		43.01
Charcoal.....	441	27.6	Indian corn.....		46.08
Clay.....	1930	120.6	Alcohol, commercial.	837	52.3
Common soil.....	1984	124	Beer, pale.....	1023	63.9
Loose earth.....			“ brown.....	1034	64.6
Brick work.....		109	Cider.....	1018	63.6
Sand.....		112.3	Milk, cow's.....	1032	64.5
Craighleith sandstone	2232	139.5	Air, atmospheric.....		.075
Dorley Dale do.....	2628	164.2	Steam.....		.037

One ton, or 2240 lbs. of	Average bulk in cubic feet.	Name of Materials used.	Shrink'gs or increase per cent.
Paving Stone,	14.835	Light sandy earth,	.12 shr.
Brick,	18.823	Yellow clayey “	.10 “
Granite.....	13.506	Gravelly “	.08 “
Marble.....	13.070	Surface or vegetable soil,	.15 “
Chalk.....	12.874	Puddled clay,	.25 “
Limestone, filled in pieces,	14	Earth filled in water,	.30 “
“ compact,	11.273	Rock broken into small pieces,	1/2 to 1 in.
Elm,	64.460	Rock broken to pass through an inch and a half ring,	105 “
Mahogany, Honduras,	64	Do. do. 2 inch ring,	90 “
“ Spanish,	42.066	Do. do. 2 1/2 do.	70 “
Fir, Mar forest,	51.650	One cubic yard of the 1 1/2 stone above weighs	2130 lb.
“ Riga,	47.782	Do. to pass through 2 inch,	2300 lb.
Beech,	51.494	Do. to pass through 2 1/2 inch ring,	2508 lb.
Ash and Dantzic oak,	47.153		
Oak, English,	36.206		
Common soil,	13.044		
Loose earth.....	20.551		
Clay,	13.574		
Sand,	20		

m2

MECHANICAL POWERS.

The Mechanical Powers are: the lever, inclined plane, wheel and axle, the wedge, pulley, and the screw.

319c. *Levers* are either straight or bent, and are of three kinds.

LEVERS CONSIDERED WITHOUT WEIGHT.

Lever of the first kind is when the power, P, and weight, W, are on opposite sides of the fulcrum, F. Then $P : W :: AF : BF$, which is true for the three kinds of levers, and from which we find $P \times BF = W \times AF$.

$$P = \frac{W \times AF}{BF}, \text{ and } W = \frac{P \times BF}{AF}. \quad (\text{See Fig. I.})$$

$$BF = \frac{W \times AF}{P}, \text{ and } AF = \frac{P \times BF}{W}.$$

Lever of the second kind is when the weight is between the fulcrum and the power, (Fig. II.) Then $P : W :: AF$ to BF , as above.

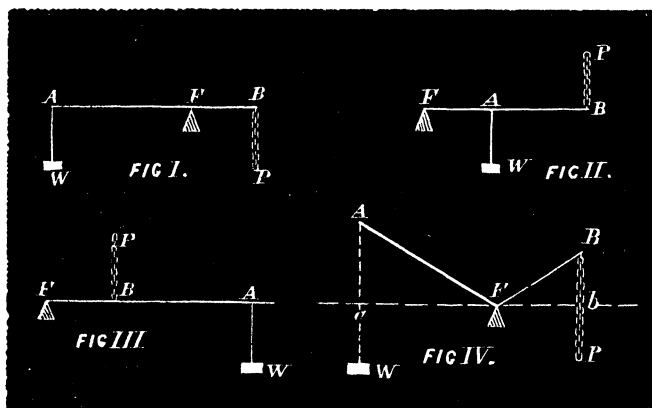
Lever of the third kind (Fig. III.) is when the power is between the fulcrum and the weight. Then $P : W :: AF$ to BF , as above.

Hence, we have the general rule: *The power is to the weight as the distance from the weight to the fulcrum, is to the distance from the power to the fulcrum.*

In a bent lever (Fig. IV.), instead of the distances AF and FB, we have to use Fa and Fb. Then $P : W :: Fa : Fb$; or, $P : W :: FA \times \cos. < A Fa : FB \times \cos. < B Fb$.

Let PABW represent a lever (see Fig. V.) Produce PA and WB to meet in C. Now the forces P and W act on C; their resultant is CR, passing through the fulcrum at F.

Let $AF = a$, $BF = b$, $\angle PAB = n$, and $\angle ABW = m$. Then $P : W :: b \sin. < m : a \sin. < n$;
And $P \cdot a \sin. n = W \cdot b \sin. m$.



LEVERS HAVING WEIGHT.

319d. When the lever is of the same uniform size and weight. Let $AB = a$ lever whose weight is w . (Fig. VI.)

Case 1. Let the centre of gravity, f , be between the fulcrum, F, and power, P; then we have, by putting $Ff = d$, $W \cdot AF = P \cdot BF + dw$,

$$P = \frac{W \cdot AF - dw}{BF}, \text{ and } W = \frac{P \cdot BF + dw}{AF}.$$

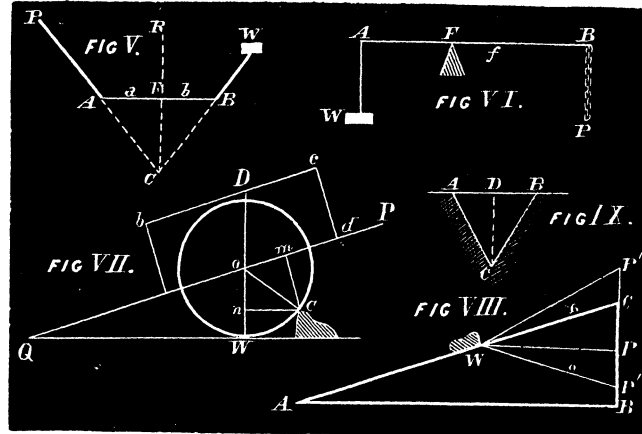
Case 2. When the centre of gravity, f , is between the fulcrum and the weight.

Then $W \cdot A F + d w = P \cdot B F$.

$$W = \frac{P \cdot B F - d w}{A F}, \text{ and } P = \frac{W \cdot A F + d w}{B F}.$$

Example from *Baker's Statics*. Let the length of the lever = 8 feet, $A F = 3$; $\therefore B F = 5$, its weight = 4 lbs., and W suspended at $A = 100$ lbs. Required the weight P suspended at B , the beam being uniform in all respects. We have the centre of gravity, a , = 4 feet from A , and at 1 foot from F towards P . Then, by case 1,

$$P = \frac{W \cdot A F - d w}{B F} = \frac{100 \cdot 3 - 1 \times 4}{5} = \frac{300 - 4}{5} = 59 \text{ 1-5 lbs.}$$



319e. Carriage wheel meeting an obstruction (see Fig. VII.) is a lever of the first kind, where the wheel must move round C .

Let $D W C = a$ wheel whose radius = r , load = $a b c d = W$. The angle of draught, $P Q W$, = a , and C , the obstruction, whose height = h .

Let $C n$ and $C m$ be drawn at right angles, to $O W$ and $O P$. Then $C m$ represents the power, and $C n$ the weight; then $P : W :: C n : C m$: sine $\angle C O n$: sine $\angle C O m$.

$D W = 2 r$; $\therefore D n = 2 r - h$; and by Euclid, B. 2, prop. 5,

$$(2 r - h) \cdot h + O n^2 = C O^2.$$

$$(2 r h - h^2)^{\frac{1}{2}} = \sqrt{(C O^2 - O n^2)} = C n; \therefore$$

$$\text{Sine } \angle C O n = \frac{C n}{C o} = \frac{\sqrt{(2 r h - h^2)}}{r} = C m.$$

When the line of draught is parallel to the road, then $C m = r - h$.

From this we have $P : W :: \sqrt{2 r h - h^2} : r - h$,

$$\text{And } P = W \cdot \frac{\sqrt{(2 r h - h^2)}}{r - h}. \text{ A general formula.}$$

Example. A loaded wagon, having a load of 3200 lbs., weight of wagon 800, meets a horse-railroad, whose rails are 3 inches above the street, the diameter of the wheel being 60 inches. Require the resistance or necessary force to overcome this obstacle.

Total weight of wagon and load, 4000 lbs. Weight on one wheel, 2000.

$$\therefore P = 2000 \times \frac{\sqrt{60 \times 3 - 9}}{30 - 3} = 968.9 \text{ lbs., which is about three times}$$

the force of a horse drawing horizontally from a state of rest.

Hence appears the injustice of punishing a man because he cannot leave a horse-railroad track at the sound of a bell, and the necessity of the local authorities obliging the railroad companies to keep their rail level with the street or road.

Of the Inclined Plane.

319f. Let the base, $AB = b$, height, $BC = h$, and length, $AC = l$. The line of traction or draught must be either parallel to the base, AB , as WP' parallel to the slant, or the inclined plane, as WP , or make an angle a with the line CW , W being a point on the plane where the centre of pressure of the load acts.

When the power P' acts parallel to the base, we have—

$$\begin{aligned} P' : W :: BC : BA :: h : b; \text{ or,} \\ P' : W :: \sin < BAC : \sin < ACB. \\ P' = \frac{W \cdot h}{b}, \text{ and } W = \frac{P' b}{h}. \end{aligned}$$

$$h = \frac{P' b}{W}, \text{ and } b = \frac{Wh}{P'}.$$

When the line of traction is parallel to the slant.

$$\begin{aligned} P : W :: h : l; \text{ hence, we have } Pl = Wh, \\ W = \frac{Pl}{h}, P = \frac{Wh}{l}, \\ h = \frac{Pl}{W}, \text{ and } l = \frac{Wh}{P}. \end{aligned}$$

When the line of traction makes an angle a with the slant, then

$P'' : W :: \sin < BAP'' : \cos < P''WC$, from which, by alternation and inversion, we can find either quantity.

Example. $W = 20000$ lbs., $< BAC = 6^\circ$, $< P''WC = 4^\circ$. Required the sustaining power, P'' .

$$\begin{aligned} P'' = \frac{\sin BAP''}{P''WC} = \frac{W \sin BAC}{\cos < P''WC} = W \cdot \frac{\sin 4^\circ}{\cos 6^\circ} = W \cdot \frac{.06976}{.99452} \\ = \frac{1395.2}{.99452} = 1413 \text{ lbs.} \end{aligned}$$

Of the Wheel and Axle.

319g. When the axle passes through the centre of the wheel at right angles to its plane, and that a weight, W , is applied to the axle, and the power, P , applied to the circumference, there will be an equilibrium, when the power is to the weight as the radius of the axle is to the radius of the wheel. Let R = radius of the wheel, and r = radius of the axle, both including the thickness of the rope; then we have

$P : W :: r : R$; from which we have

$$PR = Wr, \text{ and } P = \frac{Wr}{R}, \text{ and } W = \frac{PR}{r}. \quad (A.)$$

$$R = \frac{Wr}{P}, \text{ and } r = \frac{PR}{W}.$$

Compound Axle is that which has one part of a less radius than the other. A rope and pulley is so arranged that in raising the weight, W , the rope is made to coil on the thickest part, and to uncoil from the thinner. An equilibrium will take place, when $2P \cdot D = W(R - r)$.

D = distance of power from the centre of motion. R = radius of thicker part of axis, and r = that of the thinner.

319h. *Toothed Wheels and Axles or Pinions.* Let a , b and c be three axles or pinions, and A , B and C , three wheels.

The number of teeth in wheels are to one another as their radii.

$P : W :: abc : ABC$: that is, the power is to the weight as the product of all the radii of the pinions is to the product of all the radii of the wheels. Or, P is to W , as the product of all the teeth in the pinions is to the product of all the teeth in the wheels. (B.)

Example 1. A weight 2000 lbs. is sustained by a rope 2 inches in diameter, going round an axle 6 inches in diameter, the diameter of the wheel being 8 feet.

$$\text{From formula A, } P = \frac{Wr}{R};$$

That is, $P = \frac{2000 \times 4}{49} = 163.26 \text{ lbs.}$

Example 2. In a combination of wheels and axles there are given the radii of three pinions, 4, 6 and 8 inches, and the radii of the corresponding wheels, 20, 30 and 40 inches. What weight will $P = 100 \text{ lbs.}$ sustain at the circumference of the axle or last pinion.

By formula B, $P A B C = W a b c.$

$$W = \frac{P A B C}{W a b c} = \frac{100 \times 20 \times 30 \times 40}{4 \times 6 \times 8} = 12500 \text{ lbs.}$$

Of the Wedge. (Fig. IX.)

319i. The power of the wedge increases as its angle is acute. In tools for splitting wood, the $\angle A C B = 30^\circ$, for cutting iron, 50° , and for brass, 60° .

$P : W :: A B : A C$; or,

$P : W :: 2 \text{ sine } A C B : 1.$

Of the Pulley. (See next Fig.)

319j. The pulley is either fixed or moveable.

In a fixed pulley (Fig. I.), the power is equal to the weight.

In a single moveable pulley (Fig. II.), the rope is made to pass under the lower pulley and over the upper fixed one. Then we have $P : W :: 1 : 2.$

When the upper block or sheave remains fixed, and a single rope is made to pass over several pulleys (Fig. IV.)—for example, n pulleys—then

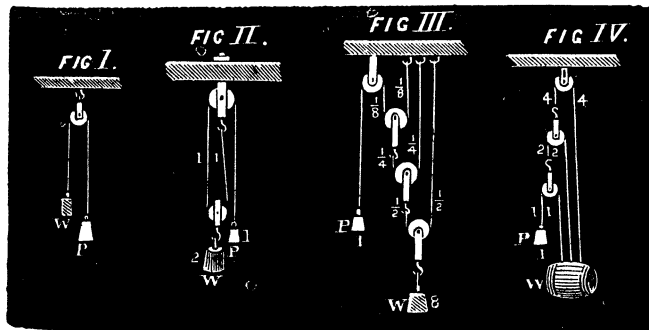
$P : W :: 1 : n$, and $P n = W$, and $P = \frac{W}{n}$, so that when $n = 6$, the

power will be one-sixth of the weight.

When there are several pulleys, each hanging by its own cord, as in Fig. III., $P : W :: 1 : 2^n$.

Here n denotes the number of pulleys.

Example. Let $W = 1600 \text{ lbs.}$, $n = 4$ pulleys. Then $P \times 2^4 = W$; that is, $P \times 16 = 1600$, and $P = 100 \text{ lbs.}$



Of the Screw.

319k. Let $L D$ = distance between the threads, and r = radius of the power from the centre of the screw. Then

$P : W :: d : 6.2832 r.$

$P r \times 6.2832 = W D.$

$$W = \frac{P r \times 6.2832}{d}, \text{ and } P = \frac{W d}{6.2832 r}.$$

Example. Given the distance, 70 inches, from the centre of the screw to a point on an iron bar at which he exerts a power of 200, the distance between the contiguous threads 2 inches, to find the weight which he can raise. Here $r = 70$, $d = 2$, and $P = 200 \text{ lbs.}$

$$W = \frac{200 \times 70 \times 6.2832}{2} = 43982.4 \text{ lbs.}$$

VIRTUAL VELOCITY.

319m. *In the Lever*, $P : W :: \text{velocity of } W : \text{velocity of } P$.

In the Inclined Plane, $\text{vel. } P : \text{vel. } W :: \text{distance drawn on the plane} : \text{the height raised in the same time}$.

Let the weight W be moved from W to a , and raised from o to a ; then $\text{vel. } P : \text{vel. } W :: W a : o a$. (Fig. VIII.)

In the Wheel and Axle, $\text{vel. } P : \text{vel. } W :: \text{radius of axle} : \text{rad. of wheel} : W : P$.

In the single Moveable Pulley, $\text{vel. } P : \text{vel. } W :: 2 : 1 :: W : P$.

In a system of Pulleys, $\text{vel. } P : \text{vel. } W :: n : 1 :: W : P$. Here n = number of ropes.

In the ARCHIMEDEAN Screw, $\text{vel. } P : \text{vel. } W$, as the radius of the power multiplied by 6.2832 is to the distance between two contiguous threads. Let R = radius of power, and d = distance between the threads; then $\text{vel. } P : \text{vel. } W :: 6.2832 R : d :: W : P$.

OF FRICTION.

319n. Friction is the loss due to the resistance of one body to another moving on it. There are two kinds of friction—the sliding and the rolling. The sliding friction, as in the inclined plane and roads; the rolling, as in pulleys, and wheel and axle.

Experiments on Friction have been made by Coulomb, Wood, Rennie, Vince, Morin, and others.

Those of Morin, made for the French Government, are the most extensive, and are adopted by engineers. When no oily substance is interposed between the two bodies, the *friction* is in proportion to their perpendicular pressures, to a certain limit of that pressure. The friction of two bodies pressed with the same weight is nearly the same without regard to the surfaces in contact. Thus, oak rubbing on oak, without unguent, gave a coefficient of friction equal to 0.44 per cent.; and when the surfaces in contact were reduced as much as possible, the coefficient was $0.41\frac{1}{2}$.

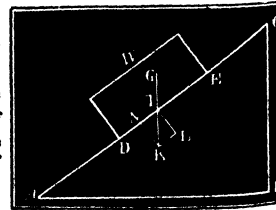
Coulomb has found that oak sliding on oak, without unguent, after a few minutes had a friction of 0.44, under a vertical pressure of 74 lbs.; and that by increasing the pressure from 74 to 2474 lbs., the coefficient of friction remained the same.

Friction is independent of the velocities of the bodies in motion, but is dependent on the unguents used, and the quantity supplied.

Morin has found that hog's lard or olive oil kept continuously on wood moving on wood, metal on metal, or wood on metal, have a coefficient 0.07 to 0.08; and that tallow gave the same result, except in the case of metals on metals, in which case he found the coefficient 0.10.

Different woods and metals sliding on one another have less friction. Thus, iron on copper has less friction than iron on iron, oak on beach has less than oak on oak, etc.

The angle of friction is $= \angle B A C$, in the annexed figure, where W represents the weight, kept on the inclined plane $A C$ by its friction. Let G = centre of gravity; then the line $I K$ represents the weight W , in direction of the line of gravity, which is perpendicular to $A B$; $I L$ = the pressure perpendicular to $A C$, and $I N = L K$ = the friction or weight sufficient to keep the weight on the plane. The two triangles, $A B C$ and $I K L$ are similar to o



another; $\therefore KL : LI :: BC : AB ::$ the altitude to the base. Also, $KL : KI :: BC : AC$.

In the first equation, we have the force of friction to the pressure of the weight W , as the height of the inclined plane is to its base.

In the second equation, we have the force of friction to the weight of the body, as the height of the plane is to its length.

Hence it appears that by increasing the height of BC from B to a certain point C , at which the body begins to slide, that the $<$ of friction or resistance is $= < BAC$.

That the *Coefficient of Friction* is the tangent of $< BAC$, and is found by dividing the height BC by the base AB .

Angle of Repose is the same as the angle of friction, or the $< BAC =$ the angle of resistance.

319o. *Friction of Plane Surfaces having been some in Contact.*

Surfaces in Contact.	Disposition of the Fibres.	State of the Surfaces.	Coeff. of Friction.	Angle of Repose.
Oak upon oak.....	Parallel.....	Without unguent	0.62	31° 48'
do.	do.	Rubbed with dry soap.....	0.44	23 45
do.	Perpendicular..	Without unguent	0.54	28 22
do.	End of one on flat of other...	do. do.	0.43	23 16
Oak upon elm.....	Parallel.....	do. do.	0.38	20 49
Elm upon oak.....	do.	With soap.....	0.41	22 18
do.	Perpendicular..	Without unguent	0.67	29 41
Ash, fir or beech on oak.....	Parallel.....	do. do.	0.53	27 56
Tanned leather upon oak.....	Leather lengthways, sideways	do. do.	0.43	23 16
Black strap leather upon oak.....	Parallel.....	do. do.	0.74	36 30
do. do. on rounded oak	Perpendicular..	do. do.	0.47	25 11
Hemp cord upon oak.....	Parallel.....	do. do.	0.80	38 40
Iron upon oak.....	do.	Steeped in water	0.65	33 02
Cast-iron upon oak.....	do.	do. do.	0.65	33 02
Copper upon oak.....	do.	Without unguent	0.62	31 48
Blk dress'd leather on iron pulley	Flat.....	do. do.	0.23	15 33
Cast iron upon cast iron.....	do.	do. do.	0.16	9 6
Iron upon cast iron.....	do.	do. do.	0.10	10 48
Oak, elm, iron, cast iron and brass, sliding two and two, on one another.....	do.	With tallow.....	0.10	5 43
do. do. do.	do.	Hog's lard.....	0.15	8 32
Common brick on common brick.....	do.	do. do.	0.67	33 50
Hard calcareous stone on the same, well dressed.....	do.	do. do.	0.70	35 00
Soft calcareous stone upon hard calcareous stone.....	do.	do. do.	0.75	36 52
do. do. on same, with fresh mortar of fine sand.....	do.	do. do.	0.74	36 30
Smooth free stone on same.....	do.	do. do.	0.71	35 23
do. do. do. with fresh mortar.....	do.	do. do.	0.66	33 26
Hard polished calcareous stone on hard polished calcareous stone.....	do.	do. do.	0.58	30 07
Well dressed granite on rough granite.....	do.	do. do.	0.66	33 26
Do., with fresh mortar.....	do.	do. do.	0.49	26 07

319p. *Friction of Bodies in Motion, one upon another.*

Surfaces in Contact.	Disposition of the Fibres.	State of the Surfaces.	Coeff. of Friction.	Angle of Repose.
Oak upon oak.....	Parallel.....	Without unguent	0.43	25° 39'
do.	do.	Rubbed with soap	0.16	9 06
do.	Perpendicular..	Without unguent	0.34	18 47
Elm upon oak.....	Parallel.....	do. do.	0.43	23 17
do.	Perpendicular..	do. do.	0.45	24 14
Iron upon oak.....	Parallel.....	Rubbed with dry soap.....	0.21	11 52
do.	do.	Without unguent	0.49	26 07
Cast iron upon oak.....	do.	Rubbed with soap	0.19	10 46
Iron upon elm.....	do.	Without unguent	0.25	14 03
Cast iron on elm.....	do.	do. do.	0.20	11 19
Tanned leather upon oak.....	Lengthways and sideways.....	do. do.	0.56	29 15
do. on cast iron and brass	do. do.	With oil.....	0.15	8 32

319*q*. *Friction of Axles in motion on their bearings.*

Cast iron axles in same bearings, greased in the usual way with hog's lard, gives a coefficient of friction of 0.14, but if oiled continuously, it gives about 0.07.

Wrought iron axles in cast iron bearings, gives as above, .07 and .05.

Wrought iron axles in brass bearings, as above, .09 and .00.

MOTIVE POWER.

319*r*. *Nominal horse power* is that which is capable of raising 33,000 pounds one foot high in one minute. The English and American engineers have adopted this as their standard; but the French engineers have adopted 32,560 lbs. Experiments have proved that both are too high, and that the average power is 22,000 lbs.

The following tables are compiled, and reduced to English measures, from Morin's *Aide Memoire*:

<i>Work done by Man and Horse moving horizontally.</i>			Hours worked per day.	Kilometres carried 1 me- tre in 1 sec.	Pounds av- oirdu- pois carried 1 foot in 1 minute.
A man unloaded.....	10	97.50		12902	
A laborer with a small two-wheel cart, going loaded and returning empty.....	10	50		6617	
Do. with a wheelbarrow as above.....	10	30		3970	
Do. walking loaded on his back.....	7	30		3970	
Do. loaded on his back, but returning unloaded.....	6	32.5		4301	
Do. carrying on a handbarrow as above.....	10	16.5		2183	
A horse with a cart at a pace continually loaded.....	10	770		101894	
Do. do. returning unloaded.....	10	420		55579	
Do. with a carriage at a constant trot.....	4.5	770		101894	
Do. loaded on the back, going at a pace.....	10	132		17467	
Do. do. at a trot.....	7	176		23290	

319 <i>s</i> . <i>Work done by Man in moving a body vertically.</i>			Hours worked per day.	Kilogrammes raised 1 met. high per sec.	Lbs. avoirdu- pois raised 1 foot high per minute.
Man ascending an inclined plane.	8	9.75		1290	
Do. raising weight with a cord and pulley, the cord descending empty.	6	3.60		476	
Do. raising weight with his hands.....	6	3.40		450	
Do. raising a weight, and carrying it on his back to the top of an easy stairway, and returning empty..	10	1.20		159	
Do. shovelling earth to a mean height of 1.60 metres.	10	1.08		143	

319 <i>t</i> . <i>Action on Machines.</i>			Hours per day.	Force in Ki- logrammes per second.	Force in pounds per minute.
A man acting on a wheel or drum at a point level with the axle.....	8	9		1191	
Do. acting at a point below the axle at an $\angle$ of 24° ..	8	8.40		1112	
Do. drawing horizontally, or driving before him.....	8	7.20		753	
Do. acting on a winch.....	8	6		794	
Do. pushing and drawing alternately in vert. position	8	5.60		728	
A horse harnessed to a carriage and going at a pace..	10	63		8337	
Do. harnessed as a riding horse, going at a pace.....	8	40.50		536	
Do. do. going at a trot.....	4.5	60		7940	

ROADS AND STREETS.

319u. Roman roads were made to connect distant cities with the Imperial Capital. In low and level grounds, they were elevated above the adjoining lands, and made as follows:

- 1st. The *Statumen*, or foundation—all soft matter was removed.
- 2d. The *Rudratio*, composed of broken stones or earthenware, etc., set in cement.
- 3d. The *Nucleus*, being a bed of mortar.

4th. The *Summa Crusta*, or outer coat, composed of bricks or stones. Near Rome, the upper coat was of granite; in other places, hard lava, so closely jointed, that it was supposed by Palladio that moulds were used for each stone or piece.

The *Curator Viarum*, or superintendent of highways, was an officer of great influence, and generally conferred on men of *consular dignity* after *Julius Cæsar*, who held that office, assisted by his colleague, *Thernus*, a noble Roman. *Victorius Marcellus*, of the prætorian order, had been selected to this office by the Emperor Domitian. These are but a few instances of the many in which men of the highest position in society became *Curator Viarum*—or, as the Americans call him, commissioner of highways, or path master.

The *Appian Way*, called also Queen of the Roman ways, was made by Censor Appius Cæcus, about 311 years before the Christian era, and built then as far as Capua, 125 miles; but subsequently to Brundisium, about the year B. C. 249. "The Appian Way was of a sufficient width (18 to 22 feet) to allow two carriages to pass; was made of hard stone, squared, and made to fit closely. After 2000 years, but little signs of wear appear."—*Eustace*.

Gravel roads, with small stones, were commonly used by the Romans. Porticos were built at convenient distances, to afford shelter to the traveler.

Roman Military roads were 36 to 40 feet wide, of which the middle 16 feet were paved. At each side there was a raised path, 2 feet wide, which again separated two sideways, each 8 feet wide.

The breadth of the Roman roads, as prescribed by the laws of the twelve tables, was but 8 feet; the width of the wheel tracks not above 3 feet. There were twenty-nine military roads made, equal in length to 48500 English miles.

The *Carthaginians*, according to Isadore, were the first who paved their public ways.

The *Greeks*, according to Strabo, neglected three objects to which the Romans paid especial attention: the *cloacæ*, or common sewers, the aqueducts, and the public highways. The Greeks made the upper part of their roads with large, square blocks of stone, whilst the Romans mostly used irregular polygons.

The *French roads* are from 30 to 60 feet wide, the middle 16 feet being paved; but once a vehicle leaves the pavement, it becomes a matter of much difficulty to extricate it from the soft surface of the sides. To obviate this difficulty, the system of using broken stones is now generally adopted, and has been used in France, under the direction of M. Turgos, a long time before *McAdam* introduced it into England.

The *German roads* resemble those of France.

The *Belgium roads* have their surfaces composed of thin brick tiles, which answer well for light work.

Sweden has long been famous for her excellent roads of stone or gravel, on which there is not a single tollgate. Each landowner is obliged to keep in repair a certain part of the road, in proportion to his property, whose limit is marked by land marks on each side of the road.

The *English, Irish and Scotch roads* are now generally made of broken stones, or macadamised; are 25 to 50 feet wide: well drained—having the centre 12 inches higher than on the sides, in a road 40 feet wide, and in proportion of 3 inches in 10 feet wide; the stones broken so as to pass through an inch-and-half ring. For the purpose of keeping them in repair, there are *depots*, or heaps of broken stones, at intervals of 600 feet. When a small hole makes its appearance, a man loosens the stones around the spot to be repaired, and then fills it up with new material, which soon becomes as when originally made.

Arthur Young states that it was not until 1660 that England took an interest in her roads. (See *Encyclopædia Britannica*, vol. xii, p. 528.) In his tour through the British Isles in 1779, he states that Ireland then had the best roads in Europe. This is not to be wondered at, when we consider that there, granite, limestone and gravel beds are abundant; that since the beginning of the reign of Charles I, the roads were under the charge of the grand jury. There, good roads must have existed at a very early date, as the stones of which the round towers are built are large, and, in some places, have been brought from a great distance.

Many of the English and Irish highways were turnpike roads; that is, roads having tollgates. Since the introduction of railways, these have been falling off in revenue. In a parliamentary inquiry into turnpike trusts in Ireland, the unanimous testimony of all the witnesses examined were against them, and in favor of having them kept in repair by presentment.

Presentment is where the grand jury receives proposals to keep road R, blank miles, from point A to point B, in repair, according to the specification of the county surveyor, during time T, at the rate of sum *s* per rod, subject to the approval of the county surveyor, who has the general supervision of all the public works, and are gentlemen of integrity and high scientific attainments. The work on hydraulics by Mr. Neville, county surveyor for Louth, and that on roads by my school-fellow, Edmund Leahy, county surveyor for Cork, are generally in the hands of every engineer.

By the parliamentary report for 1839–40, England had 21962 miles of turnpike trusts. The tolls amounted to £1,776,586; the expenditure for repairs and officers, £1,780,349, leaving a deficiency of £3,763. The same deficiency appears to take place on the Irish roads.

In England, the parish roads equal 104772 miles, costing annually for highway rates £1,168,207. The number of surveyors and deputy surveyors, or way-wardens, is 20000, or one way-warden to every $5\frac{1}{2}$ miles of road. It was then shown that the trusts had incurred debts to the enormous amount of £8,577,182.

Under the new system, one man keeping a horse is supposed to take charge of 40 miles of road.

Making and Repairing Macadamised Roads.

319c. The road bed should have a curved surface of about 1 foot rise for 40 feet wide, be a segment of a circle, and have at least 12 inches of stones on the centre, and 8 to 10 on the sides, both of which are to be on the same level. When the stones are well incorporated with one another, a layer of sand, 1 inch in thickness, is spread on top. The bed must be thoroughly drained, and the water made to flow freely in the adjoining ditches. The overseers should never allow any water to accumulate on the road, and every appearance of a rut or hole immediately checked. Where there is frost, it is liable to disintegrate the road material, unless it is built of very compact stuff. In boggy land, a soling of 12 to 18 inches of stiff clay must be laid under the broken stone. Where the bottom is sandy, and stiff clay hard to be procured, rough pavements or concrete, from 6 to 12 inches thick, under the broken stones, will be the best. In general, where the soil is well drained, broken stones will be sufficient. The road is never to have less than 8 inches on the centre and 4 on the sides. All large stones raked to the sides, and broken, so as to pass through a ring $1\frac{1}{2}$ inches in diameter. The surface always kept uniform. The English and Irish roads are generally 25 feet between the ditches, but in approaches to cities and towns, they are 40 to 50 feet. On the Irish roads, no house is allowed nearer than 30 feet of the centre of the road.

To allow for shrinkage. Mr. Leahy, in his work on roads, p. 100, says: In bog stuff, add *one-fourth* of its intended height; if the road is of clay or earth, add *one-twelfth*.

When the road passes through boggy land, the side ditches, or drains, must be dug to a depth of 4 feet below the surface of the road, and have parallel drains running along in the direction of the road, about 40 feet on each side. In this manner roads have been made over the softest bogs in Ireland. On the Milwaukee and Mississippi Railroad, near Milwaukee, a part of the road passed over the Menominee bottoms. After several weeks of filling, the company was about to relinquish that part of the route, for all the work done during the week would disappear during Sunday. The author being employed as city engineer in the neighborhood, saw the respective officers holding a consultation. He came up, and on being asked his opinion, replied: "Imitate nature; first lay on a layer of brushwood, 1 foot thick; then 2 feet of clay, and so on alternately." The plan was adopted, and has succeeded.

Where the road is wet and springy, cross drains filled with stones are to be made, to connect with the side drains or ditches; and if made within 50 or 60 feet of one another, will be sufficient to drain it.

Where the road runs along a sloping ground, catch-water drains should be run parallel with the road, so as to keep off the hill water.

Retaining walls should have a batter or slope of 3 inches to each foot in height, and the back may be parallel to the same. The thickness, $2\frac{1}{2}$ feet for 10 feet in height, and in all other cases, the thickness shall be one-fourth of the height. An offset of 8 inches should be left at front of the footing course, and the foundation cut into steps. Where such walls are along water courses, the foundation should be 15 inches below the bottom of the water, and paved along the side to a width of 18 inches or 2 feet. The filling behind is put in in layers, and rammed in.

Parapet walls should be 20 inches thick and $3\frac{1}{2}$ feet high, built of masonry laid in lime mortar, in courses of 12 or 14 inches, the top course or coping to be semicircular, and have a thorough bond at every 3 feet.

Where drains are covered, dry masonry walls, covered with flags, are preferable. Where the width of the drain is not more than 30 inches, these drains will require flags 6 inches thick; those between 18 and 24 inches are to have flags 5 inches thick; and those from 8 to 18 inches, require flags 4 inches thick.

Drainage. When the road runs along a hill, cut a drain parallel to the road, and 3 to 4 feet below the surface; then cut another of smaller dimensions near the road, and sunk below the road-bed. Again, at every 60 or 100 feet, sink cross drains, about 15 to 24 inches below the road-bed; fill with broken stones to within 6 inches of the top, which space of 6 inches is to be filled with small broken stones of the usual size in road making—these cross drains to communicate with a ditch or drain on the lower side of the road, to keep it dry.

Drain holes, about 100 feet apart; 8 inches square, and about 2 inches under the water table of the drain; may be made of 4 flag stones, draining tiles, or pipes.

Road Materials. Granite is the best.

Sienite is granite, in which hornblende is mixed. This is very durable, and resists the action of the atmosphere. This stone has a greenish color when moistened.

Sandstone, if impregnated with silica, is hard, and makes a good material. Some varieties are composed of pure silex, which makes an excellent material; but others are mixed with other substances, which make the stone porous, and unfit to be used by the action of frost, it easily disintegrates.

Limestone has a great affinity for water, which it imbibes in large quantities. If frozen in this condition, it is easily crumbled under the wheels of carriages, and becomes mud. Hence the great necessity of keeping a road made with broken limestone *thoroughly drained*, in all places where frost makes its appearance. There is nothing more injurious to roads than frosts.

Stones having fine granular appearance, and whose specific gravity is considerable, may be considered good road material.

Experiments made by Mr. Walker, civil engineer, during seventeen months of 1830 and 1831, on the Commercial Road, near London, will show the quality of the following stones: (See *Transactions Inst. Civil Engineers*, Vol. 1.)

Description of Stone.	Where procured.	Absolute wear in 17 months.	Time in which 1 inch would wear down.
Granite.....	Dartmoor.....	.207 inches.	6.8 years.
“	Guernsey.....	.060	22.5
“	Herm, near Guernsey..	.075	19.
Blue Granite.....	Peterhead.....	.131	10.8
Granite.....	Heyton.....	.141	10.
Red Granite.....	Aberdeen.....	.159	9.
Blue Granite.....	“	.225	6.33
Whinstone.....	Budle.....	.082	17.33

COMPRESSION.

<i>Lbs. avoirdupois to crush a cube of 1½ inches.</i>	<i>Lbs. avoirdupois to crush a cube of 1½ inches.</i>
Chalk..... 1127	Cornish granite.....14302
Brick, pale red color..... 1265	Dundee sandstone.....14918
Red brick, mean..... 1817	Craigleith gritstone, with the
Yellow-faced paviors..... 2254	strata.....15560
Fire brick..... 3864	Devonshire red marble, vari-
Whitby gritstone..... 5328	egated.....16712
Derby “ and friable	Compact limestone.....17354
sandstone 7070	Penryn granite.....17400
Do. from another quarry..... 9776	Peterhead “ close grained...18636
White freestone, not stratified.10264	Black compact Limerick lime-
Portland stone..... 10284	stone.....19924
Humbic gritstone.....10371	Black Brabant marble.....20742
Craigleith white freestone.....12346	Very hard freestone.....20254
Yorkshire paving, with strata.12856	White Italian veined marble...20783
Do. against the strata.....12856	Aberdeen granite, blue kind...24556
White statuary marble, not	Valencia slate.....26656
veined. 13632	Dartmoor granite.....27680
Brambyfall sandstone, near	Heyton granite.....31360
Leeds, with strata.....13632	Herm granite, near Guernsey..33600

A road made over well dried bogs or naked surface, on account of its elasticity, does not wear as fast as roads made over a hard surface. It has been found that on the road near Bridgewater, England, the part over a rocky bed wears 7, when that over a naked surface wears 5.

The covering of broken stones is, in the words of McAdam, intended to keep the road-bed dry and even.

Some of the material used on the roads near London are brought from the isle of Guernsey and Hudson Bay.

Weight of vehicles, width of tiers, and velocity, have great influence on the wear of roads. In Ireland, two-wheeled wagons or carts are generally used—the weight 6 to 8 cwt., and load 22 to 25 cwt., making a gross load of about 30 cwt. In England, four-wheeled wagons are generally used, and weigh, with their load, from 5 to 6 tons; therefore, the pressure of these vehicles is as 1660 to 3320, on any given point.

It is evident that when the vehicle is made to ascend a large stone, that in falling, it acquires a velocity which is highly injurious to the road, and that there should not be allowed any stone larger than 1½ inches square on the surface.

Table of Uniform Draught.

<i>Description of Surface.</i>	<i>Rate of Inclination.</i>
Ordinary broken stone surface.....	Level.
Close, firm stone paving.....	1 in 48.5
Timber paving.....	1 in 41.5
Timber trackway.....	1 in 31.66
Cut stone trackway.....	1 in 31.66
Iron tramway.....	1 in 29.25
Iron railway.....	1 in 28.5

Explanation. If a power of 90 lbs. will move one ton on a level, broken stone road, it will move the same weight on an iron railway having an inclined plane of 1 in 28½.

FRICTION ON ROADS.

The power required to move a wheel on a well made, level road, depends on the friction of the axles in their boxes, and to the resistance to rolling.

When the axles are well made and oiled, the friction is taken at one-eighteenth of the pressure; but in ordinary cases, it is taken at one-twelfth, $= \frac{W}{12}$, and power $= \frac{W}{12} \times \frac{a}{d} = \frac{W a}{12 d}$. Here power is that force which, if applied at the tier, would just cause the wheel to move. a = diameter of the axis, and d = diameter of the wheel.

The following is Sir John McNeill's formula, given in his evidence before a committee of the House of Lords, for the draught on common roads:

$P = \frac{W + w}{93} + \frac{w}{40} + c V$. Here W = weight of the wagon, w = weight of the load, V = velocity in feet per second, and c = a constant quantity derived from experiments on level roads.

Kind of Road.	Value of c.
For a timber surface.....	2
“ paved road.....	2
“ a well made broken-stone road, in a dry state.....	5
“ “ “ “ covered with dust.....	8
“ “ “ “ wet, and covered with mud.....	10
“ gravel or flint road, when wet.....	13
“ “ “ “ very wet, and covered with mud.....	32

Let $W = 720$, $w = 3000$, paved road; let $V = 4$ feet. Here $c = 2$, and we have—

$$P = \frac{720 + 3000}{93} + \frac{3000}{40} + 2 \times 4.$$

$P = 40 + 75 + 8 = 123$ = draught, or the force necessary to overcome the combined friction of the axle in the box and the wheel in rolling on the surface. This force is one-thirtieth of the total load of weight and wagon.

By *McNeill's Improved Dynamometer*, the following results have been obtained. Weight of wagon and load = 21 cwt.

Kind of Road.	Force in lbs.	Ratio of Draught to the Load.
Gravel road laid on earth.....	147	= 1-16th of the load.
Broken stones.....	65	= 1-36th “
“ on a paved foundation.....	46	= 1-51st “
Well-made pavement.....	33	= 1-71st “
Best stone track ways.....	$12\frac{1}{2}$	= 1-179th “
Best form of railroad.....	8	= 1-280th “

M. Poncelet gives the following value of draught or force to overcome friction:

On a road of sand and gravel.....	1-16th of the total load.
On a broken stone road, ordinary condition.....	1-25th “
“ “ in good condition.....	1-67th “
On a good pavement, at a walk.....	1-54th “
“ “ at a trot.....	1-42d “
On a road made of oak planks.....	1-98th “

Table showing the Lengths of Horizontal Lines Equivalent to several Ascending and Descending Planes, the Length of the Plane being Unity.

In calculating this table, Mr. Leahy has assumed that an ordinary horse works 8 hours per day, and draws a load of 3000 pounds, including the weight of the wagon, making the net load 1 ton.

One in	One-horse Cart.		Stage Coach.		Stage Wagon.		Angle of Elevation
	Ascend'g.	Desc'nd'g.	Ascend'g.	Desc'nd'g.	Ascend'g.	Desc'nd'g.	
5	8.32	3.27					0 1 11
10	4.16	1.65	2.85		6.07		5 42 58
15	2.90	1.06	2.23		4.39		3 48 51
20	2.08	0.83	1.93	0.07	3.54		2 51 21
25	1.66	0.70	1.74	0.26	3.04		2 17 26
30	1.55	0.74	1.62	0.39	2.70		1 54 37
35	1.45	0.77	1.53	0.47	2.46		1 38 14
40	1.40	0.79	1.46	0.54	2.27		1 25 57
45	1.35	0.81	1.41	0.59	2.13		1 16 24
50	1.31	0.83	1.37	0.63	2.02		1 8 6
55	1.29	0.84	1.34	0.66	1.93	0.07	1 2 30
60	1.26	0.85	1.31	0.69	1.85	0.15	0 57 18
65	1.24	0.86	1.29	0.71	1.78	0.22	0 52 54
70	1.22	0.87	1.27	0.72	1.72	0.27	0 49 7
75			1.68	0.32	1.25	0.75	0 45 51
80	1.19	0.88	1.64	0.36	1.23	0.77	0 42 58
85			1.60	0.40	1.22	0.78	0 40 27
90	1.17	0.89	1.57	0.43	1.21	0.79	0 38 12
95			1.54	0.46	1.20	0.80	0 36 11
100	1.15	0.90	1.51	0.49	1.19	0.81	0 34 23
110			1.45	0.55	1.17	0.83	0 31 15
120			1.43	0.58	1.15	0.85	0 28 39
130			1.39	0.61	1.14	0.86	0 26 27
140			1.36	0.64	1.13	0.87	0 24 33
150	1.10	0.92	1.34	0.66	1.12	0.88	0 22 55
160			1.32	0.68	1.12	0.88	0 21 29
170			1.30	0.70	1.11	0.89	0 20 13
180			1.28	0.72	1.10	0.90	0 19 6
190			1.27	0.73	1.10	0.90	0 18 6
200	1.07	0.93	1.26	0.75	1.09	0.91	0 17 11
210			1.24	0.76	1.09	0.91	0 16 22
220			1.23	0.77	1.08	0.92	0 15 37
230			1.22	0.78	1.08	0.92	0 14 57
240			1.21	0.79	1.08	0.92	0 14 19
250			1.20	0.80	1.07	0.93	0 13 45
260			1.20	0.80	1.07	0.93	0 13 13
270			1.19	0.81	1.07	0.93	0 12 44
280			1.18	0.82	1.07	0.94	0 12 17
290			1.18	0.82	1.06	0.94	0 11 51
300			1.17	0.83	1.06	0.94	0 11 28
350			1.15	0.85	1.05	0.95	0 9 49
400			1.13	0.87	1.05	0.95	0 8 36
450			1.11	0.89	1.04	0.96	0 7 38
500			1.10	0.90	1.04	0.96	0 6 53
550			1.09	0.91	1.03	0.97	0 6 15
600			1.09	0.92	1.03	0.97	0 5 44

Pressure of a load on an inclined plane is found by multiplying the weight of the load by the horizontal distance, and dividing the product by the length of the inclined plane.

Corollary. Hence appears that on an inclined plane, the pressure is less than the weight of the load.

M. Morin's Experiments.

Vehicle used.	Routes passed over.	Pressure	Draught in pounds.	Ratio of draught to load.
Artillery ammunition wagon,.	Broken stone,	13215	398.4	$\frac{1}{33.1}$
“ “ “	in good order,	13541	352.6	$\frac{1}{38.4}$
“ “ “	and dusty,.....	10101	250.7	$\frac{1}{40.2}$
Wagon without springs,.....		15716	306.3	$\frac{1}{51.3}$
“ “ “	Solid gravel,	12037	245.9	$\frac{1}{48.9}$
“ “ “	very dry,.....	9814	205.5	$\frac{1}{47.7}$
“ “ “		7565	150.8	$\frac{1}{50.1}$
Wagon with springs.....	Paved, in good	3528	86.6	$\frac{1}{40.8}$
“ “ “	order, with wet	7260	196.7	$\frac{1}{36.9}$
	mud,	11018	299.9	$\frac{1}{36.8}$

The greatest inclination ought not to exceed 1 in 30, and need not be less than one in 100, for a horse will draw as well on a road with a rise of 1 in 100 as on a level road. Where the road curves or bends, it should be wider, as follows: When the two lines make an angle of deflection of 90° to 120°, increase the road-bed one-fourth.

Example. Let us suppose that we ascend a hill 1 mile long at the rate of 1 foot in 30, and that we descend 1 mile with an inclination of 1 in 40. Here we have for a one-horse cart or vehicle ascending = 1.66, descending = 0.70, sum = 2.36, mean = 1.18. That is, passing over the hill of 2 miles with the above rise and fall, is equivalent to hauling over 2.36 miles of a horizontal road.

The inclined road is easily drained, and requires less material in construction and annual repair, and avoids curves.

The engineer will be able to judge which is the most economical line from the above table.

M. Morin's experiments show that—

1st. The traction is directly proportional to the load. The traction is inversely proportional to the diameter of the wheel.

2d. Upon hard roads, the resistance is independent of the width of the tire when it exceeds 3 to 4 inches.

3d. At a walking pace, the traction is the same, under the same circumstances, for carriages with and without springs.

4th. Upon hard macadamised and paved roads, the traction increases with the velocity, when above $2\frac{1}{4}$ miles per hour.

5th. Upon soft roads, the traction is independent of the velocity.

6th. Upon a pavement of hewn stones, the traction is three-fourths of that upon the best macadamised roads, at a pace but equal to it at a trot.

7th. The destruction of the road is greater as the diameter of the wheels is less, and is greater with carriages without than with springs.

TABLE C.—For Laying Out Curves. Chord A B = 200 feet or links, or any multiple of either. (See Fig. A, Sec. 319x.)

Rad. of curve.	$\frac{1}{2}$ angl. of defect 'n ° ' "	D C	F E	H G	W S	Rad. of curve.	$\frac{1}{2}$ angl. of defect 'n ° ' "	D C	F E	H G	W S
700	8 12 48	7.18	1.79	0448	0112	1900	3 01 01	2.63	0.66	0.17	.041
20	7 59 01	6.98	.747	.437	.109	20	2 59 08	.606	.652	.163	.040
40	45 59	.788	.699	.425	.106	40	57 17	.579	.642	.161	
60	33 34	.604	.653	.413	.103	60	55 28	.553	.638	.160	
80	21 57	.438	.614	.403	.101	80	53 48	.530	.633	.158	
800	10 50	.274	.570	.393	.098	2000	51 57	.503	.626	.156	.039
20	01 16	.148	.538	.385	.096	20	50 15	.477	.619	.155	
40	50 14	5.97	.495	.374	.093	40	48 38	.452	.613	.153	.038
60	6 40 39	.844	.460	.365	.091	60	46 57	.429	.607	.152	
80	31 30	.701	.426	.357	.089	80	45 20	.405	.601	.150	.037
900	22 46	.570	.394	.348	.087	2100	43 46	.382	.596	.149	
20	14 25	.436	.364	.341	.085	20	42 13	.357	.589	.147	
40	06 25	.310	.334	.334	.083	40	40 42	.339	.585	.146	.036
60	5 58 45	.222	.307	.327	.082	60	39 12	.316	.579	.145	
80	51 24	.142	.279	.320	.080	80	37 45	.296	.574	.143	
1000	44 20	.012	.254	.313	.078	2200	36 19	.275	.563	.142	.035
20	37 34	4.91	.229	.307	.077	20	34 54	.253	.558	.141	
40	31 04	.817	.205	.301	.075	40	33 31	.232	.553	.139	
60	24 48	.727	.183	.296	.074	60	32 10	.213	.549	.138	.034
80	18 46	.640	.160	.292	.073	80	30 50	.194	.544	.137	
1100	12 57	.556	.140	.285	.071	2300	29 30	.174	.542	.136	
20	07 21	.473	.117	.279	.070	20	28 14	.157	.534	.135	
40	01 57	.396	.099	.275	.069	40	26 57	.138	.530	.134	.033
60	4 56 44	.319	.080	.270	.068	60	25 42	.119	.526	.132	
80	51 41	.247	.062	.265	.066	80	24 29	.102	.521	.131	
1200	46 49	.174	.044	.261	.065	2400	23 17	.084	.517	.130	.032
20	42 06	.105	.027	.257	.064	20	22 06	.067	.513	.129	
40	37 32	.029	.010	.252	.063	40	20 56	.051	.508	.128	
60	33 07	3.98	.0994	.248	.062	60	19 44	.033	.505	.127	
80	28 51	.914	.978	.245	.061	80	18 39	.018	.500	.126	.031
1300	24 42	.853	.963	.241	.060	2500	17 33	.001	.496	.125	
20	20 41	.798	.949	.237	.059	20	16 27	1.99	.492	.124	
40	16 47	.737	.935	.234	.058	40	15 23	.969	.489	.123	.030
60	13 00	.681	.920	.230	.057	60	13 19	.954	.485	.122	
80	09 20	.628	.907	.227	.056	80	13 17	.939	.481	.121	
1400	05 46	.574	.894	.224	.055	2600	12 15	.924	.477	.120	
20	02 18	.526	.882	.221	.055	20	11 14	.909	.474	.119	
40	3 59 05	.481	.870	.218	.054	40	10 15	.895	.470	.118	.029
60	55 39	.429	.857	.214	.053	60	9 16	.880	.466	.117	
80	52 27	.382	.846	.212	.052	80	8 16	.865	.463	.117	
1500	49 20	.337	.834	.209		2700	7 22	.851	.460	.116	
20	46 20	.293	.823	.206	.051	20	6 25	.839	.456	.115	
40	43 23	.250	.813	.203	.050	40	5 29	.825	.453	.114	
60	40 31	.208	.802	.201	.049	60	4 35	.812	.450	.113	.028
80	37 43	.169	.792	.198		80	3 42	.799	.447	.113	
1600	35 00	.128	.782	.196	.048	2800	2 48	.786	.443	.112	
20	32 19	.089	.772	.193		20	1 56	.773	.440	.111	
40	29 45	.052	.763	.191	.047	40	1 04	.760	.437	.110	
60	27 13	.011	.753	.188		60	0 13	.747	.434	.109	.027
80	24 45	2.98	.745	.186	.046	80	1 59 23	.735	.431	.109	
1700	22 20	.943	.736	.184		2900	58 34	.725	.429	.108	
20	19 59	.910	.728	.182	.045	20	57 45	.714	.425	.107	
40	17 41	.876	.719	.180		40	56 57	.703	.423	.106	
60	15 26	.843	.711	.178	.044	60	56 10	.692	.420	.106	
80	13 14	.812	.703	.176		80	55 23	.681	.417	.105	.026
1800	11 05	.777	.694	.174	.043	3000	54 37	.669	.415	.104	
20	8 59	.749	.687	.172		20	53 51	.658	.412	.104	
40	6 55	.719	.680	.170	.042	40	53 07	.647	.409	.103	
60	4 55	.685	.671	.168		60	52 22	.636	.406	.102	
80	2 57	.662	.666	.167	.041	80	51 38	.625	.404	.102	

TABLE C.—For Laying Out Curves. Chord A B = 200 feet or links, or any multiple of either. (See Fig. A, Sec. 319x.)											
Rad. of curve.	½ angl. of defect'n ° / //	D C	F E	H G	W S	Rad. of curve.	½ angl. of defect'n ° / //	D C	F E	H G	W S
3100	1 50 55	1.61	.404	.101	.025	4300	1 19 57	1.16	.291	.073	.018
20	50 13	.603	.401	.100		20	19 35	.157	.289	.072	
40	49 30	.593	.398	.099		40	19 13	.152	.288	.072	
60	48 48	.583	.396	.099		60	18 51	.146	.287	.072	
80	48 07	.573	.393	.098		80	18 30	.141	.285	.071	
3200	47 27	.563	.391	.098		4400	18 08	.136	.284	.071	
20	46 47	.553	.388	.097	.024	20	17 47	.131	.283	.071	
40	46 07	.543	.386	.097		40	17 26	.126	.282	.071	
60	45 28	.534	.384	.096		60	17 05	.121	.280	.070	
80	44 50	.525	.381	.095		80	16 45	.116	.279	.070	
3300	44 11	.516	.379	.095		4500	16 24	.111	.278	.070	
20	43 34	.506	.377	.094		20	16 04	.106	.277	.069	.017
40	42 57	.497	.374	.094	.023	40	15 44	.102	.276	.069	
60	42 20	.489	.372	.093		60	15 24	.097	.274	.069	
80	41 43	.480	.370	.093		80	15 04	.092	.273	.068	
3400	41 08	.471	.368	.092		4600	14 44	.087	.272	.068	
20	40 32	.462	.366	.092		20	14 25	.082	.271	.068	
40	39 54	.453	.363	.091		40	14 06	.077	.269	.067	
60	39 22	.445	.361	.090		60	13 47	.073	.268	.067	
80	38 48	.437	.359	.090		80	13 28	.069	.267	.067	
3500	38 14	.429	.357	.089	.022	4700	13 09	.064	.266	.067	
20	37 41	.421	.355	.089		20	12 51	.059	.265	.066	
40	37 08	.413	.353	.088		40	12 32	.054	.264	.066	
60	36 35	.405	.351	.088		60	12 14	.050	.263	.066	
80	36 03	.397	.349	.087		80	11 55	.046	.262	.066	
3600	35 30	.389	.347	.087		4800	11 38	.042	.261	.065	.016
20	34 59	.381	.345	.086		20	11 20	.038	.260	.065	
40	34 27	.374	.344	.086	.021	40	11 02	.034	.259	.065	
60	33 57	.366	.342	.086		60	10 44	.030	.258	.065	
80	33 26	.358	.339	.085		80	10 27	.026	.257	.064	
3700	32 55	.351	.338	.085		4900	10 10	.022	.256	.064	
20	32 25	.344	.336	.084		20	9 53	.018	.255	.064	
40	31 56	.337	.334	.084		40	9 36	.013	.253	.063	
60	31 27	.330	.333	.083		60	9 19	.008	.252	.063	
80	30 57	.323	.331	.083		80	9 02	.004	.251	.063	
3800	30 29	.316	.329	.082		5000	8 46	1.00	.250	.063	
20	30 00	.309	.327	.082		20	8 29	.996	.249	.062	
40	29 32	.302	.326	.082		40	8 13	.992	.248	.062	
60	29 04	.295	.324	.081	.020	60	7 55	.988	.247	.062	
80	28 37	.288	.322	.081		80	7 41	.984	.246	.062	
3900	28 09	.282	.321	.080		5100	7 25	.981	.245	.061	.015
20	27 43	.276	.319	.080		20	7 09	.977	.244	.061	
40	27 16	.269	.317	.079		40	6 53	.973	.243	.061	
60	26 49	.262	.316	.079		60	6 38	.969	.242	.061	
80	26 23	.256	.314	.079		80	6 22	.965	.241	.060	
4000	25 57	.250	.312	.078	.019	5200	6 07	.962	.241	.060	
20	25 21	.243	.311	.078		20	5 52	.958	.240	.060	
40	25 06	.237	.309	.077		40	5 37	.954	.239	.060	
60	24 41	.231	.308	.077		60	5 22	.950	.238	.059	
80	24 16	.225	.306	.077		80	5 07	.947	.237	.059	
4100	23 52	.220	.305	.076		5300	4 52	.944	.236	.059	
20	23 27	.214	.304	.076		20	4 37	.940	.235	.059	
40	23 03	.208	.302	.076		40	4 23	.936	.234	.059	
60	22 39	.202	.301	.075		60	4 09	.933	.233	.058	
80	22 14	.196	.299	.075		80	3 54	.929	.232	.058	
4200	21 52	.191	.298	.075		5400	3 40	.926	.232	.058	
20	21 28	.185	.296	.074		20	3 26	.923	.231	.058	
40	21 05	.179	.295	.074		40	3 12	.919	.230	.058	
60	20 42	.173	.293	.073		60	2 58	.916	.229	.057	
80	20 20	.168	.291	.073	.018	80	2 44	.912	.228	.057	.014

TABLE C.—For Laying Out Curves. Chord A B = 200 feet or links, or any multiple of either. (See Fig. A, Sec. 319x.)

Rad. of curve.	1/2 angl. of defect'n ° ' "	D C	F E	H G	W S	Rad. of curve.	1/2 angl. of defect'n ° ' "	D C	F E	H G	W S
5500	1 2 31	.909	.227	.057	.014	6700	0 51 19	.746	.187	.047	.012
20	2 17	.905	.226	.057		20	51 10	.744	.186	.047	
40	2 03	.902	.226	.057		40	51 00	.742	.186	.047	
60	1 50	.899	.225	.056		60	50 52	.740	.185	.046	
80	1 37	.896	.224	.056		80	50 42	.738	.185	.046	
5600	1 24	.893	.223	.056		6800	50 33	.736	.184	.046	
20	1 10	.89	.222	.056		20	50 25	.733	.183	.046	
40	0 57	.86	.222	.056		40	50 16	.731	.183	.046	
60	0 44	.83	.221	.055		60	50 07	.728	.182	.046	
80	0 32	.80	.220	.055		80	49 58	.726	.182	.046	
5700	0 19	.77	.219	.055		6900	49 50	.724	.181	.045	.011
20	1 0 06	.74	.219	.055		20	49 41	.722	.181	.045	
40	0 59 54	.71	.218	.055		40	49 32	.720	.180	.045	
60	59 41	.68	.217	.054		60	49 24	.718	.179	.045	
80	59 29	.65	.216	.054		80	49 15	.716	.179	.045	
5800	59 16	.62	.216	.054		7000	49 07	.714	.179	.045	
20	59 04	.59	.215	.054		20	48 58	.712	.178	.045	
40	58 52	.56	.214	.054		40	48 50	.710	.178	.045	
60	58 40	.53	.213	.053	.013	60	48 42	.708	.177	.044	
80	58 28	.50	.213	.053		80	48 33	.706	.177	.044	
5900	58 16	.47	.212	.053		7100	48 25	.704	.176	.044	
20	58 04	.44	.211	.053		20	48 17	.702	.175	.044	
40	57 53	.42	.211	.053		40	48 09	.700	.175	.044	
60	57 41	.40	.210	.053		60	48 01	.696	.175	.044	
80	57 29	.37	.209	.052		80	47 52	.694	.174	.044	
6000	57 18	.34	.209	.052		7200	47 45	.692	.174	.044	
20	56 07	.31	.208	.052		20	47 37	.690	.173	.043	
40	56 55	.29	.207	.052		40	47 29	.688	.173	.043	
60	56 44	.26	.207	.052		60	47 21	.686	.172	.043	
80	56 33	.23	.206	.052		80	47 13	.684	.172	.043	
6100	56 22	.20	.205	.051		7300	47 06	.682	.171	.043	
20	56 11	.18	.205	.051		50	47 47	.679	.169	.042	
40	55 00	.15	.204	.051		7400	46 28	.676	.169	.042	
60	55 49	.13	.203	.051		50	46 09	.672	.168	.042	
80	55 38	.10	.203	.051		7500	45 51	.668	.167	.042	
6200	55 27	.07	.202	.051		50	45 32	.663	.166	.042	
20	55 16	.04	.201	.050		7600	45 14	.658	.165	.041	.010
40	55 06	.01	.200	.050	.012	50	44 57	.654	.164	.041	
60	54 55	.79	.200	.050		7700	44 39	.650	.163	.041	
80	54 45	.76	.199	.050		50	44 22	.646	.162	.041	
6300	54 34	.74	.199	.050		7800	44 05	.642	.160	.040	
20	54 24	.71	.198	.050		50	43 48	.638	.160	.040	
40	54 14	.78	.197	.049		7900	43 31	.634	.158	.040	
60	54 03	.76	.197	.049		50	43 15	.629	.157	.039	
80	53 53	.73	.196	.049		8000	0 42 58	.624	.157	.039	
6400	53 43	.71	.195	.049		50	42 42	.621	.155	.039	
20	53 33	.77	.195	.049		8100	42 27	.617	.154	.039	
40	53 23	.77	.194	.049		50	42 11	.614	.153	.038	
60	53 13	.77	.194	.049		8200	41 55	.611	.153	.038	
80	53 03	.77	.193	.048		50	41 40	.608	.152	.038	
6500	52 53	.76	.192	.048		8300	41 25	.605	.151	.038	.009
20	52 44	.76	.192	.048		50	41 10	.602	.150	.037	
40	52 34	.76	.191	.048		8400	40 56	.599	.150	.037	
60	52 24	.76	.191	.048		50	40 41	.596	.149	.037	
80	52 15	.76	.190	.048		8500	40 27	.593	.148	.037	
6600	52 03	.75	.189	.047		50	40 13	.589	.147	.037	
20	51 56	.75	.189	.047		8600	39 58	.585	.146	.037	
40	51 47	.75	.188	.047		50	39 45	.581	.145	.036	
60	51 37	.75	.188	.047		8700	39 31	.577	.144	.036	.009
80	51 28	.74	.187	.047							

TABLE C.—For Laying Out Curves. Chord *AB* = 200 feet or links, or any multiple of either. (See Fig. A, Sec. 319x.)

Rad. of curve.	$\frac{1}{2}$ angl. of defect'n ° ' "	D C	F E	H G	W S	Rad. of curve.	$\frac{1}{2}$ angl. of defect'n ° ' "	D C	F E	H G	W S
8750	0 39 17	.573	.143	.036	.009	14600	0 23 33	.342	.086	.022	.005
8800	39 04	.573	.143	.036		14700	23 23	.340	.085	.021	
8850	38 51	.566	.141	.035		800	23 14	.338	.085	.021	
8900	38 37	.563	.141	.035		900	23 04	.336	.083	.021	
9000	38 12	.557	.139	.035		15000	22 55	.334	.083	.021	
9100	37 47	.549	.137	.034		100	22 46	.332	.082	.021	
9200	37 22	.543	.136	.034		200	22 37	.330	.082	.021	
9300	36 58	.537	.134	.034		300	22 28	.328	.081	.020	
9400	36 35	.531	.133	.033	.008	400	22 19	.326	.081	.020	
9500	36 11	.525	.131	.033		500	22 12	.324	.080	.020	
9600	35 49	.519	.130	.033		600	22 02	.322	.080	.020	
9700	35 26	.513	.128	.032		700	21 54	.320	.079	.020	
9800	35 05	.508	.127	.032		800	21 46	.318	.079	.019	
9900	34 44	.504	.126	.032		900	21 37	.316	.078	.019	
10000	34 23	.500	.125	.031		16000	21 30	.314	.078	.019	
100	34 02	.495	.124	.031		100	21 21	.312	.078	.019	
200	33 42	.491	.123	.031		200	21 13	.310	.077	.019	
300	33 23	.486	.122	.031		300	21 05	.308	.077	.019	
400	33 03	.481	.120	.030		400	20 58	.306	.076	.019	
500	32 44	.476	.119	.030		500	20 50	.304	.076	.019	
600	32 26	.471	.118	.030		600	20 43	.302	.075	.018	
700	32 08	.467	.117	.029	.007	700	20 35	.300	.075	.018	
800	31 50	.463	.116	.029		800	20 28	.298	.074	.018	
900	31 33	.459	.115	.029		900	20 21	.296	.074	.018	
11000	31 15	.455	.114	.028		17000	20 13	.294	.073	.018	
100	30 58	.451	.113	.028		100	20 07	.292	.073	.018	
200	30 42	.447	.112	.028		200	19 59	.290	.072	.018	
300	30 25	.443	.111	.028		300	19 52	.288	.072	.018	
400	30 09	.439	.110	.028		400	19 45	.286	.072	.018	
500	29 54	.435	.109	.027		500	19 39	.284	.071	.018	
600	29 38	.431	.108	.027	.007	600	19 32	.282	.071	.017	
700	29 23	.427	.107	.027		700	19 26	.281	.071	.017	
800	29 08	.424	.106	.027		800	19 19	.280	.070	.017	
900	28 53	.421	.105	.026		900	19 12	.279	.070	.017	
12000	28 40	.418	.104	.026		18000	19 06	.278	.069	.017	.004
100	28 25	.414	.104	.026		100	0 19 00	.276	.069	.017	
200	28 11	.411	.103	.026		200	18 53	.275	.069	.016	
300	27 57	.407	.102	.026		300	18 47	.273	.068	.016	
400	27 43	.403	.101	.025		400	18 41	.272	.068	.016	
500	27 30	.399	.100	.025		500	18 35	.270	.067	.016	
600	27 17	.396	.099	.025		600	18 29	.269	.067	.016	
700	27 04	.393	.098	.025		790	18 23	.268	.067	.016	
800	26 51	.390	.098	.025		800	18 17	.267	.067	.016	
900	26 39	.387	.097	.024		900	18 11	.265	.066	.016	
13000	26 27	.385	.096	.024		19000	18 06	.264	.066	.016	
100	26 14	.382	.096	.024		100	18 00	.262	.066	.016	
200	26 03	.379	.095	.024		200	17 54	.261	.065	.015	
300	25 51	.376	.094	.024		300	17 49	.259	.065	.015	
400	25 39	.373	.093	.023		400	17 43	.258	.065	.015	
500	25 28	.370	.092	.023		500	17 38	.256	.064	.015	
600	25 17	.367	.091	.023		600	17 32	.255	.064	.015	
700	25 06	.364	.090	.023		700	17 27	.253	.063	.015	
800	24 55	.361	.090	.023		800	17 22	.252	.063	.015	
900	24 44	.358	.089	.022		900	17 17	.251	.063	.015	
14000	24 33	.356	.089	.022	.006	20000	17 11	.249	.062	.015	
100	24 23	.353	.088	.022		21000	16 21	.238	.059	.015	
200	24 13	.350	.088	.022		21120	16 16	.237	.059	.020	.004
300	24 02	.348	.087	.022		15840	21 42	.316	.079	.029	.005
400	23 52	.346	.087	.022		10560	32 33	.473	.118	.059	.007
500	23 43	.344	.086	.022	.005	5280	1 5 07	.947	.237	.119	.030

CANALS.

§20. In *locating* a canal, reference must be had to the kind of vessels to be used thereon, and the depth of water required; the traffic and resources of the surrounding country; the effect it may have in draining or overflowing certain lands; the feeders and reservoirs necessary to keep the summit level always supplied, allowing for evaporation and leakage through porous banks, etc. The canal to have as little inclination as possible, so as not to offer any resistance to the passage of boats. To be so located that its distance will be as short as possible between the cities and towns through or near which it is to pass. To have its cutting and filling as nearly equal as the nature of the case will allow. To have sufficient slopes and berms as will prevent the banks from sliding. The bottom width ought to be twice the breadth of the largest boat which is to pass through it. The depth of water 18 inches greater than the draft or depth of water drawn by a boat.

Tow-path. About 12 feet wide, being between 2 and 4 feet above the level of the water, and having its surface inclined towards the canal sufficiently to keep it dry. Vegetable soil, and all such as are likely to be washed in, are to be removed. Where there is no tow-path, a berm or bench, 2 feet wide, is left in each side, about 18 inches above the water.

Feeders may have an inclination not more than 2 feet in a mile, to be capable of supplying four or five times the necessary quantity of water to feed the *summit level*.

Reservoirs, or *basins*, may be made by excavation, or, in a hilly country, by damming the ravines. There are many instances of this on the Rideau Canal in Canada; also, on that built by the author, connecting the Chats and Chaudiere lakes, on the river Ottawa, in the same country.

This necessarily requires that an Act of the Legislature should empower them to enter on any land, and overflow it if necessary, and have commissioners to assess the benefit and damages.

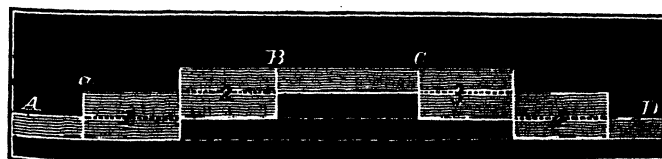
Draft is the depth of water required to float the boat.

Lift is the additional quantity required to pass the boat from one lock into another.

A boat ascending to the summit has as many lifts as there are drafts.

A boat descending from a summit to a lower level has one more lift than drafts.

Let the annexed figure represent a canal, where there are two locks ascending and two descending; there are four lifts and three drafts.



To Ascend from A to B of Lock 1. (See annexed figure.) Boat arrives at gate *a*, finds in it one prism of draft, and the other lock empty. Now, all these locks must be filled to enable the boat to arrive at the summit level B C. Let L = prism of lift, and D = prism of draft; then it is plain that to ascend from A to B requires two prisms of lift and one of draft, and putting $n = 2$, or the number of locks, the quantity required to pass the boat = $n L + (n - 1) D$.

n

To Descend from C to D = 2 locks. In lock 3, one prism of lift will be taken, and one of draft. The prism of lift passes into lock 4, together with one of draft, thus using two prisms of draft and one of lift, which is sufficient to pass the boat from C to D = $L + 2 D$. Or,

To ascend = $n L + (n - 1) D$.

To descend = $L + 2 D$. Add these two equations. The whole quantity from A to D = $(n + 1) L + (n + 1) D = (n + 1) \cdot (L + D)$.

Each additional boat passing in the same order requires two prisms of lift and two of draft; that is, the additional discharge = $2 (N - 1) (L + D)$. Here N = number of boats; therefore the whole discharge = $(n + 1) (L + D) + (2 N - 2) (L + D) = (2 N + n - 1) \cdot (L + D)$. To this must be added the loss by evaporation and leakage. Evaporation may be taken at half an inch per day. From one-third to two-thirds of the rain-fall may be collected.

The engineer will, when the channel is in slaty or porous soil, cover it with a layer of flat stones laid in hydraulic mortar, having previously covered it with fine sand.

Locks to be one foot wider than the width of beam, 18 inches deeper than draft of boat, and to be of a sufficient length to allow the rudder to be shifted from side to side.

Bottom to be an inverted arch where it is not rock. Where the bottom is not solid, drive piles, on which lay a sheeting of oak plank to receive the masonry.

The channel to have recesses to receive the lock gates.

The lock gates to make an angle of $54^\circ 44'$ with one another, being that which gives them the greatest power of resisting the pressure of the prism of water.

Reservoirs are made in natural ravines which may be found above the summit level, or they are excavated at the necessary heights above the summit.

Dams are made of solid earth or masonry. When of earth, remove the surface to the depth where a firm foundation can be had; then lay the earth in layers of eight or twelve inches; have it puddled and rammed, layer after layer, to the top. *Slope* next the water to be three or four base to one perpendicular (see sec. 147). *Outside slope* about two or two and a half base to one perpendicular. The face next the dam is faced with stone. For thickness of the top of the dam, see *Embankments* (sec. 319).

To Set Out the Section of a Canal when the Surface is Level.

321. Let the bottom width $AB = 30$ feet, height of cutting on the centre stake $HF = 20$ feet = h , ratio of slopes 2 to 1 = r —that is, for 1 foot perpendicular there is to be 2 feet base, $20 \times 2 = 40$ = base for each slope = $CG = ED$, and $20 \times 2 \times 2 = 80$ = total base for both slopes. Bottom width = 30; therefore, $80 + 30 = 110$ = width of cutting at top = CD ; and $110 + 30 \div 2 \times 20$ = sectional area = 1400. In general,

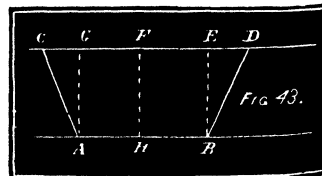
$S = (b + h r) h$ = sec'l area in ft.

$C = (b + h r) h L$ = cubic content.

Here S = transverse sectional area,

C = content of the section, b = bot

tom width, h = height, r = ratio of slope, and L = length of section.



To Set Out a Section when the Surface is an Inclined Plane, as in fig. 44.

321a. This case requires a cutting and an embankment. We will suppose the slopes to be the same in both.

Let the surface of the land be R Q, the canal A B = bottom = b = 30 feet. Height H G = 20, ratio of slopes of excavation and embankment = $1\frac{1}{2}$ base to 1 height—that is, ratio of slopes = r = $1\frac{1}{2}$ to 1.

At the centre G set up the level; set the leveling staff at N; found the height S N = 5 feet; measured G S = 20.61, and G N = 20; because the slopes being $1\frac{1}{2}$ to 1, the slope to 5 feet = $7\frac{1}{2}$; $\therefore$ G F = $12\frac{1}{2}$, and G M = $27\frac{1}{2}$ feet; and the slope corresponding to H G = $20 \times 1.5 = 35$, which added to half the bottom, gives G C = 45.

To Find G E and G Q.

G M : G S :: G C : G E; that is,
27.5 : 20.61 :: 45 : G E = 33.72 feet.

Let the top of embankment P C = 20 feet; then G P = 65.
G F : G S :: G P : G Q; that is,
 $12\frac{1}{2} : 20.61 :: 65 : G Q = 107.17$ feet.

Having G E, G Q, G S and S N, we can find the perpendicular Q V.
G S : S N :: G Q : Q V.
20.61 : 5 :: 107.17 : Q V = 26, which is perpendicular to the surface G V.
20.61 : 5 :: G E = 33.72 : E F = 8.18 feet.
 $G V^2 = G Q^2 - Q V^2$; $\therefore$ we can find G V = 103.96; and by taking 65 from the value of G V, we find $103.96 - 65 = 38.96 = P V$.

To Find the Point R.

We find, when the slope G Q continues to R, that by taking G s = 20.61, n s = 5, n t = $7\frac{1}{2}$, G t = $12\frac{1}{2}$, and s t is parallel to B R; $\therefore$ G t : G s :: G D : G R; but G D = $15 + 20 \times 1\frac{1}{2} = 45$, $\therefore$
 $12.5 : 20.61 :: 45 : G R = 74.19$.

To Find G d = H a, and Area of Cutting.

We have G s : G n :: G R : G d; that is,
20.61 : 20 :: 74.19 : G d = H a = 71.99.
G n : n s :: G d : R d; that is,
20 : 5 :: 71.99 : R d = 17.9975.
But H G = a d = 20; therefore R a = 37.998;
and H a — H B = 71.99 — 15 = B a = 56.99. Let us put 18 = 17.9975.
Area of sec. H G R a = $\frac{G H + R a}{2} \times H a = \frac{20 + 38}{2} \times 71.99 = 2087.71$
Deduct the $\triangle$ B R a = $56.99 \times 19 =$ 1082.81
Area of the figure G H B R = 1004.90
Area G H A C = $(G C + A H) \times \frac{H G}{2} = (45 + 15) \times 10,$ 600
Area of the figure C G R B A = 1604.90
Deduct triangle G E C = $45 \times \text{half of E f} = 45 \times 4.09,$ 184.05
Area of B A E G R = 1420.85

Or thus :

We have $R a$ by calculation or from the level book, 38 nearly. Also, $E g = g f - E f = 20 - 8.18 = 11.82$, which multiplied by ratio of slope, gives $A g = 17.73$, and $H g = 33.72$. But from above we have $H a = 71.99$; $\therefore 71.99 + 32.73 = a g = 104.72$.

$$\frac{104.72}{2} \times (E g + R a) = 52.36 \times (11.82 + 38) = E g a R = 2608.58$$

$$\text{Deduct } \triangle E g A + \triangle B R a; \text{ i.e., } \frac{11.82 \times 17.73}{2} + 56.99 \times 19 = 1187.59$$

$$\text{Area of the section } R E A B = 1420.99$$

Nearly the same area as above. The difference is due to calling 17.9975 = 18.

To Find the Embankment.

We have $Q V = 26$, $P V = 38.96$, $E f = 8.18$, $P C = 20$, $G F = 32.72$, and $C F = G C - G F = 45 - 32.72 = 12.28$
 $G V = 45 + 20 + 38.96 = G C + C P + P V = 103.96$
 $G S : G N :: G E : G f$; that is,
 $20.61 : 20 :: 33.72 : G f = 33.72$. This taken from $G C$ or 45 will give
 $C F = 12.28$; $\therefore f V = 12.28 + 20 + 38.96 = 71.24$
 $\frac{1}{2} (Q V + E f) = \frac{1}{2} (26 + 8.18) = 17.09$

$$\text{The product} = \text{area of } Q V F E = 1217.4916$$

$$\text{Deduct } \triangle C f E = 4.09 \times 12.28 = \frac{1}{2} E f \times C f = 50.22$$

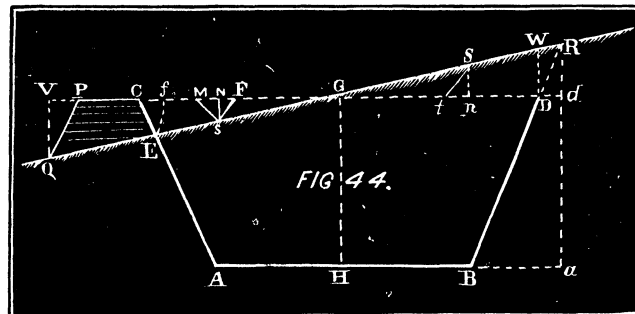
$$\text{Also deduct } \triangle Q V P = 38.96 \times 13 = 506.48$$

$$\text{Sum to be subtracted, } 556.70$$

$$\text{Area of section } Q P C E = 660.79$$

To Set Off the Boundary of a Canal or Railway.

321b. Let the width from the centre stump or stake G to boundary



line = 100 feet, if the ground is an inclined plane, as fig. 44. We can say, as $G N : G S :: G f : G E$; i. e., $20 : 20.61 :: 100 : G E = 103.05$.

Otherwise, take a length of 20 or 30 feet, and, with the assistant, measure carefully, dropping a plumb-line and bob at the lower end, and thus continue to the end. This will be sufficiently accurate.

To Find the Area of a Section of Excavation or Embankment such as A B D C.
(See Fig. 46.)

322. Let r = ratio of slopes, D = greater and d = lesser depth, and b = bottom width.

We have $d r = A E$, and $D r = B F$; $\therefore (D + d) r + b = E F$. But $E F \times (D + d) =$ twice the area of $C E F D$; i. e., $\{(D + d) r + b\} \cdot (D + d) =$ double area of $C E F D$.

$(D^2 + 2 D d + d^2) r + (D + d) b =$ double area of $C E F D$.

$d^2 r = 2 \triangle A C E$, and $D^2 r = 2 \triangle B D F$; these taken from the value of twice the area of $C D F D$, gives the required area of $A C D B = 2 D d r$. This divided by 2 will give the area of

$$A B C D = D d r + \left(\frac{D + d}{2}\right) b.$$

Rule. Multiply the heights and ratio together; to the product add the product of half the heights multiplied by the base. The sum will be the area of $A B C D$, when the slopes on both sides are equal.

Example. Let bottom $b = 30$, $d = 10$, $D = 20$, ratio of base to perpendicular $= r = 2$, to find the area of the section.

$$D d r = 10 \times 20 \times 2 = 400$$

$$\left(\frac{D + d}{2}\right) \times b = 15 \times 30 = 450$$

$$\text{Area of section } A B D C = 850$$

322a Let the slopes of $A C$ and $B D$ be unequal; let the ratio of slope for $A C = r$, and that for $B D = R$. Required area of $A B D C =$
 $\frac{b}{2} \cdot (D + d) + \frac{R + r}{2} \cdot (D d).$

Rule. Multiply the sum of the two heights by half the base, and note the product.

Multiply the product of the heights by half the sum of the ratios, and add the product to the product above noticed. The sum of the two products will be the required area.

Example. Let the heights and base be as in the last example; ratio of slope $A C = 2$, and that of slope $B D = 3$.

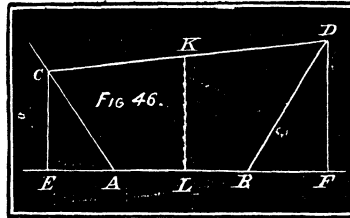
$$\frac{b}{2} (D + d) = 15 \times 30 = 450$$

$$\frac{R + r}{2} \cdot D d = 2.5 \times 200 = 500$$

$$\text{Area of } A F D C = 950$$

Let the Surface of the Side of a Hill Cut the Bottom of the Canal or Road Bed, as in Fig. 47.

322b. Here $A B$ is the bottom of the canal or road, $A C$ and $B D$ its sides, having slopes of r . $D E$ = the surface of the ground, $G F = d$ = lesser height below the bottom, and to the point where the slope $A C$ produced will meet the surface of the ground. $D H = D$ = greater height above the bottom.



Through F, draw F K parallel to A H; then D K = D + d, and A H = b + r D, and A G = r d; therefore F K = G H = b + r D - r d = b + (D - d) r, and by similar triangles.

D K : K F :: D H : M H; that is,

$$D + d : b + r D - r d :: D : M H = \frac{B D + r D^2 - r d D}{D + d}$$

But M H $\times$ D H = twice the area of $\triangle$ M D H, and twice the area of $\triangle$ B D H = B H $\times$ D H = R d $\times$ D = r D²;

$$\begin{aligned} \therefore \text{twice area of } \triangle M D B &= \frac{b D^2 + r D^3 - r d D^2}{D + d} - r D^2 \\ &= \frac{b D^2 + r D^3 - r d D^2 - r D^3 - r d D^2}{D + d} \\ &= \frac{b D^2 - 2 r d D^2}{D + d} \end{aligned}$$

$$\text{Double area} = \frac{(b - 2 r d) D^2}{D + d} \text{ that is,}$$

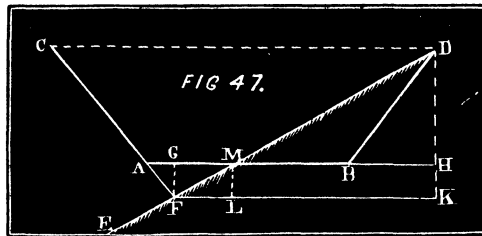
$$\text{Area of } \triangle M D B = \left(\frac{(b - 2 r d) D^2}{2 (D + d)} \right)$$

$$\text{Or} = \frac{\frac{1}{2} (b - r d) D^2}{D + d} \text{ which is that given by Sir}$$

John McNeil in his valuable tables of earthwork.

Rule. From half the base take the product of the ratio of slopes and height below the bed; multiply the difference by the square of the height above the bed of road or canal; divide this product by the sum of the two heights; the quotient will be the area of the section M D H.

Example. Let base = 40, ratio of slopes $1\frac{1}{2}$ to 1, height G F below the bed = $5\frac{1}{2}$, height D H above the bed = 20 feet, to find the area of the section M D B. (See figure 47.)



Half the base =	20
$r d = 5.5 \times 1.5 =$	8 25
	11.75
$D^2 = 20 \times 20 =$	400
	4700

Divide 4700 by $D + d = 20 + 5.5 = 25.5$

The quotient = area of M D B = 184.313 feet.

To Find the Mean Height of a Given Section whose Area = A, Base = b, Ratio of Slopes = r.

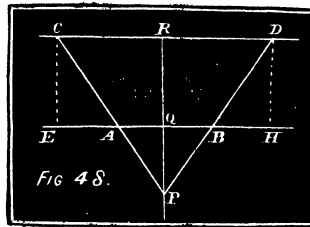
323. Let x = required mean height; then mean width = b + r x; this multiplied by the mean height, gives $b x + r x^2 = A = \text{given area}$.

$$x^2 + \frac{b}{r}x = \frac{A}{r} \quad \text{Complete the square:}$$

$$x^2 + \frac{b}{r}x + \frac{b^2}{4r^2} = \frac{A}{r} + \frac{b^2}{4r^2} =$$

$$\frac{4Ar^2 + rb^2}{4r^3} = \frac{4Ar + b^2}{4r^2}$$

$$x + \frac{b}{2r} = \frac{\sqrt{(4Ar + b^2)}}{2r}$$



Mean height = $x = \frac{(4Ar + b^2)^{\frac{1}{2}}}{2r} - \frac{b}{2r}$, and by substituting the value of A in sec. 322,

$$x = \left(\frac{(4Ddr^2 + (D+d)2br + b^2)^{\frac{1}{2}} - b}{2r} \right)$$

Rule. To the square of the base, add four times the area multiplied by the ratio of the slopes; take the square root of the product; divide this root by twice the ratio, and from the quotient take the base divided by twice the ratio. The difference will be the required mean height.

Example. Let us take the last example, where the base $b = 40$, ratio $r = 1\frac{1}{2}$, area = 184.313 square feet.

$$4Ar = 184.213 \times 4 \times 1.5 =$$

$$1105.878$$

$$b^2 = 40 \times 40 =$$

$$1600$$

$$2705.878$$

$$\text{Square root of } 2705.878 =$$

$$52.018$$

$$\text{This root divided by } 2r = 3 \text{ gives} =$$

$$17.339$$

$$\text{From this take } \frac{b}{2r} = \frac{40}{3} =$$

$$13.333$$

$$\text{Gives the mean height} = 4.006, \text{ or} =$$

$$4 \text{ feet nearly.}$$

$$4r = 6, \text{ to which add base } 40, \text{ sum} =$$

$$46$$

$$\text{Approximate mean height,}$$

$$4$$

$$184$$

Area nearly as above.

It need not be observed that if we took the mean height = 4.009, we would find 184.313 nearly. Our object here is to show the method of applying the formula to those who have no knowledge of algebraic equations.

Or by plotting the section on a large scale on cartridge paper, the area and mean depth can be computed by measurement. The mean heights are those used in using McNeil's tables of earthwork, and also in finding the middle area, necessary for applying the prismoidal formula.

Rule 2. To four times the product of the heights and ratio add the continual product of the sum of the two heights by twice the base multiplied by the ratio; to this sum add the square of the base; from the square root of this last sum subtract the base, and divide the difference by twice the ratio. The quotient will be the mean height.

Example. $D = 70, d = 30, b = 40, r = 1.$

$$70 \times 30 \times 4 \times (70 + 30) \times 80 = 16400$$

$$\text{Square of base} =$$

$$1600$$

$$18000$$

The square root = 134.164, which, divided by 2, gives 47.082, the mean height.

Another Practical Method.

324. Let $AB = \text{base} = b$, $CDBA = \text{required section}$, whose area $= A$, and mean height QR is required; ratio of slopes perpendicular to base is as 1 to r . (See fig. 48.)

We have $PQ \times 2r = AB = b$; that is,

$$PQ = \frac{b}{2r}; \text{ this } \times \text{ by the base gives twice area of } \triangle ABP = \frac{b^2}{2r};$$

therefore, area $\triangle ABP = \frac{b^2}{4r}$; consequently,

$$\text{area of } \triangle CPD = \frac{b^2}{4r} + A, \text{ or putting area of } \triangle ABP = a,$$

we have area $\triangle CPD = A + a$, and by Euclid VI, prop. 19,

$$\triangle ABP : \triangle CPD :: PQ^2 : PR^2;$$

$$\text{that is, } a : A + a :: \frac{b^2}{4r^2} : PR^2.$$

$$PR^2 = \frac{(A + a)b^2}{4ar^2} \text{ take the square root,}$$

$$PR = \frac{(A + a)^{\frac{1}{2}}}{\sqrt{a}} \cdot \frac{b}{2r}$$

$$PR = \left(\left(\frac{A + a}{a} \right)^{\frac{1}{2}} \cdot \frac{b}{2r} \right)$$

$$QR = \left(\left(\frac{A + a}{a} \right)^{\frac{1}{2}} \cdot \frac{b}{2r} - \frac{b}{2r} \right) = \text{mean height.}$$

Example. Let $AB = b = 20$, ratio $= 2$. Given area of the section 1200, which is to be equal to the section $ABCD$, whose mean height is required.

The constant area of $\triangle ABP$ is always $= \frac{b^2}{4r} = 50$.

$$\frac{(A + a)^{\frac{1}{2}}}{a} = \left(\frac{1200 + 50}{50} \right)^{\frac{1}{2}} = \left(\frac{1250}{50} \right)^{\frac{1}{2}} = \sqrt{25} = 5.$$

$$\text{Multiply by } \frac{b}{2r} = \frac{20}{4}$$

$$\frac{5.}{25, \text{ product.}}$$

$$- \frac{b}{2r}$$

$$\frac{5.}{20.}$$

$$QR = \text{mean height} =$$

$$20.$$

In this example and formula the slopes are the same on both sides.

Let $R = \text{greater}$, and $r = \text{lesser ratio}$;

$$\text{then } QR = \left(\frac{A + a}{a} \right)^{\frac{1}{2}} \cdot \frac{b}{R + r} - \frac{b}{R + r}.$$

When the Slopes are the Same on Both Sides.

325. *Rule.* To the given area above the base add the constant area below the base; divide the sum by the constant area of the $\triangle ABP$; multiply the square root of this quotient by the base divided by twice the ratio of the slope; from this product take the base divided by the ratio of slope. The difference will be the required mean height $= LR$.

When the Slopes are unequal.

Rule. To the given area above the base, add the constant area of the triangle A B P below the base, divide the sum by the constant area of $\triangle$ A B P. Multiply the square root of the quotient, by the base divided by the sum of the ratio of the slopes, from the product subtract the base divided by the sum of the ratios, the difference will be the required mean height = Q R.

Example. Let ratio R = ratio of Q B to Q P = ratio to slope B D = 3, and r = lesser ratio of A Q to P Q = 2.

$$A B = b = 20, \text{ therefore } P Q = \frac{20}{R + r} = 4.$$

Let area of A B D C = 960, and constant area of the triangle under the base = 40 = A = $\triangle$ A B P.

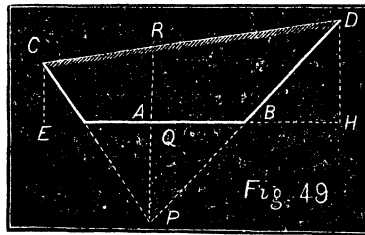
$$\left(\frac{A + a}{a}\right)^{\frac{1}{2}} \cdot \frac{b}{R + r} - \frac{b}{R + r} = \left(\frac{960 + 40}{40}\right)^{\frac{1}{2}} \cdot \frac{20}{5} - \frac{20}{5} = Q R.$$

$$Q R = 5 \times 4 - 4 = 16.$$

326. *Mean height must not be found by adding the heights on each side of the centre stump or stake, and then take half of the sum for a mean height.* This method is commonly used, and is *very erroneous*, as will appear from the following example: Let the greater height D H = 70, (see fig. 49,) the lesser C E = 30, base 40, ratio of slopes 1 to 1.

Correct Method.

$$\begin{array}{rcl} 70 = \text{greater height} & = & D \\ 30 = \text{lesser} & = & d \\ 2) 100, \text{ mean height} & = & 50 \\ 30 + 40 + 70 = \text{base E H} & = & 140 \\ \text{Sectional area of} & & \\ \quad \text{C D H E} & = & 7000 \\ \text{deduct the two triangles} & & \\ \quad \text{C E A} + \text{D B H} & = & 2900 \\ \hline \text{Area} & = & 4100 \\ & \text{Correct.} & \end{array}$$



Or, by sec. 322, we can find the area

$$\begin{array}{rcl} D d r = 70 \times 30 \times 1 & = & 2100 \\ D + d \cdot b = 50 \times 40 & = & 2000 \\ \hline 2 & & 4100, \text{ required correct area.} \end{array}$$

By the Erroneous or Common Method.

$$\begin{array}{rcl} 70 + 30 = 100 & = & \text{sum of heights.} \\ 50 & = & \text{mean height.} \\ \text{Half slope} & = & 50 \\ \hline 100 & = & \text{mean base.} \\ 50 & = & \text{mean height.} \end{array}$$

Area 5000 incorrect.

Area 4100 correct.

Difference 900 square feet.

From this great difference appears that where the mean height is required, it has to be calculated by the formula in section 323, where

$$x = \text{mean height} = \frac{(4 A r + b^2)^{\frac{1}{2}}}{2 r} - \frac{b}{2 r}$$

n2

Area found by the correct method = 4100

$$\begin{array}{r} 4 \\ 16400 = 4 A \\ 1 = r \\ 16400 = 4 A r \\ 1600 = b^2 \end{array}$$

Square root of 18000 = 134.164,

and 134.164, divided by twice the ratio, gives 67.082, from which take the base, divided by twice the ratio, leaves required mean height = 47.082.

$$\begin{array}{r} \text{By the common method} \\ \text{Difference,} \end{array} \quad \begin{array}{r} = 50 \\ 2.918 \text{ feet.} \end{array}$$

Or thus, by sec. 324: We find the mean height Q R, (fig. 49,) area of triangle A B P, having slopes 1 to 1 = 400, the perpendicular P Q = 20. And from above we have the area of the section A B D C = 4100

$$\begin{array}{r} \therefore \left(\frac{A+a}{a} \right)^{\frac{1}{2}} = \left(\frac{4100+400}{400} \right)^{\frac{1}{2}} = \sqrt{\frac{4500}{400}} = \frac{\sqrt{45}}{2} = \frac{6.7082}{2} = 3.3541 \\ \frac{b}{2r} = \frac{20}{2 \times 400} = \frac{20}{800} = 0.025 \\ \text{Less } \frac{b}{2r} = 0.025 \\ \text{Mean height Q R} = 47.082 \end{array}$$

TO FIND THE CONTENT OF AN EXCAVATION OR EMBANKMENT.

In general, the section to be measured is either a prism, cylinder, cone, pyramid, wedge, or a frustrum of a cone, pyramid, or wedge. The latter is called a prismoid.

A *Prism* is a solid, contained by plane figures, of which two are opposite, equal, similar, and having their sides parallel. The opposite, equal and similar sides are the ends. The other sides are called the lateral sides. Those prisms having regular polygons for bases, are called regular prisms.

Prismoid has its two ends parallel and dissimilar, and may be any figure.

327. *Prism. Rule.* Multiply the area of the base by the height of the section, the product = content, or $S = A l$. Here A = area of the base, and l = the length of the section, and S = sectional area.

328. *Cylinder. Rule.* Square the diameter, multiply it by .7854, then by the height, the product = content = $D^2 \times .7854$. Here D = diameter, solidity = $S = A l$. Here A = area of the base, and l = length.

329. *Cone. Rule.* Multiply the square of the diameter by .7854, and that product by one-third of the height, will give the content = $S = D^2 \times .7854 \times \frac{1}{3}$. Or, solidity = $\frac{A l}{3}$ where A and l are as above.

330. *Frustrum of a Cone. Rule.* To the areas of the two ends, add their mean proportional. Multiply their sum by one-third of the height or length, the product = content.

$$\begin{array}{l} \text{Solidity} = S = \left(A \times a \times \sqrt{A a} \right) \frac{1}{3} \\ S = (D^2 + d^2 + D d) 0.2618 \\ S = \left(\frac{D^3 - d^3}{D - d} \cdot \frac{1}{3} \right) = \left(\frac{D^3 - d^3}{D - d} \right) \times .2618 \text{ c. Here } t = 0.7854, \\ D \text{ and } d = \text{diameters, } l = \text{length, as above.} \end{array}$$

Example. Let the greater diameter of a frustrum of a cone be = $D = 2$, and the lesser = $d = 1$, and the length = 15, to find the content. Dimensions all in feet.

$$A = 4 \times 0.7854 = 3.1416 = 3.1416$$

$$a = 1 \times 0.7854 = 0.7854 = 0.7854$$

$$\text{Product} = 2.46741264, \text{ square root} = 1.5708$$

$$5.4978$$

$$\text{One-third the length, } \frac{15}{3} = 5$$

$$\text{Content or } S = 27.489$$

Or thus :

$$(\text{By sec. 330.}) \quad D^2 + d^2 + Dd = 4 + 1 + 2 = 7$$

$$l = \text{length} = 15$$

$$= \frac{15}{3}$$

$$= 5$$

$$0.7859 = \text{tabular number}$$

$$= 0.2618$$

$$\frac{3}{3}$$

$$S$$

$$= 27.489 = \text{content.}$$

Or thus :

$$\frac{D^3 - d^3}{D - d} = \frac{8 - 1}{1} = 7$$

$$t = 0.7854$$

$$5.4978$$

$$15$$

$$3 \overline{)824670}$$

$$S = 27.489 = \text{content.}$$

331. *Pyramid. Rule.* Multiply the area of the base by one-third of the length or height, and the product will be the required content. Or, solidity = $S = \frac{A l}{3}$

332. *Frustrum of a Pyramid. Rule.* To the sum of the areas of both ends add their mean proportional, multiply this sum by one-third of their height, the product will be the content, or $S = (A + a + \sqrt{Aa}) \frac{l}{3}$

Let the ends be regular polygons, whose sides are D and d , then,
 $S = \left(\frac{D^3 - d^3}{D - d} \right) \frac{t l}{3}$ Here D = greater and d = lesser side,

t = tabular area, corresponding to the given polygon, and l as above.

Rule. From the cube of the greater side take the cube of the lesser, divide this difference by the difference of the sides, multiply the quotient by the tabular number corresponding to that polygon, and that product by the length or height. One-third of this product will be the required content, the same as for the frustrum of a cone.

Example. Let 3 and 2 respectively be the sides of a square frustrum of a pyramid, and length = 15 feet.

$$A + a + \sqrt{Aa} = 9 + 4 + 6 = 19$$

$$\text{One-third the length} = \frac{15}{3} = 5$$

$$\text{Solidity} = S = 95$$

Or thus, by sec. 331 :

$$\frac{D^3 - d^3}{D - d} = \frac{27 - 8}{3 - 2} = \frac{19}{1} = 19$$

$$\text{Tabular number per Table VII.} = 5.1961$$

$$10$$

$$\text{One-third the length} = 5$$

$$S = 95$$

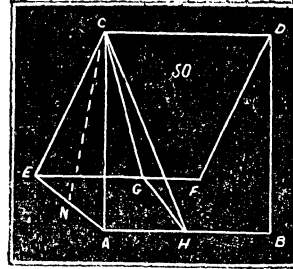
$$= \text{content.}$$

333. *Wedge* has a rectangular base and two opposite sides meeting in an edge.

Rule. To twice the length of the base add the length of the edge, multiply this sum by the breadth of the base, and the product by one-sixth of the height, the product will be the solid content, when the base has its sides parallel.

$$S = \frac{1}{6}(2L + l) \cdot b \cdot h. \text{ Here}$$

L = length of the rectangular base AB , l length of the edge CD , b = breadth of base, BH and H = height.



Example. Let $AB = 40$ feet, $BF = b = 10$, $CD = l = 30$, and let the height $NC = 50$ feet $= h$, to find the content.

$$\begin{array}{r} 2L \times l = 80 + 30 = 110 \\ b \cdot h = 10 \times 50 = 500 \\ \hline 6)55000 \end{array}$$

9166.666 cubic feet.

Let CD , the edge, be parallel to the lengths AB and EF , and AB greater than EF , HG = perpendicular width.

Rule 2. Add the three parallel edges together, multiply its one-third by half the height, multiplied by the perpendicular breadth, the product

will be the required content. Or, $S = \frac{1}{3}(L + L_1 + l) + \frac{1 \cdot h \cdot b}{2}$

Here L = greater length of base, L_1 = lesser length, l = length of the edge, h = perpendicular height, and b = perpendicular breadth.

Let us apply this to the last example:

$$\begin{array}{r} \frac{L + L_1 + l}{3} = \frac{40 + 40 + 30}{3} = \frac{110}{3} \\ \frac{h \cdot b}{2} = \frac{50 \times 10}{2} = 250 \end{array}$$

$$\text{Therefore, content} = \frac{110}{3} \times \frac{250}{1} = \frac{27500}{3} = 9166.666, \text{ as above.}$$

Example 2. Let $AB = 4$, $EF = 2.5$, $CD = 3$, height $= 12$, and width $HG = 3\frac{1}{2}$, then by Rule 2.

$$4 + 3 + 2.5 \times 12 \times 3.5 = 66\frac{1}{2} \text{ cubic feet.}$$

Note. As Rule 2 answers for any form of a wedge, whose edge is parallel to the base, the opposite sides AB and EF parallel, without any reference to their being equal.

PRISMOID.

334. The prismoid is a frustrum of a wedge, its ends being parallel to one another, and therefore similar, or the ends are parallel and dissimilar.

When the section is the frustrum of a wedge, it is made up of two wedges, one having the greater end for a base, and the other having the lesser, the content may be found by rule 2 for the wedge.

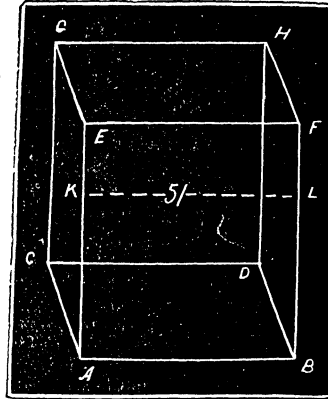
The following rule, known as the prismoidal formula, will answer for a section whose ends are parallel to one another. It is the safest and most expeditious formula now used, and has been first introduced by Sir John MacNeil in calculating his valuable tables on earth work, octavo, pp. 268. T. F. Baker, Esq., C.E., has also given a very concise formula, which, as many perhaps may prefer, I give in the next section. To Mr. Baker, of England, the world is indebted for his practical method of laying out curves.

PRISMOIDAL FORMULA.

$S = (A + a + 4M) \frac{L}{6}$ Here A = area of greater end, a = area of lesser end, M = area of middle section, and L = length of section, all in feet.

Rule. To the sum of the areas of the two ends, add four times the area of the middle section, multiply this sum by one-sixth of the length, the product will be the required content, or solidity.

Here A = area of $C A B D$,
 a = area of $G E F H$,
 and M = area of section through $K L$.



Example. Let the length L = 400 feet.
 Mean height of section $A B D C$ = 50
 Mean height of section $G E F H$ = 20
 Ratio of slopes = 2 base to 1 perpendicular, and base = 30,

50 = mean height, by sec. 326.	Height 20	50
2	2	20
Half base = 100 for slopes.	40	2)70
30	30	35
Mean br'dth, 130	Mean breadth, 70	2
Height, 50	Height, 20	70
6500	a = 1400	30
	A = 6500	100
	M = 14000	35
	21900	3500 = M .
	400 = length.	
	6)8760000	
Content in cubic feet = 9)1460000		
	3)162222.22	
	54074.07 cubic yards.	

On comparing this with Sir John MacNeil's table, we find 540.72, difference only 2 yards, which is but very little in this large section.

Baker's Method Modified. (See fig. 48.)

$$\text{Solidity} = S = \frac{r}{8} \frac{1}{l} (D^2 + Dd + d^2 - \frac{3}{4} B^2)$$

Here D = greater depth from the vertex, whose slopes meet below the base, d = lesser depth, r = ratio of slopes, B = base, l = length of section, all in feet. The depths D and d are found by adding the perpendicular $P Q$ to the mean height $q R$ of section. (See fig. 48.)

$$\text{Because } \frac{b}{22} = P Q, \therefore \frac{30}{4} = 7.5 = P Q.$$

$$\text{Consequently } D = 50 + 7.5 = 57.5$$

$$d = 20 + 7.5 = 27.5$$

$$\begin{array}{rcl}
 D^2 & = 57.5 \times 57.5 & = 3306.25 \\
 d^2 & = 27.5 \times 27.5 & = 756.25 \\
 Dd & = 57.5 \times 27.5 & = 1581.25 \\
 & & \underline{5648.75} \\
 \frac{3 B^2}{4 r^2} & = \frac{3 \times 30 \times 20}{16} = \frac{2700}{16} & = 168.75 \\
 & & \underline{\hspace{1.5cm}} \\
 D^2 + Dd + d^2 - \frac{3 B^2}{4 r^2} & & = 5475 \\
 r l & = 2 \times 400 & \underline{\hspace{1.5cm}} 800 \\
 & & \underline{81)4380000}
 \end{array}$$

54074.07, the same as that found

above by the Prismoidal formula.

The bases or road beds are, in England, for single track 20, double track 30 feet wide.

And in the United States, in embankments, single track 16, for double track 28 feet. Also in excavation, single track 24, double track 32 feet.

In laying out the line, we endeavor to have the cutting and filling equal to one another, observing to allow 10 per cent for shrinkage; for it has been found that gravel and sand shrink 8 per cent, clay 10, loam 12, and surface soil 15. Where clay is put in water, it shrinks from 30 to 33 per cent.

Rock, broken in large fragments, increases 40 per cent.; if broken into small fragments, increases 60 per cent.

The following, Table a, is calculated from a modified form of Wm. Kelly's formula.

$$\text{Content in cubic yards} = L \left\{ B \cdot \frac{1}{2} \left(\frac{H+h}{27} \right) + \left(\frac{Hh}{27} + \frac{d \cdot d}{3 \cdot 27} \right) s \right\}$$

Here L = length, B = base, H and h = greater and lesser heights, r = ratio of slope, d = difference of heights.

Rule for using Table a. Multiply tabular number of half the height by the base, and call the result = A.

2. Multiply the tabular of either height by the other height, and call the result = B.

3. Multiply the tabular number of the difference of the heights by one-third of the difference, and call the result = C.

Add results B and C together, multiply the sum by the ratio of the slopes, add the product to the result A, and multiply the sum by the length, the product will be the content in cubic yards.

Example as in section 334. Where length = 400, base = 30, heights = 50 and 20, and ratio of slopes = 2.

$$\frac{50 + 20}{2} = 35, \text{ its tabular number, by } 30 = 1.2963 \times 30 = A = 38.889.$$

$$50 \times \text{tabular } 20 = 50 \times 7.7407 = 39.0350 = B.$$

$$10 \times \text{tabular } 30 = 10 \times 1.1111 = 11.1110 = C.$$

$$48.1960 \times 2 = \begin{array}{r} 96.292 \\ 135.181 \\ \hline 400 \end{array}$$

Length,

$$54072.505 \text{ yds.}$$

By Sir John MacNeil's Table XXIII =

$$54072$$

By his prismoidal formula =

$$54074.072$$

Here we find the difference between table a and the prismoidal formula to be 1 in 36049.

Sir John's tables are calculated only to feet and 2 decimals. William Kelly's (civil engineer, for many years connected with the Ordnance Survey of Ireland) to every three inches, and to three places of decimals. Table a is arranged similar to Mr. Kelly's Table I, but calculated to tenths of a foot, and to four places of decimals. Tables b and c are the same as MacNeil's Tables LVIII and LIX, with our explanation and example.

TABLE a.—For the Computation of Prismoids, for all Bases and Slopes.

Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.
1.0	0.0037	6.1	0.2259	12.1	0.4481	18.1	0.6704	24.1	0.8926	30.1	1.1148
2	.0074	2	.2296	2	.4518	2	.6741	2	.8963	2	.1185
3	.0111	3	.2333	3	.4555	3	.6778	3	.9000	3	.1222
4	.0148	4	.2370	4	.4592	4	.6815	4	.9037	4	.1259
5	.0185	5	.2407	5	.4629	5	.6852	5	.9074	5	.1296
6	.0222	6	.2444	6	.4666	6	.6889	6	.9111	6	.1333
7	.0259	7	.2481	7	.4703	7	.6926	7	.9148	7	.1370
8	.0296	8	.2518	8	.4740	8	.6963	8	.9185	8	.1407
9	.0333	9	.2555	9	.4777	9	.7000	9	.9222	9	.1444
1.0	.0370	7.0	.2591	13.0	.4814	19.0	.7037	25.0	.9259	31.0	.1481
1	.0407	1	.2628	1	.4851	1	.7074	1	.9296	1	.1518
2	.0444	2	.2765	2	.4888	2	.7111	2	.9333	2	.1555
3	.0481	3	.2802	3	.4925	3	.7148	3	.9370	3	.1592
4	.0518	4	.2839	4	.4962	4	.7185	4	.9407	4	.1629
5	.0555	5	.2778	5	.5000	5	.7222	5	.9444	5	.1666
6	.0592	6	.2815	6	.5037	6	.7259	6	.9481	6	.1703
7	.0629	7	.2852	7	.5074	7	.7296	7	.9518	7	.1740
8	.0666	8	.2889	8	.5111	8	.7333	8	.9555	8	.1777
9	.0703	9	.2926	9	.5148	9	.7370	9	.9592	9	.1814
2.0	.0741	8.0	.2963	14.0	.5185	20.0	.7407	26.0	.9629	32.0	.1851
1	.0778	1	.3000	1	.5222	1	.7444	1	.9666	1	.1888
2	.0815	2	.3037	2	.5259	2	.7481	2	.9703	2	.1925
3	.0852	3	.3074	3	.5296	3	.7518	3	.9740	3	.1962
4	.0889	4	.3111	4	.5333	4	.7555	4	.9777	4	.1999
5	.0926	5	.3148	5	.5370	5	.7592	5	.9815	5	.2037
6	.0963	6	.3185	6	.5407	6	.7629	6	.9852	6	.2074
7	.1000	7	.3222	7	.5444	7	.7666	7	.9889	7	.2111
8	.1037	8	.3259	8	.5481	8	.7703	8	.9926	8	.2148
9	.1074	9	.3296	9	.5518	9	.7740	9	.9963	9	.2185
3.0	.1111	9.0	.3333	15.0	.5555	21.0	.7778	27.0	1.0000	33.0	.2222
1	.1148	1	.3370	1	.5592	1	.7815	1	.0037	1	.2259
2	.1185	2	.3407	2	.5629	2	.7852	2	.0074	2	.2296
3	.1222	3	.3444	3	.5666	3	.7889	3	.0111	3	.2333
4	.1259	4	.3481	4	.5703	4	.7926	4	.0148	4	.2370
5	.1296	5	.3518	5	.5741	5	.7963	5	.0185	5	.2407
6	.1333	6	.3555	6	.5778	6	.8000	6	.0222	6	.2444
7	.1370	7	.3592	7	.5815	7	.8037	7	.0259	7	.2481
8	.1407	8	.3629	8	.5852	8	.8074	8	.0296	8	.2518
9	.1444	9	.3666	9	.5889	9	.8111	9	.0333	9	.2555
4.0	.1481	10.0	.3704	16.0	.5926	22.0	.8148	28.0	.0370	34.0	.2592
1	.1518	1	.3741	1	.5963	1	.8185	1	.0407	1	.2629
2	.1555	2	.3778	2	.6000	2	.8222	2	.0444	2	.2666
3	.1592	3	.3815	3	.6037	3	.8259	3	.0481	3	.2703
4	.1629	4	.3852	4	.6074	4	.8296	4	.0518	4	.2740
5	.1667	5	.3889	5	.6111	5	.8333	5	.0555	5	.2778
6	.1704	6	.3926	6	.6148	6	.8370	6	.0592	6	.2815
7	.1741	7	.3963	7	.6185	7	.8407	7	.0629	7	.2852
8	.1778	8	.4000	8	.6222	8	.8444	8	.0666	8	.2889
9	.1815	9	.4037	9	.6359	9	.8481	9	.0703	9	.2926
5.0	.1852	11.0	.4074	17.0	.6296	23.0	.8518	29.0	.0741	35.0	.2963
1	.1889	1	.4111	1	.6333	1	.8555	1	.0778	1	.3000
2	.1926	2	.4148	2	.6370	2	.8592	2	.0815	2	.3037
3	.1963	3	.4185	3	.6407	3	.8629	3	.0852	3	.3074
4	.2000	4	.4222	4	.6444	4	.8666	4	.0889	4	.3111
5	.2037	5	.4259	5	.6481	5	.8704	5	.0926	5	.3148
6	.2074	6	.4296	6	.6518	6	.8741	6	.0963	6	.3185
7	.2111	7	.4333	7	.6555	7	.8778	7	.1000	7	.3222
8	.2148	8	.4370	8	.6592	8	.8815	8	.1037	8	.3259
9	.2185	9	.4407	9	.6629	9	.8852	9	.1074	9	.3296
6.0	.2222	12.0	.0.4444	18.0	0.6667	24.0	0.8889	30.0	1.1111	36.0	1.3333

TABLE a.—For the Computation of Prismoids, for all Bases and Slopes.

Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.
36.1	1.3270	42.1	1.5509	48.1	1.7815	54.1	2.0037	60.1	2.2259	66.1	2.4481		
2	.3407	2	.5633	2	.7852	2	.0071	2	.2296	2	.4518		
3	.3444	3	.5667	3	.7889	3	.0114	3	.2333	3	.4555		
4	.3481	4	.5704	4	.7926	4	.0148	4	.2370	4	.4592		
5	.3518	5	.5741	5	.7963	5	.0185	5	.2407	5	.4629		
6	.3555	6	.5778	6	.8000	6	.0222	6	.2444	6	.4666		
7	.3592	7	.5815	7	.8037	7	.0259	7	.2481	7	.4703		
8	.3629	8	.5852	8	.8074	8	.0296	8	.2518	8	.4740		
9	.3666	9	.5889	9	.8111	9	.0333	9	.2555	9	.4777		
37.0	.3704	43.0	.5926	49.0	.8148	55.0	.0370	61.0	.2592	67.0	.4815		
1	.3741	1	.5963	1	.8185	1	.0407	1	.2629	1	.4852		
2	.3778	2	.6000	2	.8222	2	.0444	2	.2666	2	.4889		
3	.3815	3	.6037	3	.8259	3	.0481	3	.2703	3	.4926		
4	.3852	4	.6074	4	.8296	4	.0518	4	.2740	4	.4963		
5	.3889	5	.6111	5	.8333	5	.0556	5	.2778	5	.5000		
6	.3926	6	.6148	6	.8370	6	.0593	6	.2815	6	.5037		
7	.3963	7	.6185	7	.8407	7	.0630	7	.2852	7	.5074		
8	.4000	8	.6222	8	.8444	8	.0667	8	.2889	8	.5111		
9	.4037	9	.6259	9	.8481	9	.0704	9	.2925	9	.5148		
38.0	.4073	44.0	.6295	50.0	.8518	56.0	.0741	62.0	.2963	68.0	.5185		
1	.4110	1	.6332	1	.8555	1	.0778	1	.3000	1	.5222		
2	.4147	2	.6369	2	.8592	2	.0815	2	.3037	2	.5259		
3	.4184	3	.6406	3	.8629	3	.0852	3	.3074	3	.5296		
4	.4221	4	.6443	4	.8666	4	.0889	4	.3111	4	.5333		
5	.4259	5	.6481	5	.8704	5	.0926	5	.3148	5	.5370		
6	.4296	6	.6518	6	.8741	6	.0963	6	.3185	6	.5407		
7	.4333	7	.6555	7	.8778	7	.1000	7	.3222	7	.5444		
8	.4370	8	.6592	8	.8815	8	.1037	8	.3259	8	.5481		
9	.4407	9	.6629	9	.8852	9	.1074	9	.3296	9	.5518		
39.0	.4444	45.0	.6667	51.0	.8889	57.0	.1111	63.0	.3333	69.0	.5556		
1	.4481	1	.6704	1	.8926	1	.1148	1	.3370	1	.5593		
2	.4518	2	.6741	2	.8963	2	.1185	2	.3407	2	.5630		
3	.4555	3	.6778	3	.9000	3	.1222	3	.3444	3	.5667		
4	.4592	4	.6815	4	.9037	4	.1259	4	.3481	4	.5704		
5	.4629	5	.6852	5	.9074	5	.1296	5	.3518	5	.5741		
6	.4666	6	.6889	6	.9111	6	.1333	6	.3555	6	.5778		
7	.4703	7	.6926	7	.9148	7	.1370	7	.3592	7	.5815		
8	.4740	8	.6963	8	.9185	8	.1407	8	.3629	8	.5852		
9	.4777	9	.7000	9	.9222	9	.1444	9	.3666	9	.5889		
40.0	.1814	46.0	.7037	52.0	.9259	58.0	.1481	64.0	.3704	70.0	.5926		
1	.4851	1	.7074	1	.9296	1	.1518	1	.3741	1	.5963		
2	.4888	2	.7111	2	.9333	2	.1555	2	.3778	2	.6000		
3	.4925	3	.7148	3	.9370	3	.1592	3	.3815	3	.6037		
4	.4962	4	.7185	4	.9407	4	.1629	4	.3852	4	.6074		
5	.5000	5	.7222	5	.9444	5	.1667	5	.3889	5	.6111		
6	.5037	6	.7259	6	.9481	6	.1704	6	.3926	6	.6148		
7	.5074	7	.7296	7	.9518	7	.1741	7	.3963	7	.6185		
8	.5111	8	.7333	8	.9555	8	.1778	8	.4000	8	.6222		
9	.5148	9	.7370	9	.9592	9	.1815	9	.4037	9	.6259		
41.0	.5185	47.0	.7407	53.0	.9629	59.0	.1851	65.0	.4074	71.0	.6296		
1	.5222	1	.7444	1	.9666	1	.1888	1	.4111	1	.6333		
2	.5259	2	.7481	2	.9703	2	.1925	2	.4148	2	.6370		
3	.5296	3	.7518	3	.9740	3	.1962	3	.4185	3	.6407		
4	.5333	4	.7555	4	.9777	4	.1999	4	.4222	4	.6444		
5	.5370	5	.7592	5	.9814	5	.2037	5	.4259	5	.6481		
6	.5407	6	.7629	6	.9851	6	.2074	6	.4296	6	.6518		
7	.5444	7	.7666	7	.9888	7	.2111	7	.4333	7	.6555		
8	.5481	8	.7703	8	.9925	8	.2148	8	.4370	8	.6592		
9	.5518	9	.7740	9	.9962	9	.2185	9	.4407	9	.6629		
42.0	1.5555	48.0	1.7778	54.0	2.0000	60.0	2.2222	66.0	2.4444	72.0	2.6667		

TABLE b.—For the computation of Prismoids or Earthwork.

Ft	1	2	3	4	5	6	7	8	9	10	11	12	13	Ft
0	2	8	18	32	50	72	98	128	162	200	242	288	338	0
1	6	14	26	42	62	86	114	146	182	222	266	314	366	1
2	14	24	38	56	78	104	134	168	206	248	294	344	398	2
3	26	38	54	74	98	126	158	194	234	278	326	378	434	3
4	42	56	74	96	122	152	186	224	266	312	362	416	474	4
5	62	78	98	122	150	182	218	258	302	350	402	458	518	5
6	86	104	126	152	182	216	254	296	342	392	446	504	566	6
7	114	134	158	186	218	254	294	338	386	438	494	554	618	7
8	146	168	194	224	258	296	338	384	434	488	546	608	674	8
9	182	206	234	266	302	342	386	434	486	542	602	666	734	9
10	222	248	278	312	350	392	438	488	542	600	662	728	798	10
11	266	294	326	362	402	446	494	546	602	662	726	794	866	11
12	314	344	378	416	458	504	554	608	666	728	794	864	938	12
13	366	398	434	474	518	566	618	674	734	798	866	938	1014	13
14	422	456	494	536	582	632	686	744	806	872	942	1016	1094	14
15	482	518	558	602	650	702	758	818	882	950	1022	1098	1178	15
16	546	584	626	672	722	776	834	896	962	1032	1106	1184	1266	16
17	614	654	698	746	798	854	914	978	1046	1118	1194	1274	1358	17
18	686	728	774	824	878	936	998	1064	1134	1208	1286	1368	1454	18
19	762	806	854	906	962	1022	1086	1154	1226	1302	1382	1466	1554	19
20	842	888	938	992	1050	1112	1178	1248	1322	1400	1482	1568	1658	20
21	926	974	1026	1082	1142	1206	1274	1346	1422	1502	1586	1674	1766	21
22	1014	1064	1118	1176	1238	1304	1374	1448	1526	1608	1694	1784	1878	22
23	1106	1158	1214	1274	1338	1406	1478	1554	1634	1718	1806	1898	1994	23
24	1202	1256	1314	1376	1442	1512	1586	1664	1746	1832	1922	2016	2114	24
25	1302	1358	1418	1482	1550	1622	1698	1774	1862	1950	2042	2138	2238	25
26	1406	1464	1526	1592	1662	1736	1814	1896	1982	2072	2166	2264	2366	26
27	1514	1574	1638	1706	1778	1854	1934	2018	2106	2198	2294	2393	2498	27
28	1626	1688	1754	1824	1898	1976	2058	2144	2234	2328	2426	2528	2634	28
29	1742	1806	1874	1946	2022	2102	2186	2274	2366	2462	2562	2666	2774	29
30	1862	1928	1998	2072	2150	2232	2318	2408	2502	2600	2702	2808	2918	30
31	1986	2054	2126	2202	2282	2366	2454	2546	2642	2742	2846	2954	3066	31
32	2114	2184	2256	2336	2418	2504	2594	2688	2786	2888	2994	3104	3218	32
33	2246	2318	2394	2474	2558	2646	2738	2834	2934	3038	3146	3258	3374	33
34	2382	2456	2534	2616	2702	2792	2886	2984	3086	3192	3202	3416	3534	34
35	2522	2598	2678	2762	2850	2942	3038	3138	3242	3350	3462	3578	3698	35
36	2666	2744	2822	2912	3002	3096	3194	3296	3402	3512	3626	3744	3866	36
37	2814	2894	2978	3066	3158	3254	3354	3458	3566	3678	3794	3914	4038	37
38	2966	3048	3134	3224	3318	3416	3518	3624	3734	3848	3966	4088	4214	38
39	3122	3206	3294	3386	3482	3582	3686	3794	3906	4022	4142	4266	4394	39
40	3282	3368	3458	3552	3650	3752	3858	3968	4082	4200	4322	4448	4578	40
41	3446	3534	3626	3722	3822	3926	4034	4146	4262	4382	4506	4634	4766	41
42	3614	3704	3798	3896	3998	4104	4214	4328	4446	4568	4694	4824	4958	42
43	3786	3878	3974	4074	4178	4286	4398	4514	4634	4758	4886	5018	5154	43
44	3962	4056	4154	4256	4362	4472	4586	4704	4826	4952	5082	5216	5354	44
45	4142	4238	4338	4442	4550	4662	4778	4898	5022	5150	5282	5418	5558	45
46	4326	4424	4526	4632	4742	4856	4974	5096	5222	5352	5486	5624	5766	46
47	4514	4614	4718	4826	4938	5054	5174	5298	5426	5558	5694	5834	5978	47
48	4706	4808	4914	5024	5138	5256	5378	5504	5634	5768	5906	6048	6194	48
49	4902	5006	5114	5226	5342	5462	5586	5714	5846	5982	6122	6266	6414	49
50	5102	5208	5318	5432	5550	5672	5798	5928	6062	6200	6342	6488	6638	50
Ft	1	2	3	4	5	6	7	8	9	10	11	12	13	Ft

n 3

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TABLE b.—For the computation of Prismoids or Earthwork.

Ft	14	15	16	17	18	19	20	21	22	23	24	25	26	Ft
0	392	450	512	578	648	722	800	882	968	1058	1152	1250	1352	0
1	422	482	546	614	686	762	842	926	1014	1106	1202	1302	1406	1
2	456	518	584	654	728	806	888	974	1064	1158	1256	1358	1464	2
3	494	558	626	698	774	854	938	1026	1118	1214	1314	1418	1526	3
4	536	602	672	746	824	906	992	1082	1176	1274	1376	1482	1592	4
5	582	650	722	798	878	962	1050	1142	1238	1338	1442	1550	1662	5
6	632	702	776	854	936	1022	1112	1206	1304	1406	1512	1622	1736	6
7	686	758	834	914	998	1086	1178	1274	1374	1478	1586	1698	1814	7
8	744	818	896	978	1064	1154	1248	1346	1448	1554	1664	1778	1896	8
9	806	882	962	1046	1134	1226	1322	1422	1526	1634	1746	1862	1982	9
10	872	950	1032	1118	1208	1302	1400	1502	1608	1718	1832	1950	2072	10
11	942	1022	1106	1194	1286	1382	1482	1586	1694	1806	1922	2042	2166	11
12	1016	1098	1184	1274	1368	1466	1568	1674	1784	1898	2016	2138	2264	12
13	1094	1178	1266	1358	1454	1554	1658	1766	1878	1994	2114	2238	2366	13
14	1176	1262	1352	1446	1544	1646	1752	1862	1976	2094	2216	2342	2472	14
15	1262	1350	1442	1538	1638	1742	1850	1962	2078	2198	2322	2450	2582	15
16	1352	1442	1536	1634	1736	1842	1952	2066	2184	2306	2432	2562	2696	16
17	1446	1538	1634	1734	1838	1946	2058	2174	2294	2418	2546	2678	2814	17
18	1544	1638	1736	1838	1944	2054	2168	2286	2408	2534	2664	2798	2936	18
19	1646	1742	1842	1946	2054	2166	2282	2402	2526	2654	2786	2922	3062	19
20	1752	1850	1952	2058	2168	2282	2400	2522	2648	2778	2912	3050	3192	20
21	1862	1962	2066	2174	2286	2402	2522	2646	2774	2906	3042	3182	3326	21
22	1976	2078	2184	2294	2408	2526	2648	2774	2904	3038	3176	3318	3464	22
23	2094	2198	2306	2418	2534	2654	2778	2906	3038	3174	3314	3458	3606	23
24	2216	2322	2432	2546	2664	2786	2912	3042	3176	3314	3456	3602	3752	24
25	2342	2450	2562	2678	2798	2922	3050	3182	3318	3458	3602	3750	3902	25
26	2472	2582	2696	2814	2936	3062	3192	3326	3464	3606	3752	3902	4056	26
27	2606	2718	2834	2954	3078	3206	3338	3474	3614	3758	3906	4058	4214	27
28	2744	2858	2976	3098	3224	3354	3488	3626	3768	3914	4064	4218	4376	28
29	2886	3002	3122	3246	3374	3506	3642	3782	3926	4074	4226	4382	4542	29
30	3032	3150	3272	3398	3528	3662	3800	3942	4088	4238	4392	4550	4712	30
31	3182	3302	3426	3554	3686	3822	3962	4106	4254	4406	4562	4722	4886	31
32	3336	3458	3584	3714	3848	3986	4128	4274	4424	4578	4736	4898	5064	32
33	3494	3618	3746	3878	4014	4157	4298	4446	4598	4754	4914	5078	5246	33
34	3656	3782	3912	4046	4184	4326	4472	4622	4776	4934	5096	5262	5432	34
35	3822	3950	4082	4218	4358	4502	4650	4802	4958	5118	5282	5450	5622	35
36	3992	4122	4256	4394	4536	4682	4832	4986	5144	5306	5472	5642	5816	36
37	4166	4298	4434	4574	4718	4866	5018	5174	5334	5498	5666	5838	6014	37
38	4344	4478	4616	4758	4904	5054	5208	5366	5528	5698	5864	6038	6216	38
39	4526	4662	4802	4946	5094	5246	5402	5562	5726	5894	6066	6242	6422	39
40	4712	4850	4992	5138	5288	5442	5600	5762	5928	6098	6272	6450	6632	40
41	4902	5042	5186	5334	5486	5642	5802	5966	6134	6306	6482	6662	6846	41
42	5096	5238	5384	5534	5688	5846	6008	6174	6344	6518	6696	6878	7064	42
43	5294	5438	5586	5738	5894	6054	6218	6386	6558	6734	6914	7098	7286	43
44	5496	5642	5792	5946	6104	6266	6432	6602	6776	6954	7136	7322	7512	44
45	5702	5850	6002	6158	6318	6482	6650	6822	6998	7178	7362	7550	7742	45
46	5912	6062	6216	6374	6536	6702	6872	7046	7224	7406	7592	7782	7976	46
47	6126	6278	6434	6594	6758	6926	7098	7274	7454	7638	7826	8018	8214	47
48	6344	6498	6656	6818	6984	7154	7328	7506	7688	7874	8064	8258	8456	48
49	6566	6722	6882	7046	7214	7386	7562	7742	7926	8114	8306	8502	8702	49
50	6792	6950	7112	7278	7448	7622	7800	7982	8168	8358	8552	8750	8952	50
Ft	14	15	16	17	18	19	20	21	22	23	24	25	26	Ft

TABLE b.—For the computation of Prismoids or Earthwork.

Ft	27	28	29	30	31	32	33	34	35	36	37	38	Ft
0	1458	1568	1682	1800	1922	2048	3178	2312	2450	2592	2738	2888	0
1	1514	1626	1742	1862	1986	2114	2246	2382	2522	2666	2814	2966	1
2	1574	1688	1806	1928	2054	2184	2318	2456	2598	2744	2894	3048	2
3	1638	1754	1874	1998	2126	2258	2394	2534	2678	2826	2978	3134	3
4	1706	1824	1946	2072	2202	2336	2474	2616	2762	2912	3066	3224	4
5	1778	1898	2022	2150	2282	2418	2558	2702	2850	3002	3158	3318	5
6	1854	1976	2102	2232	2366	2504	2646	2792	2942	3096	3254	3416	6
7	1934	2058	2186	2318	2454	2594	2738	2886	3038	3194	3354	3518	7
8	2018	2144	2274	2408	2546	2688	2834	2984	3138	3296	3458	3624	8
9	2106	2234	2366	2502	2642	2786	2934	3086	3242	3402	3566	3734	9
10	2198	2328	2462	2600	2742	2888	3038	3192	3350	3512	3678	3848	10
11	2294	2426	2562	2702	2846	2994	3146	3302	3462	3626	3794	3966	11
12	2394	2528	2666	2808	2954	3104	3258	3416	3578	3744	3914	4088	12
13	2498	2634	2774	2918	3066	3218	3374	3534	3698	3866	4038	4214	13
14	2606	2744	2886	3032	3182	3336	3494	3656	3822	3992	4166	4344	14
15	2718	2858	3002	3150	3302	3458	3618	3782	3950	4122	4298	4478	15
16	2834	2976	3122	3272	3426	3584	3746	3912	4082	4256	4434	4616	16
17	2954	3098	3246	3398	3554	3714	3878	4046	4218	4392	4574	4758	17
18	3078	3224	3374	3528	3686	3848	4014	4184	4358	4536	4718	4904	18
19	3206	3354	3506	3662	3822	3986	4154	4326	4502	4682	4866	5054	19
20	3338	3488	3642	3800	3962	4128	4298	4472	4650	4832	5018	5208	20
21	3474	3626	3782	3942	4106	4274	4446	4622	4802	4986	5174	5366	21
22	3614	3768	3926	4088	4254	4424	4598	4776	4958	5144	5334	5528	22
23	3758	3914	4074	4238	4406	4578	4754	4934	5118	5306	5498	5694	23
24	3906	4064	4226	4392	4562	4736	4914	5096	5282	5472	5666	5864	24
25	4058	4218	4382	4550	4722	4898	5078	5262	5450	5642	5838	6038	25
26	4214	4376	4542	4712	4886	5064	5246	5432	5622	5816	6014	6216	26
27	4374	4538	4706	4878	5054	5234	5418	5606	5798	5994	6194	6398	27
28	4538	4704	4874	5048	5226	5408	5594	5784	5978	6176	6378	6584	28
29	4706	4874	5046	5222	5402	5586	5774	5966	6162	6362	6566	6774	29
30	4878	5048	5222	5400	5582	5768	5958	6152	6350	6552	6758	6968	30
31	5054	5226	5402	5582	5766	5954	6146	6342	6542	6746	6954	7166	31
32	5234	5408	5586	5768	5954	6144	6338	6536	6738	6944	7154	7368	32
33	5418	5594	5774	5958	6146	6338	6534	6734	6938	7146	7358	7574	33
34	5606	5784	5966	6152	6342	6536	6734	6936	7142	7352	7566	7784	34
35	5798	5978	6162	6350	6542	6738	6938	7142	7350	7562	7778	7998	35
36	5994	6176	6362	6552	6746	6944	7146	7354	7562	7776	7994	8216	36
37	6194	6378	6566	6758	6954	7154	7358	7566	7778	7994	8214	8438	37
38	6398	6584	6774	6968	7166	7368	7574	7784	7998	8216	8438	8664	38
39	6606	6794	6986	7182	7382	7586	7794	8006	8222	8442	8666	8894	39
40	6818	7008	7202	7400	7602	7808	8018	8232	8450	8672	8898	9128	40
41	7034	7226	7422	7622	7826	8034	8246	8462	8682	8906	9134	9366	41
42	7254	7448	7646	7848	8054	8264	8478	8696	8918	9144	9374	9608	42
43	7478	7674	7874	8078	8286	8498	8714	8934	9158	9386	9618	9854	43
44	7706	7904	8106	8312	8522	8736	8954	9176	9402	9632	9866	10104	44
45	7938	8138	8342	8550	8762	8978	9198	9422	9650	9882	10118	10358	45
46	8174	8376	8582	8792	9006	9224	9446	9672	9902	10136	10374	10616	46
47	8414	8618	8826	9038	9254	9474	9698	9926	10158	10394	10634	10878	47
48	8658	8866	9074	9288	9506	9728	9954	10184	10418	10656	10898	11144	48
49	8906	9114	9326	9542	9762	9986	10214	10446	10682	10922	11166	11414	49
50	9158	9368	9582	9800	10022	10248	10478	10712	10950	11192	11438	11688	50
Ft	27	28	29	30	31	32	33	34	35	36	37	38	Ft

TABLE b.—For the computation of Prismoids or Earthwork.

Ft	39	40	41	42	43	44	45	46	47	48	Ft
0	3042	3200	3362	3528	3698	3872	4050	4232	4418	4608	0
1	3122	3282	3446	3614	3786	3962	4142	4326	4514	4706	1
2	3206	3368	3534	3704	3878	4056	4238	4424	4614	4808	2
3	3294	3458	3626	3798	3974	4154	4338	4526	4718	4914	3
4	3386	3552	3722	3896	4074	4256	4442	4632	4826	5024	4
5	3482	3650	3822	3998	4178	4362	4550	4742	4938	5138	5
6	3582	3752	3926	4104	4286	4472	4662	4856	5054	5256	6
7	3686	3858	4034	4214	4398	4586	4778	4974	5174	5378	7
8	3794	3968	4146	4328	4514	4704	4898	5096	5298	5504	8
9	3906	4082	4262	4446	4634	4826	5022	5222	5426	5634	9
10	4022	4200	4382	4568	4758	4952	5150	5352	5558	5768	10
11	4142	4322	4506	4694	4886	5082	5282	5486	5694	5906	11
12	4266	4448	4634	4824	5018	5216	5418	5624	5824	6048	12
13	4394	4578	4766	4958	5154	5354	5558	5766	5978	6194	13
14	4526	4712	4902	5096	5294	5496	5702	5912	6126	6344	14
15	4662	4850	5042	5238	5438	5642	5850	6062	6278	6498	15
16	4802	4992	5186	5384	5586	5792	6002	6216	6434	6656	16
17	4946	5138	5334	5534	5738	5946	6158	6374	6594	6818	17
18	5094	5288	5486	5688	5894	6104	6318	6536	6758	6984	18
19	5246	5442	5642	5846	6054	6266	6482	6702	6926	7154	19
20	5402	5600	5802	6008	6218	6432	6650	6872	7098	7328	20
21	5562	5762	5966	6174	6386	6602	6822	7046	7274	7506	21
22	5726	5928	6134	6344	6558	6776	6998	7224	7454	7688	22
23	5894	6098	6306	6518	6734	6954	7178	7406	7638	7874	23
24	6066	6272	6482	6696	6914	7136	7362	7592	7826	8064	24
25	6242	6450	6662	6878	7098	7322	7550	7782	8018	8258	25
26	6422	6632	6846	7064	7286	7512	7742	7976	8214	8456	26
27	6606	6818	7034	7254	7478	7706	7938	8174	8414	8658	27
28	6794	7008	7226	7448	7674	7904	8138	8376	8618	8864	28
29	6986	7202	7422	7646	7874	8106	8342	8582	8826	9074	29
30	7182	7400	7622	7848	8078	8312	8550	8792	9038	9288	30
31	7382	7602	7826	8054	8286	8522	8762	9006	9254	9506	31
32	7586	7808	8034	8264	8498	8736	8978	9224	9474	9728	32
33	7794	8018	8246	8478	8714	8954	9198	9446	9698	9954	33
34	8006	8232	8462	8696	8934	9176	9422	9672	9926	10184	34
35	8222	8450	8682	8918	9158	9402	9650	9902	10158	10418	35
36	8442	8672	8906	9144	9386	9632	9882	10136	10394	10656	36
37	8666	8898	9134	9374	9618	9866	10118	10374	10634	10898	37
38	8894	9128	9366	9608	9854	10104	10358	10616	10878	11144	38
39	9126	9362	9602	9846	10094	10346	10602	10862	11126	11394	39
40	9362	9600	9842	10088	10338	10592	10850	11112	11378	11648	40
41	9602	9842	10086	10334	10586	10842	11102	11366	11634	11906	41
42	9846	10088	10334	10584	10838	11096	11358	11624	11884	12148	42
43	10094	10338	10586	10838	11094	11254	11618	11886	12158	12434	43
44	10346	10592	10842	11096	11354	11616	11882	12152	12426	12704	44
45	10602	10850	11102	11358	11618	11882	12150	12422	12698	12978	45
46	10862	11112	11366	11624	11886	12152	12422	12696	12974	13256	46
47	11126	11378	11634	11894	12158	12426	12698	12974	13254	13538	47
48	11394	11648	11906	12168	12434	12704	12978	13256	13538	13824	48
49	11666	11922	12182	12446	12714	12986	13262	13542	13826	14114	49
50	11942	12200	12462	12728	12998	13272	13555	13832	14118	14408	50
Ft	39	40	41	42	43	44	45	46	47	48	Ft

TABLE c.—For calculating Prismoids.

Ft	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Ft.
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	0
1	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	1
2	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	2
3	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	3
4	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	4
5	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	5
6	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	6
7	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	7
8	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	8
9	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	9
10	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	10
11	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	11
12	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	12
13	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	13
14	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	14
15	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	15
16	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	16
17	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	17
18	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	18
19	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	19
20	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	20
21	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	21
22	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	22
23	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	23
24	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	24
25	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	25
26	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	26
27	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	27
28	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	28
29	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	29
30	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	30
31	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	31
32	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	32
33	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	33
34	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	34
35	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	35
36	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	36
37	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	37
38	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	38
39	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	39
40	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	40
41	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	41
42	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	42
43	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	43
44	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	44
45	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	45
46	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	46
47	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	47
48	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	48
49	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	49
50	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	50
Ft.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Ft.

TABLE c.—For calculating *Prismoids*.

Ft	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	Ft.
0	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	0
1	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	1
2	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	2
3	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	3
4	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	4
5	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	5
6	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	6
7	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	7
8	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	8
9	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	9
10	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	10
11	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	11
12	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	12
13	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	13
14	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	14
15	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	15
16	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	16
17	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	17
18	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	18
19	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	19
20	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	20
21	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	21
22	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	22
23	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	23
24	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	24
25	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	25
26	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	26
27	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	27
28	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	28
29	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	29
30	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	30
31	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	31
32	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	32
33	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	33
34	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	34
35	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	35
36	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	36
37	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	37
38	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	38
39	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	39
40	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	40
41	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	41
42	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	42
43	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	43
44	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	44
45	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	45
46	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	46
47	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	47
48	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	48
49	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	49
50	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	50
Ft.	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	Ft.

TABLE C.—For calculating Prismoids.

Ft.	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	Ft.
0	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	0
1	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	1
2	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	2
3	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	3
4	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	4
5	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	5
6	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	6
7	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	7
8	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	8
9	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	9
10	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	10
11	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	11
12	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	12
13	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	13
14	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	14
15	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	15
16	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	16
17	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	17
18	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	18
19	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	19
20	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	20
21	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	21
22	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	22
23	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	23
24	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	24
25	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	25
26	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	26
27	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	27
28	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	28
29	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	29
30	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	30
31	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	31
32	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	32
33	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	33
34	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	34
35	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	255	35
36	213	216	219	222	225	228	231	234	237	240	243	246	249	252	255	258	36
37	216	219	222	225	228	231	234	237	240	243	246	249	252	255	258	261	37
38	219	222	225	228	231	234	237	240	243	246	249	252	255	258	261	264	38
39	222	225	228	231	234	237	240	243	246	249	252	255	258	261	264	267	39
40	225	228	231	234	237	240	243	246	249	252	255	258	261	264	267	270	40
41	228	231	234	237	240	243	246	249	252	255	258	261	264	267	270	273	41
42	231	234	237	240	243	246	249	252	255	258	261	264	267	270	273	276	42
43	234	237	240	243	246	249	252	255	258	261	264	267	270	273	276	279	43
44	237	240	243	246	249	252	255	258	261	264	267	270	273	276	279	282	44
45	240	243	246	249	252	255	258	261	264	267	270	273	276	279	282	285	45
46	243	246	249	252	255	258	261	264	267	270	273	276	279	282	285	288	46
47	246	249	252	255	258	261	264	267	270	273	276	279	282	285	288	291	47
48	249	252	255	258	261	264	267	270	273	276	279	282	285	288	291	294	48
49	252	255	258	261	264	267	270	273	276	279	282	285	288	291	294	297	49
50	255	258	261	264	267	270	273	276	279	282	285	288	291	294	297	300	50
Ft.	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	Ft.

SPHERICAL TRIGONOMETRY.

345. A *Spherical Triangle* is formed by the intersection of three great circles on the surface of a sphere, the planes of each circle passing through the centre of the sphere.

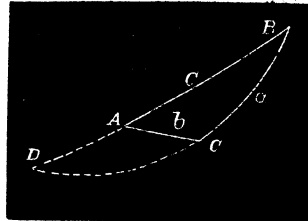
346. A *Spherical Angle* is that formed by the intersection of the planes of the great circles, and is the measure of the angles formed by the great circles.

347. The sides and angles of a spherical triangle have no affinity to those of a plane triangle, for in a spherical triangle, the sides and angles are of the same species, each being measured on the arc of a great circle.

348. As in plane trigonometry, we have isocetes equilateral oblique-angled and right-angled triangles.

349. A right-angled triangle is formed by the intersection of three great circles, two of which intersect one another at right angles, that is one great circle must pass through the centre of the sphere and the pole of another of the three circles.

Let the side of the triangle be produced to meet as at D in the annexed figure, the arc B A D and B C D are semi-circles, therefore, the side A D is the supplement of A B, and C D is the supplement of B C and the $\triangle A D C$ is the supplementary or *polar* triangle to ABC.



350. Any two sides of a $\triangle$ is greater than the third. Any side is less than the sum of the other two sides, but greater than their difference.

351. If tangents be drawn from the point B to the arcs B A and B C the angle thus formed will be the measure of the spherical angle A B C.

352. The greater angle is subtended by the greater side.

A right-angled $\triangle$ has one angle of 90° .

A quadrantal $\triangle$ has one side of 90° .

An oblique-angled $\triangle$ has no side or angle $= 90^\circ$.

The three sides of a spherical $\triangle$ are together less than 360° .

The three angles are together greater than two, and less than six right-angles.

353. The angles of one triangle if taken from 180° will give the sides of a new supplementary or polar triangle.

If the sides of a $\triangle$ be taken from 180° , it gives the angles of a polar $\triangle$.

354. If the sum of any two sides be either equal, greater or less than 180° , the sum of the opposite angles will be equal, greater or less than 180° .

355. A right-angled spherical $\triangle$ may have either,

One right angle and two acute angles.

One right angle and two obtuse angles.

One obtuse angle and two right angles.

One acute angle and two right angles.

Three right angles.

P

356. If one of the sides of the $\triangle$ be 90° , one of the other sides will be 90° , and then each side will be equal to its opposite $<$. And if any two of its sides are each $=$ to 90° , then the third side is $=$ to 90° .

357. If two of the angles are each 90° , the opposite sides are each equal to 90° .

358. If the two legs of a right-angled $\triangle$ be both acute or both obtuse, the hypotenuse will be less than a quadrant. If one be acute and the other obtuse, that is when they are of different species, the hypotenuse is greater than a quadrant.

359. In any right angled spherical $\triangle$ each of the oblique angles is of the same species as its opposite side, and the sides containing the right angle are of the same species as their opposite angles.

360. If the hypotenuse be less than 90° , the legs are of the same species as their adjacent angles, but if the hypotenuse be greater, then the legs and adjacent angles are of different species.

361. In any spherical $\triangle$ the sines of the angles are to one another as the sines of their opposite sides.

362. SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES.

$$\text{Sin. } a = \text{sin. } c \cdot \text{sin. } A, \text{ Equat. A.}$$

$$\text{tan. } a = \text{tan. } c \cdot \text{cos. } B$$

$$= \text{tan. } A \cdot \text{sin. } B, \text{ Equation B.}$$

$$\text{Sin. } b = \text{sin. } c \cdot \text{sin. } B = \frac{\text{tan. } a,}{\text{tan. } A.}$$

$$\text{Equation C.}$$

$$\text{tan. } b = \text{tan. } b \cdot \text{cos. } A$$

$$= \text{tan. } B \cdot \text{sin. } A, \text{ Equation D.}$$

$$\text{Cos. } A = \text{cos. } a \cdot \text{sin. } B,$$

$$\text{Equation E.}$$

$$\text{Cos. } B = \text{cos. } b \cdot \text{sin. } A,$$

$$\text{Equation F.}$$

$$\text{Sin. } B = \frac{\text{cos. } A.}{\text{cos. } a.}$$

$$\text{Equation G.}$$

$$\text{Cos. } c = \text{cos. } a \cdot \text{cos. } b,$$

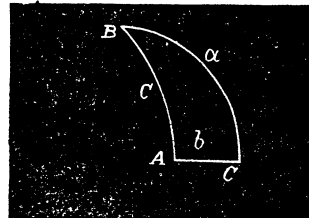
$$\text{Equation H.}$$

$$\text{Cos. } c = \text{cot. } A \cdot \text{cot. } B,$$

$$\text{Equation I.}$$

$$\text{Sin } c = \frac{\text{sin. } a.}{\text{sin. } A.} \cdot \text{Here } c = \text{hypotenuse.}$$

$$\text{Equation K.}$$



363. NAPIER'S RULES FOR THE CIRCULAR PARTS.

Lord Napier has given the following simple rules for solving right-angled spherical triangles.

The sine of the middle part = product of the adjacent parts.

The sine of the middle part = product of the cosines of the opposite parts.

In applying Napier's analogies, we take the complements of the hypotenuse and of the other angles, and reject the right angle. We will arrange Napier's rules as follows, where co. = complement of the angles or hypotenuse.

Sine of the middle part.	Is equal to the product of the tangents of the adjacent parts.	Is equal to the product of the cosines of the opposite parts.
Sine comp. A.	tan. co. c. tan. b.	Cos. co. B. . cos. a.
Sin. comp. c.	tan. co. A. . tan. co. B.	Cos. b. . cos. a.
Sin. comp. B.	tan. comp. c. . tan. a.	Cos. b. . cos. A.
Sin. a.	tan. comp. B. . tan. b.	Cos. comp. A. . cos. com. c.
Sin. b.	tan. co. A. . tan. a.	Cos. com. c. . cos. com. B.

It is easy to remember that *adjacent* requires *tangent*, and *opposite* requires *cosine*, from the letter *a* being found in the first syllable of *adjacent* and *tangent*, and *o* being in the first syllable of *opposite* and *cosine*.

Example 1. Given the $\angle A \times 23^\circ 28'$ and $c = 145^\circ$ to find the sides a and b , and the angle B .

Comp. $c = \text{comp. } 180 - 145 = 35$ and $55^\circ = \text{comp.}$

Comp. $A = 90^\circ - 23^\circ 28' = 66^\circ 32'$.

Sin. $a = \cos. 55^\circ \times \cos. 66^\circ 32' = 0.57358 \times 0.39822$ and
 $a = 13^\circ 12' 13'' = \text{natural sine of } 0.22841$.

Having a and comp. of c , we find $B = 50^\circ 81'$ and $b = 24^\circ 24'$.

Example 2. Given $b = 46^\circ 18' 23''$. $A = 34^\circ 27' 29''$ to find $\angle B$.
 Answer, $B = 66^\circ 59' 25''$.

Example 3. Given $a = 48^\circ 24' 16''$, and $b = 59^\circ 38' 27''$. We find
 $c = 79^\circ 23' 42''$.

Example 4. Given $a = 116^\circ 30' 43''$ and $b = 29^\circ 41' 32''$. We find
 $A = 103^\circ 52' 48''$

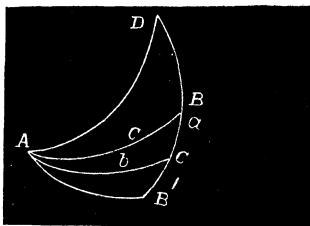
Example 5. Given $b = 29^\circ 12' 50''$, and $\angle B = 37^\circ 26' 21''$. We
 find $a = 46^\circ 55' 2''$ or $a = 133^\circ 4' 58''$.

We can use either natural or logarithmic numbers.

QUADRANTAL SPHERICAL TRIANGLES.

Let $A D = 90^\circ$, produce $D B$ to C making $D C = A D = 90^\circ$; therefore the arc $A C$ is the measure of the angle $A D B$.

If the $\angle D A B$ is less than 90° , then $D B$ is less than 90° . But if the $\angle D A B$ is greater than 90° , then the side $D B$ is greater than 90° .



Example. Let the $\angle D = 42^\circ 12' = \text{Arc } A C$ in the triangle $A B C$, and let the $\angle D A B = 54^\circ 43'$, then $90^\circ - 54^\circ 13' = 35^\circ 17' = \angle B A C = \angle A$ in the $\triangle B A C$.

By Napier's analogies, sin. comp. $A \times \text{radius} = \tan. b \times \tan. \text{comp. } c$.
 i. e., rad. cos. $A = \tan. b \cdot \cot. c$, and $\cot. c = \frac{\text{Rad. cos. } A}{\tan. b} =$

$\frac{\text{Rad. cos. } 54^\circ 43'}{\tan. 42^\circ 12'} = 48^\circ 0' 9'' = c$. And Sin. comp. $B = \cos. B =$
 $\cos. b \cdot \cos. A = \cos. b \cdot \sin. A$, and having b and A in the above, we
 have $\cos. B = \cos. 42^\circ 12' \times \sin. 48^\circ 0' 9'' = 64^\circ 39' 55'' = B$.

Again, sin. comp. $B = \tan. a \cdot \tan. \text{comp. } c$ i. e. $\cos. B = \tan. a \cdot \cot. c$.

Tan. $a = \frac{\cos. B}{\cot. c} = \frac{\cos. 64^\circ 39' 55''}{\cot. 48^\circ 0' 9''} = 25^\circ 25' 20'' = \text{value of } a$.

$\therefore 90^\circ - 25^\circ 25' 20'' = 64^\circ 34' 40'' = \text{side } D B$.—*Young's Trigonometry.*

365.

OBLIQUE-ANGLED SPHERICAL TRIANGLES.

Oblique-angled triangles are divided into six cases by Thomson and other mathematicians.

- I. When the three sides are given, to find the angles.
- II. When the three angles are given, to find the sides.
- III. When the two sides and their contained angle are given.
- IV. When one side and the adjacent angles are given.
- V. When two angles and a side opposite to one of them.
- VI. When two sides and an angle opposite to one of them.

The following formulas may be solved by logarithms or natural numbers.

366. The following is the fundamental formula, and is applicable to all spherical triangles. Puissant in his *Geodesie*, vol. I, p. 58, says: "Il serait aise de prouver que l'equation est le fondement unique de toute la Trigonometrie spherique."

$$\cos. a = \cos. b \cdot \cos. c + \sin. b \cdot \sin. c \cdot \cos. A.$$

$$\cos. b = \cos. a \cdot \cos. c + \sin. a \cdot \sin. c \cdot \cos. B.$$

$$\cos. c = \cos. a \cdot \cos. b + \sin. a \cdot \sin. b \cdot \cos. C.$$

From these we can find the following equations:

$$\cos. A = \frac{\cos. a - \cos. b \cdot \cos. c}{\sin. b \cdot \sin. c} \quad \text{Equation A.}$$

$$\cos. B = \frac{\cos. b - \cos. a \cdot \cos. c}{\sin. a \cdot \sin. c} \quad \text{Equation B.}$$

$$\cos. C = \frac{\cos. c - \cos. a \cdot \cos. b}{\sin. a \cdot \sin. b} \quad \text{Equation C.}$$

If we have a, b and A given, then side a : sine of $< A$:: side b to the sine of $< B$.

The following formulas are applicable to natural numbers and logarithms. The symbol $\frac{1}{2}$ = square root.

367. CASE I. Having the three sides given, let s = half the sum of the sides.

$$\text{Sine } \frac{1}{2} A = \left(\frac{\sin. (s-b) \sin. (s-c)}{\sin. b \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation A.}$$

$$\text{Sin. } \frac{1}{2} B = \left(\frac{\sin. (s-a) \sin. (s-c)}{\sin. a \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation B.}$$

$$\text{Sine } \frac{1}{2} C = \left(\frac{\sin. (s-a) \sin. (s-b)}{\sin. a \cdot \sin. b} \right)^{\frac{1}{2}} \quad \text{Equation C.}$$

$$\cos. \frac{1}{2} A = \left(\frac{\sin. s \cdot \sin. (s-a)}{\sin. b \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation D.}$$

$$\cos. \frac{1}{2} B = \left(\frac{\sin. s \cdot \sin. (s-b)}{\sin. a \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation E.}$$

$$\cos. \frac{1}{2} C = \left(\frac{\sin. s \cdot \sin. (s-c)}{\sin. a \cdot \sin. b} \right)^{\frac{1}{2}} \quad \text{Equation F.}$$

$$\text{Tan. } \frac{1}{2} A = \left(\frac{\sin. (s-b) \cdot \sin. (s-c)}{\sin. s \cdot \sin. (s-a)} \right)^{\frac{1}{2}} \quad \text{Equation G.}$$

$$\text{Tan. } \frac{1}{2} B = \left(\frac{\sin. (s-a) \cdot \sin. (s-c)}{\sin. s \cdot \sin. (s-b)} \right)^{\frac{1}{2}} \quad \text{Equation H.}$$

$$\text{Tan. } \frac{1}{2} C = \left(\frac{\sin. (s-a) \cdot \sin. (s-b)}{\sin. s \cdot \sin. (s-c)} \right)^{\frac{1}{2}} \quad \text{Equation I.}$$

368. CASE II. Having the three angles given, to find the sides.

$$\text{Sine } \frac{1}{2} a = \left(\frac{-\cos. S \cdot \cos. (S - A)}{\sin. B \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation A.}$$

$$\text{Sine } \frac{1}{2} b = \left(\frac{-\cos. S \cdot \cos. (S - B)}{\sin. A \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation B.}$$

$$\text{Sine } \frac{1}{2} c = \left(\frac{-\cos. S \cdot \cos. (S - C)}{\sin. A \cdot \sin. B} \right)^{\frac{1}{2}} \quad \text{Equation C.}$$

$$\text{Cos. } \frac{1}{2} a = \left(\frac{\cos. (S - B) \cdot \cos. (S - C)}{\sin. B \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation D.}$$

$$\text{Cos. } \frac{1}{2} b = \left(\frac{\cos. (S - A) \cdot \cos. (S - C)}{\sin. A \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation E.}$$

$$\text{Cos. } \frac{1}{2} c = \left(\frac{\cos. (S - A) \cdot \cos. (S - B)}{\sin. A \cdot \sin. B} \right)^{\frac{1}{2}} \quad \text{Equation F.}$$

$$\text{Tan. } \frac{1}{2} a = \left(\frac{-\cos. S \cos. (S - A)}{\cos. (S - B) \cos. (S - C)} \right)^{\frac{1}{2}} \quad \text{Equation G.}$$

$$\text{Tan. } \frac{1}{2} b = \left(\frac{-\cos. S \cos. (S - B)}{\cos. (S - A) \cos. (S - C)} \right)^{\frac{1}{2}} \quad \text{Equation H.}$$

$$\text{Tan. } \frac{1}{2} c = \left(\frac{-\cos. S \cos. (S - C)}{\cos. (S - A) \cos. (S - B)} \right)^{\frac{1}{2}} \quad \text{Equation I.}$$

369. CASE III. When two sides and the angle contained by them are given to find the remaining parts.

Let us suppose the two sides a and b and the contained $\angle = C$.

By Napier's analogies,

$$\text{Cos. } \frac{1}{2} (a + b) : \cos. \frac{1}{2} (a \infty b) :: \cot. \frac{1}{2} C : \tan. \frac{1}{2} (A + B) \quad \text{Equat. J.}$$

$$\text{Sin. } \frac{1}{2} (a + b) : \sin. \frac{1}{2} (a \infty b) :: \cot. \frac{1}{2} C : \tan. \frac{1}{2} (A \infty B) \quad \text{Equat. K.}$$

Tan. of half the sum of the unknown angles =

$$\frac{\cos. \frac{1}{2} (a \infty b) \cdot \cot. \frac{1}{2} C}{\cos. \frac{1}{2} (a + b)} \quad \text{Equation L.}$$

$$\text{Tan. of half the difference of same} = \frac{\sin. \frac{1}{2} (a \infty b) \cdot \cot. \frac{1}{2} C}{\sin. \frac{1}{2} (a + b)} \quad \text{Equation M.}$$

∞ signifies the difference between a and b .

Having determined half the sum and half the difference of the angles, we find the angles A and B .

Then the side c may be found from (Equation F.)

$\sin. B : \sin. b :: \sin. C : \sin. c$, from which c is found.

370. Napier's analogies for finding the side from the angle.

$$\cos. (A + B) : \cos. (A \infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a + b) \quad \text{Equation N.}$$

$$\text{or } \sin. (A + B) : \sin. (A \infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a - b) \quad \text{Equation O.}$$

$$\text{or } \tan. \frac{1}{2} c = \frac{\cos. (A + B) \cdot \tan. \frac{1}{2} (a + b)}{\cos. (A \infty B)} \quad \text{Equation P.}$$

$$\text{or } \tan. \frac{1}{2} c = \frac{\sin. (A + B) \cdot \tan. \frac{1}{2} (a - b)}{(\sin. A \infty B)} \quad \text{Equation Q.}$$

The value may be found from the general equation.

371. CASE IV. When one side and the adjacent angles are given.

Given A and B and the adjacent side c.

$$\cos. \frac{1}{2} (A + B) : \cos. (A \infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a + b)$$

$$\sin. \frac{1}{2} (A + B) : \sin. A (\infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a - b)$$

From these we have the sides a and b.

$$\tan. \frac{1}{2} (a + b) = \frac{\cos. (A \infty B) \cdot \tan. \frac{1}{2} c}{\cos. \frac{1}{2} (A + B)} \quad \text{Equation R.}$$

$$\tan. \frac{1}{2} (a - b) = \frac{\sin. \frac{1}{2} (A \infty B) \cdot \tan. \frac{1}{2} c}{\sin. \frac{1}{2} (A + B)} \quad \text{Equation S.}$$

And to find $\angle C$, we have

$$\cot. \frac{1}{2} C = \frac{\cos. \frac{1}{2} (a + b) \cdot \tan. \frac{1}{2} (A + B)}{\cos. \frac{1}{2} (a \infty b)} \quad \text{Equation T.}$$

$$\cot. \frac{1}{2} C = \frac{\sin. \frac{1}{2} (a + b) \cdot \tan. \frac{1}{2} (A - B)}{\sin. \frac{1}{2} (a \infty b)} \quad \text{Equation U.}$$

372. CASE V. When two sides and an angle opposite to one of them are given, as, a, b and the angle A.

$$\sin. a : \sin. b :: \sin. A : \sin. B = \frac{\sin. b \cdot \sin. A}{\sin. a} \therefore \text{we have B.}$$

To find C and c, as we have now a, b and A and B.

$$\text{We have from (Eq. T) } \cot \frac{1}{2} C = \frac{\cos. \frac{1}{2} (a + b) \cdot \tan. \frac{1}{2} (A + B)}{\cos. \frac{1}{2} (a \infty b)} \quad \text{(V)}$$

and from (R) we have the value of c, for

$$\tan. \frac{1}{2} c = \frac{\cos. \frac{1}{2} (A + B) \cdot \tan. \frac{1}{2} (a + b)}{\cos. \frac{1}{2} (A \infty B)} \cdot \text{(W)} \quad \text{Having the angles}$$

A, B and C, and the sides a and b, we can find c, because $\sin. B : \sin. C :: \sin. b : \sin. c$.

NOTE. As the value determined by proportion admits sometimes of a double value, because two arcs have the same sine. It is therefore better to use Napier's analogies.

373. CASE VI. When two angles A and B and the side a opposite to one of them are given to find the other parts.

$$\sin. A : \sin. B :: \sin. a : \sin. b \therefore \text{we have side b.}$$

By Eq. (V) we find the $\angle C$.

By Eq. (W) we find c, which may be found by proportion.

NOTE. If cosine A is less than cosine B, B and b will be of the same species, (i. e.,) each must be more or less than 90° in the above proportion. If cos. B is less than cos. A, then b may have two values.

374. Examples with their answers for each case.

CASE I. Ex. 1. Given $c = 79^\circ 17' 14''$, $b = 58^\circ$ and $a = 110^\circ$ to find A.

$$\text{Answer. } A = 121^\circ 54' 56''.$$

$$\text{Ex. 2. Given } a = 100^\circ, b = 37^\circ 18', \text{ and } c = 62^\circ 46'.$$

$$\text{Answer. } A = 176^\circ 15' 46''.$$

Ex. 3. Given $a = 61^\circ 32' 12''$, $b = 83^\circ 19' 42''$, $c = 23^\circ 27' 46''$ to find A.

$$\text{Answer. } A = 20^\circ 39' 48''.$$

$$\text{Ex. 4. Given } a = 46^\circ, b = 72^\circ, \text{ and } c = 68^\circ.$$

$$\text{Answer. } A = 48^\circ 58', B = 85^\circ 48', C = 76^\circ 28'.$$

CASE II. Ex. 1. Given $A = 90^\circ$, $B = 95^\circ 6'$, $C = 71^\circ 86'$, to find the sides.

Answer. $a = 91^\circ 42'$, $b = 95^\circ 22' 30''$, $c = 71^\circ 31' 30''$.

Ex. 2. $A = 89^\circ$, $B = 5^\circ$, $C = 88^\circ$.

Answer. $a = 53^\circ 10'$, $b = 4^\circ$, $c = 53^\circ 8'$.

Ex. 3. $A = 103^\circ 59' 57''$, $B = 46^\circ 18' 7''$, $C = 36^\circ 7' 52''$.

Answer. $a = 42^\circ 8' 48''$.

CASE III. Ex. 1. Given $a = 38^\circ 30'$, $b = 70^\circ$, and $C = 31^\circ 34' 26''$.

Answer. $B = 130^\circ 3' 11''$, $A = 30^\circ 28' 11''$.

Ex. 2. Given $a = 78^\circ 41'$, $b = 153^\circ 30'$, $C = 140^\circ 22'$.

Answer. $A = 133^\circ 15'$, $B = 160^\circ 39'$, $c = 120^\circ 50'$.

Ex. 3. Given $a = 13$, $c = 9^\circ$, $B = 176^\circ$ to find other parts.

Answer. $A = 2^\circ 24'$, $C = 1^\circ 40'$.

CASE IV. Ex. 1. Given $a = 71^\circ 45'$, $B = 104^\circ 5'$, $C = 82^\circ 18'$ to find etc.

Answer. $A = 70^\circ 31'$, $b = 102^\circ 17'$, $c = 86^\circ 41'$.

Ex. 2. $A = 30^\circ 28' 11''$, $B = 130^\circ 3' 11''$, $c = 40^\circ$ to find etc.

Answer. $a = 38^\circ 30'$, $b = 70^\circ$, $C = 31^\circ 34' 26''$.

Ex. 3. Given $B = 125^\circ 37'$, $C = 98^\circ 44'$, $a = 45^\circ 54'$ to find etc.

Answer. $A = 61^\circ 55'$, $b = 138^\circ 34'$, $c = 126^\circ 26'$.

CASE V. Ex. 1. $a = 136^\circ 25'$, $c = 125^\circ 40'$, $C = 100^\circ$ to find etc.

Answer. $A = 123^\circ 19'$, $B = 62^\circ 6'$, $b = 46^\circ 48'$.

Ex. 2. Given $a = 84^\circ 14' 29''$, $b = 44^\circ 13' 45''$, $A = 130^\circ 5' 22''$ to

Answer. $B = 32^\circ 26' 7''$, $C = 36^\circ 45' 28''$, $c = 51^\circ 6' 12''$.

Ex. 3. Given $a = 54^\circ$, $c = 22^\circ$, $C = 12^\circ$ to find etc.

Answer. $b = 73^\circ 16'$, $B = 147^\circ 53'$, $A = 26^\circ 41'$, or

$b = 33^\circ 32'$, $B = 17^\circ 51'$, $A = 153^\circ 19'$.—*Peirce's Trigonometry.*

CASE VI. Ex. 1. Given $A = 103^\circ 16'$, $B = 76^\circ 44'$, $b = 30^\circ 7'$ to find etc.

Answer. $a = 149^\circ 53'$, $c = 164^\circ 50'$, $C = 149^\circ 30'$.—*Thomson.*

Ex. 2. Given $A = 104^\circ$, $C = 95^\circ$, $a = 138^\circ$ to find etc.

Answer. $b = 17^\circ 21'$, $c = 136^\circ 36'$, $B = 25^\circ 37'$, or

$b = 171^\circ 37'$, $c = 43^\circ 24'$, $B = 167^\circ 47'$.—*Peirce.*

Ex. 3. Given $A = 17^\circ 46' 16\frac{1}{2}''$, $B = 151^\circ 43' 52''$, $a = 37^\circ 48'$ to find etc.

Answer. $b = 180^\circ$, $c = 74^\circ 30'$.—*Young's Trigonometry.*

SPHERICAL ASTRONOMY

375. *Meridians*, are great circles passing through the celestial poles and the place of the observer, and are perpendicular to the equinoctial. They are called hour lines, and circles of right ascension.

Altitude of a Celestial Object, is its height above the horizon, measured on the meridian or vertical circle.

Zenith Distance, is the complement of the altitude, or the altitude taken from 90° .

Azimuth or Vertical Circles, pass through the zenith and nadir, and cut the horizon at right angles.

Azimuth or Bearing of a celestial object, is the arc intercepted between the North and South points and a circle of altitude passing through the

place of the body, and is the same as the angle formed at the zenith by the intersection of the celestial meridian and circle of altitude.

Greatest Azimuth or Elongation of a celestial object, is that at which during a short time the azimuth or bearing appears to be stationary, and at which point the object moves rapidly in altitude, but appears stationary in azimuth. When the celestial object is at this point, it is the most favorable situation for determining the true time, and variation of the compass, and consequently the astronomical bearing of any line in surveying. See Table XXII.

Parallax, is the difference of the angles as taken from the surface and centre of the earth. It increases from the horizon to the zenith, and is to be always added to the observed altitude. (See Table XVIII.)

Dip, is the correction made for the height of the eye above the horizon when on water, and is always to be subtracted. When on land using an artificial horizon, half the observed altitude will be used. (See Table XVI.)

Refraction in altitude, is the difference between the apparent and true altitude, and is always to be subtracted. (See Table XVII.)

As the greatest effect of refraction is near the horizon, altitudes less than 26° ought to be avoided as much as possible.

Prime Vertical, is the azimuth circle cutting the East and West points.

Elevation of the Pole, is an arc of the meridian intercepted between the elevated pole and the horizon.

Declination, is that portion of its meridian between the equinoctial and centre of the object, and is either North or South as the celestial object is North or South of the equinoctial.

Polar distance, is the declination taken from 90° .

Right Ascension is the arc of the equinoctial between its meridian and the vernal equinox, and is reckoned eastward.

Latitude of a celestial object is an arc of celestial longitude between the object and the ecliptic, and is North or South latitude according as the object is situated with respect to the ecliptic between the first points of Aries and a circle of longitude passing through that point.

Mean Time, is that shown by a clock or chronometer. The mean day is 24 hours long.

Apparent Solar Days, are sometimes more or less than 24 hours.

Equation of Time, is the correction for changing mean time into apparent time and *visa versa*, and is given in the nautical almanacs each year.

Sidereal Time. A sidereal day is the interval between two successive transits of the same star over the meridian, and is always of the same length; for all the fixed stars make their revolutions in equal time. The sidereal is shorter than the mean solar day by $3' 56\frac{1}{2}''$. This difference is owing to the sun's annual motion from *West* to *East*, by which he leaves the star as if it were behind him.

The star culminates $3' 56.5554''$ earlier every day than the time shown by the clock.

Civil Time, begins at midnight and runs to 12 or noon, and then from noon again 12 hours to midnight.

Astronomical or Solar Day, is the time between two successive transits of the sun's centre over the same meridian. It begins at noon and is

reckoned on 24 hours to the next noon, without regarding the civil time. This is always known as apparent time.

Nautical or Sea Day, begins 12 hours earlier than the astronomical.

Example. Civil time, April 8th, 12h. = Ast. 8d. 0h.

Example. Civil time, April 9th, 10h. = Ast. 8d. 22h.

If the civil time be after noon of the given day, it agrees with the astronomical; but when the time is before noon, add 12 hours to the civil time, and put the date one day back for the astronomical. The *nautical* or *sea day* is the same as the *civil time*, the noon of each is the beginning of the astronomical day.

376. To find at what time a heavenly body will culminate, or pass the meridian of a given place. (See 264E, p. 69.)

From the Nautical Almanac take the star's right ascension, also the R. A. of the mean sun, or sidereal time. From the star's R. A., increased by 24 if necessary, subtract the sidereal time above taken, the difference will be the approximate sidereal time of transit at the station. Apply the correction for the longitude in time to the approximate, by adding for E. longitude, and subtracting for W. longitude, the sum or difference will be the Greenwich date or time of transit. The correction is 0.6571s. for each degree.

Ex. At what time did α Scorpie (Anteres) pass the meridian of Copenhagen, in longitude $12^{\circ} 35'$ E. of Greenwich, on the 20th August, 1846?

Star's R. A.	=	16 20 02
Sun's R. A. from sid. col.	=	9 53 45.5
Sidereal interval, at station,	=	6 26 16.5
Cor. for long. = $12^{\circ} 35' \times 0.6571s.$	=	+ 8.27
(Here 3m. 56.55s. divided by $360^{\circ} = 0.6571s.$)	=	6 26 24.77
This reduced to mean time,	=	6 25 21.46

The correction for long. is added in east and subtracted in west long.

NOTE. The sidereal columns of the Nautical Almanac, are found by adding or subtracting the equation of time, to or from the sun's R. A. at mean noon. What we have given in sec. 264E, will be sufficiently near for taking a meridian altitude.

377.

LATITUDE BY OBSERVATION OF THE SUN.

RULE. Correct the sun's altitude of the limb for index error. Subtract the dip of the horizon. The difference = apparent altitude. From the apparent altitude, take the refraction corresponding to the altitude; the difference = true altitude of the observed limb. To this altitude, add or subtract the sun's semi-diameter, taken from p. 2 of the Nautical Almanac, the sum or difference = true altitude of the sun's centre. Add the sun's semi-diameter when the lower limb is observed, and subtract for the upper.

From 90, subtract the true altitude, the difference will be the zenith distance, which is north, if the zenith of the observer is north of the sun, and south, if his zenith is south of the sun.

From the Nautical Almanac, take the sun's declination, which correct for the longitude of the observer; then if the corrected declination and the zenith distance be of the same name, that is, both north or south, their sum will be the latitude; but if one is north and the other south, their difference will be the latitude.

Example. From Norie's Epitome of Navigation, August 30, 1851, in long. 129° W., the meridian altitude of the sun's lower limb was $57^{\circ} 18' 30''$, the observer's zenith north of the sun. Height of the eye above the horizon, 18 feet. Require the latitude.

	° / ''
Observed altitude,	57 18 30
Dip of the horizon, correction from Table XVI,	— 4 08
Apparent altitude of sun's lower limb =	57 14 22
Correction from Tables XVII and XVIII for refraction and parallax,	— 32
True altitude of the sun's lower limb =	57 13 50
Sun's semi-diameter from N. A. for the given day +	15 52
True altitude of sun's centre =	57 29 42
Zenith distance = $90 - \text{alt.}$	= 32 30 18 N.
Declination on 30th August, is	N. 9 08 30
Declination on 31st August, is	N. 8 46 58
Decrease in 24 hours,	0 21 32
$360^{\circ} : 21' 32'' :: 129^{\circ} : 7' 43''$.	

	° / ''
Declination, 30th August, 1851,	= N. 9 08 30
Correction for W. longitude 129° =	— 7 43
Correct declination at station	= 9 00 47 N.
From above, the zenith distance	= 32 30 18 N.
North latitude	= 41 31 05

Norie gives $41^{\circ} 30' 53''$, because he does not use the table of declination in the N. A., but one which he considers approximately near.

As the Nautical Almanacs are within the reach of every one, and the expense is not more than one dollar, it is presumed that each of our readers will have one for every year.

Example 2. On the 17th November, 1848, in longitude 80° E., meridian altitude of sun's lower limb was $50^{\circ} 6'$ south of the observer, (that is, south of his zenith) the eye being 17 feet above the level of the horizon. Required the latitude. Answer, $20^{\circ} 32' 58''$.

NOTE. On land we have no correction for dip.

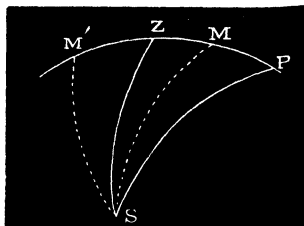
378. To find the latitude when the celestial object is off the meridian, by having the hour angle between the place of the object and meridian, the altitude and declination or polar distance.

Let S = place of the star. P the elevated pole. Z = the zenith.

Here P S = p = codeclination = polar distance.

Z S = z = zenith distance and P Z is the colatitude = P, and the hour angle, Z P S = h.

By case VI, we have p, z, and the hour angle Z P S = h, to find P Z. Let fall the perpendicular S M. Let it fall within the $\triangle$ S P Z, then we have



Tan. $PM = \cos. h \times \cotan. \text{declination} = \cos. h \cdot \tan. \text{pol. dist.}$

Cos. $ZM = \cos. PM \times \sin. \text{alt.} \times \text{cosecant of declination.}$

Colatitude $= PM + ZM$ when the perp. falls within $\triangle PSZ$.

Colatitude $= PM - ZM$ when the perp. falls without the same.

It is to be observed that there may be an ambiguity whether the point M would fall inside or out of the $\triangle PSZ$. This can only happen when the object is near the prime vertical, that is due E. or W. As the observation should be made near the meridian, the approximate latitude will show whether M is between the pole, P and zenith, Z or not.

Having the two sides p and z , and the $\angle h = \angle SPZ$, we find PZ the colat. by sec. 372.

379. *Latitude from a double altitude of the sun, and the elapsed time.*

The altitudes ought to be as near the meridian as possible, and the elapsed time not more than two hours. When not more than this time, we may safely take the mean of the sun's polar distance at the two altitudes.

Let S and S' be the position of the object at the time of observations.

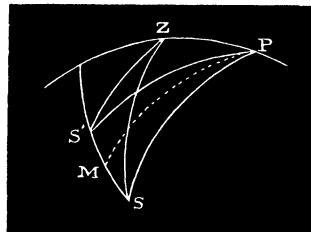
ZS and $ZS' =$ zenith distances.

PS and PS' , the polar distances.

Angle $SPS' =$ elapsed time.

To find the colatitude $= PZ$.

Various rules are published for the solution of this problem, but we will follow the immortal *Delambre*.



Delambre, who has calculated more spherical triangles than any other man, found, after investigating the many formulas, that the direct method of resolving the triangle was the best and most accurate method. We now have the following:

PS and $PS' =$ polar distances. ZS and $ZS' =$ colatitudes. Hour angle $= \angle SPS'$.	}	To find colat. PZ .
--	---	-----------------------

Half of $PS + PS' =$ mean polar distance $= p$.

One-half the elapsed time in space $= h$.

Draw the perpendicular PM , then we have

Log. sin. $SM = \log. \sin. \text{mean polar distance} + \log. \sin. \text{one-half hour angle in space}$, and having $SM = S'M$, we have the base, $SM S'$.

Consequently, in the $\triangle SZS'$, we have the three sides given to find the angles, and also the three sides of the triangle $PS S'$. By sec. 367, we find the angles $PS S'$ and $ZS S'$ $\therefore$ the $\angle PSZ$ is found, and the sides PS and ZS is found by observation, then we have in the triangle PSZ the two sides PS , SZ and the angle PSZ , to find the colat. PZ , which can be found by sec. 369.

380. *To find the latitude by a meridian altitude of Polaris, or any other circumpolar star.*

Take the altitude of the object above and below the pole, where great accuracy is required. Let their apparent zenith distances be z and z' respectively, and also, r and r' , the refractions due to the altitudes, then

Colatitude $= \text{correct zenith distance} = \frac{1}{2}(z + z' + r + r')$

Let A and A' be the correct altitudes, then we have

Colatitude $= \frac{1}{2}(180 - (A + A') + (r + r'))$

NOTE. Here we do not require to know the declination of the object.

By this method, we observe several stars, from a mean of which the latitude may be found with great accuracy. The instrument is to be placed in the plane of the meridian as near as possible. The altitude will be the least below the pole, and greatest above it, at the time of its meridian transit or passage.

381. *To find the latitude by a meridian altitude of a star above the pole.*

Correct the altitude as above for the sun. From this, take the polar distance, the difference = the required latitude.

Let A and A' = corrected altitudes above and below the pole.

p = polar distance of the object. Then

Latitude = $A - p$ when $*$ is above the pole.

Latitude = $A + p$ when $*$ is below the pole.

382. *To find the latitude by the pole star, at any time of the day.*

The following formula is given in the British Nautical Almanacs since 1840, and is the same in Schumacher's Ephemeris :

$$L = a - p \cdot \cos. h + \frac{1}{2} \sin. 1'' (p \sin. h)^2 \tan. a.$$

$$- \frac{1}{3} \sin. ^2 1'' (p \cos. h) (p \sin. h)^2.$$

If we reject the fourth term, it will never cause an error more than half a second. Then we have

$$L = a - p \cdot \cos. h + \frac{1}{2} \sin. 1'' (p \sin. h)^2 \cdot \tan. a.$$

Here L = latitude, a = true altitude of the star.

p = apparent polar distance, expressed in seconds.

h = star's hour angle = $S - r$.

S = sidereal time of observation.

r = right ascension of the star.

p is *plus* when the $*$ is W. of the meridian, and negative when E.

Example. In 1853, Jan. 21, in longitude 80° W., about 2 hours after the upper transit of Polaris, its altitude, cleared of index error, refraction and parallax, was observed = $40^\circ 10'$. Star's declination = $88^\circ 31' 47''$. Mean time of observation by chronometer = 7h. 0m. 32.40s. To find the latitude.

	h	m	s
1853, Jan. 21, Polaris' R. A.,	1	5	36.79
Sidereal time, mean noon, Greenwich,	20	3	2.73
Sid. interval from mean noon at Greenwich =	5	2	34.06
Cor. $80^\circ \times 0.6571$, to be subtracted in W. long.			52.57
Sidereal interval of meridian passage at station,	5	1	41.49
Mean time of observation, 7h. 0m. 32.40s. which,			
reduced to sidereal time by Table XXXI, =	7	1	41.49
Hour angle h in arc = 30° =			
in time,	2	0	00
$p = 5292.6''$, its log. = 3.7236691			
$h = 30^\circ$ its log. cosine, 9.9375306			

$$\text{Log. of } p \cos. h = 3.6611997 = 4583.5 = \text{first correction.}$$

$$4583.5'' = 1^\circ 16' 23.5'' = \text{negative} = -1^\circ 16' 23.5'' = \text{first cor.}$$

To find the second correction.

$$\begin{array}{l} \text{Log. sin. } h = 30^\circ = 9.6989700 \\ \text{Polar dis. } p = 5292.6, \text{ log} = 3.7236691 \end{array} \left. \vphantom{\begin{array}{l} \text{Log. sin. } h = 30^\circ = 9.6989700 \\ \text{Polar dis. } p = 5292.6, \text{ log} = 3.7236691 \end{array}} \right\} = 3.4226291$$

$(p \sin. h)^2$	$= 3.4226291 \times 2 = 6.8452782$
$\frac{1}{2} \sin. 1''$	$= 4.3845449$
$\tan. \text{ of alt. } 40^\circ 10'$	$= 9.9263778$
$\frac{1}{2} \sin. 1'' (p \cdot \sin. h)^2 \cdot \tan \alpha$	$= 1.1562009$
	$= + 14.31'' = \text{second cor.}$
Altitude,	40 10 00
First correction	— 1 16 23.50
	38 53 36.50
Second correction	+ 0 0 14.31
	38 53 50.81 = required latitude.

NOTE. Here we rejected the fourth term as of no consequence.

The longitude may be assumed approximately near; for an error of one degree in longitude, makes but an error of 0.63s. in the hour angle.

383. *To find the variation of the compass by an azimuth of a star.*

At sec. 264c and 264H, we have shown how to find the azimuth, when the star was at its greatest elongation. To find the azimuth at any other time, we take the altitude, and know the polar distance of the star and the colatitude of the place; that is, we have the

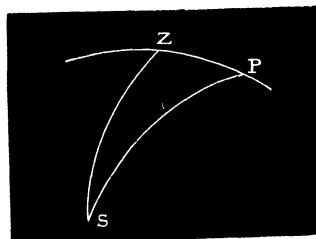
Polar distance, P S

Colatitude, P Z

Zenith distance, Z S

To find the

Azimuth angle P Z S.



We find the required angle P Z S by sec. 367.

By Table XXIII, we can find the azimuth from the greatest elongation of certain circumpolar stars.

384. *To find at what time Polaris or any other star will be at its greatest eastern or western elongation or azimuth. Its true altitude and greatest azimuth at that time. Also to determine the error of the chronometer or watch.*

In the following example, let P = polar distance, L = latitude, R. A. = right ascension, and G. A. = greatest azimuth.

Given the latitude of observatory house in Chicago = $41^\circ 50' 30''$ N. longitude, $87^\circ 34' 7''$ W. on the 1st December, 1866, to find the above.

Polaris, polar distance = $1^\circ 24' 4''$.

NOTE. In determining the greatest azimuth, we select a star whose polar distance does not exceed 16° , and for determining the true mean time, we take a star whose polar distance will be greater than 16° or about 20 to 30° , and which can be used early in the night. *Calculating the altitude and time of the star's greatest azimuth, is claimed by us as new, simple and infallibly true, and can be found by any ordinary surveying instrument whose vertical arc reads to minutes.*

It is generally believed by surveyors, that when Polaris, Alioth in Ursa Majoris, or Gamma in Cassiopeae, are in the same plane or vertical line, Polaris is then on the meridian.

It is to be much regretted that the above two last named stars so much used by surveyors, have not found place in the British or American Ephemeris. However, *we have calculated the R. A. and declination of them till 1940. See Table XXV.*

NOTE. We will send a copy of this part of our work to the respective Nautical Almanac offices above named, urging the necessity of giving the right ascension and declination of these two stars. With what success, our readers will hereafter see.

Time from Merid. Passage.		Altitude at G. A.		Greatest Azimuth.	
Tan. p	8.388437	Radius,	10.000000	Radius =	10.000000
Tan. L +	9.951023	Sine L +	9.824174	Sine p = +	8.388307
	18.339460		19.824174		18.388307
Less	10.	Cos. p —	9.999870	Cos. L —	9.872151
Cosine =	8.339460	Sine =	9.824304	Sine =	8.516156
88° 44' 53''		True alt.	41° 51' 25''	1° 52' 51''	
Sid. 5h. 54m. 59.53s.		Cor. tab. XII +	1 8	Greatest azimuth.	
		Appt. alt.	41° 52' 33''		

Polaris R. A. =	1h. 10m. 54.30s.
Sun's R. A. = sid. column,	16 41 25.04
	8 29 29.26
Cor. for 87° 34' 7'' at 0.6571s. for each deg. —	57.54
Upper transit in sidereal time =	8 28 31.72
Time from meridian passage to G. E. A. =	5 54 59.53
This would be in day time, for G. E. A.,	2 33 32.19
This is after midnight, for G. W. A.,	14 23 21.25
Or, December 2d,	2 23 21.25
Which, if reduced to mean time, gives	2 22 57.70

385. *To find the azimuth or bearing of Polaris from the meridian, when Polaris and Alioth (Epsilon in Ursa Majoris) are on the same vertical line.*

Example. The latitude of observatory house in Chicago, (corner of 26th and Halsted streets,) is 41° 50' 30''. Required the azimuth of Polaris when vertical with Alioth, on the first day of January, 1867.

Right Ascension.	Ann. variation.	N. P. D.	Ann. variation.
Polaris, 1h. 10m. 17s.	+ 19.664s.	1° 23' 59''	— 19.12''
Alioth, 12h. 48m. 10s.	+ 2.661s.	33° 19' 05''	— 19.67''
Gamma, 0h. 48m. 42s.	+ 3.561s.	30° 0' 15''	— 19.613''

Latitude, 41° 50' 30'' ∴ colatitude = 48° 9' 30''.

Polaris N. P. D. 1° 24' and colat. less polar distance = Z.

Altitude above the pole = 43° 14' 29''

48° 9' 30'' — 1° 24' = 46° 45' 30'', zenith dist. of Polaris.

To find Alioth's zenith distance.

Latitude,	41° 50' 30''
Alioth below the pole,	33° 19' 05'' = polar distance.
Alioth's altitude,	8° 31' 25'' under transit.
Alioth's zenith distance,	81° 28' 35''
Polaris' upper transit, 1st January, 1867,	1h. 10m. 17s.
Alioth's upper transit, 12h. 48m. 10s.	Under at 0h. 48m. 10s.
Hour angle in space = 5° 31' 45'', in time =	22m, 07s.

Here we find that Alioth passes the meridian below the pole 22m, 7s, earlier than Polaris will pass above it, consequently, they will be vertical E. of the meridian.

As Polaris moves about half a minute of a degree in one minute of time, it is evident that we may take the zenith distances of both stars the same as if taken on the meridian without any sensible error.

We have in the $\triangle P Z S$, fig. in sec. 383, the sides

$P S$ = polar distance. $Z S$ = zenith distance. And the hour angle $S P Z$, in space, to find the azimuth angle $S Z P$. By sec. 372,

$$\text{we have } \sin. < S Z P = \frac{\sin. < S P Z \cdot \sin. P S}{\sin. Z S} = \frac{\sin. h \times \sin. p}{\sin. z}$$

$$\sin. < S Z P = \frac{\sin. 5^{\circ} 31' 45'' \times \sin. 1^{\circ} 24'}{\sin. 46^{\circ} 45' 30''} = 0^{\circ} 11'.$$

That is, the azimuth of Polaris is $11'$ E. of the meridian, when Alioth is on it below the pole. Alioth is going E. and Polaris going W., therefore, they meet E. of the meridian. Their motions are

$$\begin{array}{l} \text{As } \frac{\text{sine polar distance of Polaris}}{\text{sine of its zenith distance}} \cdot \frac{\text{sine polar distance of Alioth.}}{\text{sine of its zenith distance.}} \text{ Or} \\ \text{As } \frac{\text{sine } 1^{\circ} 24'}{\text{sine } 46^{\circ} 45' 30''} \cdot \frac{\text{sine } 33^{\circ} 19' 05''}{\text{sine } 81^{\circ} 28' 35''} \cdot .0244 \cdot .5468 \\ \text{Or as } 0.0244 \times 0.9899 : 0.5468 \times 0.7285. \text{ Or } 1 : 16. \end{array}$$

And $17 : 11' : 1$: Polaris' space moved west = $39''$ nearly.

Therefore, $11' - 39'' = N. 10' 21'' E.$ = required azimuth.

386. To find the azimuth of Polaris when on the same vertical plane with γ in *Ursa Majoris*, in *Chicago*, on the 1st Jan., 1867: Lat. $41^{\circ} 50' 30''$.

R. A. of Polaris at upper transit, 1h, 10m, 17s.

R. A. of γ Urs. Maj. at upper transit, 11h, 46m, 49s.

“ “ “ “ under transit, 23h, 46m, 49s.

Hour angle in space, $20^{\circ} 52' =$ in sidereal time to, 1h, 23m, 28s.

Polaris' polar dist. above the pole = $1^{\circ} 24'$ $\therefore$ its alt. = $43^{\circ} 14' 30''$, and the altitude taken from 90° , gives the zenith dist. = $46^{\circ} 45' 30''$. Gamma's polar distance, from Nautical Almanac, $35^{\circ} 34'$ below the pole $\therefore$ its altitude = $41^{\circ} 50' 30'' - 35^{\circ} 34' = 6^{\circ} 16' 30''$, and its zenith distance, $83^{\circ} 43' 30''$.

In the $\triangle S P Z$, we have the hour $< S P Z = h$, equal to $20^{\circ} 52'$, $P S = 1^{\circ} 24'$, and $Z P = 43^{\circ} 14' 30''$. By sec. 372,

$$\sin. < S Z P = \frac{\sin. 20^{\circ} 52' \times \sin. 1^{\circ} 24'}{\sin. 46^{\circ} 45' 30''} \quad \text{By using Table A,}$$

$$\text{we have } \sin. S Z P = \frac{.35619 \times .02443}{.72837} = .01195 = 41'$$

Angular motion of Polaris is to the angular motion of γ nearly

$$\begin{array}{l} \text{as } \frac{\text{sine polar dist. of Polaris}}{\text{sine of its zenith dist.}} \cdot \frac{\text{sine polar dist. of } \gamma}{\text{sine of its zenith dist.}} \\ \text{that is, } \frac{\sin. p}{\sin. z} \cdot \frac{\sin. P}{\sin. Z} \cdot \frac{\sin. P \times \sin. z}{\sin. p \times \sin. Z} \cdot 1. \text{ By Table A,} \end{array}$$

$$\sin. P = \sin. 35^{\circ} 34' = .5817$$

$$\sin. z = \sin. 46^{\circ} 45' 30'' = .7284. \text{ Their product} = .42371028 = B.$$

$\sin. p \times \sin. Z = \sin. 1^{\circ} 24' \times \sin. 83^{\circ} 43' 30'' = .0244 \times .294 = .02428342 = C$, divided into B, gives the value of the 4th number = 27. As γ moves E. 27' and Polaris moves W. 1' in the same time, making a total distance of 28' $\therefore 28 : 41' :: 1 : 1' 28''$, which, taken from the above 41', leaves the azimuth of Polaris N. $39' 32''$ E. of the meridian.

Table XXIII gives the greatest azimuths of certain stars near the North and South Poles; by which the true bearing of a line and variation of the compass can be found several times during the night. There are several bright stars near the North Pole. The nearest one to the South Pole is β Hydri, which is now about 12° from it. This circumstance led us to ask frequently why there should not be the same means given those south of the Equator as to those north of it. It was on the night of the 18th January, 1867, as we revelled in a pleasant starry dream, that we heard the words—*God has given the Cross to man the emblem of and guide to salvation. He has also made the Southern Cross a guide in Surveying and Navigation.* Not a moment was lost in seeing if this was so. We found from our British Association's Catalogue of Stars, that when α' (a star of the first magnitude) in the foot of the Southern Cross was vertical with β (a bright star) in the tail of the Serpent, that then, in lat. 12° , they were within $1' 12''$ of the true meridian, and that their annual variations are so small as to require about 50 years to make a change of half a minute in the azimuth or bearing of any line.

We rejoice at the valuable discovery, but struck with awe at the forethought of the Great Creator in ordaining such an infallible guide, and brought once more to mind the expression of Capt. King, of the Royal Engineers, who, after taking the time according to our new method, in 1846, near Ottawa, Canada, and seeing the perfect work of the heavens, said—“*Who dares say there is no God?*”

Our readers will perceive that Tables XXIII, XXVI, XXVII and XXVIII are original, and the result of much time and labor.

Table XXVI gives the azimuth of α' Crucis when vertical with β Hydra in the southern hemisphere until the year 2150.

Table XXVII gives the azimuth of Polaris when vertical with Alioth in Ursa Majoris until the year 1940.

Table XXVIII, when Polaris is vertical with γ in Cassiopeae till 1940.

387.

TO DETERMINE THE TRUE TIME.

The true time may be obtained by a meridian passage of the sun or star. When the telescope is in the plane of the meridian, as in observatories, we find the meridian transit of both limbs of the sun, the mean of which will be the apparent noon, which reduce to mean time by adding or subtracting the equation of time. If we observe the meridian passage of a star, we compare it with the calculated time of transit, and thereby find the error of the chronometer or watch.

388. *By equal altitudes* of a star, the mean of both will be the apparent time of transit, which, compared with the calculated time of transit, will give the error of the watch, if any.

389. *By equal altitudes of the sun*, taken between 9 A. M. and 3 P. M. In this method we will use Baily's Formula, and that part of his Table XVI, from 2 to 8 hours elapsed time between the observations.

$X = \pm A \delta \tan. L + B \delta \tan. D$. Here

T = time in hours, L = latitude of place, *minus when south*.

D = dec. at noon, also *minus when south*.

δ = double variation of dec. in seconds, deduced from the noon of the preceding day to that of the following. *Minus when the sun is going S.*

X = correction in seconds. A is *minus* if the time for noon is required, and *plus* when midnight is required. The values of A and B for time T , may be found from Table XXVIII, which is part of Baily's Table XVI, and agrees with Col. Frome's Table XIV, in his Trigonometrical Surveying, and also with Capt. Lee's Table of Equal Altitudes. We give the values of A and B but for 6 hours of elapsed time or interval, for before or after this time, (that is, before 9 A. M. or after 3 P. M.) it will be better to take an altitude when the sun is on or near the prime vertical, which time and altitude may be found from Tables XXI and XXII of this work.

390. To determine the time at Tasche in lat. $45^\circ 48'$ north, on the 9th of August, 1844, by equal altitudes of the sun.

		Chronometer Time.						Elap. time T.	Value of X.
		A. M.			P. M.				
° ′		h	m	s	h	m	s	h m	s
Alt. U. L.	78 50	1	28	23	8	03	16.5	6 33	10.63
“	79 19.30	1	29	52.8	8	01	46.5		
“	85 36.00	1	49	33	7	42	18	5 48	10.1
“	87 02.10	1	53	53.5	7	37	46.2		

Here the sun is going south, therefore D is *minus*. The lat. is north, $\therefore L$ is *plus*. Also δ is *minus*. We want the time of noon, $\therefore$ the value of A is *minus*, and $-A \times -\delta \times +L$, will be positive or *plus*, and also, $B \times -\delta \times -D$, will be *plus* in the following calculation, where we find $\delta = 2094''$ —from the Nautical Almanac:

$T = 6h. 3m.$ its log. $A = -7.7793$, and log. $B = -7.5951$.

$\delta = 2094''$, its log. $= -3.3210$, log. $\delta = -3.310$.

$L = 45^\circ 48'$ log. tan. $= +0.0121$, log. tan. $D = -9.4433$.

First correction $+ 12.95s. = 1.1124$. $2.32s. = -0.3654$.

Second correction 2.32

$X = 10.63$

Time A. M. $= t = 1h. 28m. 23.0s.$

Time P. M. $= t' = 8 \quad 03 \quad 16.5$

$t + t' = 9 \quad 31 \quad 39.5$

$t + t' = 4 \quad 45 \quad 49.75$

$X = + 10.63$

4 46 00.38 chronometer time of app't noon.

05 09.09 equat. time from Naut. Almanac.

4h. 40m. 51.29s. chronom. fast of mean time, at
app't noon, August 9, 1844.

Correct this for the daily rate of loss or gain by the chronometer, the result will be the true mean time of chronometer at apparent noon. This time converted into space, will give the long. W. of the meridian, whose mean time the chronometer is supposed to keep. The above is one of Col. J. D. Graham's observations, as given by Captain Lee, U. S. T. E. in his Tables and Formulas.

Time by Equal Altitudes. (See sec. 388.)

We set the instrument to a given altitude to the nearest minute in advance of the star, and wait till it comes to that altitude.

Example from Young's Nautical Astronomy.

Observations made on the star Arcturus, Nov. 29, 1858, in longitude $98^{\circ} 30'$ E. to find the time :

Altitudes E. and W. of the Meridian.	Times shown by Chronometer.	Sum of Times.
43 10	h. m. s.	30 7 42
	{ 11 55 47 }	
	{ 18 11 55 }	
43 30	h. m. s.	30 7 42
	{ 11 57 57 }	
	{ 18 9 45 }	
43 50	h. m. s.	30 7 42
	{ 12 0 7 }	
	{ 18 7 35 }	

From the sum of the times, we get the chronometer time of the star's meridian passage, or transit, equal to 15h. 3m. 51s.

Arcturus, R. A. Nov. 29,	h. m. s.	
R. A. of mean sun, sid. col.,	14 9 13	
Mean time of transit at station,	16 20 48	Diff. for 1h. = + 10.76s.
Long. $98^{\circ} 30'$ E. in time,	21 48 25 nearly.	15½ hours.
Mean time at Greenwich,	6 24 00 subtract.	164.09
Cor. for 15½ hours,	15 14 25 nearly.	or 2m. 44s.
Mean time at Greenwich,	— 2 44 subtract, because R. A. is	increasing.
Mean time by chronometer,	15 11 41	
Error on mean time,	15 3 51	
	— 7 50 at station.	

Mean time of transit at place,	h. m. s.	
Cor. for increase in R. A.,	21 48 25 nearly.	
	— 2 44	
Mean time as shown by chronometer,	21 45 41	
Error of chronometer on mean time,	15 3 51	
	6 41 50 at station.	

By sec. 388. Set the altitude to a given minute in advance, and wait till the star comes to this, and note the mean time.

Time before Midnight.	Altitudes of Star.	Time after Midnight.
h. m. s.	° ' "	h. m. s.
9 50 10	50 0	2 7 40
9 50 20	50 10	2 7 30
9 50 21	50 20	7 19
9 50 20.3		2 7 29.7 Mean.
14 7 29.7		+ 12
2) 23 57 50.0		14 7 29.7

11 58 55 Mean time by clock at station.

390.*

True time by a Horizontal Dial.

This dial is made on slate or brass, well fastened on the top of a post or column, and the face engraved like a clock. (See fig. 49.) It may be set by finding the true mean time and reducing it to the apparent, by means of the equation of time, found in all almanacs. Having the correct apparent noon by clock, set the dial.

Otherwise. Near the dial make a board fast to some horizontal surface, on which paste some paper, and draw thereon several cocentric circles. Perpendicular to this, at the common centre, erect a piece of fine steel wire, and watch where the end of its shadow falls on the circles between the hours of 9 and 3. Find the termini on two points of the same or more circles; bisect the spaces between them, through which, and the centre of the circles, draw a line, which will be the 12 o'clock hour line, from which, at any future time, we may find the apparent, and hence the true mean time.

A brass plate may be fastened to an upper window sill, in which set a perpendicular wire as gnomon, and draw the meridian.

Calculation. We have the latitude, hour angle and radius to find the hour arc from the meridian.

Rule. Rad. : sin. lat. :: tan. hour angle : tan. of the hour arc from the meridian.

Example. Lat. 41° . Hour angle between 10 and 12 = 2 hours = 30° .
As 1 : .65606 :: .57735 : tan. hour arc = .37878, whose arc is = $20^\circ 44' 55''$.

In like manner we calculate the arc from 12 to each of the hours, 1, 3 and 5, which are the same on both sides. The morning and evening hours are found by drawing lines (see fig. 49) from 3, 4 and 5 through the centre or angle of the style at *c*. These will give the morning hours. For the evening hours, draw the lines through 7, 8, 9, and centre *d*, at the angle of the style. The half and quarter hours are calculated in like manner. The slant of the gnomon, *d f*, must point to the elevated pole, and the plate or dial be set horizontal for the lat. for which it is made. The $\angle$ of the gnomon is equal the latitude. A horizontal dial made for one latitude may be made to answer for any other, by having the line *d f* point to the elevated pole. *Example.* One made for lat. 41° may be used in lat. 50° , by elevating the north end of the dial plate 9° , and *vice versa*.

The following table shows the hour arcs at four places:

LAT. 41° .	LAT. 49° .	LAT. $54^\circ 36'$. BELFAST, IRELAND	LAT. $55^\circ 52'$. GLASGOW, SCOTL'D.
1h. = $9^\circ 58'$	$11^\circ 26'$	$12^\circ 19'$	$12^\circ 30'$
2 20 45	23 33	25 12	25 32½
3 33 16	37 03	39 11	39 37½
4 48 39	52 35	54 41	55 06½
5 67 47	70 27	71 48	72 04
6 90 00	90 00	90 00	90 00

To set off these hour arcs, we may, from *c*, set off on line *c n* the chord of 60° and describe a quadrant, in which set off from the line *c n* the hour arcs above calculated.

In our early days we made many dials by the following simple method:

We draw the lines, *c n* and *g h*, so that *c g* will be 5 inches, and described the quadrants, *c, g, k*.

We have, by using a scale of 20 parts to the inch, a radius *c k* = 100.

As the chord of an arc is twice the sine of that arc, we find the sines of half the above hour arcs in Table A; double it; set the decimal mark two places ahead; those to the left will be divisions on the scale to be set off from *k* in the arc *k g*. *Example*—

Let half of the hour arc = $4^\circ 59'$, twice its sine = .17374, which give 17.4 parts for the chord to be set off.

391. *By our new method, we select one of the bright circumpolar stars given in the N. A., whose polar distance is between 15 and 30 degrees. (See our Time Stars in Table XXIV.)*

By sec. 264c, we find the sidereal time of its meridian passage = T .

By sec. 264b, we find its hour angle from ditto = t .

By sec. 264f, we have its true altitude A , when at its greatest azimuth or elongation from the meridian.

Example. Star, S , on a given day, in latitude, L , passed the meridian at time, T , and took time, t , to come to its greatest azimuth, east or west.

We now reduce the sidereal time to mean time.

Greatest eastern azimuth was at time $T - t$. Mean time.

Greatest western ditto, $T + t$. Ditto.

True altitude of its greatest azimuth = A .

Let r = refraction and i index error, then App. alt. = $A + r \pm i$.

We now set the instrument a few minutes before the calculated sidereal time reduced to mean time, and elevate the telescope to the alt. = $A + r \pm i$, and observe when the star comes to the cross hairs at time T' .

The difference between mean time, $T \pm t$ and T' gives the error of time as shown by the watch or chronometer.

This method is extremely accurate, because the star changes its altitude rapidly when near its greatest elongation. As we may take several stars on the same night, we can have one observation to check another.

Now having the true time at station and an approximate longitude, we can find a new longitude, and with it as a basis, find a second, and so on to any desired degree of accuracy.

392. *To find the difference of Longitude.*

1. By rockets sent up at both stations, the observers having previously compared their chronometers and noted the time of breaking.

2. As the last, but instead of rockets, flashes of gunpowder on a metal plate is used. This signal can be seen under favorable circumstances, a distance of forty miles.

3. By the electric telegraph.

4. By the Heliostat.

5. By the Drummond light.

6. By moon culminating stars.

7. By lunar observations.

In 7, we require the altitudes of the moon and star, and the angular distance between the moon's bright limb and the star at the same time, thus requiring three observers. If one has to do it alone, he takes the altitudes first, then the lunar distance, note the times, and repeat the observations in reverse order, and find the mean reduced altitude, also the mean lunar distance.

8. By occultation or eclipse of certain stars by the moon.

393. *By the Electric Telegraph.*

The following example and method used by the late Col. Graham is so very plain, that we can add nothing to it. No man was more devoted to the application of astronomy to Geodesy than he:

LONGITUDE OF CHICAGO AND QUEBEC.

The following interesting letter of Col. Graham, Superintendent of U. S. Works on the Northern Lakes, is in reference to the observations made by him, in conjunction with Lieut. Ashe, R. N., in charge of the observatory at Quebec, to ascertain the difference of longitude between this city and Quebec:

CHICAGO, June 5, 1857.

To the Editor of the Chicago Times: A desire having been expressed by some of the citizens of Chicago for the publication of the results of the observations made conjointly by Lieut. E. D. Ashe, Royal Navy, and myself, on the night of the 15th of May, ult., for ascertaining by telegraphic signals the difference of longitude between Chicago and Quebec, I herewith offer them for your columns, in case you should think them of sufficient interest to be announced. All the observations at Quebec were made under the direction of Lieut. Ashe, who has charge of the British observatory there, while those at this place were made under my direction.

The electric current was transmitted *via* Toledo, Cleveland, Buffalo, Toronto and Montreal, a distance, measured along the wires, of 1,210 miles, by one entire connection between the two extreme stations, and without any intermediate repetition; and yet all the signals made at the end of this long line were distinctly heard at the other, thus making the telegraphic comparisons of the *local time* at the two stations perfectly satisfactory.

This "local time" was determined (also on the night of the 15th ultimo) by observations of the meridian transits of stars, by the use of transit instruments and good clocks or chronometers at the two stations. The point of observation for the "time" at Quebec was the citadel, and at Chicago the Catholic church on Wolcott street, near the corner of Huron.

The following is the result:

1. CHICAGO SIGNALS RECORDED AT BOTH STATIONS. ELECTRIC FLUID TRANSMITTED FROM WEST TO EAST.		
Correct Chicago sidereal time of signals.	Correct Quebec sidereal time of signals.	Difference of longitude. Electric fluid transmitted from west to east.
h. m. s.	h. m. s.	h. m. s.
16 11 13.19	16 16 54.83	1 05 41.64
15 42 18.28	16 47 59.83	1 05 41.55
Mean; electric fluid transmitted from west to east,		1 05 41.595
2. QUEBEC SIGNALS RECORDED AT BOTH STATIONS—ELECTRIC FLUID TRANSMITTED FROM EAST TO WEST.		
Correct Quebec sidereal time of signals.	Correct Chicago sidereal time of signals.	Difference of longitude. Electric fluid transmitted from east to west.
h. m. s.	h. m. s.	h. m. s.
16 24 15.83	15 18 34.40	1 05 41.43
16 54 45.83	15 49 04.39	1 05 41.41
Mean; electric fluid transmitted from east to west.		1 05 41.435
Mean; electric fluid transmitted from west to east, as above,		1 05 41.595
RESULT—Chicago west, in longitude from Quebec,		1 05 41.515
Difference between results of electric fluid transmitted east and west = 0.16 and half diff. = 0.08.		

From which it would appear that the electric fluid was transmitted along the wires between Chicago and Quebec in 8-100ths of a second of time. At this rate it would be only $1\frac{1}{2}$ seconds of time in being transmitted around the circumference of the earth.

I will now proceed to a deduction of the longitude of Chicago, west of the meridian of Greenwich, by combining the above result with a determination of the longitude of Quebec made by myself in the year 1842, while serving as commissioner and chief astronomer on the part of the United States for determining our northwestern boundary, which will be found published at pages 368-369 of the American Almanac for the year 1848. That determination gave for the longitude of the centre of the citadel of Quebec west of Greenwich:

	h.	m.	s.
Difference of longitude between the same point and the Catholic Church on Wolcott street, near the intersection of Huron street, Chicago, by the above described operations,	4	44	49.65
Longitude west of Greenwich, of the Catholic Church on Wolcott street, street, near Huron street, Chicago, Illinois,	1	05	41.51
	5	50	31.16

That is to say, five hours, fifty minutes, thirty-one and sixteen-hundredth seconds of time, or in arc, 87deg. 37min. 47 4-10sec.

J. D. GRAHAM,
Major Topographical Engineers, Brevet Lieut. Col. U. S. Army.

By the Heliostat.

This instrument consists of a mirror, pole, Jacob staff or rod, and a brass ring with cross wires. The brass ring used in our Heliostat, is $\frac{3}{4}$ of an inch thick and $3\frac{1}{4}$ inches diameter. In this is fixed a steel point 2 inches long. There are 4 holes in the ring for to receive cross wires or silk threads made fast by wax. The flag-staff is bored at every 6 inches on both sides to receive the ring, which ought to be at a sufficient distance from the side of the pole so as not to obstruct the direction of the reflected rays of the sun. The pole and ring are set in direction of station B, about 30 to 40 feet in advance of the mirror placed over station A, and the centre of the ring in direction of B, as near as possible. The ring can be raised or lowered to get an approximate direction to B. It will be well to remove the rings from side to side, till the observer at B sees the flash given at A, when B sends a return flash to A.

The mirror is of the best looking-glass material, $3\frac{3}{4}$ inches in diameter, set in bronzed brass frame or ring, $4\frac{1}{2}$ inches outer diameter, $3\frac{3}{4}$ inches inner diameter, and three-tenths of an inch thick. This is set into a semicircular ring, four-tenths of an inch thick, leaving a space between it and the mirror of two-tenths of an inch; both are connected by two screws, one of which is a clamping screw. Both rings are attached to a circular piece of the same dimensions as the outer piece, $1\frac{1}{2}$ inches long; and to this is permanently fixed a cylindrical piece, $\frac{1}{2}$ inch in diameter and $1\frac{1}{2}$ inches long, into which there is a groove to receive the clamping screw from the tube or socket.

The socket or tube, is 8 inches long, and $\frac{1}{2}$ inch inner diameter, having two clamping screws, one to clamp the whole to the rod or Jacob staff, and the other to allow of the mirror being turned in any direction.

By these three clamping screws, the mirror is raised to any required height, and turned in any direction. The back of the mirror is lined with brass, in the centre of which there is a small hole, opposite to which the silvering is removed. The observer at A sets the centre of the mirror over station A, looks through the hole and through the centre of the cross, and elevates one or both, till he gets an approximate direction of the line, A, B. Our Heliostat, with pouch, weighs but $3\frac{3}{4}$ pounds.

A mirror of 4 inches will be seen at a distance of 40 miles. One of 8 to 10 inches will be seen at a distance of 100 miles.

We use a mirror of 4 inches diameter, fitted up in a superior style by Mr. B. Kratzenstein, mathematical instrument maker, Chicago. Like all his work, it reflects credit on him. We have found it of great use in large surveys, such as running long lines on the prairies, where it is often required to run a line to a given point, call back our flagman,

or make him move right or left. We are indebted to Mr. James Reddy, now of Chicago, formerly civilian on the Ordnance Surveys of Ireland, England and Scotland, for many hints respecting the construction and application of the Heliostat.

Example. Let A be the east and B the west station. Observer A shuts off the reflection at 2h. p. m.—2h. 1m.—2h. 2m., etc., which B observes to agree with his local time 1h.—1h. 1m.—1h. 2m., etc., showing a difference in time of 1h. or 15 degrees of longitude.

The Drummond Light.

This light was invented by Captain Drummond, of the Royal Engineers, when employed on the Irish Ordnance Survey. It is made by placing a ball of lime, about a quarter of an inch in diameter, in the focus of a parabolic reflector. On this ball a stream of oxy-hydrogen gas is made to burn, raising the lime to an intense heat, and giving out a brilliant light. This has been used in Ireland, where a station in the barony of Ennishowen was made visible in hazy weather, at the distance of 67 miles. Also, on the 31st December, 1843, at half-past 3 p. m., a light was exhibited on the top of Slieve Donard, in the County Down, which was seen from the top of Snowdon, in Wales, a distance of 108 miles. On other mountains, it has been seen at distances up to 112 miles. As the apparatus is both burdensome and expensive, and the manipulation dangerous, unless in the hands of an experienced chemist, we must refer our readers to some laboratory in one of the medical colleges. The Heliostat is so simple and so easily managed, that it supersedes the Drummond light in sunny weather. (See *Trigonometrical Surveying*.)

To find the Longitude by Moon Culminating Stars.

394. We set the instrument in the plane of the meridian by Polaris at its upper or lower transit, or its greatest eastern or western elongation, or azimuth. If we cannot use Polaris, take one of the stars in Ursa Minoris at its greatest azimuth, as calculated in Table XXIII. When the instrument is thus set, let there be a permanent mark made at a distance from the station, so as to check the instrument during the time of making the observations. If the instrument be within a few minutes of the meridian, it will be sufficiently correct for our purpose; but by the above, it can be exactly placed in the meridian.

Moon culminating stars are those which differ but little in declination from the moon, and appear generally in the field of view of the telescope along with the moon. We observe the time of meridian passage of the moon's bright limb and one of the moon culminating stars, selected from the Nautical Almanac for the given time.

Let L = longitude of Greenwich or any other principal meridian.

l , longitude of the station.

A, the observed difference of R. A. between the moon's bright limb, and star at L, from Nautical Almanac.

a , observed difference R. A. between the same at the station.

d , difference of longitude.

h , mean hourly difference in the moon's R. A. in passing from L to l .

Then we have

$$d = \frac{A - a}{h}$$

The following example and solution is from Colonel Frome's Trigonometrical Surveying, p. 238. London, 1862.

At Chatham, March 9, 1838, the transit of α Leonis was observed by chronometer at 10h. 20m. 7s.; the daily gaining rate of chronometer being 1.5s. to find the longitude.

Eastern Meridian, Chatham. Observed transits.

		h.	m.	s.
α Leonis,		10	52.46	
Moon's bright limb,		11	20	7.5
		0	27	21.5
On account of rate of chronometer,	—	0	0	0.03
As 24h : 1.5s. : $\frac{1}{2}$ h. : 0.03s.				
		0	27	21.47
Equivalent in sidereal time,	— a ,	0	27	25.96

Western Meridian, Greenwich. Apparent right ascension.

		h.	m.	s.
α Leonis,		9	59	46.18
Moon's bright limb,		10	27	16.76
	A,	0	27	30.58
Observed transits,	a,	0	27	25.96

Difference of sidereal time between the intervals = $A - a = 0 \ 0 \ 4.62$

Due to change in time of moon's semidiameter passing the

meridian, (N. A., Table of Moon's Culminating Stars,) + $0 \ 0 \ 0.01$

Difference in moon's right ascension, $0 \ 0 \ 4.63$

Variation of moon's right ascension in 1 hour of terrestrial longitude is, by the Nautical Almanac, 112.77 seconds.

Therefore, As 112.77 : 1h. : : 4.63s. : : 147.80 = 2m. 27.8s., the difference of longitude.

When the difference of longitude is considerable, instead of using the figures given in the list of moon culminating stars for the variation of the moon's right ascension in one hour of longitude, the right ascension of her centre at the time of observation should be found by adding to or subtracting from the right ascension of her bright limb at the time of Greenwich transit, the observed change of interval, and the sidereal time in which her semidiameter passes the meridian. The Greenwich mean time corresponding to such R. A., being then taken from the N. A. and converted into sidereal time, will give, by its difference from the observed R. A., the difference of longitude required. From above:

		h.	m.	s.
Moon's R. A. at Greenwich transit,		10	27	16.76
Sidereal time of semidiameter passing the meridian	+	0	1	2.26
Moon's R. A. at Greenwich transit,		10	28	19.02
Observed difference,		0	0	4.62
Moon's R. A. at the time, and sid. time at station,		10	28	14.40
Greenwich mean time, corresponding to the above R. A.,				
taken from Nautical Almanac, (Table, Moon's R. A. and				
Dec.,) 11h. 17m. 0.5s., or sidereal time,		10	25	46.5
Difference of longitude,		0	2	27.9

Longitude by Lunar Distances.—Young's Method.

395. In this method we take the altitudes of the moon and sun, or one of the following bright stars, and the distance between their centres. In the Northern Hemisphere we have

α Arietes, α Tauri (*Aldebaran*,) β Geminorum (*Pollux*,) α Leonis (*Regulus*,) α Virginis (*Spica*,) α Scorpii (*Antares*,) α Aquilae (*Altair*,) α Piscis Australis (*Fomalhaut*,) and α Pegasi (*Markab*,)

We observe the moon's bright limb, and add the semidiameter of the moon, sun, or planet, and thereby find the apparent distance between their centres. This has to be corrected so as to find the true altitude and distance of the centres.

The following formula by Professor Young, formerly of Belfast, Ireland, appears to us to be easily applied, by either using the tables of logarithms, or natural sines and cosines, given in Table A.

Let a , a' , and d represent the apparent altitudes and distance of the moon and star. A , A' , and D the true altitudes and distance.

D is the required lunar distance and ∞ = symbol for difference,

$$D = \left\{ \cos. d + \cos. (a + a') \right\} \frac{\cos. (A + A') + \cos. A \infty A'}{\cos. (a + a') + \cos. a \infty a'} - \cos. (A + A')$$

Example from Young's Nautical Astronomy:—

Let the apparent altitude of the moon's centre,	$24^{\circ} 29' 44'' = a'$
The true altitude,	$25^{\circ} 17' 45'' = A$
The apparent altitude of the star = a' ,	$45^{\circ} 9' 12'' = a'$
Its true altitude,	$45^{\circ} 8' 15'' = A'$
The apparent distance of the star and centre of the moon,	$63^{\circ} 35' 14'' = d$

Here we have,

Cos. d = cos. $63^{\circ} 35' 14''$,	nat. cos. 444835
Cos. $(a + a') = \cos. 69^{\circ} 38' 56''$	“ “ 347772
Cos. $d + \cos. (a + a')$	= sum, 792607 = S
Cos. $(A \infty A') = \cos. 19^{\circ} 50' 30'' = \text{nat. cos. } 940634$	
Cos. $(A + A') = \cos. 70^{\circ} 26' 0'' = \text{nat. cos. } 334903$	
Cos. $(A + A') + \cos. (A \infty A')$	sum, 1275537 = S'
and S multiplied by $S' = 127537 \times 792607 =$	P
Cos. $(a + a') = \text{from above, } 347772$	
Cos. $(a \infty a') = \cos. 20^{\circ} 29' 28'' = 935704$	

Cos. $(a + a') + \cos. (a \infty a') = 1283476 = S''$. Divide P by S'' , and it will give .45280, which is the nat. cos. of $63^{\circ} 4' 45'' = D$

396. *Example.* September 2, 1858, at 4h. 50m. 11s., as shown by the chronometer, in Lat. $21^{\circ} 30' N.$, the following lunar observations were taken:—

Height of the eye above the horizon, 24 feet.

Alt. Sun's L.L.	Obs. Alt. Moon's L.L.	Dist. of Near Limbs.
$58^{\circ} 40' 30''$	$32^{\circ} 52' 20''$	$65^{\circ} 32' 10''$
Index cor. + 2 10	+ 3 40	- 1 10
Sun's noon, Dec., at Greenwich, $7^{\circ} 56' 46'' 5 N.$	Diff. for 1 hour, = $-54'' 96$	
Cor. for 4h. 50m.,	- 4 26	5
Dec.	$7^{\circ} 52' 21$	For 5 hours = 27480
	90	For 10 m. = 916
Polar dist.	$82^{\circ} 7' 39$	$60) 26 5 64$
		- $4' 26''$

72H*34

REQUIRED THE LONGITUDE.

Sun's semidiam.	15' 53", 8	Moon's semidiam.	16' 17"
Equa. of time,	25s. 35	Diff. for 1h.,	+ 0' 796
Cor. for 4h. 50m.,	3 85		5
Corrected eq. of time,	29 2	Sub.	
		For 5 hours,	3980
		For 10 m.,	133
			+ 3 847
Moon's Hor. Parallax,	59' 35" 1	Diff. for 12h., = 5" 7	
Cor. for 5 hours,	2"	Diff. for 5h., = 2"	
Hor. Parallax corrected,	59 37		

Minutes and seconds may be easily obtained, but there is a table for furnishing this difference in the Nautical Almanac, p. 520.

The difference between the moon's R. A. at 23h., and at the following noon is by (Naut. Alm.) + 2m. 5s., the proportional part of which, for 7m. 42s., is + 16s.

Also, the difference between the two declinations is - 8' 1", the proportional part of which is 7m. 42s., is 1' 2".

1. For the Apparent and True Altitudes.

SUN.		MOON.	
Obs. Alt. L.L.	58° 42' 40"	Obs. Alt. L.L.,	32° 56' 0"
Dip - 4' 49"	- 4' 49" } + 11 5	Dip, - 4 49	
Semidiam. + 15 54		Semidiam., + 16 17	+ 11 37
Apparent Alt.,	58 53 45	Augment. n + 9	
Refrac.—less parallax,	- 30	Apparent Alt.,	33 7 37
True Alt.,	58 53 15	Cor. for Alt.,	+ 48 26
		True Alt.,	35 56 3

2. For the Mean Time at Ship.

Sun's Alt.,	58° 53' 15"	Tab.	Parts
Lat.,	21 30 0	Compliment of cosine, 0.0312 32	diff. for secants
Pol. dist.,	82 7 39	" " 0.041 24 29 -	1131

2) 162 30 54

½ sum, =	81 15 27	cosine,	9.182196	1369 -	36962
½ sum - alt.	22 22 12	sine,	9.580392	511 +	6132
			18.798034		31962
			320		

2) 18.797714

½ hour angle 14° 30' 31½"	sine,	9.398857	
Hour angle, 29 1 3	=	1h. 56m. 4s.,	apparent time at ship.
Equa. of time,		29	
Mean time at ship,		1h. 55m. 35s.	

3. For the True Distance, the G. Time, and the Longitude.

Obs. dist.	65° 01' 0"	Appt. dist.	66° 3' 20" nat. cos.	403850 = y
Sun's semi,	+ 15 54	Appt. alt.	58 53 45	
Moon's + Augm. +	16 26		33 7 37	

Sum, 92 1 22 nat. cos. - 035297 = x

Multiplier = y - x = 370553

True Alt., $\left\{ \begin{array}{l} 58^{\circ} 53' 15'' \\ 33 \ 56 \ 3 \end{array} \right.$ Diff. $25^{\circ} 46' 18''$ nat. cos. $900556 + = W$
 $w - x = 865259 = \text{Divisor,}$

Sum, $92 \ 49 \ 18$ nat. cos. $- 049228 = v$
 Diff. $24 \ 57 \ 12$ nat. cos. 906652

Multiplier, 370553 , inverted = 857424 Multiplicand.
 355073 Multiplier.

NOTE.—This rapid method is } 2672272
 done by throwing off a figure } 600197
 in each line as we proceed. } 4287
 429
 26

Divisor, 865259 3177211 { $367198 = \text{Quotient.}$
 2595777 { $+ 049228 = v$

NOTE.—The division is abridged } 581434 416426
 by rejecting a figure each time, } 519155
 in the divisor. } 62279 nat cos. $65^{\circ} 23' 27''$.
 60568
 1711
 865
 846
 779
 67
 69

True distance, $65^{\circ} 23' 27''$
 Dist. at 3h. (Naut. A.) $66 \ 24 \ 23$ Proportional Log. of diff. 2537
 $1 \ 0 \ 56$ " " 4704
 Interval of time, $1\text{h. } 49\text{m. } 18\text{s.}$ P L = 2167
 $+ 1$

Mean time at Green., $3\text{h. } + 1 \ 49 \ 19$
 $1 \ 55 \ 35$

Long. W. in time, $2 \ 53 \ 44$ Long. = $43^{\circ} 26' W$.

And the error of the chronometer is 52s. fast on Greenwich mean time.

A base line is selected as level as can be found, and as long as possible, this is lined, leveled, and measured with rods of Norway pine, with platinum plates and points to serve as indices to connect the rods. They are daily examined by a standard measure, reference being had to the change of temperature. (See p. 165.) At each extremity stones are buried, and at the trig. points are put discs of copper or brass, with a centre point in them. From these extreme points angles are taken to points selected on high places, thus dividing the country into large triangles, and their sides calculated.

These are again subdivided into smaller triangles, whose sides may range from one mile to two miles. These lines are chained, horizontally, by the chain and plumb-line; or, as on the ordnance survey of Ireland, the lines of slopes are measured, and the angles of elevation and depression taken. Spires of churches, angles of towers and of public buildings are observed.

On the main lines of the triangles, the heights of places are calculated from the field book, and marked on the lines. When inaccessible points are observed from other points, we must take a station near the inaccessible one, and reduce it to the centre by (sec. 244.) On the second or third pages of the field book, we sketch a diagram of the main triangle, and all chain lines, with their numbers written on the respective lines, in the direction in which the lines were run. The main triangle may be subdivided in any manner that the locality will allow. See Fig. 64 is the best.

Here we have three check-lines, D F, D E, and F E, on the main triangle, and having the angles at A, B, and C, with the distances, A D, D C, C E, B E, B F, and F D, we can calculate F D, D E, and F E, insuring perfect accuracy. We chain as stated in Section 211.

In keeping our field book we prefer the ordnance system of beginning at the bottom, and enter toward the top the offsets and inlets, stating at what line and distance we began, and on what; we note every fence and object that we pass over or near; leave a mark at every 10 chains, or 500 feet, and a small peg, numbered as in the field book.

398. See the diagram (figure 65).

Here we began 114 feet farther on line 1 than where we met our picket and peg at 3500 feet, and closed on line 3 at 870, where we had a peg and a long Isoceles' triangle dug out of the ground.

We write the bearings of lines as on line 3, and also take the angles, and mark them as above.

When there are Woods. Poles are fastened to trees, and made to project over the tops of all the surrounding ones. The position of these are observed or *Triggered*. The roads, walks, lakes, etc., in these woods can be surveyed by traversing, closing, from time to time, on the principal stations or Trig. points, but we require one line running to one of the forest poles, on which to begin our traverse, and continue, closing occasionally on the main lines and Trig. points.

399. *Traverse Surveying.* See Secs. 216, 217, 255.

The bearing of the most westerly station is taken. At Sec. 216 is given a good example where we begin at the W. line of the estate, making its bearing 0, and the land is kept on the right. There we began with zero and closed with 180, showing the work to close on the assumed bearing.

400. *To Protract these Angles at Sec. 216.* Draw the line A B through the sheet; let A be S, and B, N. On this lay of other lines parallel to A B, according to the number of bearings, size of protractor and scale. We lay down A B, then from B set off four, five, or more angles, L, K, I, and H. Lay the parallel ruler from A to L, draw a line and mark the distance A L of the second line on it. Lay the ruler from A to K, move one edge to pass through L, draw a line, mark the third line L K on it. Lay the ruler on A I, move the other edge to pass through K, draw the line K I, equal to the fourth line. Lay the ruler on A to H, make the other edge pass through I, and mark the fifth line, I H. Now, we suppose that we are getting too far from our first meridian, A B. We now remove the protractor to the next meridian, and select a point opposite H, and then lay off the bearings, G, F, E, D, etc.

Now, from this new station, which we will call X, we lay the parallel ruler to F and make the other edge pass through H, and set off the sixth line H G. Lay the parallel ruler from X to F, and move the other edge through G, and mark the seventh line, G F, and so proceed.

We have used a heavy circular protractor made by Troughton & Simms, of London, it is 12 inches diameter, with an arm of 10 inches, this, with a parallel ruler 4 feet long, enabled us to lay down lines and angles with facility and extreme accuracy.

401. By a table of tangents we lay off on one of the lines, A B, the distance, 20 inches, on a scale of 20 parts to the inch. Then find the nat. tangent to the required angle, and multiply it by 400 divisions of the scale, it will give the perp., B C, at the end of the base. Join A and C, and on A C lay off the given distance, and so proceed.

By this means we can, without a protractor, lay off any required angle.

REGISTERED SHEET FOR COMPUTATION.

Plans and Plats.	Triangles and Trapeziums.	1st side.	2d side.	3d side.	Contents in Chains.
Plat 1 Division K of Thos. Linskey's Farm.	Triangle A C B,	4454 lks	3398	4250	679.5032
	" A F D,	2234	1766	1684	143.0516
	On line D F,	2234	10	98	0.0490
	Additives,	90	70	400	3.2000
		70	50	900	5.4000
			50	600	1.5000
			Total Additives,		158.2006
	Div. K, Negatives, D F,		20	140	1400
		20	100	260	9600
		100	80	500	4.5000
			80	500	2.0000
					7.6000
					150.6006

Area, 15.06006 Acres.

There is always a content plat or plan made, which is lettered and numbered, and the Register Sheet made to correspond with it.

403. *Computation by Scale.* Where the plats or maps for content are drawn on a large scale, of 2 or 3 chains to the inch, we double up the sheet by bringing the edges together. Draw a line about an inch from the margin; on this line mark off every inch, and dot through; now open the sheet and draw corresponding lines through these dots; make a small circle around every fifth one, and number them in pencil mark.

Lines are now drawn through the part to be computed. Where every pair of lines meet the boundaries, the outlines are then equated with a piece of thin glass having a perpendicular line cut on it, or, better, with a piece of transparent horn. When all the outlines of the figure are thus equated, we measure the length in chains, which, multiplied by the chains to one inch, will give the content in square chains. This gives an excellent check on the contents found by triangulation or traversing. It will be very convenient to have a strip of long drawing paper, on the edge of which a scale of inches is made. We apply zero to the left-hand side of the first parallel, and make a mark, a, at the other end; then bring mark a to the left side of the second parallelogram, and make a mark, b, at the other end, and so continue to the end. Then apply the required scale to the fractional part, to find the total distance.

The English surveyors compute by triangulation on paper, and sometimes by parallels having a long scale, with a movable vernier and cross-hairs, to

equate the boundaries. We do not wish to be understood as favoring computation from paper.

The Irish surveyors always draw the parallel lines on the content plat or map, and mark the scale at three or four places, to test the expansion or contraction of the sheet during the construction or calculation. We prefer, when possible, 3 chains, or 200 feet, to an inch for estates in the country, and 40 feet for city property.

403a. *Division of Land.*

When the area A is to be cut off from a rectangular tract, the sides of which are a and b . Then corresponding sides of the tract,

$$S = \left\{ \frac{A}{a} \text{ and } \frac{A}{b} \right\} \text{ respectively, the required side, } S.$$

404. When the area A, = triangle A D E, is to be cut off from the triangle A C B, by a line parallel to one of its sides. (Fig. 66.)

Then triangle A B C : triangle A D E :: A B² : A D².

405. From a given point, D, in the triangle, A B C, to draw a line, dividing it into two parts, as A and B. (See Fig. 66.) We find the angle A B C. By (Sec. 29,) A D × A E × $\frac{1}{2}$ sin. A = area B

$$(i. e.) A D \times A E, \sin. A = 2 B$$

$$A E = \left\{ \frac{2 B}{A D. \sin. A} \right\}$$

NOTE.—We prefer this to any other complicated formula, in cutting off a given area from a quadrilateral or triangular field.

406. When the area B or A is to be cut off by the line D E, (Fig. 66,) making a given angle, C, with the line A B, let area = S.

Let the angle at A = b , that at D = c , and that at E = d , and A D, the required side.

$$A D = x, \text{ and } A E = \frac{\sin. c. x}{\sin. d}$$

$$D E = \frac{\sin. b. x}{\sin. d} \text{ but } A D \times D E \times \frac{1}{2} \sin. c = \text{Area} = B$$

$$\frac{\sin. b. x}{\sin. d} \cdot \sin. c. x = 2 B$$

$$x^2 \sin. c. \sin. b = 2 B \sin. d$$

$$A D = x = \left\{ \frac{2 B, \sin. d}{\sin. c, \sin. b} \right\}^{\frac{1}{2}}$$

From the value of x we find A E and D E from above.

Having A D and A E from these formulas, let us assume A D = 10 chains, and having found the value of A E by substituting 10 chains for x .

Multiply the numerical value of A E by 10 chains, and again by $\frac{1}{2}$ the natural sine of the angle D A B, let its area = s , L, Then $s : S :: A D^2 : \text{the required } A E^2$,

$$s : S :: 100 : A D^2.$$

As s , S , and 100 are given, we have

$$A D = \left\{ \frac{100 S}{s} \right\}^{\frac{1}{2}}$$

This useful problem was proposed to us in Dublin, at our examination for Certified Land Surveyor, September, 1835, by W. Longfield, Esq., Civil Engineer and Surveyor.

NOTE.—When the given area is to be cut off by the shortest line, D E, in the triangle A D E, (Fig. 66.) then A D = D E.

407. When the area B is to be cut off by the line D E, starting from the point D. (Fig. 66.)

$$A D = \frac{2 B}{A E \sin. A} \quad A E = \frac{2 B}{A D \sin. A}$$

408. From the quadrilateral, (Fig. 67,) A B C D, to cut off the area A by the line F E, parallel to the side B C.

Produce the lines B A and C D to meet at G. Take the angles at B, C, D and A, and, as a check, take the angle G. Measure G D and G A. We have the area of the quadrilateral = A + B, and of the triangle G D A = C, and the line G B is given. By Sec. 404 we find the line A F or G E. For triangle G C B : triangle G F E :: G B² : G F² or :: G C² : G E².

By taking the square roots we find G F and G E.

409. To divide any quadrilateral figure into any number of equal parts, by lines dividing one of the sides into equal parts.

Let A B C D be the required figure, (see Fig. 70,) whose angles, sides, and areas are given, produce the the sides C D and B A to meet in E. As the angles at A and D are given, we find the angle E, and consequently the sides A E and D E, and area B of the triangle A E D. We have the distances E A, E F, and E G, and areas B + A = triangle E F K, and B + 2 A = triangle E G H : and by Sec. 29.

$$E K = \frac{F E \cdot \frac{1}{2} \times s}{B + A} \quad \text{and} \quad E H = \frac{B + 2 A}{G E \cdot \frac{1}{2} \sin. E}$$

410. If, in the last problem, it were required to have the sides B A and C D proportionally divided so as to give equal areas,

Let B A = a, C D = n a, A E = b, D E = c, and $\frac{1}{2} \sin. E = S$, and x = A F, then we have, by Sec.

$$(b + x)(c + n x) = \frac{A}{S} \quad \text{from which we have}$$

$$b c + (b n + c) x + n x^2 = \frac{A}{S}$$

$$x^2 + \left\{ \frac{b n + c}{n} \right\} x = \frac{A - b c s}{s} \quad \text{put } \frac{b n + c}{n} = 2 m, \text{ and complete}$$

the square, and find the square root.

$$x + 2 m x + m^2 = \frac{A - b c s + m^2}{s}$$

$$x = -m \pm \sqrt{\frac{A - b c s + m^2}{s}} = A F \text{ and } n \times A F = K D.$$

In like manner we find the points G and H.

411. *Contouring.* (Fig. 70a.)

Three points forming the vertexes of a triangle, A B C, whose altitudes above the sea, or *datum* line, are given. Lines are chained from A to B, B to C, and C to A, and stations marked at given distances, and contour points made at every change of altitude equal to 10, 20, or 30 feet. Lines are chained down the side of the hill, and connected with check-lines. The level of station *a* is carried around the hill, showing where the contour line intersects each chain line, to the place of beginning. Begin again at the next station, *b*, below, and proceed as in the above, and so to the lowest station. The contour lines will be the same as if water raised to different heights around the hill, leaving flood-line marks on the hill. The plotting is similar to triangular surveying. The shading of the hill requires practice.

Final Examination. When a plan is ready for final examination, tracings are taken, of such size as to cover a sheet of letter paper, or white card-board of that size, made to fit an ordinary portfolio. In the field, the examiner puts himself in the direction of two objects, such as fences or houses, and paces the distance to the nearest fixed corner, and, by applying his scale, he can find if it is correct; by these means he will detect all omissions and errors. He will be able to put on the topography of the survey. He generally finds pacing near enough to discover errors, but where errors occur, he chains the required distances.

412. In plotting in detail we use two scales, one flat, 12 inches long, but having the same scale on both sides, such as one chain to an inch, or three chains to an inch. The other scale is 2 inches long, for plotting the offsets graduated on both sides of the index in the middle, ends not beveled. If the index is one inch from each end, we draw a line parallel to the chain line, one inch distant. If the index is two inches, we draw it two inches from the line. On each end of the small scale we have, at two chains' distance, lines marked on it to check the reading on the large scale. At each end of the chain line, perpendiculars are drawn to find the point of beginning. The large scale in position, the small one slides along its edge to the respective distances where the offset can be set off on either side of the chain line.

413. *Finishing the Plans or Map.*

Indian ink, made fresh, to which add a little Prussian blue, expose to the sun or heat for a short time, to increase its blackness.

1 and 2. FORESTS AND WOODS.—*Jaunne jonquille*, composed of gum gamboge, 8 parts; Prussian blue, 3 parts; water, 8 parts. The woods have not the trees sketched as heavily as forests.

3. BRAMBLES, BRIARS, BRUSHWOOD.—Same as No. 1, but lighter, by adding 4 parts of water.

4. TURF-PIT.—The water pits by Prussian blue, and the bog by sepia and blue.

5. MEADOWS OR PRAIRIES.—Prussian blue, 6 parts; gamboge, 2 parts; and water, 8 parts.

6. SWAMP.—In addition to dashes of water, we pass a light tint of Prussian blue.

7. CULTIVATED LAND.—Sepia, 6 parts; carmine, 1 part; gamboge, $\frac{1}{2}$ part.

8. CULTIVATED LAND, BUT WET.—Same as above, except that dashes of water are marked with blue.

9. TREES.—Same as 1 and 2; sketched on, and shaded with sepia.
10. HEATH, FURZE.—Une teinte panachée, nearly green, and light carmine.
Teinte panachée is where two colors are taken in two brushes, and laid on carefully, coupled together.
11. MARSH.—The blue of water, with horizontal spots of grass green, or to No. 5 add 2 parts of water.
12. PASTURES.—To No. 5 add 4 parts of water.
13. VINEYARDS.—Carmine and Prussian blue in equal parts.
14. ORCHARDS.—Prussian blue and gamboge in equal parts.
15. UNCULTIVATED LAND, FILLED WITH WEEDS.—Same as No. 3.
16. FIELDS OR ENCLOSURES.—Walled in are traced in carmine, and if boarded, in sepia. Hedges, same as for forests, to which is added 2 parts of green meadow.
17. HABITATIONS.—A fine, pale tint of carmine, light, for massive buildings, and heavier for house of less importance.
18. VEGETABLE GARDENS.—Each ridge or square receives a different color of carmine, sepia, gamboge—the color for woods and meadows.
19. PLEASURE GARDENS, FLOWER GARDENS.—Are colored with meadow color, and wood color for massive trees; the alley, or walks, are white, or gamboge with a small point of carmine.
20. The colors used are, generally, Indian Ink, Carmine, Gamboge, Prussian Blue, Sepia, Minum, Vermillion, Emerald Green, Cobalt Blue, Indian Yellow.

414. *Levelling.*

THE ENGLISH AND IRISH BOARDS OF WORKS METHODS.

DISTANCES.					Height of Collimation.	Reduced Level.	Required Grade.	Cut.	Fill.	REMARKS.
Left.	Centre.	Right.	Staff.	Soundings.						
60	10.00		2.44		97.03	94.59	90.60	3.99		Bench Mark, 94.59, at Station, 900 ft.
	10.50		8.84		97.03	88.19				
	11.00		2.83		97.03	94.20	90.50	3.70		
174	11.50		0.74		94.94					Bank of Creek, Middle of Creek.
	12.00		2.18		94.94	92.76		2.36		
	13.00		5.35	8.30	94.94	89.59				
	14.00		6.77		96.36					B.M., Peg and Stake in Meadow.
	15.00		3.57		96.36	92.79	90.10		2.69	
	15.00	120	6.32		96.36	90.04				
	15.00	136	8.27		96.36	88.09				
	15.70	136	2.63		96.36	93.73				

This method of keeping a field-book was used by the English and Irish Board of Works. Size of books 8 by 6¼ inches.

Many Engineers there kept their books thus: ruled from left to right, Back Sights, Fore Sights, Rise, Fall, Reduced Level, Distance, Permanent Reduced Levels, and Remarks. Book, 7½ by 5 inches.

414a. Colonel Frome, Royal English Engineer, in his Treatise on Surveying, gives, from left to right, Distances, B. S., F. S., +, -, Rise, Fall, Remarks. The columns Rise and Fall show the elevation at any station above *datum*, that assumed at the beginning.

Sir John McNeill's plan of showing the route for the road, and a profile of the cutting and filling on the same: the line is not less than a scale of 4 inches to 1 mile, and the vertical sections not less than 100 feet to an inch.

415. *Our Method.*

Left	Dist. Centre	Right.	Bench.	B. S.	Height of Collimation.	F. S.	Elevation.	Grade.	Cut.	Fill.	REMARKS.
	00		50.0	10.0	60.00	4.00	56.00	60.00		4.00	Bench Mark, nail in roof of ash tree at the S.-W. corner of Thos. King's garden.
	100					4.50	55.50	60.30		4.80	
	200					9.00	51.00	60.60		9.60	
	300					11.50	48.50	60.90		11.40	
	400		48.50	8.00	56.50	0.10	56.40	61.20		4.80	NOTE.—At 697 came to bank of Flaskagh river, 2 feet above level of water; depth, 8 feet, width, 67 feet. (Here make a sketch showing depth of water for every 10 feet.)
	500					0.20	56.30	61.50		5.20	
	600					0.30	56.20	61.80		5.60	
	600	30				1.10	55.40	61.80		6.40	
	600	90				2.40	54.10	61.80		7.70	
	600					0.40	56.10	61.80		5.70	
	600					0.70	55.80	61.80		6.00	
	697					30.90	25.60	62.10		36.50	
40	764					22.40	34.10	62.20		28.10	B. M. at elevation 38.50, opposite distance, 870 feet, on the S.-W. corner of John McCabe's residence.
80	800					20.10	36.40	62.40		26.00	
	850					18.00	38.50				
	900		38.50	6.50	45.00	5.40	39.60	62.70		23.10	
	1175		38.50			7.10	37.90	63.22		25.32	B. M., 38.00, door-step of Mr. Jas. Roger's school-house.
	1175	170	38.50			7.00	38.00				

B. M. + Back sights = 50.0 + 10. + 8. + 6.50 = 74.50
 F. sights at turning points 11.50 + 18.00 + 7.00 = 36.50

Proof. 38.00

416. *Levelling by Barometrical Observations.*

BAROMETRICAL MEASUREMENT OF HEIGHTS. — BAILY.

TABLE A.

THERMOMETERS IN OPEN AIR.

$t+t'$	A	$t+t'$	A	$t+t'$	A	$t+t'$	A	$t+t'$	A
1	4.74914	37	4.76742	73	4.78497	109	4.80183	145	4.81807
2	966	38	792	74	544	110	229	6	851
3	4.75017	39	842	75	592	1	275	7	895
4	069	40	891	76	640	2	321	8	939
5	120	41	941	77	688	3	367	9	983
6	172	42	990	78	735	4	412	150	4.82027
7	223	43	4.77039	79	783	5	458	1	071
8	274	44	089	80	830	6	504	2	115
9	326	45	138	81	878	7	550	3	159
10	377	46	187	82	925	8	595	4	203
11	428	47	236	83	972	9	641	5	247
12	479	48	285	84	4.79019	120	687	6	291
13	531	49	334	85	066	1	732	7	335
14	582	50	383	86	113	2	777	8	379
15	633	51	432	87	160	3	822	9	423
16	684	52	481	88	207	4	867	160	466
17	735	53	530	89	254	5	912	1	510
18	786	54	579	90	301	6	957	2	553
19	837	55	628	91	348	7	4.81002	3	596
20	888	56	677	92	395	8	047	4	640
21	938	57	726	93	442	9	092	5	683
22	989	58	774	94	488	130	137	6	727
23	4.76039	59	823	95	535	1	182	7	770
24	090	60	871	96	582	2	227	8	813
25	140	61	919	97	629	3	272	9	857
26	190	62	968	98	675	4	317	170	900
27	241	63	4.78016	99	722	5	362	1	943
28	291	64	065	100	768	6	407	2	986
29	342	65	113	101	814	7	452	3	4.83030
30	392	66	161	102	860	8	496	4	073
31	442	67	209	103	907	9	541	5	116
32	492	68	257	104	953	140	585	6	159
33	542	69	305	105	999	1	630	7	201
34	592	70	352	106	4.80045	2	675	8	244
35	642	71	400	107	091	3	719	9	287
36	4.76692	72	4.78449	108	4.80137	144	763	180	329

NOTE. t = temperature of the air at the lower station; t' = that at the upper station; A = correction for temperature, dependent on $t + t'$.

And for Table B.: r = temperature of mercury at the lower station; r' = that at the upper station; B = correction due to the mercury dependent on $r - r'$; C = correction for the latitude of the place; D = latitude; R = height of barometer at lower station; R' = height of barometer at upper station. For Table B. see next page.

BAROMETRICAL MEASUREMENT OF HEIGHTS.

TABLE B.

417. ATTACHED THERMOMETERS.

$r - r'$	B	$r - r'$	B	$r - r'$	B	Lat.	C
0	0.00000	20	0.00087	40	0.00174	0	0.00117
1	04	21	91	41	78	5	115
2	09	22	96	42	82	10	110
3	13	23	100	43	87	15	100
4	17	24	104	44	91	20	090
5	22	25	0.00109	45	95	25	075
6	26	26	13	46	0.00200	30	058
7	30	27	17	47	04	35	040
8	35	28	22	48	08	40	020
9	39	29	26	49	13	45	0.00000
10	43	30	30	50	17	50	9.99980
11	48	31	35	51	21	55	62
12	52	32	39	52	26	60	42
13	56	33	43	53	30	65	25
14	0.00061	34	48	54	34	70	10
15	65	35	52	55	39	75	9.99900
16	69	36	56	56	43	80	890
17	74	37	61	57	47	85	85
18	78	38	65	58	52	90	9.99883
19	0.00083	39	0.00169	59	0.00256		

418. *Example from Colonel Frome's* TRIGONOMETRICAL SURVEYING.

SURVEYING p. 110.				Correction for Capacity.	Corrected Barometer.	Diff. of Level.	Remarks.
Stations.	A F	D F	Bar.				
High Water Mark	61°	58°	30.405	.004	30.409		
Parade, Brompton Barracks, . . .	60°	57°	30.276	.002	30.278	116.6	

$t + t' = 58 + 57 = 115$. From Table A = 4.80458

$r - r' = 61 - 60 = 1$. B = 0.00004 = m

Lat. $51^{\circ} 24'$ 9.99974 = n

Log. of R = Log. 30.409 1.48300 = p

Log. of R' = Log. 30.278 1.48117 = q

+ B 00004 {

D = 0.00183 = $p - q$

Log. D = 3.26245

A = 4.80458

C = 9.99974

2.06677 = 116.6 = altitude in feet, which was found by the spirit level to be 115 feet.

These Tables are from the Smithsonian Meteorological and Physical Tables, published in Washington, 1858.

In 1844, in Ottawa, Canada, Mrs. McDermott, in my absence, kept a record of numerous observations of the state of thermometer and mountain barometer, for Sir William Logan, Provincial Geologist, then making a tour of the valley of the River Ottawa and its tributaries. (See his Geological Reports.) The observations were made at the hours of 7, 9, noon, 3, and 6, to be used for the lower Station, at Montreal.

419. *To find the Altitude of one Station above another, from the Temperature of the Boiling of Water.*

This method is not so reliable as that by barometrical observations, although Colonel Sykes, in Australia, has found altitudes above the sea agree with those found by triangulation closer than he had anticipated.

There are very valuable tables in the Smithsonian Institute's Meteorological and Physical Tables—Tables XXIV, XXV, and XXVI—for finding the altitudes by this method.

Take any tin pot and lay a piece of board across the top, having groove to receive the thermometer, and a button or slide to keep it steady, at about two inches from the bottom. Take several observations, carefully noting them, and at the same time the temperature of the surrounding air. Use Fahrenheit's thermometer.

TABLE A.

TABLE B.

Boiling point of Water.	Pressure of Barometer in inches.	Altitude above the Sea, (from 30 in.)	Temperature of the air.	Multiplier.	Temperature of the air.	Multiplier.
185°	17.048	14.548	32°	1.000	62°	1.062
86	.423	13.977	33	2	3	64
87	.809	.408	34	4	4	66
88	18.195	12.843	35	6	5	69
89	.592	.280	6	8	6	71
190	.996	11.719	7	10	7	73
91	19.407	.161	8	12	8	75
92	.825	10.606	9	15	9	77
93	20.251	10.053	40	17	70	79
94	.685	9.502	1	1.019	71	81
95	21.126	8.953	2	21	2	1.083
96	.576	8.407	3	23	3	85
97	22.033	7.864	4	25	4	87
98	.498	7.324	45	27	75	89
99	.971	6.786	6	29	6	91
200	23.453	6.250	7	31	7	94
1	.943	5.716	8	33	8	96
2	24.442	5.185	9	35	9	98
3	.949	4.657	50	37	80	1.100
4	25.465	4.131	1	1.039	81	102
5	.990	3.607	2	42	2	104
6	26.523	3.085	3	44	3	106
7	27.066	2.566	4	46	4	108
8	27.618	2.049	55	48	85	110
9	28.180	1.534	6	50	6	112
10	28.751	1.021	7	52	7	114
11	29.331	.509	8	54	8	116
12	29.922	0	9	56	9	117
13	30.522	.507	60	58	90	121
214	31.194	1.013	61	1.060	91	123

NOTE.—The first two columns of Table B are from the Smithsonian Tables referred to above. The third column and Table B is from Col. Frome's Treatise on Trigonometrical Surveying, 3d edition, 1862.

Example.—Boiling point, upper station, 209°, lower, 202°; temperature of the air at upper station, 72°, lower, 84°, mean temperature, 78°.

From Table A, 209°. Alt., 1534 ft.
202. " 5185

Approximate height, 3651
Mean temperature, 78°. Multiplier from Table B, 1096

Product, 4001 ft.

Where the degrees are taken to tenths, then we interpolate.

419a.—*Continued from Sec. 410. Having one side, A B, and the adjacent angles,—to find the area*—Let the triangle A B C (Fig. 68,) be the triangle; the side A B = s , and the angles A and B are given, also the angle C.

$$\text{Sin. C} : S :: \text{sin. A} : B C = \frac{S \cdot \text{Sin. A}}{\text{Sin. C}}, \text{ and } A C = \frac{S \cdot \text{Sin. B}}{\text{Sin. C}}$$

$$\text{By Sec. 29. } S \cdot \frac{S \cdot \text{Sin. A}}{2 \text{ Sin. C}} \cdot \text{Sin. B} = \frac{S^2 \cdot \text{Sin. A} \cdot \text{Sin. B}}{2 \text{ Sin. C}} = \text{area.}$$

420. *From a point, P, within a given figure, to draw a line cutting off any part of it by the line F G.*—Let the figure I G B A E = the required area. (See Fig. 69.)

Let the A B C D E F the tract be plotted on a scale of ten feet to an inch, from which we can find the position of the required line very nearly, with reference to the points B and E. Run the assumed line, A S, through P, finding the distances A P = m and P S = n , also the angles P T A, P S G, and that the tract A S B A T is too great, by the area d . Hence the true line, T P G, must be such that the triangle P S G - P A T = d .

Assume the angle S P G = P, then we find the angles T and G, and by Sec. 409 we find the areas of the triangles P S G and P A T. If the difference is not = d , again, calculate the sides P G and P T.

420a. *From the triangle A B C to cut off a given area (say one-third,) by a line drawn through the given point, D.* (Fig. 69a.)

Through D draw the line D G parallel to A C.

Now all the angles at A, B, and C are given, and the line D G is given to find the point I or H, through which, and the given point D, the line I D H will cut off the triangle A H I = to one-third the area of the triangle A B C. (Fig. 69a.) Make A F one-third of A C, then the triangle A B F = one-third of the triangle A B C, which is to be = to triangle A I H.

The triangle A H I = A H × A I × $\frac{1}{2}$ Nat. Sin. angle A.

The triangle A B F = A B × A F × $\frac{1}{2}$ Nat. Sin. A.

$$A H \times A I = A B \times A F, \text{ and } A I = \frac{A B \times A F}{A H} \text{ and as the triangles H G D and H A I are similar.}$$

$$H G : G D :: H A : A I :: H A : \frac{A B \times A F}{H A}$$

H G : G D :: H A² : A B × A F, and by Euclid, 6-16,

$$G D \times H A^2 = H G \times A B \times A F = (H A - A G) \cdot A F \cdot A B \\ = H A \cdot A F \cdot A B - A G \cdot A F \cdot A B$$

$$\text{and } H A^2 = \frac{A B \cdot A F}{G D} \cdot H A - \frac{A B \cdot A F}{G D} \cdot A G. \text{ Let } P = \frac{A B \cdot A F}{G D}$$

Now we have P and A G given, to find A H or A I,

$$A H^2 = P \times H A - P \times A G$$

$$H A^2 = P \times H A - P \times A G. \text{ Complete the square}$$

$$H A^2 - P \times H A + \frac{P^2}{4} = \frac{P^2}{4} - P \times A G.$$

$$H A - \frac{P}{2} = \left\{ \frac{P^2}{4} - A G \times P \right\}^{\frac{1}{2}} \text{ Whence}$$

$$A H = \frac{1}{2} P + \left(\frac{1}{4} P^2 - A G \times P \right)^{\frac{1}{2}}, \text{ when D is inside the triangle.}$$

$$A H = \frac{1}{2} P + \left(\frac{1}{4} P^2 + A G \times P \right)^{\frac{1}{2}}, \text{ when } d \text{ is outside.}$$

421. Through the point D to draw the line G D E so that the triangle B G E will be the least possible. Through D draw H D I parallel to B C, make B H = H G, and draw G D E, which is the required line. Fig. 69a.

Geodetical Jurisprudence, p. 72, B.

Chief Justice Caton's opinion adds the following in support of established lines and monuments:—

Dreer *v.* Carskaddan, 48 Penn. State, 28.

Bartlett *v.* Hubert, 21 Texas, 8.

Thomas *v.* Patten, 13 Maine, 329.

To Divide pro rata.

After Bailey *v.* Chamblin, 20 Indiana, 33, add

Jones *v.* Kemble, 19 Wisconsin, 429.

Francoise *v.* Maloney, Illinois, April Term, 1871.

Withham *v.* Cutts, 4 Greenleaf R., Maine, 9.

309Me. After English Reports, 42, p. 307, add

Knowlton *v.* Smith, 36 Missouri, 620.

Jordan *v.* Deaton, 23 Arkansas, 704.

United States Digest, Vol. 27—where an owner points out a boundary, and allows improvements to be made according to it, cannot be altered when found incorrect by a survey.

For Laying Out Curves.

Example after p. 72. Let radius = 2000 feet; chord, 200; then tangential angle = $2^{\circ} 51' 57''$; versed sine at the middle, 2,503 feet. If the ground does not admit of laying off long chord of 200 feet, make $200 = 200$ half feet = 100, then for radius 4000 find the versed sine = 1,251 and the tang. angle = $1^{\circ} 25' 57''$. If we use the chord of 200 feet, half feet, or links, then we are to take the ordinates in Table C as feet, half feet, or links.

Canals.

The Illinois and Michigan locks are 128 feet long, 18 feet wide, and 6 feet deep, bottom 36, surface 60, tow-path 15, berm 7, tow-path above water, 3 feet.

The New York Canals.—Eric Canal, 363 miles long, when first built, 40 feet at top, 28 at bottom, 4 feet deep, 84 locks, each 90x15, lockage 688, 8 large feeders, 18 aqueducts. The aqueduct across the Mohawk is 1188 feet in length.

The Pennsylvania Canal—top 40, bottom 28, depth 4, locks 90x15, and some, 90x17.

The Ohio and Erie Canal—40 feet at top, 4 feet deep.

Rideau Canal, in Canada—129½ miles long, 53 locks, each 134x33.

Welland Canal, in Canada—locks, large enough to admit large vessels. It is now in progress of widening and deepening, so as to admit of the largest vessels that may sail on the lakes, and to correspond with the canals and lakes at Lachine, and on the River St. Lawrence.

CORRECTIONS.

Page 43, example 2, read the polygon a b c d e f g h, Fig. 38.

Page 72B53, *soda* No O read *soda* Na O.

72B55, 4th line, read felspathic.

72B111, after the 8th line insert Sir William Bland makes it as 17 to 13, egg-shaped.

72s, begin at 8th line from bottom and put mean base = $50 + 40 = 90$

50

4500

4100

Difference, square feet, 400

72r, in 4th equation from bottom read solidity $s = (A \times a + \sqrt{Aa}) \frac{1}{3}$

$$s = (D^2 + d^2 + Dd) \cdot 0.2618 \cdot h$$

$$s = \frac{D^3 - d^3}{D - d} \times \frac{1}{3} = \left\{ \frac{D^3 - d^3}{D - d} \right\} \times 0.2618 \cdot h$$

72w, in 3d equation from the bottom read Because $\frac{b}{2r}$

72H*, at 16th line from bottom, for $rS - bA$, read $rS + bQ$.

72H*10, at 14th from bottom, for product of the adjacent parts, read product of tan of adjacent parts.

72H*24, Sec. 388, for apparent, read mean.

72H*30, by the Helio-stat, insert after Helio-stat Fig. II.

72R*, under 82°, opposite 48, for 2921 put 9921.

104, under 2, opposite 12, make it 1.9300, and opposite 13 make it 1.9199.

110, under 2, opposite 12, put 1.8999.

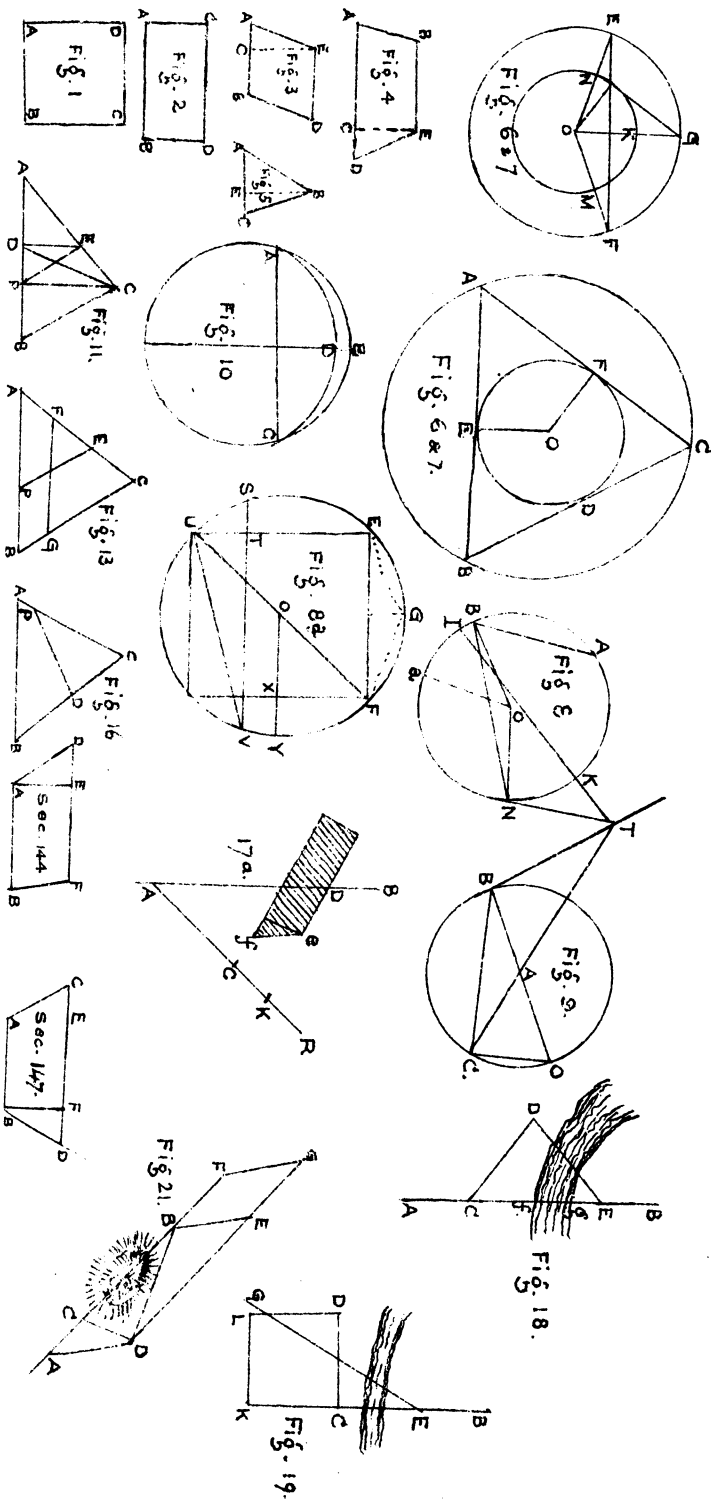
113, under 9 put 2.9722.

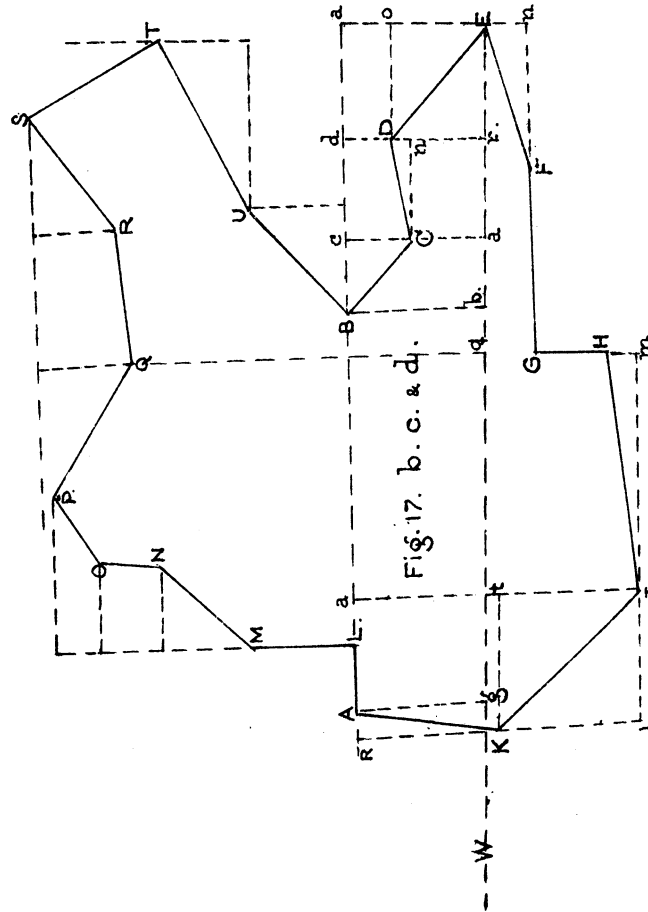
264, after the words *From the above we have* insert

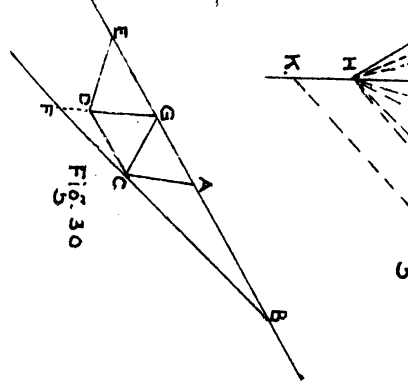
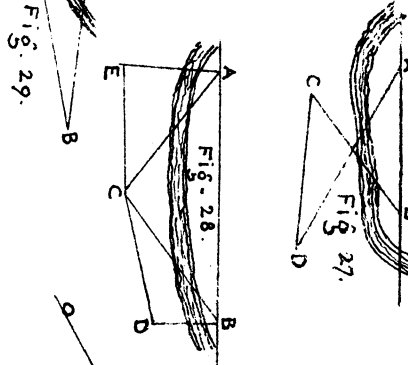
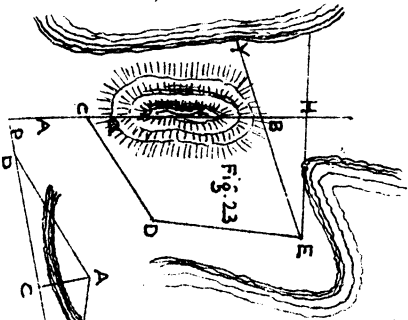
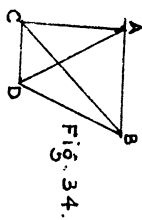
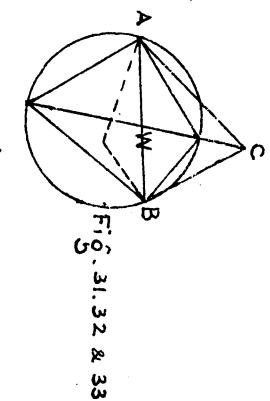
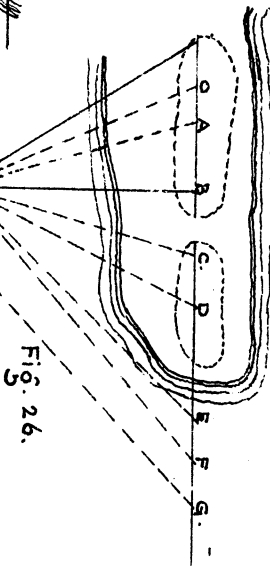
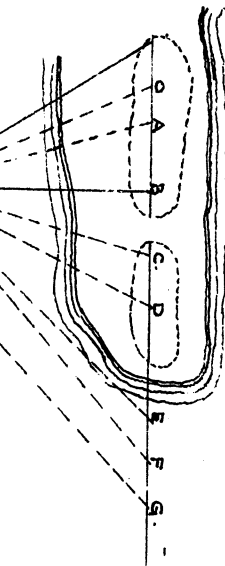
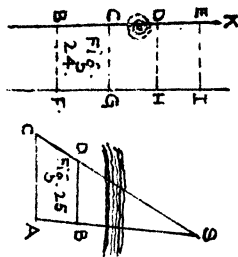
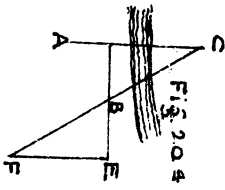
$$V \left\{ \frac{Ri}{0.000507 + \frac{0.00000647}{R}} \right\}^{\frac{1}{2}} \quad \text{in metres in terms of its radius.}$$

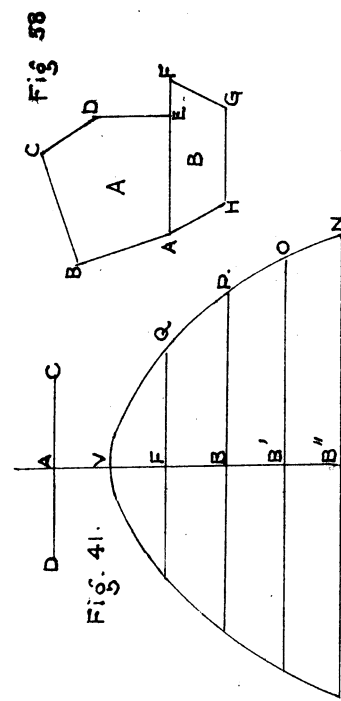
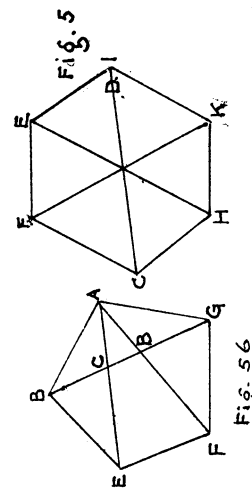
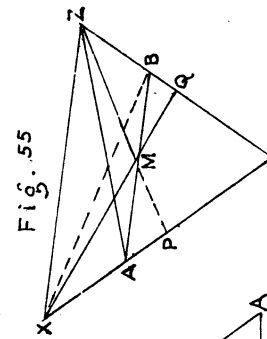
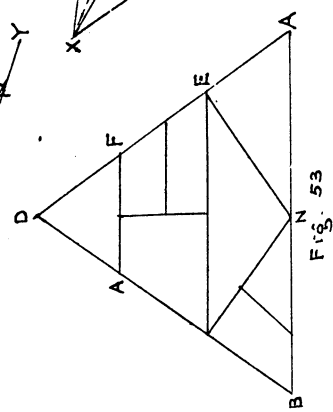
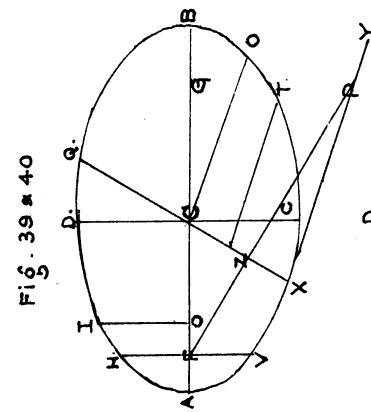
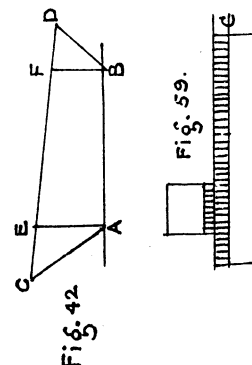
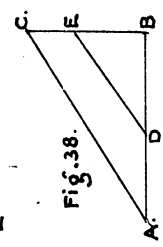
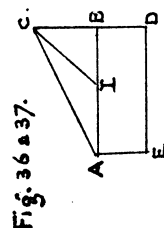
$$V \left\{ \frac{Di}{0.000309 + \frac{0.00000648}{D}} \right\}^{\frac{1}{2}} \quad \text{in terms of English feet.}$$

$$V \left\{ \frac{ri}{0.00007726 + \frac{0.00000162}{r}} \right\}^{\frac{1}{2}} \quad \text{in terms of its hydraulic mean depth.}$$

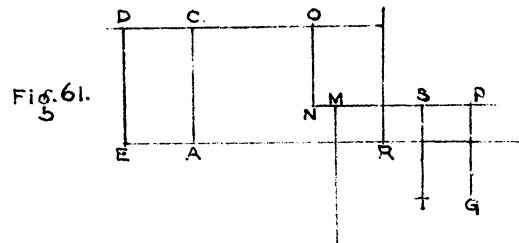
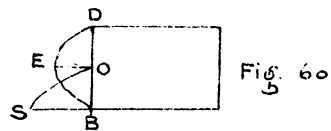


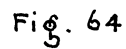
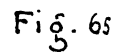


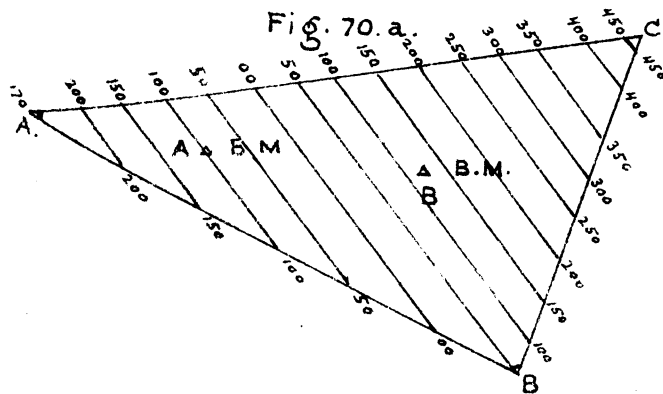
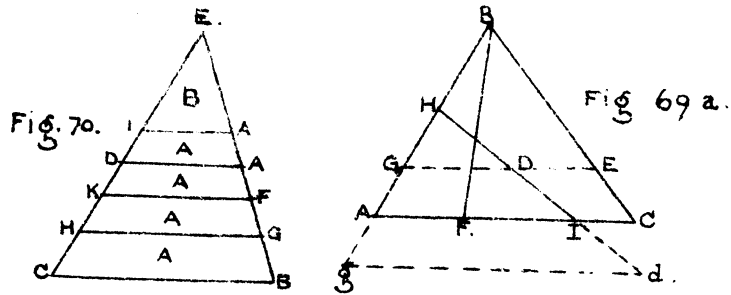
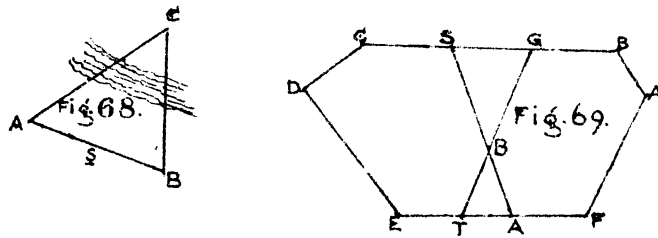
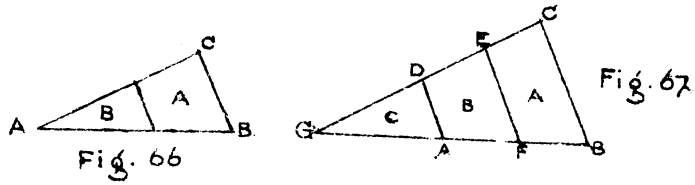




Page 5.







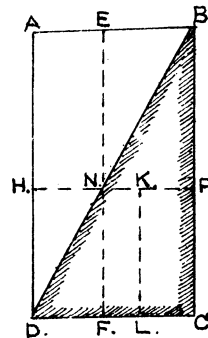
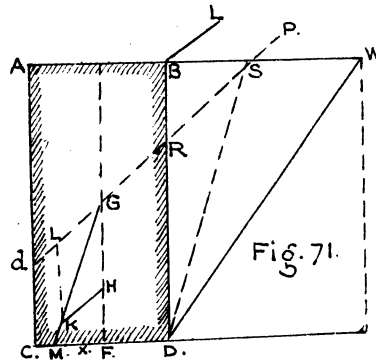
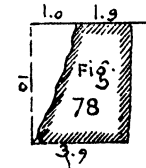
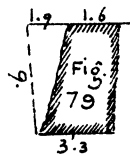
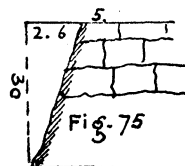
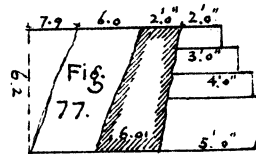
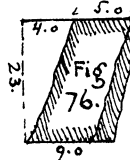
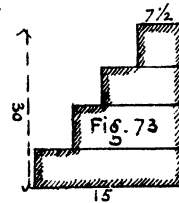
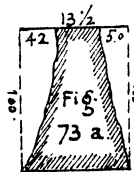
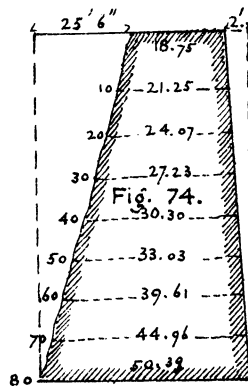
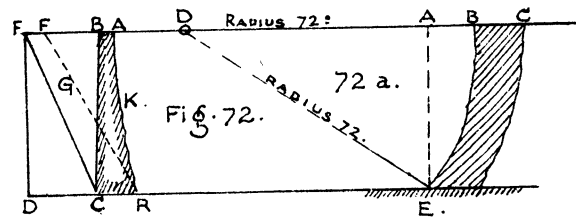


Fig 70.b.
P. 72. B. 112.



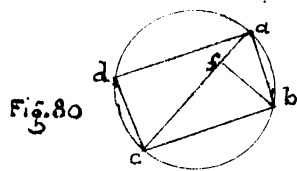
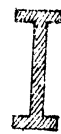


Fig. 80

Fig. 80 a.



Threefold flanges
of equal areas.

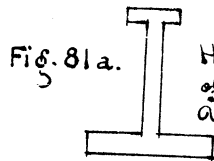


Fig. 81 a.

Hodgkinson's section
of greatest strength.
Area of lower flange 6 times
that of its upper one.

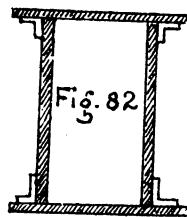


Fig. 82

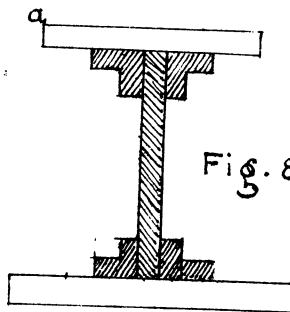


Fig. 83.

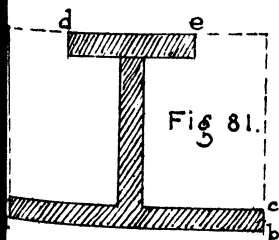


Fig. 81.

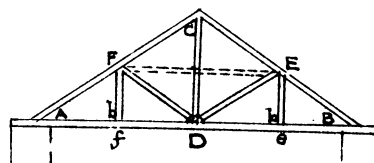


Fig. 83 a.

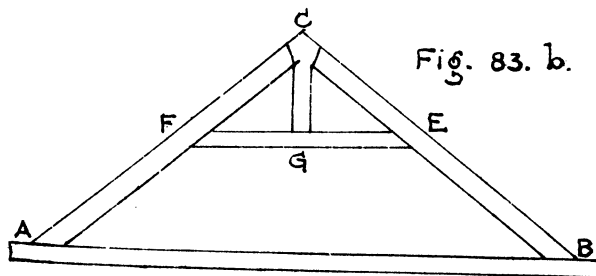
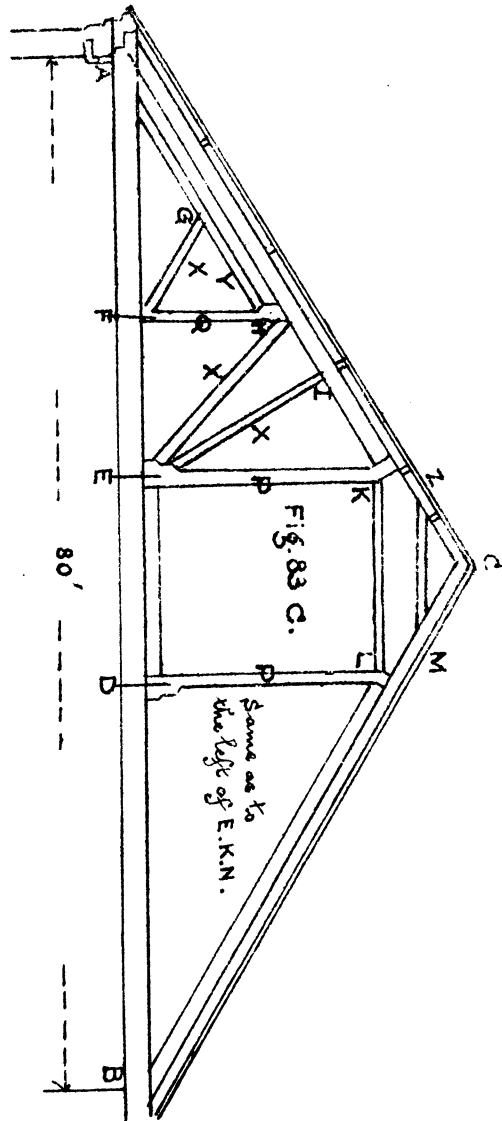
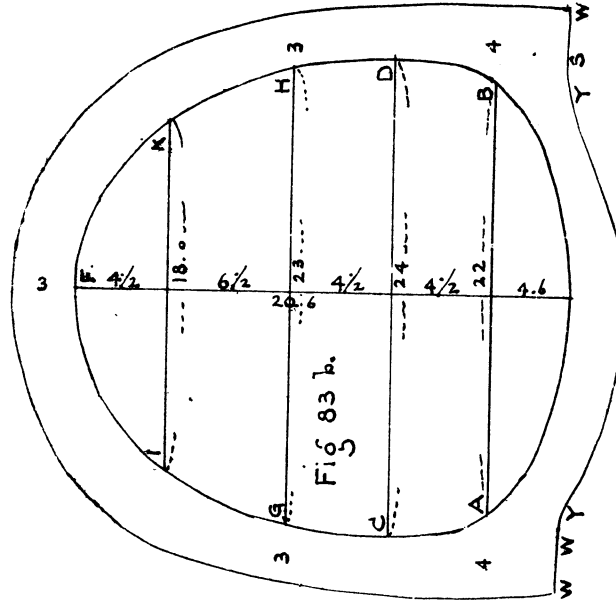


Fig. 83. b.





TUNNEL FOR ONE TRACK

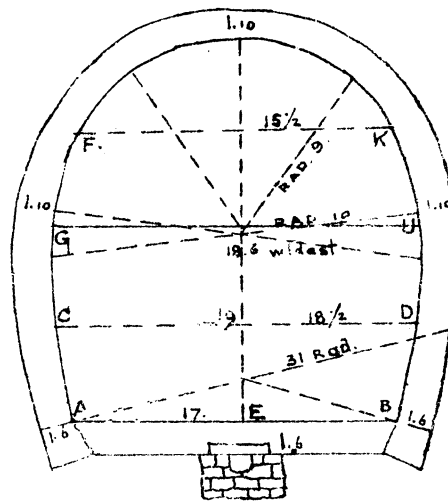


Fig. 83 d.

HOOSAIC TUNNEL

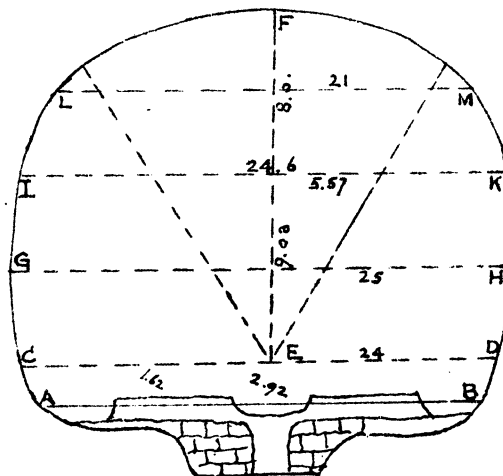
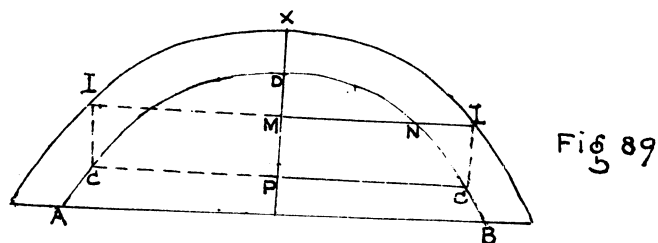
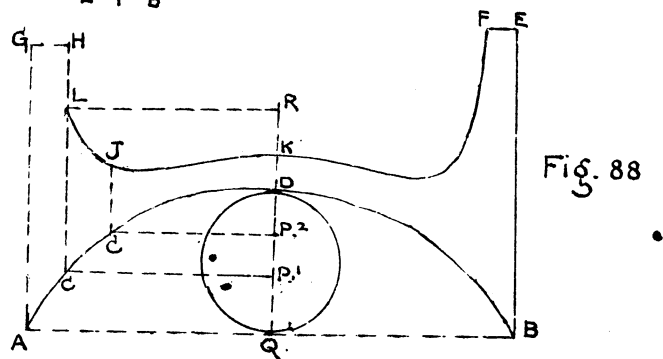
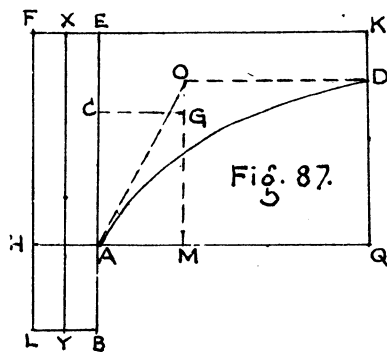
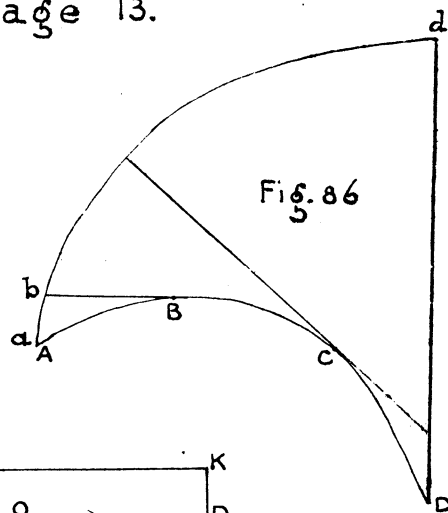
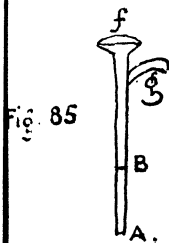
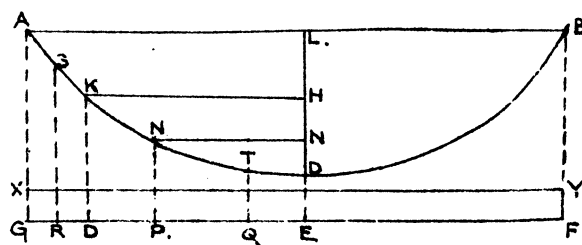
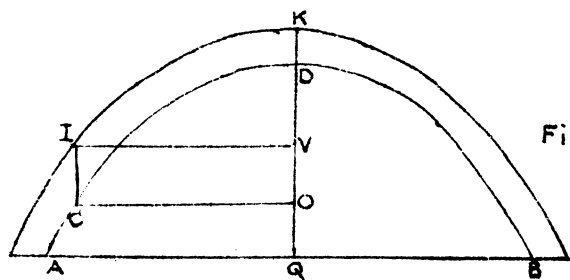
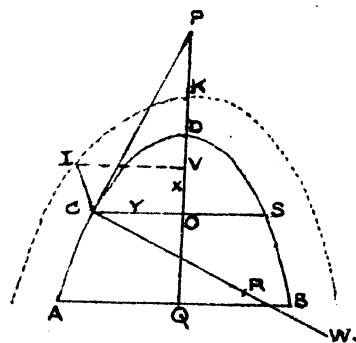


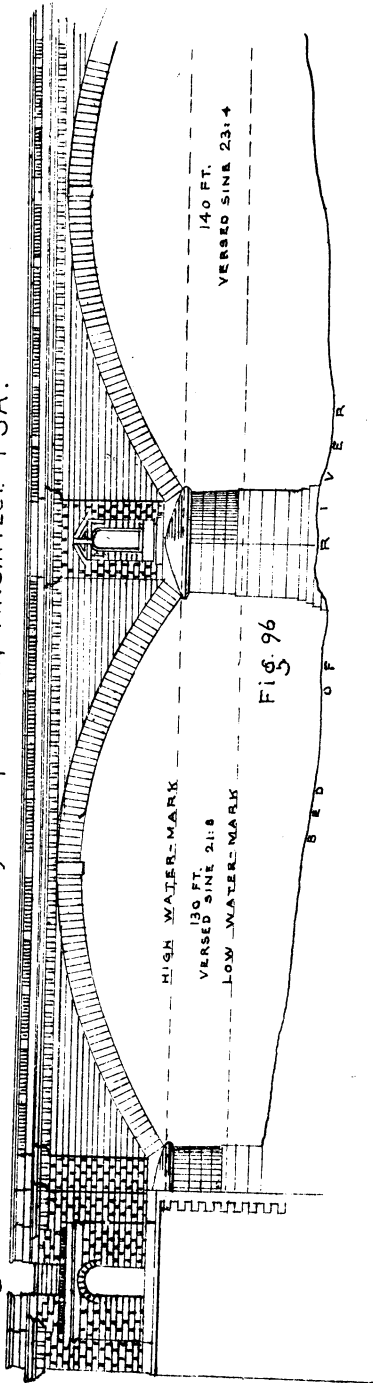
Fig. 83 c.



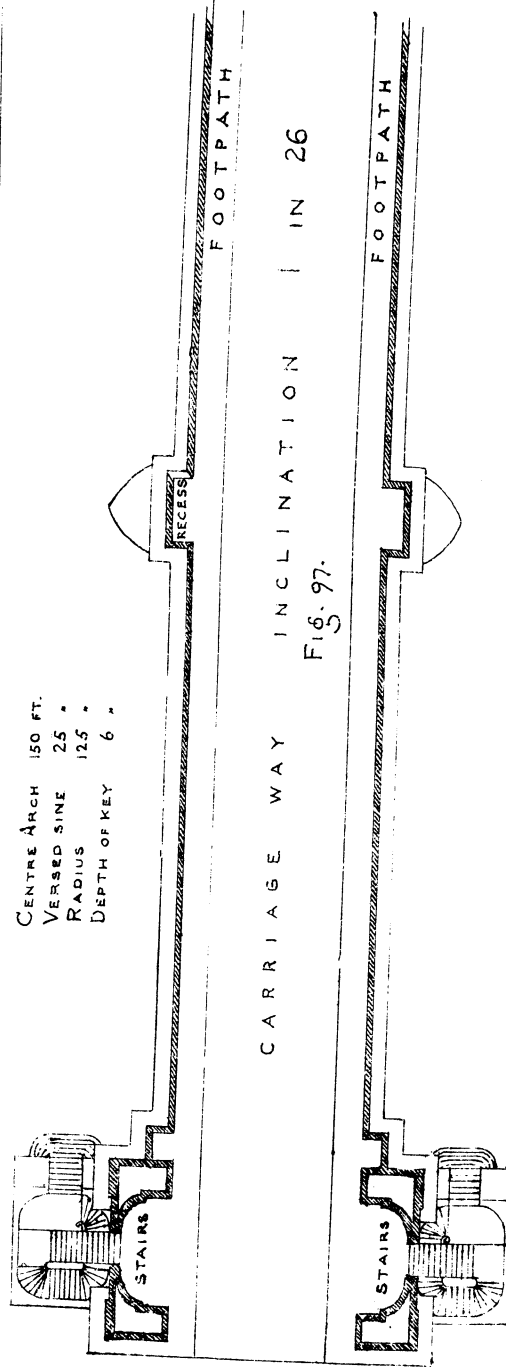


DESIGN FOR THE NEW LONDON BRIDGE

by Joseph Gwilt, Architect. FSA.



CENTRE ARCH 150 FT.
VERSED SINE 25 "
RADIUS 125 "
DEPTH OF KEY 6 "



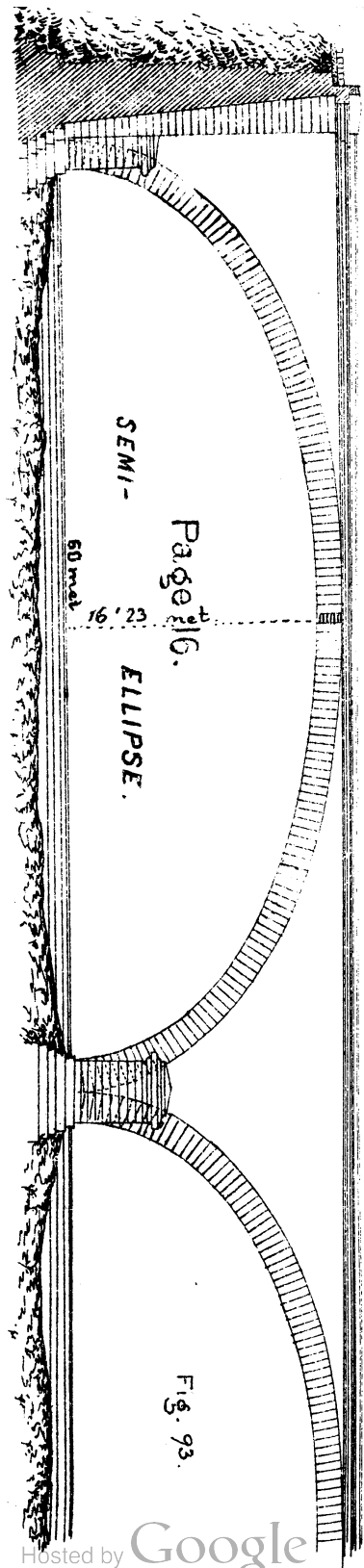


Fig. 93.

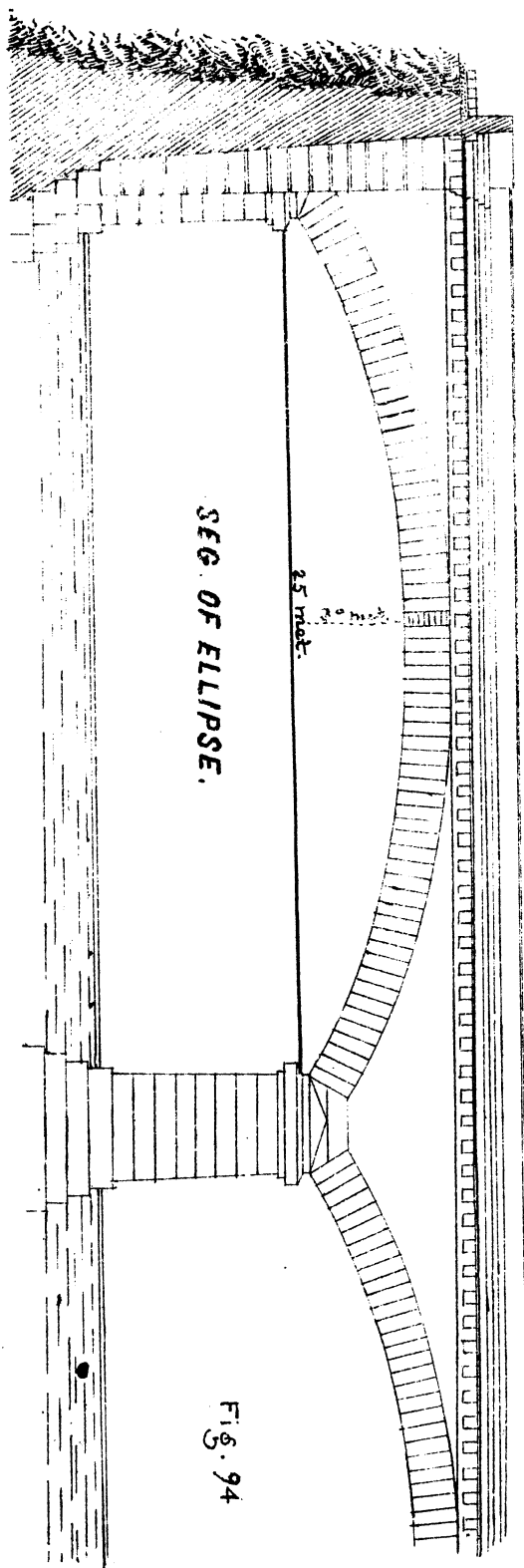


Fig. 94

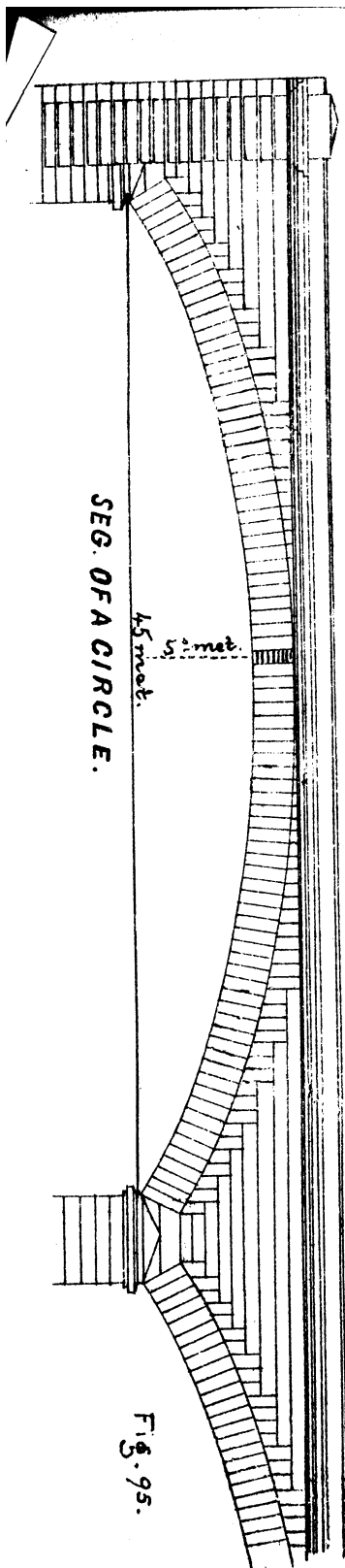


Fig. 95.

TABLE A. NATURAL SINE. 721*												
1	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
0	000 00	017 45	034 90	052 34	069 76	087 16	104 53	121 87	139 17	156 54	174 29	191 53
1	29	74	085 19	63	070 05	45	82	122 16	46	59	75	58
2	58	018 03	48	92	34	74	105 11	45	75	58	45	57
3	87	32	77	053 21	63	088 03	40	74	140 04	57	33	56
4	001 16	62	036 06	50	92	31	69	123 02	31	61	55	55
5	45	91	35	79	071 21	60	97	131	60	54	54	54
6	75	019 20	64	054 08	50	89	106 26	60	90	54	54	54
7	002 04	49	93	37	79	089 18	55	89	141 19	53	48	52
8	33	78	037 23	66	072 08	47	84	124 18	47	51	51	51
9	62	020 07	52	95	37	76	107 13	47	77	51	45	50
10	91	36	81	055 24	66	090 05	42	76	142 05	50	49	49
11	003 20	65	038 10	53	95	34	71	125 04	34	49	49	49
12	49	94	39	82	073 24	63	108 00	33	63	48	47	47
13	78	021 23	68	056 11	53	92	29	62	92	47	47	47
14	004 07	52	97	40	82	091 21	58	91	143 20	46	45	45
15	36	81	039 26	69	074 11	50	87	126 20	49	44	43	43
16	65	022 11	56	98	40	79	109 16	49	78	44	43	43
17	95	40	81	057 27	69	092 08	45	78	144 07	43	42	42
18	005 24	69	040 18	56	98	37	73	127 06	36	42	41	41
19	53	98	42	85	075 27	66	110 02	35	64	41	40	40
20	82	023 27	71	058 14	56	95	31	64	93	40	39	39
21	006 11	56	041 00	44	85	093 24	60	93	145 22	39	38	38
22	40	85	29	73	076 14	53	89	128 22	38	37	37	37
23	69	024 14	59	059 02	43	82	111 18	51	80	36	35	35
24	98	43	88	31	72	094 11	47	80	146 08	36	35	35
25	007 27	72	042 17	60	077 01	40	76	129 08	37	34	33	33
26	56	025 01	46	89	30	69	112 05	37	66	34	33	33
27	85	30	75	060 18	59	98	34	66	95	33	32	32
28	008 14	60	043 04	47	88	095 27	63	95	147 23	32	31	31
29	44	89	33	76	078 17	56	91	130 24	52	31	30	30
30	73	026 18	62	061 05	46	81	113 20	53	81	30	29	29
31	009 02	47	91	34	75	096 14	49	81	148 10	29	28	28
32	31	76	044 20	63	079 04	42	78	131 10	38	28	27	27
33	60	027 05	49	92	33	71	114 07	39	67	27	26	26
34	90	34	78	062 21	62	097 00	36	68	96	26	25	25
35	010 18	63	045 07	50	91	29	65	97	149 25	25	24	24
36	47	92	36	79	080 20	58	94	132 26	54	24	23	23
37	76	028 21	65	063 08	49	87	115 23	54	82	23	22	22
38	011 05	50	94	37	78	098 16	52	83	150 11	22	21	21
39	34	79	046 23	66	081 07	45	80	133 12	40	21	20	20
40	64	029 08	53	95	36	74	116 09	41	69	20	19	19
41	93	38	82	064 24	65	099 03	38	70	97	19	18	18
42	012 22	67	047 11	53	94	32	67	99	151 26	18	17	17
43	51	96	40	82	082 23	61	96	134 27	55	17	16	16
44	80	030 25	69	065 11	52	90	117 25	56	84	16	15	15
45	013 09	54	98	40	81	100 19	54	85	152 12	15	14	14
46	38	81	048 27	69	083 10	48	83	135 14	41	14	13	13
47	67	031 12	56	98	39	77	118 12	43	70	13	12	12
48	96	41	85	066 27	68	101 06	40	72	99	12	11	11
49	014 25	70	049 14	56	97	35	69	136 00	153	11	10	10
50	54	99	43	85	084 26	64	98	29	56	10	9	9
51	83	032 28	72	067 14	55	92	119 27	58	85	9	8	8
52	015 13	57	050 01	43	84	102 21	56	87	154 14	8	7	7
53	42	86	30	73	085 13	50	85	137 16	42	7	6	6
54	71	033 16	59	068 02	42	79	120 14	44	71	6	5	5
55	016 00	45	88	31	71	103 08	43	73	155 00	5	4	4
56	29	74	051 17	60	086 00	37	71	138 02	29	4	3	3
57	58	034 03	46	89	29	66	121 00	31	57	3	2	2
58	87	32	76	069 18	58	95	29	60	86	2	1	1
59	017 16	61	052 05	47	87	104 24	58	89	156 15	1	0	0
60	45	90	54	76	087 16	53	87	139 17	43	0	0	0
7	89°	88°	87°	86°	85°	84°	83°	82°	81°	80°	79°	78°
NATURAL COSINE.												

72.5*		NATURAL SINE.										TABLE A.	
	9°	10°	11°	12°	13°	14°	15°	16°	17°				
0	156 43	173 65	190 81	207 91	224 95	241 92	258 82	275 64	292 37	60			
1	72	93	191 09	208 20	225 23	242 20	259 10	276 20	293 58	59			
2	157 01	174 22	192 38	209 48	226 52	243 49	260 38	277 48	294 51	57			
3	30	51	67	83	99	115	131	147	163	179			
4	58	79	95	111	127	143	159	175	191	207			
5	87	103	119	135	151	167	183	199	215	231			
6	158 16	175 37	192 52	209 62	226 65	243 62	260 50	277 31	294 04	54			
7	45	65	81	97	113	129	145	161	177	193			
8	73	94	110	126	142	158	174	190	206	222			
9	159 02	176 23	193 38	210 47	227 50	244 46	261 35	278 15	295 51	50			
10	31	51	66	82	98	114	130	146	162	178			
11	59	80	95	111	127	143	159	175	191	207			
12	88	104	120	136	152	168	184	199	215	231			
13	160 17	177 37	194 52	211 61	228 63	245 59	262 47	279 27	296 49	49			
14	46	66	81	97	113	129	145	161	177	193			
15	74	94	110	126	142	158	174	190	206	222			
16	161 08	178 23	195 38	212 46	229 48	246 44	263 31	280 11	297 43	48			
17	32	52	66	82	98	114	130	146	162	178			
18	60	80	95	111	127	143	159	175	191	207			
19	89	105	121	137	153	169	185	199	215	231			
20	162 18	179 37	196 52	213 60	230 62	247 58	264 45	281 25	298 47	47			
21	47	66	80	96	112	128	144	160	176	192			
22	75	95	110	126	142	158	174	190	206	222			
23	163 04	180 23	197 38	214 46	231 48	248 44	265 31	282 11	299 43	46			
24	33	52	66	82	98	114	130	146	162	178			
25	61	81	94	110	126	142	158	174	190	206			
26	90	106	122	138	154	170	186	199	215	231			
27	164 19	181 38	198 51	215 59	232 60	249 56	266 43	283 23	300 45	45			
28	47	66	80	96	112	128	144	160	176	192			
29	76	95	110	126	142	158	174	190	206	222			
30	565 05	182 24	199 37	216 44	233 45	250 41	267 28	284 08	301 30	44			
31	33	52	65	81	97	113	129	145	161	177			
32	62	81	94	110	126	142	158	174	190	206			
33	91	107	123	139	155	171	187	199	215	231			
34	166 20	183 39	200 51	217 58	234 59	251 55	268 42	285 22	302 44	43			
35	48	67	81	97	113	129	145	161	177	193			
36	77	95	110	126	142	158	174	190	206	222			
37	167 06	184 24	201 36	218 43	235 44	252 40	269 27	286 07	303 29	42			
38	34	52	65	81	97	113	129	145	161	177			
39	63	81	93	109	125	141	157	173	189	205			
40	92	108	124	140	156	172	188	199	215	231			
41	168 20	185 39	202 50	219 57	236 58	253 54	270 41	287 21	304 43	41			
42	49	67	80	96	112	128	144	160	176	192			
43	78	95	110	126	142	158	174	190	206	222			
44	169 06	186 24	203 36	220 43	237 44	254 40	271 27	288 07	305 29	40			
45	35	52	64	80	96	112	128	144	160	176			
46	64	81	93	109	125	141	157	173	189	205			
47	92	107	123	139	155	171	187	199	215	231			
48	170 21	187 39	204 50	221 57	238 58	255 54	272 41	289 21	306 43	39			
49	50	67	80	96	112	128	144	160	176	192			
50	78	95	110	126	142	158	174	190	206	222			
51	171 07	188 24	205 35	222 42	239 43	256 39	273 26	290 06	307 28	38			
52	36	52	63	79	95	111	127	143	159	175			
53	64	81	92	108	124	140	156	172	188	204			
54	93	109	125	141	157	173	189	199	215	231			
55	172 22	189 39	206 50	223 57	240 58	257 54	274 41	291 21	308 43	37			
56	37	52	63	79	95	111	127	143	159	175			
57	65	81	92	108	124	140	156	172	188	204			
58	94	110	126	142	158	174	190	206	222	238			
59	173 08	190 24	207 34	224 41	241 42	258 38	275 25	292 05	309 27	36			
60	38	52	63	79	95	111	127	143	159	175			
61	66	81	91	107	123	139	155	171	187	203			
62	95	111	127	143	159	175	191	207	223	239			
63	174 09	191 24	208 34	225 41	242 42	259 38	276 25	293 05	310 27	35			
64	39	52	63	79	95	111	127	143	159	175			
65	67	81	91	107	123	139	155	171	187	203			
66	96	112	128	144	160	176	192	208	224	240			
67	175 10	192 24	209 34	226 41	243 42	260 38	277 25	294 05	311 27	34			
68	40	52	63	79	95	111	127	143	159	175			
69	68	81	91	107	123	139	155	171	187	203			
70	97	113	129	145	161	177	193	209	225	241			
71	176 11	193 24	210 34	227 41	244 42	261 38	278 25	295 05	312 27	33			
72	41	52	63	79	95	111	127	143	159	175			
73	69	81	91	107	123	139	155	171	187	203			
74	98	114	130	146	162	178	194	210	226	242			
75	177 12	194 24	211 34	228 41	245 42	262 38	279 25	296 05	313 27	32			
76	42	52	63	79	95	111	127	143	159	175			
77	70	81	91	107	123	139	155	171	187	203			
78	99	115	131	147	163	179	195	211	227	243			
79	178 13	195 24	212 34	229 41	246 42	263 38	280 25	297 05	314 27	31			
80	43	52	63	79	95	111	127	143	159	175			
81	71	81	91	107	123	139	155	171	187	203			
82	100	116	132	148	164	180	196	212	228	244			
83	179 14	196 24	213 34	230 41	247 42	264 38	281 25	298 05	315 27	30			
84	44	52	63	79	95	111	127	143	159	175			
85	72	81	91	107	123	139	155	171	187	203			
86	101	117	133	149	165	181	197	213	229	245			
87	180 15	197 24	214 34	231 41	248 42	265 38	282 25	299 05	316 27	29			
88	45	52	63	79	95	111	127	143	159	175			
89	73	81	91	107	123	139	155	171	187	203			
90	102	118	134	150	166	182	198	214	230	246			
91	181 16	198 24	215 34	232 41	249 42	266 38	283 25	300 05	317 27	28			
92	46	52	63	79	95	111	127	143	159	175			
93	74	81	91	107	123	139	155	171	187	203			
94	103	119	135	151	167	183	199	215	231	247			
95	182 17	199 24	216 34	233 41	250 42	267 38	284 25	301 05	318 27	27			
96	47	52	63	79	95	111	127	143	159	175			
97	75	81	91	107	123	139	155	171	187	203			
98	104	120	136	152	168	184	200	216	232	248			
99	183 18	200 24	217 34	234 41	251 42	268 38	285 25	302 05	319 27	26			
100	48	52	63	79	95	111	127	143	159	175			
101	76	81	91	107	123	139	155	171	187	203			
102	105	121	137	153	169	185	201	217	233	249			
103	184 19	201 24	218 34	235 41	252 42	269 38	286 25	303 05	320 27	25			
104	49	52	63	79	95	111	127	143	159	175			
105	77	81	91	107	123	139	155	171	187	203			
106	106	122	138	154	170	186	202	218	234	250			
107	185 20	202 24	219 34	236 41	253 42	270 38	287 25	304 05	321 27	24			
108	50	52	63	79	95	111	127	143	159	175			
109	78	81	91	107	123	139	155	171	187	203			
110	107	123	139	155	171	187	203	219	235	251			

TABLE A. NATURAL SINE.													72k*
	18°	19°	20°	21°	22°	23°	24°	25°	26°				
0	809 02	825 57	842 02	858 37	874 61	890 73	906 74	922 62	938 37	60			
1	29	84	29	64	88	391 00	407 00	88	63	59			
2	57	326 12	57	91	375 15	27	27	423 15	89	68			
3	85	39	84	359 18	42	53	53	41	439 16	57			
4	310 12	67	343 11	45	69	80	80	67	42	56			
5	40	94	39	73	95	392 07	408 06	94	68	55			
6	68	327 22	66	360 00	376 22	34	33	424 20	94	54			
7	95	49	93	27	49	60	60	46	440 20	53			
8	311 23	77	344 21	54	76	87	86	73	46	52			
9	51	328 04	48	81	377 03	393 14	409 13	99	72	51			
10	78	32	75	361 08	30	41	39	425 25	98	50			
11	312 06	59	345 03	35	57	67	66	52	441 24	49			
12	33	87	30	62	84	94	92	78	51	48			
13	61	329 14	57	90	378 11	394 21	410 19	426 04	77	47			
14	89	42	84	362 17	38	48	45	31	442 03	46			
15	313 16	69	346 12	44	65	74	72	57	29	45			
16	44	97	39	71	92	395 01	98	83	55	44			
17	72	330 24	66	98	379 19	28	411 25	427 09	81	43			
18	99	51	94	363 25	46	55	51	36	443 07	42			
19	314 27	79	347 21	52	73	81	78	62	33	41			
20	54	331 06	48	79	99	396 08	412 04	88	59	40			
21	82	34	75	364 06	380 26	35	31	428 15	86	39			
22	315 10	61	348 03	34	53	61	57	41	444 11	38			
23	37	89	80	61	80	88	84	67	87	37			
24	65	332 16	57	88	381 07	397 15	413 10	94	64	86			
25	93	44	84	365 15	34	41	37	429 20	90	35			
26	316 20	71	349 12	42	61	68	63	46	445 16	34			
27	48	98	39	69	88	95	90	72	42	33			
28	75	333 26	66	96	382 15	398 22	414 16	99	68	32			
29	317 03	53	93	366 23	41	48	43	430 25	94	31			
30	30	81	350 21	50	68	75	69	51	446 20	30			
31	58	334 08	48	77	95	399 02	96	77	46	29			
32	86	36	75	367 04	383 22	28	415 22	431 04	72	28			
33	318 13	63	351 02	31	49	55	49	30	98	27			
34	41	90	30	58	76	82	75	56	447 24	26			
35	68	335 18	57	85	384 03	400 08	416 02	82	50	25			
36	96	45	84	368 12	30	35	28	432 09	76	24			
37	319 24	73	352 11	39	56	62	55	35	448 02	23			
38	51	336 00	39	67	83	88	81	61	28	22			
39	79	27	68	94	385 10	401 15	417 07	87	54	21			
40	320 06	55	93	369 21	37	42	34	433 13	80	20			
41	84	82	353 20	48	64	68	60	40	449 06	19			
42	61	337 10	47	75	91	95	87	66	32	18			
43	89	37	75	370 02	386 17	402 21	418 13	92	58	17			
44	321 16	64	354 02	29	44	48	40	434 18	84	16			
45	44	92	29	56	71	75	66	45	450 10	15			
46	71	338 19	56	83	98	403 01	92	71	86	14			
47	99	46	84	371 10	387 25	28	419 19	97	62	13			
48	322 27	74	355 11	37	52	55	45	435 23	88	12			
49	54	339 01	38	64	78	81	72	49	451 14	11			
50	82	29	65	91	388 05	404 08	98	75	40	10			
51	323 09	56	92	372 18	32	34	420 24	436 02	66	9			
52	37	83	356 19	45	59	61	51	28	92	8			
53	64	340 11	47	72	86	88	77	54	452 18	7			
54	92	38	74	99	389 12	405 14	421 04	80	43	6			
55	324 19	65	357 01	373 26	39	41	30	437 06	69	5			
56	47	93	28	53	66	67	56	33	95	4			
57	74	341 20	55	80	93	94	83	59	453 21	3			
58	325 02	47	82	374 07	390 20	406 21	422 09	85	47	2			
59	29	75	358 10	34	46	47	65	438 11	78	1			
60	57	342 02	37	61	73	74	62	37	99	0			
	71°	70°	69°	68°	67°	66°	65°	64°	63°				
NATURAL COSINE.													

72L*		NATURAL SINE.										TABLE A.	
°	'	27°	28°	29°	30°	31°	32°	33°	34°	35°	'	°	'
0	453 99	469 47	484 81	500 00	515 04	529 92	544 64	559 19	573 58	586 60			
1	454 25	73 485 06	25 29	530 17	545 13	559 43	574 05	588 58	603 51				
2	51 99	32 50	54 79	66 91	78 03	89 15	99 27	109 39	119 51				
3	77 470 24	57 76	66 91	78 03	89 15	99 27	109 39	119 51	129 63				
4	455 03	50 83	501 01	516 04	531 07	546 10	561 13	576 16	591 19				
5	29 76	486 08	26 28	531 15	546 18	561 21	576 24	591 27	606 30				
6	54 471 01	34 51	53 40	546 10	561 13	576 16	591 19	606 22	621 25				
7	80 27	59 76	65 85	561 13	576 16	591 19	606 22	621 25	636 28				
8	456 06	53 84	502 01	517 03	532 06	547 09	562 12	577 15	592 18				
9	32 78	487 10	27 28	532 14	547 17	562 20	577 23	592 26	607 29				
10	58 472 04	35 52	53 38	547 08	562 11	577 14	592 17	607 20	622 23				
11	84 29	61 77	68 86	592 17	607 20	622 23	637 26	652 29	667 32				
12	457 10	55 86	503 02	518 03	533 06	548 09	563 12	578 15	593 18				
13	36 81	488 11	27 28	533 12	548 15	563 18	578 21	593 24	608 27				
14	62 473 06	37 52	52 61	548 05	563 08	578 11	593 14	608 17	623 20				
15	87 32	62 77	69 88	593 14	608 17	623 20	638 23	653 26	668 29				
16	458 13	58 88	504 03	519 02	534 05	549 08	564 11	579 14	594 17				
17	39 83	489 13	28 29	534 11	549 14	564 17	579 20	594 23	609 26				
18	65 474 09	38 53	52 61	549 02	564 05	579 08	594 11	609 14	624 17				
19	91 34	64 79	71 82	604 11	619 14	634 17	649 20	664 23	679 26				
20	459 17	60 89	505 03	520 02	535 05	550 08	565 11	580 14	595 17				
21	42 86	490 14	28 29	535 09	550 12	565 15	580 18	595 21	610 24				
22	68 475 11	40 53	51 60	550 00	565 03	580 06	595 09	610 12	625 15				
23	94 37	65 78	72 83	605 08	620 11	635 14	650 17	665 20	680 23				
24	460 20	62 90	506 03	521 01	536 04	551 07	566 10	581 13	596 16				
25	46 88	491 16	28 29	536 07	551 10	566 13	581 16	596 19	611 22				
26	72 476 14	41 54	51 60	551 01	566 04	581 07	596 10	611 13	626 16				
27	97 39	66 79	73 84	606 09	621 12	636 15	651 18	666 21	681 24				
28	461 23	65 92	507 04	522 00	537 03	552 06	567 09	582 12	597 15				
29	49 90	492 17	29 30	537 05	552 08	567 11	582 14	597 17	612 20				
30	75 477 16	42 54	50 61	552 01	567 04	582 07	597 10	612 13	627 16				
31	462 01	41 68	79 84	607 09	622 12	637 15	652 18	667 21	682 24				
32	26 67	93 508 04	523 00	538 03	553 06	568 09	583 12	598 15	613 18				
33	52 93	493 18	29 30	538 04	553 07	568 10	583 13	598 16	613 19				
34	78 478 18	44 54	49 50	553 01	568 04	583 07	598 10	613 13	628 16				
35	463 04	44 69	79 84	608 09	623 12	638 15	653 18	668 21	683 24				
36	30 69	94 509 04	99 524 23	539 02	554 05	569 08	584 11	599 14	614 17				
37	55 95	494 19	29 30	539 02	554 05	569 08	584 11	599 14	614 17				
38	81 479 20	45 54	48 49	554 01	569 04	584 07	599 10	614 13	629 16				
39	464 07	46 70	79 84	609 09	624 12	639 15	654 18	669 21	684 24				
40	33 71	95 510 04	98 525 22	540 00	555 03	570 06	585 09	600 12	615 15				
41	58 97	495 21	29 30	540 00	555 03	570 06	585 09	600 12	615 15				
42	84 480 22	46 54	47 48	555 01	570 04	585 07	600 10	615 13	630 16				
43	465 10	48 71	79 84	610 09	625 12	640 15	655 18	670 21	685 24				
44	36 86	73 96	511 04	526 01	541 04	556 07	571 10	586 13	601 16				
45	61 99	496 22	29 30	526 01	541 04	556 07	571 10	586 13	601 16				
46	87 481 24	47 54	46 47	541 01	556 04	571 07	586 10	601 13	616 16				
47	466 13	50 72	79 84	611 09	626 12	641 15	656 18	671 21	686 24				
48	39 75	97 512 04	96 527 20	542 00	557 03	572 06	587 09	602 12	617 15				
49	64 482 01	49 23	29 30	542 00	557 03	572 06	587 09	602 12	617 15				
50	90 26	48 54	45 46	557 01	572 04	587 07	602 10	617 13	632 16				
51	467 16	52 73	79 84	612 09	627 12	642 15	657 18	672 21	687 24				
52	42 77	98 513 04	94 528 19	543 00	558 03	573 06	588 09	603 12	618 15				
53	67 483 03	49 24	29 30	543 00	558 03	573 06	588 09	603 12	618 15				
54	93 28	49 54	44 45	558 01	573 04	588 07	603 10	618 13	633 16				
55	468 19	54 74	79 84	613 09	628 12	643 15	658 18	673 21	688 24				
56	44 79	99 514 04	93 529 18	544 00	559 03	574 06	589 09	604 12	619 15				
57	70 484 05	49 24	29 30	544 00	559 03	574 06	589 09	604 12	619 15				
58	96 80	50 54	43 44	559 01	574 04	589 07	604 10	619 13	634 16				
59	469 21	56 75	79 84	614 09	629 12	644 15	659 18	674 21	689 24				
60	47 81	500 00	515 04	529 92	544 64	559 19	573 58	588 51	603 54				
°	62°	61°	60°	59°	58°	57°	56°	55°	54°	°			
NATURAL COSINE.													

TABLE A. NATURAL SINE. 72m*										
	86°	87°	88°	89°	40°	41°	42°	43°	44°	
0	587 79	601 82	615 86	629 82	642 79	656 06	669 13	682 00	694 66	60
1	588 02	602 06	616 12	630 00	643 01	657 28	670 35	683 21	696 08	59
2	26	28	35	42	50	56	62	68	74	58
3	49	51	58	65	72	78	84	90	96	57
4	73	74	81	88	94	99	105	111	117	56
5	96	98	104	110	116	121	127	132	138	55
6	589 20	603 21	617 04	631 18	644 12	657 38	670 43	683 27	696 12	54
7	43	44	49	55	60	65	70	75	80	53
8	67	67	72	77	82	87	92	97	102	52
9	90	90	95	100	105	110	115	120	125	51
10	590 14	604 14	618 18	632 03	645 01	658 03	671 07	684 12	697 17	50
11	37	37	41	46	51	56	61	66	71	49
12	61	60	64	69	74	79	84	89	94	48
13	84	83	87	92	97	102	107	112	117	47
14	591 08	605 06	619 09	633 16	646 12	659 13	672 15	685 18	698 21	46
15	31	29	32	38	43	48	53	58	63	45
16	54	53	57	62	67	72	77	82	87	44
17	78	76	80	85	90	95	100	105	110	43
18	592 01	606 22	620 01	633 38	647 01	660 00	673 01	686 03	699 04	42
19	25	24	28	33	38	43	48	53	58	41
20	48	45	49	54	59	64	69	74	79	40
21	72	68	72	77	82	87	92	97	102	39
22	95	91	95	100	105	110	115	120	125	38
23	593 18	607 14	621 15	634 25	648 12	661 10	674 09	687 09	699 08	37
24	42	38	42	47	52	57	62	67	72	36
25	65	61	65	70	75	80	85	90	95	35
26	89	84	88	93	98	103	108	113	118	34
27	594 12	608 07	622 06	635 18	648 40	662 18	675 16	688 14	701 12	33
28	36	30	34	39	44	49	54	59	64	32
29	59	53	57	62	67	72	77	82	87	31
30	82	76	80	85	90	95	100	105	110	30
31	595 06	609 22	623 20	636 08	649 01	662 84	675 80	688 75	701 69	29
32	29	22	26	31	36	41	46	51	56	28
33	52	45	49	54	59	64	69	74	79	27
34	76	68	72	77	82	87	92	97	102	26
35	99	91	95	100	105	110	115	120	125	25
36	596 22	610 15	624 11	637 20	650 11	663 06	676 02	689 00	702 15	24
37	46	38	42	47	52	57	62	67	72	23
38	69	61	65	70	75	80	85	90	95	22
39	93	84	88	93	98	103	108	113	118	21
40	597 16	611 07	625 02	638 10	651 00	664 14	677 09	690 04	703 19	20
41	39	30	34	39	44	49	54	59	64	19
42	63	53	57	62	67	72	77	82	87	18
43	86	76	80	85	90	95	100	105	110	17
44	598 09	612 22	626 15	639 22	652 10	665 01	678 16	691 09	704 01	16
45	32	22	26	31	36	41	46	51	56	15
46	56	45	49	54	59	64	69	74	79	14
47	79	68	72	77	82	87	92	97	102	13
48	599 02	613 14	627 06	640 11	653 23	666 38	679 43	692 48	705 53	12
49	26	14	18	23	28	33	38	43	48	11
50	49	37	41	46	51	56	61	66	71	10
51	72	60	64	69	74	79	84	89	94	9
52	95	83	87	92	97	102	107	112	117	8
53	600 19	614 06	628 19	641 00	654 08	667 18	680 08	693 19	706 08	7
54	42	29	33	38	43	48	53	58	63	6
55	65	51	55	60	65	70	75	80	85	5
56	89	74	78	83	88	93	98	103	108	4
57	601 12	615 20	629 09	642 12	655 18	668 27	681 15	694 03	707 11	3
58	35	20	24	29	34	39	44	49	54	2
59	58	43	47	52	57	62	67	72	77	1
60	82	66	70	75	80	85	90	95	100	0
	58°	59°	60°	61°	62°	63°	64°	65°	66°	
NATURAL COSINE.										

72N*			NATURAL SINE.										TABLE A.		
/	45°	46°	47°	48°	49°	50°	51°	52°	53°	/					
0	707 11	719 34	731 35	743 14	754 71	766 04	777 15	788 01	798 64	60					
1		31	54	55	34	90	23	33	19	81	59				
2		52	74	75	53	755 09	42	51	37	99	58				
3		72	95	95	73	28	61	69	56	799	16	57			
4		93	720 15	732 15	92	47	79	88	73	34	56				
5	708 13	35	34	744 12	66		98	778 06	91	51	55				
6		34	55	54	31	85	767 17	24	789 08	68	54				
7		55	75	74	51	756 04	35	43	26	86	53				
8		75	95	94	70	23	54	61	44	800	03	52			
9		96	721 16	733 14	89	42	72	79	62	21	51				
10	709 16	36	33	745 09	61	91	97	80	38	50					
11		37	56	53	28	81	768 10	779 16	98	56	49				
12		57	76	73	48	757 00	28	34	790 16	73	48				
13		78	96	93	67	19	47	52	33	91	47				
14		98	722 16	734 13	86	38	66	70	51	801	08	46			
15	710 19	36	32	746 06	57	84	88	69	25	45					
16		39	57	52	25	75	769 03	780 07	87	43	44				
17		59	77	72	44	94	21	25	791 05	60	43				
18		80	97	91	64	758 13	40	43	22	78	42				
19	711 00	723 17	735 11	83	32	59	61	40	95	41					
20		21	37	31	747 03	51	77	79	58	302	12	40			
21		41	57	51	22	70	96	98	76	80	39				
22		62	77	70	41	89	770 14	781 16	93	47	38				
23		82	97	90	60	759 08	33	34	792 11	64	37				
24	712 03	724 17	736 10	80	27	51	52	29	82	36					
25		23	37	29	99	46	70	70	47	99	35				
26		43	57	49	748 18	65	88	88	64	803	16	34			
27		64	77	69	38	84	771 07	782 06	82	34	33				
28		84	97	88	57	760 03	25	25	793 00	51	32				
29	713 05	725 17	737 08	76	22	44	43	18	68	31					
30		25	37	28	96	41	62	61	35	86	30				
31		45	57	47	749 15	59	81	79	53	804	03	29			
32		66	77	67	34	78	99	97	71	20	28				
33		86	97	87	53	97	772 18	783 15	88	38	27				
34	714 07	726 17	738 06	73	761 16	36	33	794 06	55	26					
35		27	37	26	92	35	55	51	24	72	25				
36		47	57	46	750 11	54	73	69	41	89	24				
37		68	77	65	30	73	92	87	59	805	07	23			
38		88	97	85	50	92	773 10	784 05	77	24	22				
39	715 08	727 17	739 04	69	762 10	29	24	94	41	21					
40		29	37	24	88	29	47	42	795 12	58	20				
41		49	57	44	751 07	48	66	60	30	76	19				
42		69	77	63	26	67	84	78	47	93	18				
43		90	97	83	46	86	774 02	96	65	806	10	17			
44	716 10	728 17	740 02	65	763 04	21	785 14	83	27	16					
45		30	37	22	84	23	39	32	796 00	44	15				
46		50	57	41	752 03	42	58	50	18	62	14				
47		71	77	61	22	61	76	68	35	79	13				
48		91	97	80	41	80	94	86	53	96	12				
49	717 11	729 17	741 00	61	98	775 13	786 04	71	807	13	11				
50		32	37	20	80	764 17	31	22	88	30	10				
51		52	57	39	99	36	50	40	797 06	48	9				
52		72	76	59	753 18	55	68	58	23	65	8				
53		92	96	78	37	73	86	76	41	82	7				
54	718 13	730 16	98	56	92	776 05	94	58	99	6					
55		33	36	742 17	75	765 11	23	787 11	76	808	16	5			
56		53	56	37	95	30	41	29	93	33	4				
57		73	76	56	754 14	48	60	47	798 11	50	3				
58		94	96	76	33	67	78	65	29	67	2				
59	719 14	731 16	95	52	86	96	83	46	85	1					
60		34	35	743 14	71	766 04	777 15	788 01	64	809	02	0			
/	44°	43°	42°	41°	40°	39°	38°	37°	36°	35°	34°	33°			
NATURAL COSINE.															

TABLE A. NATURAL SINE. 720*										
	54°	55°	56°	57°	58°	59°	60°	61°	62°	
0	809 02	819 15	829 04	838 67	848 05	857 17	866 03	874 62	882 95	60
1	19	32	20	83	20	32	17	76	883 08	59
2	36	49	36	99	36	47	32	90	22	58
3	53	65	53	839 15	51	62	46	875 04	36	57
4	70	82	69	30	66	77	61	18	49	56
5	87	99	85	46	82	92	75	32	63	55
6	810 04	820 15	830 01	62	97	858 06	90	46	77	54
7	21	32	17	78	849 13	21	867 04	61	90	53
8	38	48	34	94	28	36	19	75	884 04	52
9	55	65	50	840 09	43	51	33	89	17	51
10	72	82	66	25	59	66	48	876 03	31	50
11	89	98	82	41	74	81	62	17	45	49
12	811 06	821 15	98	57	89	96	77	31	58	48
13	23	32	831 15	72	850 05	859 11	91	45	72	47
14	40	48	31	88	20	26	868 05	59	85	46
15	57	65	47	841 04	35	41	20	73	99	45
16	74	81	63	20	51	56	34	87	885 12	44
17	91	98	79	35	66	70	49	877 01	26	43
18	812 08	822 14	95	51	81	85	63	15	89	42
19	25	31	832 12	67	96	860 00	78	29	53	41
20	42	48	28	82	851 12	15	92	43	66	40
21	59	64	44	98	27	30	869 06	56	80	39
22	76	81	60	842 14	42	45	21	70	93	38
23	93	97	76	30	57	59	35	84	886 07	37
24	813 10	823 14	92	45	73	74	49	98	20	36
25	27	30	833 08	61	88	89	64	878 12	34	35
26	44	47	24	77	852 03	861 04	78	26	47	34
27	61	63	40	92	18	19	93	40	61	33
28	78	80	56	843 08	34	33	870 07	54	74	32
29	95	96	73	24	49	48	21	68	88	31
30	814 12	824 13	89	39	64	63	36	82	887 01	30
31	28	29	834 05	55	79	78	50	96	15	29
32	45	46	21	70	94	92	64	879 09	28	28
33	62	62	37	86	853 10	862 07	79	23	41	27
34	79	78	53	844 02	25	22	93	37	55	26
35	96	95	69	17	40	37	871 07	51	68	25
36	815 13	825 11	85	83	55	51	21	65	82	24
37	30	28	835 01	48	70	66	36	79	95	23
38	46	44	17	64	85	81	50	93	888 08	22
39	63	61	33	80	854 01	95	64	880 06	22	21
40	80	77	49	95	16	863 10	78	20	35	20
41	97	93	65	845 11	81	25	93	34	48	19
42	816 14	826 10	81	26	46	40	872 07	48	62	18
43	31	26	97	42	61	64	21	62	75	17
44	47	43	836 13	57	76	69	35	75	88	16
45	64	59	29	73	91	84	50	89	889 02	15
46	81	75	45	88	855 06	98	64	881 03	15	14
47	98	92	61	846 04	21	864 13	78	17	28	13
48	817 14	827 08	76	19	36	27	92	80	42	12
49	31	24	92	35	51	42	873 06	44	55	11
50	48	41	837 08	50	67	57	21	58	68	10
51	65	57	24	66	82	71	35	72	81	9
52	82	73	40	81	97	86	49	85	95	8
53	98	90	56	97	856 12	865 01	63	99	890 08	7
54	818 15	828 06	72	847 12	27	15	77	882 13	21	6
55	82	22	88	28	42	30	91	26	85	5
56	48	39	838 04	43	57	44	874 06	40	48	4
57	65	55	20	59	72	59	20	54	61	3
58	82	71	85	74	87	73	84	67	74	2
59	99	87	51	89	857 02	88	48	81	87	1
60	819 15	829 04	67	848 05	17	866 03	62	95	891 01	0
	55°	54°	53°	52°	51°	50°	49°	48°	47°	
NATURAL COSINE.										

72P*		NATURAL SINE.										TABLE A.	
	68°	64°	60°	66°	67°	68°	69°	70°	71°				
0	891 01	898 79	906 81	913 55	920 50	927 18	933 58	939 69	945 52	50			
1	14	92	43	66	62	29	68	79	61	59			
2	27	899 05	55	78	73	40	79	89	71	58			
3	40	18	68	90	85	51	89	99	80	57			
4	53	30	80	914 02	96	62	934 00	940 09	90	56			
5	67	43	92	14	921 07	73	10	19	99	55			
6	80	56	907 04	25	19	84	20	29	946 09	54			
7	93	68	17	37	30	94	31	39	18	53			
8	892 06	81	29	49	41	928 05	41	49	27	52			
9	19	94	41	61	52	16	52	58	37	51			
10	32	900 07	53	72	64	27	62	68	46	50			
11	45	19	66	84	75	38	72	78	56	49			
12	59	32	78	96	86	49	83	88	65	48			
13	72	45	90	915 08	98	59	93	98	74	47			
14	85	57	908 02	19	922 09	70	935 03	941 08	84	46			
15	98	70	14	31	20	81	14	18	93	45			
16	893 11	82	26	43	31	92	24	27	947 02	44			
17	24	95	39	55	43	929 03	34	37	12	43			
18	37	901 08	51	66	54	13	44	47	21	42			
19	50	20	63	78	65	24	55	57	30	41			
20	63	33	75	90	76	35	65	67	40	40			
21	76	46	87	916 01	87	45	75	76	49	39			
22	89	58	89	13	99	56	85	86	58	38			
23	894 02	71	909 12	25	923 10	67	96	96	68	37			
24	15	83	24	36	21	78	936 06	942 06	77	36			
25	28	96	36	48	32	88	16	16	86	35			
26	41	902 08	48	60	43	99	26	25	95	34			
27	54	21	60	71	55	930 10	37	35	948 05	33			
28	67	33	72	83	66	20	47	45	14	32			
29	80	46	84	94	77	31	57	54	23	31			
30	93	59	96	917 06	88	42	67	64	32	30			
31	895 06	71	910 08	18	99	52	77	74	42	29			
32	19	84	20	29	924 10	63	88	84	51	28			
33	32	96	32	51	21	74	98	93	60	27			
34	45	903 09	44	52	32	84	937 08	943 03	69	26			
35	58	21	56	64	44	95	18	13	78	25			
36	71	34	68	75	55	931 06	28	22	88	24			
37	84	46	80	87	66	16	38	42	97	23			
38	97	58	92	99	77	27	48	42	949 06	22			
39	896 10	71	911 04	918 10	88	37	59	51	15	21			
40	23	83	16	22	99	48	69	61	24	20			
41	36	96	28	33	925 10	59	79	70	33	19			
42	49	904 08	40	45	21	69	89	80	43	18			
43	62	21	52	56	32	80	99	90	52	17			
44	74	33	64	68	43	90	938 09	99	61	16			
45	87	46	76	79	54	932 01	19	944 09	70	15			
46	897 00	58	88	91	65	11	29	18	79	14			
47	13	70	912 00	919 02	76	22	39	28	88	13			
48	26	83	12	14	87	32	49	38	97	12			
49	39	95	24	25	98	43	59	47	950 06	11			
50	52	905 07	36	36	926 09	53	69	57	15	10			
51	64	20	48	48	20	64	79	66	24	9			
52	77	32	60	59	31	74	89	76	33	8			
53	90	45	72	71	42	85	99	85	43	7			
54	898 08	57	83	82	53	95	939 09	95	52	6			
55	16	69	95	94	64	933 06	19	945 04	61	5			
56	28	82	913 07	920 05	75	16	29	14	70	4			
57	41	94	19	16	86	27	39	23	79	3			
58	54	906 06	31	28	97	37	49	33	88	2			
59	67	18	43	39	927 07	48	59	42	97	1			
60	79	31	55	50	18	58	69	52	951 06	0			
	28°	25°	24°	23°	22°	21°	20°	19°	18°				
NATURAL COSINE.													

TABLE A. NATURAL SINE. 720*										
	72°	73°	74°	75°	76°	77°	78°	79°	80°	
0	951 06	956 32	961 26	965 93	970 30	974 37	978 15	981 63	9848 1	60
1	15	39	34	996 00	37	44	21	68		659
2	24	47	42	08	44	50	27	74	9849	158
3	33	56	50	15	51	57	33	79		657
4	42	64	58	23	58	63	39	85	9850	156
5	51	73	66	30	65	70	45	90		655
6	59	81	74	38	72	76	51	96	9851	154
7	68	90	82	45	79	83	57	982 01		653
8	77	98	90	53	86	89	63	07	8852	152
9	86	957 07	98	60	93	96	69	12		651
10	95	15	962 06	67	971 00	975 02	75	18	9853	150
11	952 04	24	14	75	06	08	81	23		649
12	13	32	22	82	13	15	87	29	9854	148
13	22	40	30	90	20	21	93	34		647
14	31	49	38	97	27	28	99	40	9855	146
15	40	57	46	967 05	34	34	979 05	45		645
16	48	66	53	12	41	41	10	50	9856	144
17	57	74	61	19	48	47	16	56		543
18	66	82	69	27	55	53	22	61	9857	042
19	75	91	77	34	62	60	28	67		541
20	84	99	85	42	69	66	34	72	9858	040
21	93	958 07	93	49	76	73	40	77		539
22	953 01	16	963 01	56	82	79	46	83	9859	038
23	10	24	08	64	89	85	52	88		537
24	19	32	16	71	96	92	58	94	9860	036
25	28	41	24	78	972 03	98	63	99		435
26	37	49	32	86	10	976 04	69	983 04		934
27	45	57	40	93	17	11	75	10	9861	433
28	54	65	47	00	23	17	81	15		932
29	63	74	55	968 07	30	23	87	20	9862	431
30	72	82	63	15	37	30	92	25		930
31	80	90	71	22	44	36	98	31	9863	329
32	89	98	79	29	51	42	980 04	36		828
33	98	959 07	86	37	57	48	10	41	9864	327
34	954 07	15	94	44	64	55	16	47		826
35	15	23	964 02	51	71	61	21	52	9865	225
36	24	31	10	58	78	67	27	57		724
37	33	40	17	66	84	73	33	62	9866	223
38	41	48	25	73	91	80	39	68		722
39	50	56	33	80	98	86	44	73	9867	121
40	59	64	40	87	973 04	92	50	78		620
41	67	72	48	94	11	98	56	83	9868	119
42	76	81	56	969 02	18	977 05	61	89		618
43	85	89	63	09	25	11	67	94	9869	017
44	93	97	71	16	31	17	73	99		516
45	955 02	960 05	79	23	38	23	79	984 04	9870	015
46	11	13	86	30	45	29	84	09		414
47	19	21	94	37	51	35	90	14		913
48	28	29	965 02	45	58	42	96	20	9871	412
49	36	37	09	52	65	48	981 01	25		811
50	45	46	17	59	71	54	07	30	9872	310
51	54	54	24	66	78	60	12	35		89
52	62	62	32	73	84	66	18	40	9873	28
53	71	70	40	80	91	72	24	45		77
54	79	78	47	87	98	78	29	50	8874	16
55	88	86	55	94	974 04	84	35	55		65
56	96	94	62	970 01	11	91	40	61	9875	14
57	956 05	961 02	70	08	17	97	46	66		53
58	13	10	78	15	24	978 03	52	71	9876	02
59	22	18	85	23	30	09	57	76		41
60	30	26	93	30	37	15	63	81		90
	17°	16°	15°	14°	13°	12°	11°	10°	9°	
NATURAL COSINE.										

r

72R* NATURAL SINE. TABLE A.									
7	81°	82°	83°	84°	85°	86°	87°	88°	89°
0	9876	9902	7	9925	5	9946	2	9961	9
1	9877	3	9903	1	8	5	9962	2	8
2	8	5	9926	2	8	5	9976	0	6
3	9878	2	9	5	9946	1	7	2	7
4	7	9904	3	9	4	9963	0	4	9
5	9879	1	7	9927	2	7	2	6	9987
6	6	9905	1	6	9947	0	5	8	2
7	9880	0	5	9	3	7	9977	0	3
8	5	9	9928	3	6	9	2	5	7
9	9	9906	3	6	9	9964	2	4	6
10	9881	4	7	9929	0	9948	2	4	6
11	8	9907	1	3	5	7	8	9	9995
12	9882	3	5	7	8	9	9978	0	9988
13	7	9	9930	0	9949	1	9965	2	2
14	9883	2	9908	3	3	4	4	3	2
15	6	7	7	7	7	7	6	5	3
16	9884	1	9909	1	9931	0	9950	0	9
17	5	4	4	4	3	9966	1	9979	0
18	9	8	7	6	4	2	9	6	8
19	9885	4	9910	2	9932	0	8	6	3
20	8	6	4	9951	1	8	5	2	8
21	9886	3	9911	0	7	4	9967	1	7
22	7	4	9933	1	7	3	9	4	9996
23	9887	1	8	4	9952	0	6	9980	1
24	6	9912	2	7	3	8	3	7	1
25	9888	0	5	9934	1	6	9968	0	5
26	4	9	4	8	3	3	6	9990	0
27	9	9913	3	7	9953	1	5	8	1
28	9889	3	7	9935	1	4	7	9981	0
29	7	9914	1	4	7	9	2	4	5
30	9890	2	4	7	9954	0	9969	2	3
31	6	8	9936	0	2	4	5	6	6
32	9891	0	9915	2	4	5	6	7	7
33	4	6	7	8	9	9	9	9	8
34	9	9916	0	9937	0	9955	1	9970	1
35	9892	3	3	4	3	3	2	1	9
36	7	7	7	6	5	4	2	9997	0
37	9893	1	9917	1	9938	0	9	8	6
38	6	5	3	9956	2	9971	0	7	5
39	9894	0	8	6	4	2	9	6	2
40	4	9918	2	9939	0	7	4	9983	1
41	8	6	3	9957	0	6	3	7	4
42	9895	3	9	6	2	9	4	9	4
43	7	9919	3	9	5	9972	1	6	9992
44	9896	1	7	9940	2	8	3	8	2
45	5	9920	0	6	9958	0	5	9	3
46	9	4	9	3	7	9984	1	4	7
47	9897	3	8	9941	2	6	9	5	7
48	8	2921	1	5	8	9973	1	4	6
49	9898	2	5	8	9959	1	4	6	7
50	6	9	9942	1	4	6	7	9	9
51	9899	0	9922	2	4	6	8	9	9993
52	4	6	8	9	9974	0	9985	1	1
53	8	9923	0	9943	1	9960	2	2	2
54	9900	2	3	4	4	4	4	3	2
55	6	7	7	7	7	6	5	4	2
56	9901	1	9924	0	9944	0	9	8	7
57	5	4	3	9961	2	9975	0	8	6
58	9	8	6	4	2	9986	0	7	4
59	9902	3	9925	1	9	7	4	1	8
60	7	5	9945	2	9	6	3	9	5
7	8°	7°	6°	5°	4°	3°	2°	1°	0°
NATURAL COSINE.									

TABLE B. NATURAL TANGENT.														72s*
1	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°
0	000 00	017 46	034 92	052 41	069 93	087 49	105 10	122 78	140 54	158 60	176 18	193 50	211 17	228 59
1	001 29	018 04	035 21	053 00	070 22	088 07	105 40	123 08	141 13	159 18	177 07	194 50	212 17	229 59
2	002 58	019 04	036 09	054 16	071 10	089 25	106 46	124 26	142 26	160 26	178 26	196 26	214 26	232 26
3	003 87	020 07	037 25	055 03	072 27	090 13	107 48	125 48	143 48	161 48	179 48	197 48	215 48	233 48
4	004 76	021 11	038 12	056 20	073 14	091 01	108 31	126 31	144 31	162 31	180 31	198 31	216 31	234 31
5	005 65	022 15	039 00	057 08	074 02	092 18	110 11	128 11	146 11	164 11	182 11	200 11	218 11	235 11
6	006 54	023 19	040 16	058 16	075 19	093 06	111 05	129 05	147 05	165 05	183 05	201 05	219 05	236 05
7	007 43	024 23	041 04	059 12	076 07	094 23	112 17	130 17	148 17	166 17	184 17	202 17	220 17	237 17
8	008 32	025 27	042 20	060 29	077 24	095 11	113 05	131 05	149 05	167 05	185 05	203 05	221 05	238 05
9	009 21	026 31	043 08	061 16	078 12	096 00	114 23	132 23	150 23	168 23	186 23	204 23	222 23	239 23
10	010 10	027 35	044 24	062 04	079 29	097 17	115 11	133 11	151 11	169 11	187 11	205 11	223 11	240 11
11	011 00	028 39	045 12	063 21	080 17	098 05	116 00	134 00	152 00	170 00	188 00	206 00	224 00	241 00
12	012 00	029 43	046 28	064 08	081 04	099 23	117 18	135 18	153 18	171 18	189 18	207 18	225 18	242 18
13	013 00	030 47	047 16	065 25	082 22	100 11	118 06	136 06	154 06	172 06	190 06	208 06	226 06	243 06
14	014 00	031 51	048 33	066 13	083 09	101 28	119 23	137 23	155 23	173 23	191 23	209 23	227 23	244 23
15	015 00	032 55	049 20	067 00	084 27	102 16	120 13	138 13	156 13	174 13	192 13	210 13	228 13	245 13
16	016 00	033 59	050 07	068 08	085 14	103 05	121 01	139 01	157 01	175 01	193 01	211 01	229 01	246 01
17	017 00	034 63	051 24	069 05	086 02	104 22	122 19	140 19	158 19	176 19	194 19	212 19	230 19	247 19
18	018 00	035 67	052 12	070 03	087 20	105 10	123 17	141 17	159 17	177 17	195 17	213 17	231 17	248 17
19	019 00	036 71	053 00	071 10	088 28	106 18	124 25	142 25	160 25	178 25	196 25	214 25	232 25	249 25
20	020 00	037 75	054 16	072 27	089 36	107 28	125 32	143 32	161 32	179 32	197 32	215 32	233 32	250 32
21	021 00	038 79	055 03	073 14	090 45	108 37	126 36	144 36	162 36	180 36	198 36	216 36	234 36	251 36
22	022 00	039 83	056 20	074 02	091 54	109 48	127 40	145 40	163 40	181 40	199 40	217 40	235 40	252 40
23	023 00	040 87	057 08	075 19	092 63	110 59	128 49	146 49	164 49	182 49	200 49	218 49	236 49	253 49
24	024 00	041 91	058 16	076 07	093 72	111 69	129 59	147 59	165 59	183 59	201 59	219 59	237 59	254 59
25	025 00	042 95	059 12	077 24	094 81	112 79	130 69	148 69	166 69	184 69	202 69	220 69	238 69	255 69
26	026 00	043 99	060 29	078 12	095 90	113 88	131 79	149 79	167 79	185 79	203 79	221 79	239 79	256 79
27	027 00	044 03	061 16	079 29	096 99	114 97	132 88	150 88	168 88	186 88	204 88	222 88	240 88	257 88
28	028 00	045 07	062 04	080 17	097 08	115 10	133 97	151 97	169 97	187 97	205 97	223 97	241 97	258 97
29	029 00	046 11	063 21	081 04	098 17	116 19	134 10	152 10	170 10	188 10	206 10	224 10	242 10	259 10
30	030 00	047 15	064 08	082 22	099 26	117 28	135 19	153 19	171 19	189 19	207 19	225 19	243 19	260 19
31	031 00	048 19	065 25	083 09	100 35	118 37	136 28	154 28	172 28	190 28	208 28	226 28	244 28	261 28
32	032 00	049 23	066 13	084 27	101 44	119 46	137 37	155 37	173 37	191 37	209 37	227 37	245 37	262 37
33	033 00	050 27	067 00	085 14	102 53	120 55	138 46	156 46	174 46	192 46	210 46	228 46	246 46	263 46
34	034 00	051 31	068 08	086 02	103 62	121 64	139 55	157 55	175 55	193 55	211 55	229 55	247 55	264 55
35	035 00	052 35	069 05	087 20	104 71	122 73	140 64	158 64	176 64	194 64	212 64	230 64	248 64	265 64
36	036 00	053 39	070 03	088 28	105 80	123 82	141 75	159 75	177 75	195 75	213 75	231 75	249 75	266 75
37	037 00	054 43	071 10	089 36	106 89	124 91	142 86	160 86	178 86	196 86	214 86	232 86	250 86	267 86
38	038 00	055 47	072 27	090 45	107 98	125 93	143 93	161 93	179 93	197 93	215 93	233 93	251 93	268 93
39	039 00	056 51	073 14	091 54	108 107	126 102	144 102	162 102	180 102	198 102	216 102	234 102	252 102	269 102
40	040 00	057 55	074 02	092 63	109 116	127 111	145 111	163 111	181 111	199 111	217 111	235 111	253 111	270 111
41	041 00	058 59	075 19	093 72	110 125	128 120	146 120	164 120	182 120	200 120	218 120	236 120	254 120	271 120
42	042 00	059 63	076 07	094 81	111 134	129 129	147 129	165 129	183 129	201 129	219 129	237 129	255 129	272 129
43	043 00	060 67	077 24	095 90	112 143	130 138	148 138	166 138	184 138	202 138	220 138	238 138	256 138	273 138
44	044 00	061 71	078 12	096 99	113 152	131 147	149 147	167 147	185 147	203 147	221 147	239 147	257 147	274 147
45	045 00	062 75	079 29	097 08	114 161	132 156	150 156	168 156	186 156	204 156	222 156	240 156	258 156	275 156
46	046 00	063 79	080 17	098 17	115 170	133 165	151 165	169 165	187 165	205 165	223 165	241 165	259 165	276 165
47	047 00	064 83	081 04	099 26	116 179	134 174	152 174	170 174	188 174	206 174	224 174	242 174	260 174	277 174
48	048 00	065 87	082 22	100 35	117 188	135 183	153 183	171 183	189 183	207 183	225 183	243 183	261 183	278 183
49	049 00	066 91	083 09	101 44	118 197	136 192	154 192	172 192	190 192	208 192	226 192	244 192	262 192	279 192
50	050 00	067 95	084 27	102 53	119 206	137 201	155 201	173 201	191 201	209 201	227 201	245 201	263 201	280 201
51	051 00	068 99	085 14	103 62	120 215	138 210	156 210	174 210	192 210	210 210	228 210	246 210	264 210	281 210
52	052 00	069 03	086 02	104 71	121 224	139 219	157 219	175 219	193 219	211 219	229 219	247 219	265 219	282 219
53	053 00	070 07	087 20	105 80	122 233	140 228	158 228	176 228	194 228	212 228	230 228	248 228	266 228	283 228
54	054 00	071 11	088 28	106 89	123 242	141 237	159 237	177 237	195 237	213 237	231 237	249 237	267 237	284 237
55	055 00	072 15	089 36	107 98	124 251	142 246	160 246	178 246	196 246	214 246	232 246	250 246	268 246	285 246
56	056 00	073 19	090 45	108 107	125 260	143 255	161 255	179 255	197 255	215 255	233 255	251 255	269 255	286 255
57	057 00	074 23	091 54	109 116	126 269	144 264	162 264	180 264	198 264	216 264	234 264	252 264	270 264	287 264
58	058 00	075 27	092 63	110 125	127 278	145 273	163 273	181 273	199 273	217 273	235 273	253 273	271 273	288 273
59	059 00	076 31	093 72	111 134	128 287	146 282	164 282	182 282	200 282	218 282	236 282	254 282	272 282	289 282
60	060 00	077 35	094 81	112 143	129 296	147 291	165 291	183 291	201 291	219 291	237 291	255 291	273 291	290 291
61	061 00	078 39	095 90	113 152	130 305	148 300	166 300	184 300	202 300	220 300	238 300	256 300	274 300	291 300
62	062 00	079 43	096 99	114 161	131 314	149 309	167 309	185 309	203 309	221 309	239 309	257 309	275 309	292 309
63	063 00	080 47	097 08	115 170	132 323	150 318	168 318	186 318	204 318	222 318	240 318	258 318	276 318	293 318
64	064 00	081 51	098 17	116 179	133 332	151 327	169 327	187 327	205 327	223 327	241 327	259 327	277 327	294 327
65	065 00	082 55	099 26	117 188	134 341	152 336	170 336	188 336	206 336	224 336	242 336	260 336	278 336	295 336
66	066 00	083 59	100 35	118 197	135 350	153 345	171 345	189 345	207 345	225 345	243 345	261 345	279 345	296 345
67	067 00	084 63	101 44	119 206	136 359	154 354	172 354	190 354	208 354	226 354	244 354	262 354	280 354	297 354
68	068 00	085 67	102 53	120 215	137 368	155 363	173 363	191 363	209 363	227 363	245 363	263 363	281 363	298 363
69	069 00	086 71	103 62	121 224	138 377	156 372	174 372	192 372	210 372	228 372	246 372	264 372	282 372	299 372
70	070 00	087 75	104 71	122 233	139 386	157 381	175 381	193 381	211 381	229 381	247 381	265 381	283 381	3

72T*		NATURAL TANGENT.										TABLE B.	
	9°	10°	11°	12°	13°	14°	15°	16°	17°				
0	158 38	176 33	194 88	212 56	230 87	249 33	267 95	286 74	305 73	03			
1	68	93	68	86	231 17	64	268 26	287 06	306 05	69			
2	98	63	98	213 16	48	95	57	38	37	58			
3	159 28	177 23	195 29	47	79	250 26	88	69	69	57			
4	58	53	59	77	232 09	56	269 20	288 01	307 00	56			
5	88	83	89	214 08	40	87	51	32	32	55			
6	160 17	178 13	196 19	38	71	251 18	82	64	64	54			
7	47	43	49	69	233 01	49	270 13	95	96	53			
8	77	73	80	99	32	80	44	289 27	308 28	52			
9	161 07	179 03	197 10	215 29	63	252 11	76	58	60	51			
10	37	33	40	60	93	42	271 07	90	91	50			
11	67	63	70	90	234 24	73	38	290 21	309 23	49			
12	96	93	198 01	216 21	55	253 04	69	53	55	48			
13	162 26	180 23	31	51	85	35	272 01	84	87	47			
14	56	53	61	82	235 16	66	32	291 16	310 19	46			
15	86	83	91	217 12	47	97	63	47	51	45			
16	163 16	181 13	199 21	43	78	254 28	94	79	83	44			
17	46	43	52	73	236 08	59	273 26	292 10	311 15	43			
18	76	73	82	218 04	39	90	57	42	47	42			
19	164 06	182 03	200 12	34	70	255 21	88	74	78	41			
20	35	33	42	64	237 00	52	274 19	293 05	312 10	40			
21	65	63	73	95	31	83	51	37	42	39			
22	95	93	201 03	219 25	62	256 14	82	68	74	38			
23	165 25	183 23	33	56	93	45	275 13	294 00	313 06	37			
24	55	53	64	86	238 23	76	45	32	38	36			
25	85	84	94	220 17	54	257 07	76	63	70	35			
26	166 15	184 14	202 24	47	85	38	276 07	95	314 02	34			
27	45	44	54	78	239 16	69	39	295 26	343	33			
28	74	74	85	221 08	46	258 00	70	58	66	32			
29	167 04	185 04	203 15	39	77	31	277 01	90	98	31			
30	34	34	45	69	240 08	62	32	296 21	315 30	30			
31	64	64	76	222 00	39	93	64	53	62	29			
32	94	94	204 06	31	69	259 24	95	85	94	28			
33	168 24	186 24	36	61	241 00	55	278 26	297 16	316 26	27			
34	54	54	66	92	31	86	58	48	58	26			
35	84	84	97	223 22	62	260 17	89	80	90	25			
36	169 14	187 14	205 27	53	93	48	279 21	298 11	317 22	24			
37	44	45	57	83	242 23	79	52	43	54	23			
38	74	75	88	224 14	54	261 10	83	75	86	22			
39	170 04	188 05	206 18	44	85	41	280 15	299 06	318 18	21			
40	33	35	48	75	243 16	72	46	38	50	20			
41	63	65	79	225 05	47	262 03	77	70	82	19			
42	93	95	207 09	36	77	35	281 09	300 01	319 14	18			
43	171 23	189 25	39	67	244 08	66	40	33	46	17			
44	53	55	70	97	39	97	72	65	78	16			
45	83	86	208 00	226 28	70	263 28	282 03	97	320 10	15			
46	172 13	190 16	30	58	245 01	59	34	301 28	42	14			
47	43	46	61	89	32	90	66	60	74	13			
48	73	76	91	227 19	62	264 21	97	92	321 06	12			
49	173 03	191 06	209 21	50	93	52	283 29	302 24	39	11			
50	33	36	52	81	246 24	83	60	55	71	10			
51	63	66	82	228 11	55	295 15	91	87	322 03	9			
52	93	97	210 13	42	86	46	284 23	303 19	35	8			
53	174 23	192 27	43	72	247 17	77	54	51	67	7			
54	53	57	73	229 03	48	266 08	86	82	99	6			
55	83	87	211 04	34	78	39	285 17	304 14	323 31	5			
56	175 13	193 17	34	64	248 09	70	49	46	63	4			
57	43	47	64	95	40	267 01	80	78	96	3			
58	73	78	95	230 26	71	33	286 12	305 09	324 28	2			
59	176 03	194 08	212 25	56	239 02	64	43	41	60	1			
60	33	38	56	87	33	95	74	73	92	0			
	80°	79°	78°	77°	76°	75°	74°	73°	72°				
NATURAL COTANGENT.													

TABLE B. NATURAL TANGENT.												72u*
	18°	19°	20°	21°	22°	23°	24°	25°	26°			
0	324 92	344 33	363 97	383 86	404 03	424 47	445 23	466 31	487 73	60		
1	325 24	65 364 30	384 20		36	82	58	66	488 09	59		
2	56	98	63	53	70	425 16	93	467 02	45	58		
3	88	345 30	96	87	405 04	51	446 27	37	81	57		
4	326 21	63	365 29	385 20	38	85	62	73	489 17	56		
5	53	96	62	53	72	426 19	97	468 08	53	55		
6	85	346 28	95	87	406 06	54	447 32	43	89	54		
7	327 17	61	366 28	386 20	40	88	67	79	490 26	53		
8	49	93	61	54	74	427 22	448 02	469 14	62	52		
9	82	347 26	94	87	407 07	57	37	50	98	51		
10	328 14	58	367 27	387 21	41	91	72	85	491 34	50		
11	46	91	60	54	75	428 26	449 07	470 21	70	49		
12	78	348 24	93	87	408 09	60	42	56	492 06	48		
13	329 11	56	368 26	388 21	43	94	77	92	42	47		
14	43	89	59	54	77	429 29	450 12	471 28	78	46		
15	75	349 22	92	88	409 11	63	47	63	493 15	45		
16	330 07	54	369 25	389 21	45	98	82	99	51	44		
17	40	87	58	55	79	430 32	451 17	472 34	87	43		
18	72	350 20	91	88	410 13	67	52	70	494 23	42		
19	331 04	52	370 24	390 22	47	431 01	87	473 05	59	41		
20	36	85	57	55	81	36	452 22	41	95	40		
21	69	351 18	90	89	411 15	70	57	77	495 32	39		
22	332 01	50	371 23	391 22	49	432 05	92	474 12	68	38		
23	33	83	57	56	83	39	453 27	48	496 04	37		
24	66	352 16	90	90	412 17	74	62	83	40	36		
25	98	48	372 23	392 23	51	433 08	97	475 19	77	35		
26	333 30	51	56	57	85	43	454 32	55	497 13	34		
27	63	353 14	89	90	413 19	78	67	90	49	33		
28	95	46	373 22	393 24	53	434 12	455 02	476 26	86	32		
29	334 27	79	55	57	87	47	38	62	498 22	31		
30	60	354 12	88	91	414 21	81	73	98	58	30		
31	92	45	374 22	394 25	55	435 16	456 08	477 33	94	29		
32	335 24	77	55	58	90	50	43	69	499 31	28		
33	57	355 10	88	92	415 24	85	78	478 05	67	27		
34	89	43	375 21	395 26	58	436 20	457 13	40	500 04	26		
35	336 21	76	54	59	92	54	48	76	40	25		
36	54	356 08	88	93	416 26	89	84	479 12	76	24		
37	86	41	376 21	396 26	60	437 24	458 19	48	501 13	23		
38	337 18	74	54	60	94	58	54	84	49	22		
39	51	357 07	87	94	417 28	93	89	480 19	85	21		
40	83	40	377 20	397 27	63	438 28	459 24	55	502 22	20		
41	338 16	72	54	61	97	62	60	91	58	19		
42	48	358 05	87	95	418 31	97	95	481 27	95	18		
43	81	38	378 20	398 29	65	439 32	460 30	63	508 31	17		
44	339 13	71	53	62	99	66	65	98	68	16		
45	45	359 04	87	96	419 33	440 01	461 01	482 34	504 04	15		
46	78	37	379 20	399 30	68	36	36	70	41	14		
47	340 10	69	53	63	420 02	71	71	483 06	77	13		
48	43	360 02	86	97	36	441 05	462 06	42	505 14	12		
49	75	35	380 20	400 31	70	40	42	78	50	11		
50	341 08	68	53	65	421 05	75	77	484 14	87	10		
51	40	361 01	86	98	39	442 10	463 12	50	506 23	9		
52	73	34	381 20	401 32	73	44	48	86	60	8		
53	342 05	67	53	66	422 07	79	83	485 21	96	7		
54	38	99	86	402 00	42	443 14	464 18	57	507 33	6		
55	70	362 32	382 20	34	76	49	54	93	69	5		
56	343 03	65	53	67	423 10	84	89	486 29	508 06	4		
57	35	98	86	403 01	45	444 18	465 25	65	43	3		
58	68	363 31	383 20	35	79	53	60	487 01	79	2		
59	344 00	64	53	69	424 13	88	95	37	509 16	1		
60	33	97	86	404 03	47	445 23	466 31	73	53	0		
	71°	70°	69°	68°	67°	66°	65°	64°	63°			
NATURAL COTANGENT.												

72v* NATURAL TANGENT. TABLE B.										
	27°	28°	29°	30°	31°	32°	33°	34°	35°	
0	509 53	531 71	554 31	577 35	600 86	624 87	649 41	674 51	700 21	60
1	89	532 08	556 69	579 74	601 26	625 27	650 82	675 93	701 43	59
2	510 26	46	555 07	578 13	602 65	626 68	651 24	676 36	702 47	58
3	63	83	45	51	602 05	626 08	651 65	676 78	702 89	57
4	99	533 20	557 83	580 90	603 45	627 49	652 06	677 20	703 31	56
5	511 36	58	556 21	579 29	604 84	628 89	653 48	678 63	704 74	55
6	73	95	59	68	605 24	629 30	654 89	679 05	705 16	54
7	512 09	534 32	558 97	581 07	606 64	630 70	655 31	680 48	706 59	53
8	46	70	557 36	580 46	607 03	631 11	656 72	681 90	707 01	52
9	83	535 07	559 74	582 85	608 43	632 52	657 14	682 32	708 43	51
10	513 20	45	558 12	581 24	609 83	633 92	658 55	683 75	709 55	50
11	56	82	50	62	610 22	634 33	659 97	684 17	710 66	49
12	93	536 20	560 88	583 01	611 62	635 73	660 38	685 60	711 70	48
13	514 30	57	559 26	583 40	612 02	636 14	661 80	686 02	712 81	47
14	67	94	64	79	613 42	637 55	662 21	687 45	713 92	46
15	515 03	537 32	561 03	584 18	614 81	639 95	663 63	688 88	714 03	45
16	40	69	41	57	615 21	631 36	656 04	681 30	709 17	44
17	77	538 07	562 79	585 96	616 61	632 77	657 46	682 78	710 28	43
18	516 14	44	561 17	584 35	617 01	633 17	658 88	683 15	711 39	42
19	51	82	56	74	618 41	634 58	660 29	684 85	712 50	41
20	88	539 20	563 94	586 13	619 81	636 99	661 01	685 31	713 61	40
21	517 24	57	562 32	587 52	620 21	638 40	662 31	686 43	714 72	39
22	61	95	70	91	621 60	640 80	663 54	687 86	715 83	38
23	98	540 32	565 09	588 31	622 00	641 21	664 96	688 29	716 94	37
24	518 35	70	47	70	623 40	642 62	666 38	689 71	718 05	36
25	72	541 07	85	587 09	624 80	643 03	667 80	690 14	719 16	35
26	519 09	45	564 24	589 48	625 20	644 44	668 21	691 75	720 27	34
27	46	83	62	87	626 60	645 84	669 63	692 00	721 38	33
28	83	542 20	565 01	588 26	627 00	646 25	670 05	693 42	722 49	32
29	520 20	58	39	65	628 40	647 66	671 47	694 85	723 60	31
30	57	96	77	589 05	629 80	648 07	672 89	695 28	724 71	30
31	94	543 33	566 16	590 44	630 20	649 48	673 30	696 71	725 82	29
32	521 31	71	54	83	631 60	650 89	674 72	697 14	726 93	28
33	68	544 09	93	590 22	632 00	651 30	675 14	698 57	728 04	27
34	522 05	46	567 31	591 61	633 40	652 71	676 56	699 90	729 15	26
35	42	84	69	591 01	634 80	653 12	677 98	700 42	730 26	25
36	79	545 22	568 08	592 40	635 20	654 53	679 40	701 85	731 37	24
37	523 16	60	46	79	636 61	655 94	680 82	702 28	732 48	23
38	53	97	85	592 18	637 01	656 35	681 24	703 71	733 59	22
39	90	546 35	569 23	593 58	638 41	657 76	682 66	704 14	734 70	21
40	524 27	73	62	97	639 81	658 17	683 08	705 57	735 81	20
41	64	547 11	570 00	594 36	640 21	659 58	684 50	706 92	736 92	19
42	525 01	48	39	76	641 61	660 99	685 92	707 83	738 03	18
43	38	86	78	594 15	642 01	661 40	686 34	708 76	739 14	17
44	75	548 24	571 16	595 54	643 42	662 81	687 76	709 69	740 25	16
45	526 13	62	55	94	644 82	663 22	688 18	710 72	741 36	15
46	50	549 00	93	595 33	645 22	664 62	689 60	711 61	742 47	14
47	87	38	572 32	73	646 62	665 04	690 02	712 59	743 58	13
48	527 24	75	71	596 12	647 02	666 46	691 44	713 52	744 69	12
49	61	550 13	573 09	51	648 43	667 87	692 86	714 45	745 80	11
50	98	51	48	91	649 83	668 28	693 28	715 88	746 91	10
51	528 36	89	86	597 30	650 24	669 69	694 71	716 81	748 02	9
52	73	551 27	574 25	70	651 64	670 10	695 13	717 75	749 13	8
53	529 10	65	64	598 09	652 04	671 25	696 31	718 69	750 24	7
54	47	552 03	575 03	49	653 45	672 66	697 69	719 61	751 35	6
55	85	41	41	88	654 85	673 07	698 72	720 54	752 46	5
56	530 22	79	80	599 28	655 25	674 47	699 82	721 74	753 57	4
57	59	553 17	576 19	67	656 66	675 88	700 24	722 71	754 68	3
58	96	55	57	600 07	657 06	676 28	701 66	723 74	755 79	2
59	531 34	93	96	46	658 47	677 69	702 09	724 77	756 90	1
60	71	554 31	577 35	86	659 87	678 11	703 51	725 70	758 01	0
	62°	61°	60°	59°	58°	57°	56°	55°	54°	
NATURAL COTANGENT.										

TABLE B. NATURAL TANGENT. 72w*																
	36°	37°	38°	39°	40°	41°	42°	43°	44°							
0	726 54	753 55	781 29	809 78	839 10	869 29	900 40	932 52	965 69	06						
1	99	754 01	75	810 27	60	80	93	933 06	966 25	59						
2	727 43	47	782 22	75	840 09	870 31	901 46	60	81	58						
3	88	92	69	811 23	59	82	99	934 15	967 38	57						
4	728 32	755 38	783 16	71	841 08	871 33	902 51	69	94	56						
5	77	84	63	812 20	58	84	903 04	935 24	968 50	55						
6	729 21	756 29	784 10	68	842 08	872 36	57	78	969 07	54						
7	66	75	57	813 16	58	87	904 10	936 33	63	53						
8	730 10	757 21	785 04	64	843 07	873 38	63	88	970 20	52						
9	55	67	51	814 13	57	89	905 16	937 42	76	51						
10	731 00	758 12	98	61	844 07	874 41	69	97	971 33	50						
11	44	58	786 45	815 10	57	92	906 21	938 52	89	49						
12	89	759 04	92	58	845 07	875 43	74	939 06	972 46	48						
13	732 34	50	787 39	816 06	56	95	907 27	61	973 02	47						
14	78	96	86	55	846 06	876 46	81	940 16	59	46						
15	733 23	760 42	788 34	817 03	56	98	908 34	71	974 16	45						
16	68	88	81	52	847 06	877 49	87	941 25	72	44						
17	734 13	761 34	789 28	818 00	56	878 01	909 40	80	975 29	43						
18	57	80	75	49	848 06	52	93	942 35	86	42						
19	735 02	762 26	790 22	98	56	879 04	910 46	90	976 43	41						
20	47	72	70	819 46	849 06	55	99	943 45	977 00	40						
21	92	763 18	791 17	95	56	880 07	911 53	944 00	56	39						
22	736 37	64	64	820 44	850 06	59	912 06	55	978 13	38						
23	81	764 10	792 12	92	57	881 10	59	945 10	70	37						
24	737 26	56	59	821 41	851 07	62	913 13	65	979 27	36						
25	71	765 02	793 06	90	57	882 14	66	946 20	84	35						
26	738 16	48	54	822 38	852 07	65	914 19	76	980 41	34						
27	61	94	794 01	87	57	883 17	73	947 31	98	33						
28	739 06	766 40	49	823 36	853 07	69	915 26	86	981 55	32						
29	51	86	96	85	58	884 21	80	948 41	982 13	31						
30	96	767 33	795 44	824 34	854 08	73	916 33	96	70	30						
31	740 41	79	91	83	58	885 24	87	949 52	983 27	29						
32	86	768 25	796 39	825 31	855 09	76	917 40	950 07	84	28						
33	741 31	71	86	80	59	886 28	94	62	984 41	27						
34	76	769 18	797 34	826 29	856 10	80	918 47	951 18	99	26						
35	742 21	64	81	78	60	887 32	919 01	73	985 56	25						
36	67	770 10	798 29	827 27	857 10	84	55	952 29	986 13	24						
37	743 12	57	77	76	61	888 36	920 08	84	71	23						
38	57	771 03	799 24	828 25	858 11	88	62	953 40	987 28	22						
39	744 02	49	72	74	62	889 40	921 16	95	86	21						
40	47	96	800 20	829 23	859 12	92	70	954 51	988 43	20						
41	92	772 42	67	72	63	890 45	922 24	955 06	989 01	19						
42	745 38	89	801 15	830 22	860 14	97	77	62	58	18						
43	83	773 35	63	71	64	891 49	923 31	956 18	990 16	17						
44	746 28	82	802 11	831 20	861 15	892 01	85	73	73	16						
45	74	774 28	58	69	66	53	924 39	957 29	991 31	15						
46	747 19	75	803 06	832 18	862 16	893 06	93	85	89	14						
47	64	775 21	54	68	67	58	925 47	958 41	992 47	13						
48	748 10	68	804 02	833 17	863 18	894 10	926 01	97	993 04	12						
49	55	776 15	50	66	68	63	55	959 52	62	11						
50	749 00	61	98	834 15	864 19	895 15	927 09	960 08	994 20	10						
51	46	777 08	805 46	65	70	67	63	64	78	9						
52	91	54	94	835 14	865 21	896 20	928 17	961 20	995 36	8						
53	750 37	778 01	806 42	64	72	72	72	76	94	7						
54	82	48	90	836 13	866 23	897 25	929 26	962 32	996 52	6						
55	751 28	95	807 38	62	74	77	80	88	997 10	5						
56	73	779 41	86	837 12	867 25	898 30	930 34	963 44	68	4						
57	752 19	88	808 34	61	76	83	88	964 00	998 26	3						
58	64	780 35	82	838 11	868 27	899 35	931 43	57	84	2						
59	753 10	82	809 30	60	78	88	97	965 13	999 42	1						
60	55	781 29	78	839 10	869 29	900 40	932 52	69	1.000 00	0						
	59°	52°	51°	50°	49°	48°	47°	46°	45°							
NATURAL COTANGENT.																

72x*		NATURAL TANGENT.										TABLE B.	
	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°		
0	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175	1.23490	1.27994	1.32694	1.37594	1.42694	60	
1	058	618	299	126	104	246	563	1.28071	1.32871	1.37871	1.43071	59	
2	116	674	362	191	172	316	637	1.32871	1.37871	1.43071	1.48371	58	
3	175	734	425	256	240	387	710	1.37871	1.43071	1.48371	1.53771	57	
4	233	794	487	321	308	457	784	1.43071	1.48371	1.53771	1.59271	56	
5	291	855	550	387	375	528	858	1.48371	1.53771	1.59271	1.64871	55	
6	350	915	613	452	443	599	931	1.53771	1.59271	1.64871	1.70571	54	
7	408	976	676	517	511	669	1.24005	1.59271	1.64871	1.70571	1.76371	53	
8	467	1.04036	738	582	579	740	079	1.64871	1.70571	1.76371	1.82271	52	
9	525	097	801	648	647	811	153	1.70571	1.76371	1.82271	1.88271	51	
10	583	158	864	713	715	882	227	1.76371	1.82271	1.88271	1.94371	50	
11	642	218	927	778	783	953	301	1.82271	1.88271	1.94371	2.00571	49	
12	701	279	990	844	851	1.20024	375	1.88271	1.94371	2.00571	2.06871	48	
13	759	340	1.08053	909	919	095	449	1.94371	2.00571	2.06871	2.13271	47	
14	818	401	116	975	987	166	523	2.00571	2.06871	2.13271	2.19771	46	
15	876	461	179	1.12041	1.16056	237	597	2.06871	2.13271	2.19771	2.26371	45	
16	935	522	243	106	124	308	672	2.13271	2.19771	2.26371	2.33071	44	
17	994	583	306	172	192	379	746	2.19771	2.26371	2.33071	2.39871	43	
18	1.01058	644	369	238	261	451	820	2.26371	2.33071	2.39871	2.46771	42	
19	112	705	432	303	329	522	895	2.33071	2.39871	2.46771	2.53771	41	
20	170	766	496	369	398	593	969	2.39871	2.46771	2.53771	2.60871	40	
21	229	827	559	435	466	665	1.25044	2.46771	2.53771	2.60871	2.68071	39	
22	288	888	622	501	535	736	118	2.53771	2.60871	2.68071	2.75371	38	
23	347	949	686	567	603	808	193	2.60871	2.68071	2.75371	2.82771	37	
24	406	1.05010	749	633	672	879	268	2.68071	2.75371	2.82771	2.90271	36	
25	465	072	813	699	741	951	343	2.75371	2.82771	2.90271	2.97871	35	
26	524	133	876	765	809	1.21023	417	2.82771	2.90271	2.97871	3.05571	34	
27	583	194	940	831	878	094	492	2.90271	2.97871	3.05571	3.13371	33	
28	642	255	1.09003	897	947	166	567	2.97871	3.05571	3.13371	3.21271	32	
29	702	317	067	963	1.17016	238	642	3.05571	3.13371	3.21271	3.29271	31	
30	761	378	131	1.18029	085	310	717	3.13371	3.21271	3.29271	3.37371	30	
31	820	439	195	096	154	382	792	3.21271	3.29271	3.37371	3.45571	29	
32	879	501	258	162	223	454	867	3.29271	3.37371	3.45571	3.53871	28	
33	939	562	322	228	292	526	943	3.37371	3.45571	3.53871	3.62271	27	
34	998	624	386	295	361	598	1.26.018	3.45571	3.53871	3.62271	3.70771	26	
35	1.02057	685	450	361	430	670	093	3.53871	3.62271	3.70771	3.79371	25	
36	117	747	514	428	500	742	169	3.62271	3.70771	3.79371	3.88071	24	
37	176	809	578	494	569	814	244	3.70771	3.79371	3.88071	3.96871	23	
38	236	870	642	561	638	887	320	3.79371	3.88071	3.96871	4.05771	22	
39	295	932	706	627	708	959	395	3.88071	3.96871	4.05771	4.14771	21	
40	355	994	770	694	777	1.22031	471	3.96871	4.05771	4.14771	4.23871	20	
41	414	1.06056	834	761	846	104	546	4.05771	4.14771	4.23871	4.33071	19	
42	474	117	899	828	916	176	622	4.14771	4.23871	4.33071	4.42371	18	
43	533	179	963	894	986	249	698	4.23871	4.33071	4.42371	4.51771	17	
44	593	241	1.10027	961	1.18055	321	774	4.33071	4.42371	4.51771	4.61271	16	
45	653	303	091	1.14028	125	394	849	4.42371	4.51771	4.61271	4.70871	15	
46	713	365	156	095	194	467	925	4.51771	4.61271	4.70871	4.80571	14	
47	772	427	220	162	264	539	1.27001	4.61271	4.70871	4.80571	4.90371	13	
48	832	489	285	229	334	612	077	4.70871	4.80571	4.90371	5.00271	12	
49	892	551	349	296	404	685	153	4.80571	4.90371	5.00271	5.10271	11	
50	952	613	414	363	474	758	230	4.90371	5.00271	5.10271	5.20371	10	
51	1.03012	676	478	430	544	831	306	5.00271	5.10271	5.20371	5.30571	9	
52	072	738	543	498	614	904	382	5.10271	5.20371	5.30571	5.40871	8	
53	132	800	608	565	684	977	458	5.20371	5.30571	5.40871	5.51271	7	
54	192	862	672	632	754	1.23050	535	5.30571	5.40871	5.51271	5.61771	6	
55	252	925	737	699	824	123	611	5.40871	5.51271	5.61771	5.72371	5	
56	312	987	802	767	894	196	688	5.51271	5.61771	5.72371	5.83071	4	
57	372	1.07049	867	834	964	270	764	5.61771	5.72371	5.83071	5.93871	3	
58	433	112	931	902	1.19035	343	841	5.72371	5.83071	5.93871	6.04771	2	
59	493	174	996	969	105	416	917	5.83071	5.93871	6.04771	6.15771	1	
60	553	237	1.11061	1.15037	175	490	994	5.93871	6.04771	6.15771	6.26871	0	
	44°	43°	42°	41°	40°	39°	38°	37°					
NATURAL COTANGENT.													

TABLE B.		NATURAL TANGENT.							72y*
	58°	54°	55°	56°	57°	58°	59°	60°	
0	1.32704	1.37638	1.42815	1.48256	1.53987	1.60033	1.66428	1.73205	60
1	785	722	908	349	1.54085	137	538	321	59
2	865	807	992	442	183	241	647	438	58
3	946	891	1.48080	536	281	345	757	555	57
4	1.33026	976	169	629	379	449	867	671	56
5	107	1.38060	258	722	478	553	978	788	55
6	188	145	347	816	576	657	1.67088	905	54
7	268	229	436	909	675	761	198	1.74022	53
8	349	314	525	1.49003	774	865	309	140	52
9	430	399	614	097	873	970	419	257	51
10	511	484	703	190	972	1.61074	530	375	50
11	592	568	792	284	1.55071	179	641	492	49
12	673	653	881	378	170	283	752	610	48
13	754	738	970	472	269	388	863	728	47
14	835	824	1.44060	566	368	493	974	846	46
15	916	909	149	661	467	598	1.68085	964	45
16	998	994	239	755	567	703	196	1.75082	44
17	1.34079	1.39079	329	849	666	809	308	200	43
18	160	165	418	944	766	914	419	319	42
19	242	250	508	1.50038	866	1.62019	531	437	41
20	323	336	598	133	966	125	643	556	40
21	405	421	688	228	1.56065	230	754	675	39
22	487	507	778	322	165	336	866	794	38
23	568	592	868	417	265	442	979	913	37
24	650	679	968	512	366	548	1.69091	1.76032	36
25	732	764	1.45049	607	466	654	203	151	35
26	814	850	139	702	566	760	316	271	34
27	896	936	229	797	667	866	428	390	33
28	978	1.40022	320	893	767	972	541	510	32
29	1.35060	109	410	988	868	1.63079	653	630	31
30	142	195	501	1.51084	969	185	766	749	30
31	224	281	592	179	1.57069	292	879	869	29
32	307	367	682	275	170	398	992	990	28
33	389	454	773	370	271	505	1.70106	1.77110	27
34	472	540	864	466	372	612	219	230	26
35	554	627	955	562	474	719	332	351	25
36	637	714	1.46046	658	575	826	446	471	24
37	719	800	137	754	676	934	560	592	23
38	802	887	229	850	778	1.64041	673	713	22
39	885	974	320	946	879	148	787	834	21
40	968	1.41061	411	1.52043	981	256	901	955	20
41	1.36051	148	503	139	1.58083	363	1.71015	1.78077	19
42	134	235	595	235	184	471	129	198	18
43	217	322	686	332	286	579	244	319	17
44	300	409	778	429	388	687	358	441	16
45	383	497	870	525	490	795	473	563	15
46	466	584	962	622	593	903	588	685	14
47	549	672	1.47054	719	695	1.65011	702	807	13
48	633	759	146	816	797	120	817	929	12
49	716	847	238	913	900	228	932	1.79051	11
50	800	934	330	1.53010	1.59002	337	1.72047	174	10
51	883	1.42022	422	107	105	445	163	296	9
52	967	110	514	205	208	554	278	419	8
53	1.37050	198	607	302	311	663	393	542	7
54	134	286	699	400	414	772	509	665	6
55	218	374	792	497	517	881	625	788	5
56	302	462	885	595	620	990	741	911	4
57	386	550	977	693	723	1.66099	857	1.80034	3
58	470	638	1.48070	791	826	209	973	158	2
59	554	726	163	888	930	818	1.73089	281	1
60	638	815	256	987	1.60033	428	205	405	0
	36°	36°	34°	33°	32°	31°	30°	29°	
NATURAL COTANGENT.									

NATURAL TANGENT.									TABLE B.	
72z*	61°	62°	63°	64°	65°	66°	67°	68°		
0	1.80405	1.88073	1.96261	2.05080	2.14451	2.24604	2.35585	2.47509	60	
1	529	205	402	182	614	780	776	716	59	
2	653	337	544	333	777	956	967	924	58	
3	777	469	685	485	940	2.25132	2.36158	2.48132	57	
4	901	602	827	637	2.15104	309	349	340	56	
5	1.81025	734	969	790	268	486	541	549	55	
6	150	867	1.97111	942	432	663	733	758	54	
7	274	1.89000	253	2.06094	596	840	925	967	53	
8	399	133	395	247	760	2.26018	2.37118	2.49177	52	
9	524	266	538	400	925	196	311	386	51	
10	649	400	681	553	2.16090	374	504	597	50	
11	774	533	823	706	255	552	697	807	49	
12	899	667	966	860	420	730	891	2.50018	48	
13	1.82025	801	1.98110	2.07014	585	909	2.38084	229	47	
14	150	935	253	167	751	2.27088	279	440	46	
15	276	1.90069	396	321	917	267	473	652	45	
16	402	203	540	476	2.17083	447	668	864	44	
17	528	337	684	630	249	626	863	2.51076	43	
18	654	472	828	785	416	806	2.39058	289	42	
19	780	607	972	939	582	987	253	502	41	
20	906	741	1.99116	2.08094	749	2.28167	449	715	40	
21	1.83033	876	261	250	916	348	645	929	39	
22	159	1.91012	406	405	2.18084	528	841	2.52142	38	
23	286	147	550	560	251	710	2.40038	357	37	
24	413	282	695	716	419	891	235	571	36	
25	540	418	841	872	587	2.29073	432	786	35	
26	667	554	986	2.09028	755	254	629	2.53001	34	
27	794	690	2.00131	184	923	437	827	217	33	
28	922	826	277	341	2.19092	619	2.41025	432	32	
29	1.84049	962	423	498	261	801	223	648	31	
30	177	1.92098	569	654	430	984	421	865	30	
31	305	235	715	811	599	2.30167	620	2.54082	29	
32	433	371	862	969	769	351	819	299	28	
33	561	508	2.01008	2.10126	938	534	2.42019	516	27	
34	689	645	155	284	2.20108	718	218	734	26	
35	818	782	302	442	278	902	418	952	25	
36	946	920	449	600	449	2.31086	618	2.55170	24	
37	1.85075	1.93057	596	758	619	271	819	389	23	
38	204	195	743	916	790	456	2.43019	608	22	
39	333	332	891	2.11075	961	641	220	827	21	
40	462	470	2.02039	233	2.21132	826	422	2.56046	20	
41	591	608	187	392	304	2.32012	623	266	19	
42	720	746	335	552	475	197	825	487	18	
43	850	885	483	711	647	383	2.44027	707	17	
44	979	1.94023	631	871	819	570	230	928	16	
45	1.86109	162	780	2.12030	992	756	433	2.57150	15	
46	239	301	929	190	2.22164	943	636	371	14	
47	369	440	2.03078	350	337	2.33130	839	593	13	
48	499	579	227	511	510	317	2.45043	815	12	
49	630	718	376	671	683	505	246	2.58038	11	
50	760	858	526	832	857	693	451	261	10	
51	891	997	675	993	2.23030	881	655	484	9	
52	1.87021	1.95137	825	2.13154	204	2.34069	860	708	8	
53	152	277	975	316	378	258	2.46065	932	7	
54	283	417	2.04125	477	553	447	270	2.59166	6	
55	415	557	276	639	727	636	476	381	5	
56	546	698	426	801	902	825	682	606	4	
57	677	838	577	963	2.24077	2.35015	888	831	3	
58	809	979	728	2.14125	252	205	2.47095	2.60057	2	
59	941	1.96120	879	288	428	395	302	283	1	
60	1.88073	261	2.05030	451	604	585	509	509	0	
	28°	27°	26°	25°	24°	23°	22°	21°		
NATURAL COTANGENT.										

TABLE B. NATURAL TANGENT.										72A**
	69°	70°	71°	72°	73°	74°	75°	76°		
0	2.60509	2.74748	2.90421	3.07768	3.27085	3.48741	3.73205	4.01078	60	
1	736	997	696	3.08073	426	3.49125	640	576	59	
2	963	2.75246	971	379	767	509	3.74075	4.02074	58	
3	2.61190	496	2.91246	685	3.28109	894	512	574	57	
4	418	746	523	991	452	3.50279	950	4.03076	56	
5	646	996	799	3.09298	795	666	3.75388	578	55	
6	874	2.76247	2.92076	606	3.29139	3.51053	828	4.04081	54	
7	2.62103	498	354	914	483	441	3.76268	589	53	
8	332	750	632	3.10223	829	829	709	4.05092	52	
9	561	2.77002	910	532	3.30174	3.52219	3.77152	599	51	
10	791	254	2.93189	842	521	609	595	4.06107	50	
11	2.63021	507	468	3.11153	868	3.53001	3.78040	616	49	
12	252	761	748	464	3.31216	393	485	4.07127	48	
13	483	2.78014	2.94028	775	565	785	931	639	47	
14	714	269	309	3.12087	914	3.54179	3.79378	4.08152	46	
15	945	523	591	400	3.32264	573	827	666	45	
16	2.64177	778	872	713	614	968	3.80276	4.09182	44	
17	410	2.79033	2.95155	3.13027	965	3.55364	726	699	43	
18	642	289	437	341	3.33317	761	3.81177	4.10216	42	
19	875	545	721	656	670	3.56169	630	736	41	
20	2.65109	802	2.96004	972	3.34023	557	3.82083	4.11256	40	
21	342	2.80059	288	3.14288	377	957	537	778	39	
22	576	316	573	605	732	3.57357	992	4.12301	38	
23	811	574	858	922	3.35087	758	3.83449	825	37	
24	2.66046	833	2.97144	3.15240	443	3.58160	906	4.13350	36	
25	281	2.81091	430	558	800	562	3.84364	877	35	
26	516	350	717	877	3.36158	966	824	4.14405	34	
27	752	610	2.98004	3.16197	516	3.59370	3.85284	934	33	
28	989	870	292	517	875	775	745	4.15465	32	
29	2.67225	2.82130	580	838	3.37234	3.60181	3.86208	997	31	
30	462	391	869	3.17159	594	588	671	4.16530	30	
31	700	653	2.99158	481	955	996	3.87136	4.17064	29	
32	937	914	447	804	3.38317	3.61405	601	600	28	
33	2.68175	2.83176	738	3.18127	679	814	3.88068	4.18137	27	
34	414	439	3.00028	451	3.39042	3.62224	536	675	26	
35	653	702	319	775	406	636	3.89004	4.19215	25	
36	892	965	611	3.19100	771	3.63048	474	756	24	
37	2.69131	2.84229	903	426	3.40136	461	945	4.20298	23	
38	371	494	3.01196	752	502	874	3.90417	842	22	
39	612	758	489	3.20079	869	3.64289	890	4.21387	21	
40	853	2.85023	733	406	3.41236	705	3.91364	933	20	
41	2.70094	289	3.02077	734	604	3.65121	839	4.22481	19	
42	335	555	372	3.21063	973	538	3.92316	4.23030	18	
43	577	822	667	392	3.42343	957	793	580	17	
44	819	2.86089	963	722	713	3.66376	3.93271	4.24132	16	
45	2.71062	356	3.03260	3.22053	3.43084	796	751	685	15	
46	305	624	556	384	456	3.67217	3.94232	4.35239	14	
47	548	892	854	715	829	638	713	795	13	
48	792	2.87161	3.04152	3.23048	3.44202	3.68061	3.95196	4.26352	12	
49	2.72036	430	450	381	576	485	680	911	11	
50	281	700	749	714	951	909	3.96165	4.27471	10	
51	526	970	3.05049	3.24049	3.45327	3.69335	651	4.28032	9	
52	771	2.88240	349	383	703	761	3.97139	595	8	
53	2.73017	511	649	719	3.46080	3.70188	627	4.29159	7	
54	263	783	950	3.25055	458	616	3.98117	724	6	
55	509	2.89055	3.06252	392	837	3.71046	607	4.30291	5	
56	756	327	544	729	3.47216	476	3.99099	860	4	
57	2.74004	600	857	3.26067	596	907	592	4.31430	3	
58	251	873	3.07160	406	977	3.72338	4.00086	4.32001	2	
59	499	2.90147	464	745	3.48359	771	582	573	1	
60	748	421	768	3.27085	741	3.73205	4.01078	4.33148	0	
	20°	19°	18°	17°	16°	15°	14°	13°		
NATURAL COTANGENT.										

NATURAL TANGENT.										TABLE B.
72B**	77°	78°	79°	80°	81°	82°	83°	84°	7	
0	4.33148	4.70463	5.14455	5.67128	6.31375	7.11537	8.14435	9.51436	60	
1	723	4.71137	5.15256	8094	2566	3042	6398	4106	59	
2	4.34300	813	5.16058	9064	3761	4553	8370	6791	58	
3	879	4.72490	863	5.70037	4961	6071	8.20352	9490	57	
4	4.35459	4.73170	5.17671	1013	6165	7594	2344	9.62205	56	
5	4.36040	851	5.18480	1992	7374	9125	4345	4935	55	
6	623	4.74534	5.19293	2974	8587	7.20661	6355	7680	54	
7	4.37207	4.75219	5.20107	3960	9804	2204	8376	9.70441	53	
8	793	906	925	4949	6.41026	3754	8.30406	3217	52	
9	4.38381	4.76595	5.21744	5941	2253	5310	2446	6009	51	
10	969	4.77286	5.22566	6937	3484	6873	4496	8817	50	
11	4.39560	978	5.23391	7936	4720	8442	6555	9.81641	49	
12	4.40152	4.78673	5.24218	8938	5961	7.30018	8625	4482	48	
13	745	4.79370	5.25048	9944	7206	1600	8.40705	7338	47	
14	4.41340	4.80068	880	5.80953	8456	3190	2795	9.90211	46	
15	936	769	5.26715	1966	9710	4786	4896	3101	45	
16	4.42534	4.81471	5.27553	2982	6.50970	6389	7007	6007	44	
17	4.43134	4.82175	5.28393	4001	2234	7999	9128	8931	43	
18	735	882	5.29235	5024	3503	9616	8.51259	10.0187	42	
19	4.44338	4.83590	5.30080	6051	4777	7.41240	3402	0483	41	
20	942	4.84300	928	7080	6055	2871	5555	0780	40	
21	4.45548	4.85013	5.31778	8114	7339	4509	7718	1080	39	
22	4.46155	727	5.32631	9151	8627	6154	9893	1381	38	
23	764	4.86444	5.33487	5.90191	9921	7806	8.62078	1633	37	
24	4.47374	4.87162	5.34345	1236	6.61219	9465	4275	1988	36	
25	986	882	5.35206	2283	2523	7.51132	6482	2294	35	
26	4.48600	4.88005	5.36070	3335	3831	2806	8701	2602	34	
27	4.49215	4.89330	936	4390	5144	4487	8.70931	2913	33	
28	832	4.90056	5.37805	5448	6463	6176	3172	3224	32	
29	4.50451	785	5.38677	6510	7787	7872	5425	3538	31	
30	4.51071	4.91516	5.39552	7576	9116	9575	7689	3854	30	
31	693	4.92249	5.40429	8646	6.70450	7.61287	9964	4172	29	
32	4.52316	984	5.41309	9720	1789	3005	8.82252	4491	28	
33	941	4.93721	5.42192	6.00797	3133	4732	4551	4813	27	
34	4.53568	4.94460	5.43078	1878	4483	6466	6862	5136	26	
35	4.54196	4.95201	966	2962	5838	8208	9185	5462	25	
36	826	945	5.44857	4051	7199	9957	8.91520	5789	24	
37	4.55458	4.96690	5.45751	5143	8564	7.71715	3867	6118	23	
38	4.56091	4.97438	5.46648	6240	9936	3480	6227	6450	22	
39	726	4.98188	5.47548	7340	6.81312	5254	8598	6783	21	
40	4.57363	940	5.48451	8444	2694	7035	9.00983	7119	20	
41	4.58001	4.99695	5.49356	9552	4082	8825	3379	7457	19	
42	641	5.00451	5.50264	6.10664	5475	7.80622	5789	7797	18	
43	4.59283	5.01210	5.51176	1779	6874	2428	8211	8139	17	
44	927	971	5.52090	2899	8278	4242	9.10646	8483	16	
45	4.60572	5.02734	5.53007	4023	9688	6064	8093	8829	15	
46	4.61219	5.03499	927	5151	6.91104	7895	5554	9178	14	
47	868	5.04267	5.54851	6283	2525	9734	8028	9529	13	
48	4.62518	5.05037	5.55777	7419	3952	7.91582	9.20516	9882	12	
49	4.63171	809	5.56706	8559	5385	3438	3016	11.0237	11	
50	825	5.06584	5.57638	9703	6823	5802	5530	0594	10	
51	4.64480	5.07360	5.58573	6.20851	8268	7176	8058	0954	9	
52	4.65138	5.08139	5.59511	2003	9718	9058	9.30599	1316	8	
53	797	921	5.60452	3160	7.01174	8.00948	3165	1691	7	
54	4.66458	5.09794	5.61397	4321	2637	2848	5724	2048	6	
55	4.67121	5.10490	5.62344	5486	4105	4756	8307	2417	5	
56	786	5.11279	5.63295	6655	5579	6674	9.40904	2789	4	
57	4.68452	5.12069	5.64248	7829	7059	8600	3515	3163	3	
58	4.69121	862	5.65205	9007	8546	8.10536	6141	3540	2	
59	791	5.13658	5.66165	6.30189	7.10038	2481	8781	3919	1	
60	4.70463	5.14455	5.67128	1375	1537	4435	9.51436	4301	0	
	12°	11°	10°	9°	8°	7°	6°	5°		
NATURAL COTANGENT.										

TRAVERSE TABLE;
OR,
LATITUDES AND DEPARTURES
TO
EVERY MINUTE:
AND
CALCULATED TO FOUR PLACES OF DECIMALS.

BY MICHAEL McDERMOTT,
CIVIL ENGINEER AND SURVEYOR.

A

74 LATITUDE 0 DEGREES.										
/	1	2	3	4	5	6	7	8	9	/
0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000	60
1	00	00	00	00	00	00	00	00	00	59
2	00	00	00	00	00	00	00	00	00	58
3	00	00	00	00	00	00	00	00	00	57
4	00	00	00	00	00	00	00	00	00	56
5	00	00	00	00	00	00	00	00	00	55
6	00	00	00	00	00	00	00	00	00	54
7	00	00	00	00	00	00	00	00	00	53
8	00	00	00	00	00	00	00	00	00	52
9	00	00	00	00	00	00	00	00	00	51
10	00	00	00	00	00	00	00	00	8.9999	50
11	00	00	00	00	00	5.9999	6.9999	7.9999	99	49
12	00	00	00	00	00	99	99	99	99	48
13	00	00	00	00	00	99	99	99	99	47
14	00	00	00	00	00	99	99	99	99	46
15	00	00	00	00	00	99	99	99	99	45
16	00	00	00	00	00	99	99	99	99	44
17	00	00	00	00	00	99	99	99	99	43
18	00	00	00	00	00	99	99	99	99	42
19	00	00	00	00	00	99	99	99	99	41
20	00	00	2.9999	3.9999	4.9999	99	99	99	99	40
21	00	00	99	99	99	99	99	99	98	39
22	00	00	99	99	99	99	99	99	98	38
23	00	00	99	99	99	99	99	99	98	37
24	00	00	99	99	99	99	99	99	98	36
25	00	1.9999	99	99	99	98	98	98	97	35
26	00	99	99	99	99	98	98	98	97	34
27	00	99	99	99	99	98	98	98	97	33
28	00	99	99	98	99	98	98	98	97	32
29	00	99	99	98	98	98	97	98	97	31
30	00	99	99	98	98	98	97	97	96	30
31	00	99	99	98	98	97	97	97	96	29
32	00	99	99	98	98	97	97	97	96	28
33	00	99	99	98	98	97	97	97	96	27
34	00	99	99	98	98	97	97	96	96	26
35	00	99	99	98	98	97	97	96	96	25
36	99	99	99	98	98	97	97	96	96	24
37	0.9999	99	98	98	97	96	97	95	95	23
38	99	99	98	98	97	96	96	95	95	22
39	99	99	98	98	97	96	96	95	95	21
40	99	99	98	97	97	96	96	95	94	20
41	99	99	98	97	97	96	96	94	94	19
42	99	99	98	97	97	96	95	94	94	18
43	99	98	98	97	96	95	95	94	93	17
44	99	98	98	97	96	95	94	94	93	16
45	99	98	98	97	96	95	94	93	92	15
46	99	98	97	96	96	95	94	93	92	14
47	99	98	97	96	96	94	94	93	92	13
48	99	98	97	96	96	94	93	92	91	12
49	99	98	97	96	95	94	93	92	91	11
50	99	98	97	96	95	93	92	91	90	10
51	99	98	97	96	95	93	92	91	90	9
52	99	98	97	96	95	93	92	91	90	8
53	99	98	96	95	95	93	92	90	89	7
54	99	97	96	95	94	93	92	90	89	6
55	99	97	96	95	94	92	91	90	89	5
56	99	97	96	95	94	92	91	90	88	4
57	99	97	96	94	93	92	91	89	88	3
58	99	97	96	94	93	92	90	89	87	2
59	99	97	96	94	93	91	90	89	87	1
60	0.9999	9.9997	2.9996	3.9994	4.9993	5.9991	6.9990	7.9988	8.9987	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 89 DEGREES.										

DEPARTURE 0 DEGREES.										75
1	2	3	4	5	6	7	8	9	10	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	60
1	03	06	09	12	15	17	20	23	26	59
2	06	12	17	23	29	35	41	46	52	58
3	09	17	26	35	44	52	61	70	78	57
4	12	23	35	46	58	70	81	93	0.0104	56
5	15	29	44	58	73	89	0.0102	0.0116	30	55
6	18	35	53	70	88	0.0105	23	40	58	54
7	20	41	61	82	0.0102	22	43	63	84	53
8	23	47	70	93	17	40	63	86	0.0210	52
9	26	52	79	0.0105	31	57	83	0.0210	36	51
10	29	58	87	16	46	75	0.0204	33	62	50
11	32	65	96	28	60	92	24	56	88	49
12	35	70	0.0105	40	75	0.0209	44	79	0.0314	48
13	38	76	13	51	89	27	65	0.0302	40	47
14	41	81	22	63	0.0204	44	85	26	66	46
15	44	87	31	74	18	62	0.0305	49	92	45
16	47	93	40	86	33	79	26	72	0.0419	44
17	50	99	48	98	48	96	47	96	46	43
18	52	0.0105	67	0.0210	62	0.0314	67	0.0419	72	42
19	55	11	66	21	77	32	87	42	98	41
20	58	16	75	33	91	49	0.0407	66	0.0524	40
21	61	22	83	44	0.0306	67	28	89	50	39
22	64	28	92	56	20	84	48	0.0512	76	38
23	67	34	0.0201	68	35	0.0401	68	35	0.0602	37
24	70	40	09	79	49	19	89	58	28	36
25	73	45	18	91	64	36	0.0509	82	54	35
26	76	51	27	0.0302	78	54	29	0.0605	80	34
27	79	57	36	14	93	71	50	28	0.0707	33
28	81	63	44	26	0.0407	88	70	51	33	32
29	84	69	53	38	22	0.5006	91	75	60	31
30	87	75	62	49	37	23	0.0611	98	86	30
31	89	81	71	61	51	41	31	0.0722	0.0802	29
32	93	86	79	72	66	59	52	45	38	28
33	96	92	88	84	80	76	72	68	64	27
34	99	98	97	96	95	93	92	91	90	26
35	0.0102	0.0204	0.0305	0.0407	0.0509	0.0611	0.0713	0.0814	0.0916	25
36	05	09	14	19	24	28	33	38	42	24
37	08	15	23	30	38	46	53	61	68	23
38	11	21	32	42	53	63	74	84	95	22
39	13	27	40	54	67	80	94	0.0907	0.1021	21
40	16	33	49	66	82	98	0.0815	31	48	20
41	19	39	58	77	97	0.0716	35	54	74	19
42	22	44	67	89	0.0611	33	55	78	0.1100	18
43	25	50	75	0.0500	26	51	76	0.1001	26	17
44	28	56	84	12	40	68	96	24	52	16
45	31	62	93	24	55	85	0.0916	47	78	15
46	34	68	0.0401	35	69	0.0803	37	70	0.1204	14
47	37	73	10	47	84	20	57	94	30	13
48	40	79	19	58	98	38	77	0.1117	56	12
49	43	85	28	70	0.0723	55	98	40	83	11
50	45	91	36	82	27	72	0.1018	63	0.1309	10
51	48	97	45	93	42	90	38	86	35	9
52	51	0.0303	54	0.0605	57	0.0908	59	0.1210	62	8
53	54	08	63	16	71	25	79	86	88	7
54	57	14	71	28	86	43	0.1100	57	0.1414	6
55	60	20	80	40	0.0800	60	20	80	40	5
56	63	26	89	52	15	77	40	0.1303	66	4
57	66	32	97	63	29	95	61	26	92	3
58	69	37	0.0506	75	44	0.1012	81	50	0.1518	2
59	72	43	15	86	58	29	0.1201	73	44	1
60	0.0175	0.0349	0.0524	0.0698	0.0873	0.1047	0.1222	0.1396	0.1571	0
1	2	3	4	5	6	7	8	9		
LATITUDE 89 DEGREES.										

76 LATITUDE 1 DEGREE.										
	1	2	3	4	5	6	7	8	9	
0	0.9999	1.9997	2.9996	3.9994	4.9993	5.9991	6.9990	7.9988	8.9987	60
1	99	97	95	94	92	91	89	88	86	59
2	98	97	95	94	92	90	89	87	86	58
3	98	97	95	93	92	90	88	86	86	57
4	98	97	95	93	92	90	88	86	85	56
5	98	96	95	93	91	89	87	86	84	55
6	98	96	95	93	91	89	87	86	84	54
7	98	96	94	92	90	88	86	85	82	53
8	98	96	94	92	90	88	86	84	82	52
9	98	96	94	92	90	87	85	83	81	51
10	98	96	94	92	90	87	85	83	81	50
11	98	96	93	91	89	87	85	82	80	49
12	98	96	93	91	89	87	85	82	80	48
13	98	95	93	91	89	86	84	82	79	47
14	98	95	93	91	89	86	84	82	79	46
15	98	95	93	90	88	86	83	81	78	45
16	98	95	93	90	88	86	83	81	78	44
17	97	95	92	90	87	85	82	80	77	43
18	97	95	92	90	87	85	82	79	77	42
19	97	95	92	89	87	84	81	79	76	41
20	97	95	92	89	87	84	81	79	76	40
21	97	94	92	89	86	83	80	78	75	39
22	97	94	92	89	86	83	80	78	75	38
23	97	94	91	88	85	82	79	76	74	37
24	97	94	91	88	85	82	79	76	73	36
25	97	94	91	88	85	81	78	75	72	35
26	97	94	91	88	85	81	78	75	72	34
27	97	93	90	87	84	81	77	74	70	33
28	97	93	90	87	84	80	77	74	70	32
29	97	93	90	86	83	80	76	73	69	31
30	97	93	90	86	83	80	76	73	69	30
31	97	93	89	86	82	79	76	72	68	29
32	96	93	89	86	82	78	75	71	68	28
33	96	93	89	85	82	78	75	71	67	27
34	96	93	89	85	82	78	74	70	67	26
35	96	92	88	84	81	77	74	70	66	25
36	96	92	88	84	81	77	73	69	65	24
37	96	92	88	84	80	76	72	68	64	23
38	96	92	88	84	80	75	71	67	63	22
39	96	92	87	83	79	75	71	67	63	21
40	96	92	87	83	79	75	71	66	62	20
41	96	91	87	83	78	74	70	66	61	19
42	96	91	87	82	78	74	69	65	60	18
43	95	91	86	82	77	73	69	64	60	17
44	95	91	86	82	77	72	68	63	59	16
45	95	90	86	81	76	72	67	63	58	15
46	95	90	86	81	76	71	66	62	57	14
47	95	90	85	80	76	71	66	62	57	13
48	95	90	85	80	76	71	66	61	56	12
49	95	90	85	80	75	70	65	60	55	11
50	95	90	85	80	75	69	64	59	54	10
51	95	90	84	79	74	69	64	59	53	9
52	95	89	84	79	74	68	63	58	52	8
53	95	89	84	78	73	67	63	57	51	7
54	95	89	83	78	73	67	62	56	51	6
55	94	89	83	77	72	66	61	55	50	5
56	94	89	83	77	72	66	60	54	49	4
57	94	89	82	76	71	65	60	54	48	3
58	94	88	82	76	71	65	59	53	47	2
59	94	88	82	76	70	64	58	52	46	1
60	0.9994	1.9988	2.9982	3.9976	4.9970	5.9963	6.9957	7.9951	8.9945	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 88 DEGREES.										

DEPARTURE 1 DEGREE.										77
1	2	3	4	5	6	7	8	9	10	
0	0.0175	0.0349	0.0524	0.0698	0.0873	0.1047	0.1222	0.1396	0.1571	60
1	77	55	32	0.0710	87	64	42	0.1419	97	59
2	80	61	41	21	0.0902	82	62	42	0.1623	58
3	83	66	50	33	16	99	82	66	49	57
4	86	72	59	45	31	0.1117	0.1303	90	76	56
5	89	78	67	56	46	35	24	0.1513	0.1702	55
6	92	84	76	68	60	52	44	36	28	54
7	95	90	85	80	75	69	64	59	54	53
8	98	96	93	91	89	87	84	82	80	52
9	0.0201	0.0401	02	0.0803	0.1004	0.1204	0.1405	0.1606	0.1806	51
10	04	07	11	14	18	22	25	29	32	50
11	06	13	20	26	33	39	46	52	59	49
12	09	19	28	38	47	56	66	75	85	48
13	12	25	37	49	62	74	86	98	0.1911	47
14	15	30	46	61	76	91	0.1506	0.1722	37	46
15	18	36	54	72	91	0.1309	27	45	63	45
16	21	43	63	84	0.1106	27	48	69	90	44
17	24	48	72	96	20	44	68	92	0.2016	43
18	27	54	81	0.0908	35	61	88	0.1815	42	42
19	30	60	89	19	49	80	0.1609	38	68	41
20	33	65	98	31	64	96	29	62	94	40
21	36	71	0.0707	42	78	0.1414	49	85	0.2120	39
22	38	77	16	54	93	31	70	0.1908	47	38
23	41	83	24	66	0.1207	48	90	31	73	37
24	44	89	33	77	22	66	0.1710	54	99	36
25	47	94	42	89	36	83	30	78	0.2225	35
26	50	0.0500	50	0.1000	51	0.1501	51	0.2001	51	34
27	53	06	59	12	65	18	71	24	77	33
28	56	12	68	24	80	36	92	48	0.2304	32
29	59	18	77	36	95	53	0.1808	71	30	31
30	62	24	85	47	0.1309	61	33	94	56	30
31	65	29	94	59	24	88	53	0.2118	82	29
32	68	35	0.0803	70	38	0.1606	73	41	0.2408	28
33	71	41	12	82	53	23	93	64	35	27
34	73	47	20	94	67	40	0.1914	87	61	26
35	76	53	29	0.1105	82	58	34	0.2210	87	25
36	79	58	38	17	96	75	54	34	0.2513	24
37	82	64	46	28	0.1411	93	74	57	39	23
38	85	70	55	40	25	0.1710	95	80	65	22
39	88	76	64	52	40	27	0.2015	0.2303	91	22
40	91	82	72	63	54	45	36	26	0.2617	20
41	94	88	81	75	69	63	57	50	44	19
42	97	93	90	87	84	80	77	74	70	18
43	0.0300	99	99	98	98	98	97	97	96	17
44	03	0.0605	0.0908	0.1210	0.1513	0.1815	0.2118	0.2420	0.2722	16
45	05	11	62	22	27	32	38	43	49	15
46	08	17	25	33	42	50	58	66	75	14
47	11	22	34	45	56	67	78	90	0.2801	13
48	14	28	42	56	71	85	99	0.2513	27	12
49	17	34	51	68	85	0.1902	0.2219	36	53	11
50	20	40	60	80	0.1600	19	39	59	79	10
51	23	46	68	91	14	37	60	82	0.2905	9
52	26	51	77	0.1303	29	54	80	0.2606	31	8
53	29	57	86	14	43	72	0.2300	29	57	7
54	32	63	95	26	58	90	21	53	84	6
55	35	69	0.1004	38	73	0.2007	42	76	0.3011	5
56	37	75	12	50	87	24	62	99	37	4
57	40	81	21	61	0.1702	42	82	0.2722	63	3
58	43	86	30	73	16	59	0.2402	46	89	2
59	46	92	38	84	31	77	23	69	0.3115	1
60	0.0349	0.0698	0.1047	0.1396	0.1745	0.2094	0.2443	0.2792	0.3141	0
1	2	3	4	5	6	7	8	9		
LATITUDE 88 DEGREES.										

78									
LATITUDE 2 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.9994	1.9988	2.9982	3.9976	4.9970	5.9963	6.9957	7.9951	8.9945
1	94	88	81	75	69	63	57	50	44
2	94	87	81	75	69	62	56	50	43
3	94	87	81	74	68	62	55	49	42
4	94	87	81	74	68	61	55	48	42
5	93	87	80	74	67	60	54	48	41
6	93	87	80	73	67	60	53	46	40
7	93	87	80	73	66	59	52	46	39
8	93	87	79	72	66	58	52	45	38
9	93	87	79	72	65	57	51	44	37
10	93	86	79	71	65	57	50	43	36
11	93	85	78	70	64	56	49	42	34
12	93	85	78	70	63	55	48	41	33
13	93	85	78	70	63	55	48	40	33
14	92	85	77	69	62	54	47	39	32
15	92	85	77	69	62	54	46	38	31
16	92	84	77	68	61	53	45	38	30
17	92	84	76	68	61	53	45	37	29
18	92	84	76	67	60	51	43	35	27
19	92	84	75	67	60	51	43	34	26
20	92	83	75	67	59	50	42	34	25
21	92	83	75	66	59	50	41	33	24
22	92	83	75	66	58	49	41	32	24
23	91	83	74	65	58	48	39	30	22
24	91	82	74	65	56	47	38	30	21
25	91	82	73	64	56	47	38	29	20
26	91	82	73	64	55	46	37	28	19
27	91	82	73	64	55	45	36	27	18
28	91	81	72	63	54	44	35	26	16
29	91	81	72	62	54	44	35	25	15
30	91	81	72	62	53	43	34	24	15
31	90	81	71	61	52	42	33	23	14
32	90	80	71	61	51	41	31	22	12
33	90	80	70	60	51	41	31	21	10
34	90	80	70	60	50	40	30	20	10
35	90	80	69	59	50	39	29	18	08
36	90	79	69	59	49	38	28	18	07
37	90	79	69	58	48	38	28	17	06
38	89	79	68	58	47	36	26	15	05
39	89	79	68	57	47	36	25	14	04
40	89	78	68	57	46	35	24	14	03
41	89	78	67	56	45	34	23	12	01
42	89	78	67	56	45	33	22	11	00
43	89	78	66	55	44	33	22	10	8.9899
44	89	77	66	54	43	32	20	09	97
45	89	77	66	54	43	31	20	08	97
46	88	77	65	53	42	30	18	06	95
47	88	76	65	53	41	29	17	05	94
48	88	76	64	52	41	29	17	05	93
49	88	76	64	52	40	27	15	03	91
50	88	76	63	51	39	27	15	02	90
51	88	75	63	50	38	26	13	01	88
52	88	75	63	50	38	25	13	00	88
53	88	75	62	49	37	24	12	7.9898	86
54	87	74	62	49	36	23	10	98	85
55	87	74	61	48	35	22	09	96	83
56	87	74	61	48	35	21	08	95	82
57	87	73	60	47	34	20	07	94	80
58	87	73	60	46	33	20	06	93	79
59	87	73	59	46	32	18	05	91	78
60	0.9986	1.9973	2.9959	3.9945	4.9932	5.9918	6.9904	7.9890	8.9878
1	2	3	4	5	6	7	8	9	10
DEPARTURE 87 DEGREES.									

DEPARTURE 2 DEGREES.										79
1	2	3	4	5	6	7	8	9	10	
0	0.3490	0.0698	0.1047	0.1396	0.1745	0.2094	0.2443	0.2792	0.3141	60
1	52	0.0704	56	0.1408	60	0.2111	63	0.2815	67	59
2	55	10	64	19	74	29	84	38	93	58
3	58	15	69	31	89	48	0.2504	62	0.3219	57
4	61	21	82	42	0.1803	64	24	85	45	56
5	64	27	91	54	18	81	45	0.2908	72	55
6	66	33	99	66	32	98	65	31	98	54
7	69	39	0.1108	77	47	0.2216	85	54	0.3324	53
8	72	45	17	89	62	34	0.2606	78	51	52
9	75	50	26	0.1500	76	51	26	0.3002	77	51
10	78	56	34	12	91	69	47	25	0.3403	50
11	81	62	43	24	0.1905	86	67	48	29	49
12	84	68	52	36	20	0.2303	87	71	55	48
13	87	74	60	47	34	21	0.2708	94	81	47
14	90	79	69	59	49	38	28	0.3118	0.3507	46
15	93	85	78	70	63	56	48	41	33	45
16	96	91	87	82	78	73	69	64	60	44
17	98	97	95	94	94	90	89	87	86	43
18	0.0401	0.0803	0.1204	0.1605	0.2007	0.2408	2.2809	0.3210	0.3612	42
19	04	08	13	17	21	25	29	34	18	41
20	07	14	21	28	36	43	50	57	64	40
21	10	20	30	40	50	60	70	80	90	39
22	13	26	39	52	65	77	90	0.3303	0.3716	38
23	16	31	48	64	79	95	0.2911	27	43	37
24	19	38	56	75	94	0.2513	32	50	69	36
25	21	43	65	87	0.2109	30	52	74	95	35
26	25	49	74	98	23	48	72	97	0.3821	34
27	28	55	83	0.1710	38	65	93	0.3420	48	33
28	30	61	91	22	52	82	0.3013	43	74	32
29	33	67	0.1300	33	67	0.2600	33	66	0.3900	31
30	36	72	09	45	0.2181	17	53	90	26	30
31	39	78	17	56	96	35	74	0.3513	52	29
32	42	84	26	68	0.2210	52	94	36	78	28
33	45	90	35	80	25	69	0.3114	59	0.4004	27
34	48	96	43	91	39	87	35	82	30	26
35	51	0.0901	52	0.1803	54	0.2704	55	0.3606	66	25
36	54	07	61	14	68	22	75	29	82	24
37	56	13	69	26	82	38	95	52	0.4108	23
38	59	19	78	38	97	56	0.3216	75	35	22
39	62	25	87	49	0.2312	74	36	98	61	21
40	65	31	96	61	27	92	57	0.3722	88	20
41	68	36	0.1405	73	41	0.2809	77	46	0.4214	19
42	71	42	13	84	56	27	98	69	40	18
43	74	48	22	96	70	44	0.3318	92	66	17
44	77	54	31	0.1908	85	62	38	0.3815	92	16
45	80	60	39	19	99	79	59	88	0.4318	15
46	83	65	48	31	0.2414	96	79	62	44	14
47	86	71	57	42	28	0.2914	99	85	70	13
48	89	77	66	54	43	31	0.3420	0.3908	97	12
49	91	83	74	66	57	48	40	31	0.4423	11
50	94	89	83	77	72	66	60	54	49	10
51	97	94	92	89	86	83	80	78	75	9
52	0.0500	0.1000	0.1500	0.2000	0.2501	0.3001	0.3501	0.4001	0.4501	8
53	03	06	09	12	15	18	21	24	27	7
54	06	12	18	24	30	35	41	47	53	6
55	09	18	26	35	44	53	62	70	79	5
56	12	23	35	47	59	70	82	94	0.4605	4
57	15	29	44	58	73	88	0.3602	0.4117	31	3
58	18	35	53	70	88	0.3105	23	40	58	2
59	21	41	62	82	0.2603	23	44	64	85	1
60	0.0523	0.1047	0.1570	0.2094	0.2617	0.3140	0.3664	0.4187	0.4711	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 87 DEGREES.										

LATITUDE 3 DEGREES.										
80	1	2	3	4	5	6	7	8	9	10
0	0.9986	1.9973	2.9959	3.9945	4.9932	5.9918	6.9904	7.9890	8.9877	60
1	86	72	58	44	31	17	03	89	75	59
2	86	72	58	44	30	16	02	88	74	58
3	86	72	57	43	29	15	01	86	72	57
4	86	71	57	43	29	14	00	86	71	56
5	86	71	57	42	28	13	6.9899	84	70	55
6	85	71	56	42	27	12	98	83	69	54
7	85	70	55	41	26	11	96	82	67	53
8	85	70	55	40	26	11	96	81	66	52
9	85	70	54	40	25	09	94	79	64	51
10	85	69	54	39	24	08	93	78	62	50
11	85	69	54	38	23	08	92	77	61	49
12	84	69	53	38	22	06	91	75	60	48
13	84	69	53	37	21	05	89	74	58	47
14	84	68	52	36	21	05	89	73	57	46
15	84	68	52	36	20	03	87	71	55	45
16	84	68	51	35	19	03	87	70	54	44
17	84	67	51	34	18	02	85	69	52	43
18	83	67	50	34	17	00	84	67	51	42
19	83	67	50	33	17	5.9900	83	66	50	41
20	83	66	49	32	16	5.9899	82	65	48	40
21	83	66	49	32	15	97	80	63	46	39
22	82	65	48	31	14	96	79	62	44	38
23	82	65	48	30	13	96	78	61	42	37
24	82	65	47	30	12	94	77	59	42	36
25	82	64	47	29	11	93	75	58	40	35
26	82	64	46	28	11	93	75	57	39	34
27	82	64	46	28	10	91	73	55	37	33
28	82	63	45	27	09	90	72	54	35	32
29	82	63	45	26	08	88	70	52	34	31
30	81	63	44	25	07	88	69	50	32	30
31	81	62	44	25	06	87	68	50	31	29
32	81	62	43	24	05	86	67	48	29	28
33	81	62	42	23	04	85	66	46	27	27
34	81	61	42	22	03	84	64	45	25	26
35	80	61	41	22	02	82	63	43	24	25
36	80	61	41	21	02	82	62	42	23	24
37	80	60	40	20	01	81	61	41	21	23
38	80	60	40	20	00	79	59	39	19	22
39	80	59	39	19	4.9899	78	58	38	17	21
40	80	59	39	18	98	77	57	36	16	20
41	79	59	38	17	97	76	55	34	14	19
42	79	58	38	17	96	75	54	34	13	18
43	79	58	37	16	95	74	53	32	11	17
44	79	58	36	15	94	73	52	30	09	16
45	79	57	36	14	93	72	50	29	07	15
46	78	57	35	14	92	70	49	27	06	14
47	78	56	35	13	91	69	47	26	04	13
48	78	56	34	12	90	68	46	24	02	12
49	78	56	33	11	89	67	45	22	00	11
50	78	55	33	10	88	66	43	21	8.9798	10
51	77	55	32	10	87	64	42	19	97	9
52	77	54	32	09	86	63	40	18	95	8
53	77	54	31	08	85	62	39	16	93	7
54	77	54	30	07	84	61	38	14	91	6
55	77	53	30	06	83	60	36	13	89	5
56	76	53	29	06	82	58	35	11	88	4
57	76	52	29	05	81	57	33	10	86	3
58	76	52	28	04	80	56	32	08	84	2
59	76	52	27	03	79	55	31	06	82	1
60	0.9976	1.9951	2.9927	3.9902	4.9878	5.9854	6.9829	7.9806	8.9780	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 86 DEGREES.										

DEPARTURE 8 DEGREES.										81
1	2	3	4	5	6	7	8	9	10	11
0	0.0523	0.1047	0.1570	0.2094	0.2617	0.3140	0.3664	0.4187	0.4711	60
1	26	53	79	0.2105	32	58	84	0.4210	37	59
2	29	58	88	17	46	75	0.3704	34	63	58
3	32	64	96	28	61	93	25	57	89	57
4	35	70	0.1605	40	75	0.3210	45	80	0.4815	56
5	38	76	14	52	90	27	65	0.4303	41	55
6	41	82	22	63	0.2704	45	86	26	67	54
7	44	77	31	75	19	62	0.3806	50	93	53
8	47	93	40	86	33	80	26	73	0.4919	52
9	50	99	49	98	48	97	47	96	46	51
10	52	0.1105	57	0.2210	62	0.3314	67	0.4419	72	50
11	55	11	66	21	77	22	87	42	98	49
12	58	16	75	33	91	49	0.3907	66	0.5024	48
13	61	22	83	44	0.2806	67	28	89	50	47
14	64	28	92	56	20	84	48	0.4512	76	46
15	67	34	0.1701	67	35	0.3401	68	35	0.5102	45
16	70	40	09	79	49	19	89	58	18	44
17	73	45	18	81	64	36	0.4009	82	44	43
18	76	51	27	0.2302	78	54	29	0.4605	80	42
19	79	57	36	14	93	71	50	28	0.5207	41
20	81	63	44	26	0.2907	88	70	51	33	40
21	84	69	53	38	22	0.3506	91	75	60	39
22	87	75	62	49	37	24	0.4111	98	86	38
23	90	80	71	61	51	41	81	0.4722	0.5312	37
24	93	86	79	72	66	59	52	25	38	36
25	96	92	88	84	80	76	72	48	64	35
26	99	98	97	96	95	93	92	91	90	34
27	0.0602	0.1204	0.1805	0.2407	0.3009	0.3611	0.4213	0.4814	0.5416	33
28	05	09	14	19	24	28	33	38	42	32
29	08	15	23	30	38	46	53	61	68	31
30	11	21	32	42	53	63	74	84	95	30
31	13	27	40	54	67	80	94	0.4907	0.5521	29
32	16	33	49	65	82	98	0.4314	30	47	28
33	19	38	58	77	96	0.3715	34	54	73	27
34	22	44	66	88	0.3111	33	55	77	99	26
35	24	50	75	0.2500	25	50	75	0.5000	0.5625	25
36	28	56	84	12	40	67	95	23	51	24
37	31	62	92	23	54	85	0.4416	46	77	23
38	34	67	0.1901	35	69	0.3802	36	70	0.5703	22
39	37	73	10	46	83	20	56	93	29	21
40	40	79	19	58	98	37	77	0.5116	56	20
41	42	85	27	70	0.3212	54	97	39	82	19
42	45	91	36	81	27	72	0.4517	62	0.5808	18
43	48	96	45	92	41	89	37	86	34	17
44	51	0.1302	53	0.2604	56	0.3907	58	0.5209	60	16
45	54	08	62	16	71	24	78	32	86	15
46	57	14	71	28	85	41	98	55	0.5912	14
47	60	20	79	39	99	69	0.4619	78	38	13
48	63	25	88	51	0.3314	76	39	0.5302	64	12
49	66	31	97	62	28	94	59	25	80	11
50	69	37	0.2006	74	43	0.4011	80	48	0.6017	10
51	71	43	14	86	57	28	0.4700	71	43	9
52	74	49	23	97	72	46	20	94	69	8
53	77	55	31	0.2709	87	64	40	0.5422	96	7
54	80	60	41	21	0.3401	81	61	42	0.6122	6
55	83	66	49	32	16	99	82	65	48	5
56	86	72	58	44	30	0.4116	0.4802	88	74	4
57	89	78	67	56	45	33	23	0.5511	0.6200	3
58	92	84	75	67	59	51	43	34	26	2
59	95	89	84	79	74	78	63	58	52	1
60	0.0698	0.1395	0.2093	0.2790	0.3488	0.4186	0.4883	0.5551	0.6278	0
1	2	3	4	5	6	7	8	9		
LATITUDE 86 DEGREES.										

LATITUDE 4 DEGREES.										
	1	2	3	4	5	6	7	8	9	10
0	0.9976	1.1951	2.9927	3.9902	3.9878	5.9854	6.9829	7.9805	8.9780	60
1	75	51	26	02	77	52	28	03	79	59
2	75	50	26	01	76	51	26	02	77	58
3	75	50	25	00	75	50	25	00	75	57
4	75	50	24	3.9899	74	49	24	7.9798	73	56
5	75	49	24	98	73	48	22	97	71	55
6	74	49	23	98	72	46	21	95	70	54
7	74	48	22	97	71	45	19	94	68	53
8	74	48	21	96	70	44	18	92	66	52
9	74	48	21	95	69	43	17	90	64	51
10	74	47	20	94	68	42	15	89	62	50
11	73	47	20	94	67	40	14	87	51	49
12	73	46	19	92	66	39	12	85	58	48
13	73	46	19	92	65	37	10	83	56	47
14	73	46	18	91	64	36	09	82	54	46
15	73	45	18	90	63	35	08	80	53	45
16	72	45	17	89	62	34	06	78	51	44
17	72	44	16	88	61	33	05	77	49	43
18	72	44	16	88	60	31	03	75	47	42
19	72	43	14	86	58	30	01	73	44	41
20	71	43	14	86	57	28	00	71	43	40
21	71	42	13	85	56	27	6.9798	70	41	39
22	71	42	12	84	55	26	97	68	39	38
23	71	42	12	83	54	25	96	66	37	37
24	71	41	11	82	53	23	94	64	35	36
25	70	41	10	81	52	22	92	62	33	35
26	70	40	10	80	51	21	91	61	31	34
27	0.9970	40	09	80	40	19	89	59	29	33
28	70	40	08	78	48	18	87	57	26	32
29	70	39	08	78	47	16	86	55	25	31
30	69	38	08	77	46	15	84	54	23	30
31	69	38	07	76	45	13	82	51	20	29
32	69	37	06	75	44	12	81	50	18	28
33	69	37	06	74	43	11	80	48	17	27
34	68	37	05	73	42	10	78	46	15	26
35	68	36	04	72	40	08	76	44	12	25
36	68	36	03	71	39	07	75	42	10	24
37	68	35	03	70	38	06	73	41	08	23
38	68	35	02	69	37	04	71	38	06	22
39	67	34	01	68	36	03	70	37	04	21
40	67	34	00	67	34	01	68	34	01	20
41	66	33	00	66	33	00	66	33	8.9699	19
42	66	33	2.9899	66	32	5.9798	65	31	98	18
43	66	32	98	64	31	97	63	29	95	17
44	66	32	98	64	30	95	61	27	93	16
45	66	31	97	63	29	94	60	26	91	15
46	65	31	96	62	27	92	58	23	89	14
47	65	30	96	61	26	91	57	22	87	13
48	65	30	95	60	25	89	54	19	85	12
49	65	29	94	59	24	87	53	18	82	11
50	64	29	93	58	22	86	51	15	80	10
51	64	28	93	57	21	85	49	14	78	9
52	64	28	92	56	20	83	47	11	75	8
53	64	27	91	55	19	82	46	10	73	7
54	64	27	91	54	18	81	45	08	72	6
55	63	26	90	53	16	79	42	06	69	5
56	63	26	89	52	15	78	41	04	67	4
57	63	25	88	51	14	76	39	02	64	3
58	63	25	88	50	13	75	38	00	63	2
59	62	24	87	49	11	73	35	7.9698	60	1
60	0.9962	1.9924	2.9886	3.9848	4.9810	5.9771	6.9733	7.9695	8.9657	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 85 DEGREES.										

DEPARTURE 4 DEGREES.										88
1	2	3	4	5	6	7	8	9	10	
0	0.0698	0.1395	0.2093	0.2790	0.3488	0.4186	0.4883	0.5581	0.6278	60
1	0.0701	0.1401	0.2102	0.2802	0.3503	0.4203	0.4904	0.5604	0.6305	59
2	03	07	10	14	17	20	24	27	31	58
3	06	13	19	25	32	38	44	50	57	57
4	09	18	28	37	46	55	64	74	83	56
5	12	24	36	48	61	73	85	97	0.6409	55
6	15	30	45	60	75	90	0.5005	0.5720	35	54
7	18	36	54	72	89	0.4307	25	43	61	53
8	21	42	62	83	0.3604	25	46	66	87	52
9	24	47	71	95	19	42	66	90	0.6513	51
10	27	53	80	0.2906	33	60	86	0.5813	39	50
11	30	58	89	18	48	77	0.5107	86	66	49
12	32	65	97	30	62	94	27	59	92	48
13	35	71	0.2206	41	77	0.4412	47	82	0.6618	47
14	38	76	15	53	91	29	67	0.5906	44	46
15	41	82	23	64	0.3706	47	88	29	70	45
16	44	88	32	76	20	64	0.5208	52	96	44
17	47	94	41	88	35	81	28	75	0.6722	43
18	50	0.1500	49	99	49	99	49	98	48	42
19	53	05	58	0.3001	64	0.4516	69	0.6022	74	41
20	56	11	67	22	78	34	89	45	0.6800	40
21	59	17	76	34	93	51	0.5309	68	27	39
22	61	23	84	46	0.3807	68	30	91	53	38
23	64	29	93	57	22	85	50	0.6111	78	37
24	67	34	0.2302	69	36	0.4603	70	38	0.6905	36
25	70	40	10	80	51	21	91	61	31	35
26	73	46	19	92	65	38	0.5411	84	57	34
27	76	52	28	0.3104	80	55	31	0.6207	83	33
28	79	58	36	15	94	73	52	30	0.7009	32
29	82	63	45	27	0.3909	90	72	54	35	31
30	85	69	54	38	23	0.4708	92	77	61	30
31	88	75	63	50	38	25	0.5513	0.6300	98	29
32	90	81	71	62	52	42	33	23	0.7114	28
33	93	87	80	73	67	60	53	46	40	27
34	96	92	89	85	81	77	73	70	66	26
35	99	98	97	96	96	95	94	93	92	25
36	0.0802	0.1604	0.2406	0.3208	0.4010	0.4812	0.5614	0.6416	0.7218	24
37	05	10	15	20	25	29	34	39	44	23
38	08	16	23	31	39	47	55	62	70	22
39	11	21	32	43	54	64	75	86	96	21
40	14	27	41	54	68	82	95	0.6509	0.7322	20
41	17	33	50	66	83	99	0.5716	32	49	19
42	19	39	58	78	97	0.4916	36	55	75	18
43	22	45	67	89	0.4112	34	56	78	0.7401	17
44	25	50	76	0.3301	26	51	76	0.6602	27	16
45	28	56	84	12	41	69	97	25	58	15
46	31	62	93	24	55	86	0.5817	48	79	14
47	34	68	0.2502	36	70	0.5003	37	71	0.7505	13
48	37	74	10	47	84	21	58	94	31	12
49	40	79	19	59	99	38	78	0.6718	57	11
50	43	85	28	70	0.4213	56	98	41	83	10
51	46	91	37	82	28	73	0.5919	64	0.7610	9
52	48	97	45	94	42	90	39	87	86	8
53	51	0.1703	54	0.3405	57	0.5108	59	0.6810	62	7
54	54	08	63	17	71	25	79	34	88	6
55	57	14	71	28	86	43	0.6000	57	0.7714	5
56	60	20	80	40	0.4300	60	20	80	40	4
57	63	26	89	52	15	17	40	0.6903	66	3
58	66	32	98	63	29	95	61	26	92	2
59	69	37	0.2606	75	44	0.5212	81	50	0.7818	1
60	0.0872	0.1740	0.2615	0.3486	0.4358	0.5230	0.6101	0.6973	0.7844	0
1	2	3	4	5	6	7	8	9		
LATITUDE 85 DEGREES.										

84									
LATITUDE 5 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.9962	1.4924	2.9886	3.9848	4.9810	5.9771	6.9733	7.9695	8.9657
1	62	23	85	47	09	69	31	94	55
2	61	23	84	46	07	68	30	91	53
3	61	22	84	45	06	67	28	90	51
4	61	22	83	44	05	65	26	87	48
5	60	21	82	43	04	64	24	86	46
6	60	21	81	42	02	62	23	83	44
7	60	20	81	41	01	61	21	82	42
8	60	20	80	40	00	59	19	79	39
9	60	19	79	38	4.9798	58	17	77	37
10	59	19	78	38	97	56	16	75	35
11	59	18	77	36	96	55	14	73	32
12	59	18	76	35	94	53	12	70	29
13	59	17	76	34	93	52	10	69	27
14	58	17	75	33	92	50	08	66	25
15	58	16	74	32	90	48	06	64	22
16	58	16	73	31	89	47	05	62	20
17	58	15	73	30	88	45	03	60	18
18	57	14	72	29	86	43	6.9600	58	15
19	57	14	71	28	85	42	99	56	13
20	57	13	70	27	84	40	97	54	10
21	56	13	69	26	82	38	95	52	08
22	56	12	69	25	81	37	93	50	06
23	56	12	68	24	80	35	91	47	03
24	56	11	67	22	78	34	89	45	00
25	56	11	66	21	77	32	88	42	8.9598
26	55	10	65	20	76	31	86	41	96
27	55	00	64	19	74	29	84	48	93
28	55	09	64	18	73	27	82	36	91
29	55	08	63	17	71	25	79	34	88
30	54	08	62	16	70	24	78	32	86
31	54	07	61	15	69	22	76	30	83
32	53	07	60	14	67	20	74	27	81
33	53	06	59	12	66	19	72	25	78
34	53	06	58	11	64	17	70	22	75
35	53	05	58	10	63	16	68	20	73
36	52	05	57	09	62	14	66	18	71
37	52	04	56	08	60	12	64	16	68
38	52	03	55	07	59	10	62	14	65
39	52	03	54	06	57	08	60	11	63
40	51	02	53	04	56	07	58	09	60
41	51	02	52	03	54	05	56	07	57
42	51	01	52	02	53	04	54	05	55
43	51	01	51	01	52	02	52	02	53
44	50	00	50	00	50	00	50	00	50
45	50	1.9899	59	3.9799	49	3.9698	48	7.9598	47
46	49	99	58	98	47	96	46	95	45
47	49	98	57	96	46	95	44	93	42
48	49	98	56	95	44	93	42	90	39
49	48	97	56	94	43	91	40	88	37
50	48	96	55	93	41	89	37	86	34
51	48	96	53	92	40	87	35	83	31
52	48	95	53	90	38	86	33	81	28
53	48	95	52	89	37	84	31	78	26
54	47	94	51	88	35	82	29	76	23
55	47	93	50	87	34	80	27	74	20
56	46	93	49	86	32	78	25	71	18
57	46	92	48	84	31	77	23	79	15
58	46	92	47	83	29	75	21	66	12
59	46	91	47	82	28	73	19	64	10
60	0.9945	1.9890	2.9836	3.9781	4.9726	5.9671	6.9616	7.9562	8.9507
1	2	3	4	5	6	7	8	9	10
DEPARTURE 84 DEGREES.									

DEPARTURE 5 DEGREES.										85
1	2	3	4	5	6	7	8	9	10	
0	0.0872	0.1743	0.2615	0.3486	0.4358	0.5230	0.6101	0.6973	0.7844	60
1	75	49	24	98	73	47	22	96	71	59
2	77	55	32	0.3510	87	64	42	0.7019	97	58
3	80	61	41	21	0.4402	82	62	42	0.7923	57
4	83	66	49	32	16	99	82	65	48	56
5	85	72	58	34	30	0.5316	0.6202	88	74	55
6	89	78	67	56	45	33	22	0.7111	0.8000	54
7	92	84	75	67	59	51	43	34	26	53
8	95	89	84	79	74	68	63	58	52	52
9	98	95	93	90	88	86	83	81	78	51
10	0.0901	0.1801	0.3702	0.3602	0.4503	0.5403	0.6304	0.7204	0.8105	50
11	03	07	10	14	17	20	24	27	31	49
12	06	13	19	25	32	38	44	50	57	48
13	09	18	28	37	46	55	64	74	83	47
14	12	24	36	48	61	73	85	97	0.8209	46
15	15	30	45	60	75	90	0.6405	0.7320	35	45
16	18	36	54	72	90	0.5507	25	43	61	44
17	21	42	62	83	0.4604	25	46	66	87	43
18	24	47	71	95	19	42	66	90	0.8313	42
19	27	53	80	0.3706	33	60	86	0.7413	39	41
20	30	59	88	18	48	77	0.6507	36	66	40
21	32	65	97	20	62	94	27	59	92	39
22	35	71	0.3806	41	77	0.5612	47	82	0.8418	38
23	38	76	15	53	91	29	67	0.7506	44	37
24	41	82	23	64	0.4706	47	88	29	70	36
25	44	88	32	76	20	64	0.6608	52	96	35
26	47	94	41	88	35	81	28	75	0.8522	34
27	50	0.1900	49	99	49	0.5709	49	98	48	33
28	53	05	58	0.3811	64	16	69	0.7622	74	32
29	56	11	67	22	78	34	89	45	0.8600	31
30	59	17	76	34	93	51	0.6710	68	27	30
31	61	23	84	46	0.4807	85	30	91	53	29
32	64	28	93	57	21	0.5803	49	0.7714	78	28
33	67	34	0.2901	68	36	20	70	37	0.8704	27
34	70	40	10	80	50	37	90	60	30	26
35	73	46	19	92	65	55	0.6810	83	56	25
36	75	52	27	0.3903	79	72	31	0.7806	82	24
37	79	57	36	15	94	81	51	80	0.8808	23
38	82	63	45	26	0.4908	90	71	53	34	22
39	85	69	54	38	23	0.5907	92	76	61	21
40	87	75	62	50	37	24	0.6912	99	87	20
41	90	81	71	61	52	42	32	0.7922	0.8913	19
42	93	86	80	73	66	59	52	46	89	18
43	96	92	88	84	81	77	73	69	65	17
44	99	98	97	96	95	94	93	92	91	16
45	0.1002	0.2004	0.3006	0.4008	0.5010	0.6011	0.7013	0.8015	0.9017	15
46	05	10	14	19	24	29	34	38	43	14
47	08	15	23	31	39	46	54	62	69	13
48	11	21	32	42	53	64	74	85	95	12
49	14	27	41	54	68	81	95	0.8108	0.9122	11
50	16	33	49	66	82	98	0.7115	31	48	10
51	19	38	58	77	96	0.6115	34	54	73	9
52	22	44	66	89	0.5111	33	55	77	99	8
53	25	50	75	0.4100	25	50	75	0.8200	0.9225	7
54	28	56	84	12	40	67	95	23	51	6
55	31	62	92	23	54	85	0.7216	46	77	5
56	34	67	0.3101	35	69	0.6202	36	70	0.9802	4
57	37	73	10	46	83	20	56	93	29	3
58	40	79	19	58	98	37	77	0.8316	56	2
59	42	85	27	70	0.5212	54	97	39	82	1
60	0.1045	0.2091	0.3136	0.4181	0.5226	0.6272	0.7317	0.8362	0.9408	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 84 DEGREES.										

LATITUDE 6 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.9945	1.9890	2.9836	3.9781	4.9726	5.9671	6.9616	7.9562	8.9507	06
1	45	90	33	79	24	69	14	59	04	59
2	45	89	34	78	23	68	12	57	01	58
3	44	89	33	77	21	66	10	54	8.9499	57
4	44	88	32	76	20	64	08	52	96	56
5	43	88	31	75	18	62	06	50	93	55
6	43	87	30	74	17	60	04	47	91	54
7	43	87	29	72	15	59	02	45	98	53
8	43	86	28	71	14	57	00	42	85	52
9	42	85	27	69	12	54	6.9597	39	82	51
10	42	84	26	68	11	53	95	37	79	50
11	41	83	25	67	09	51	93	34	76	49
12	41	83	25	66	08	49	91	32	74	48
13	41	82	24	65	03	47	88	30	71	47
14	41	82	23	64	05	45	86	27	68	46
15	40	80	22	62	03	43	84	25	65	45
16	40	80	21	61	01	41	81	22	62	44
17	40	79	20	59	4.9699	39	79	19	59	43
18	40	79	19	58	98	38	77	17	56	42
19	39	78	18	57	96	36	75	14	54	41
20	38	78	17	56	95	34	73	12	51	40
21	38	77	16	54	93	32	70	09	48	39
22	38	77	15	53	92	30	68	06	45	38
23	38	76	14	52	90	28	66	04	42	37
24	38	75	13	51	89	26	64	02	39	36
25	37	74	12	49	87	24	62	7.9499	36	35
26	37	74	11	48	85	22	59	96	33	34
27	36	73	10	47	83	20	57	94	30	33
28	36	73	09	46	82	18	55	91	28	32
29	36	72	08	44	80	16	52	88	24	31
30	36	71	07	43	79	14	50	86	21	30
31	35	70	06	41	77	12	48	83	19	29
32	35	70	05	40	76	11	46	81	16	28
33	35	69	04	39	74	08	43	78	18	27
34	34	69	03	38	72	06	41	75	10	26
35	34	68	02	36	70	04	38	73	07	25
36	34	67	01	35	69	02	36	70	03	24
37	33	66	00	33	67	00	34	67	01	23
38	33	66	2.9799	32	66	5.9599	32	65	8.9398	22
39	33	65	98	31	64	96	29	62	95	21
40	32	65	97	30	62	94	27	59	92	20
41	32	64	96	28	60	92	24	56	88	19
42	32	63	95	27	59	90	22	54	85	18
43	31	62	94	25	57	88	20	51	82	17
44	31	62	93	24	55	86	17	48	79	16
45	31	61	92	22	53	84	15	46	76	15
46	30	61	91	21	52	82	12	42	73	14
47	30	60	90	20	50	80	10	40	70	13
48	30	59	89	19	49	78	08	38	67	12
49	29	58	88	17	47	76	05	34	64	11
50	29	58	87	16	45	74	03	32	61	10
51	29	57	86	14	43	72	00	29	58	9
52	28	57	85	13	42	70	6.9498	26	55	8
53	28	56	84	11	40	68	95	23	51	7
54	28	55	83	10	38	66	93	21	48	6
55	27	54	82	09	36	63	90	18	45	5
56	27	54	81	08	35	61	88	15	42	4
57	27	53	80	06	33	59	86	14	39	3
58	26	52	79	05	31	57	83	10	36	2
59	26	51	78	03	29	55	81	06	33	1
60	0.9926	1.9851	2.9777	3.9702	4.9628	5.9553	6.9479	7.9404	8.9330	0
1	2	3	4	5	6	7	8	9	10	11
DEPARTURE 83 DEGREES.										

RETURN 6 DEGREES.										87
1	2	3	4	5	6	7	8	9	10	11
0	0.1045	0.2091	0.3136	0.4181	0.5227	0.6272	0.7317	0.8362	0.9408	60
1	0.0748	97	43	93	41	89	37	86	34	59
2	51	0.2102	53	0.4204	56	0.6307	58	0.8409	60	58
3	54	08	62	16	70	24	78	32	86	57
4	57	14	71	28	85	41	98	55	0.9512	56
5	60	19	79	39	99	58	0.7418	78	37	55
6	63	25	88	50	0.5313	76	38	0.8501	63	54
7	66	31	97	62	28	93	59	24	90	53
8	68	37	0.3205	74	42	0.6410	79	47	0.9616	52
9	71	43	14	85	57	28	99	70	42	51
10	74	48	23	97	71	35	0.7519	94	68	50
11	77	54	31	0.4308	86	53	40	0.8617	94	49
12	80	60	40	20	0.5400	80	60	40	0.9720	48
13	83	66	49	32	15	0.6507	80	63	46	47
14	86	72	57	43	29	15	0.7601	86	72	46
15	88	77	66	55	44	32	21	0.8710	98	45
16	92	83	75	66	58	60	41	33	0.9824	44
17	95	89	94	78	73	77	62	56	51	43
18	97	95	92	89	87	84	81	78	76	42
19	0.1100	0.2200	81	0.4401	0.5501	0.6601	0.7701	0.8802	0.9902	41
20	03	06	0.3309	12	16	19	22	25	28	40
21	06	12	18	24	30	36	42	48	54	39
22	09	18	27	36	45	53	62	71	80	38
23	12	24	35	43	59	71	83	94	1.0006	37
24	15	29	44	59	74	88	0.7803	0.8918	32	36
25	18	35	53	70	88	0.6706	23	41	58	35
26	21	41	62	82	0.5603	23	44	64	85	34
27	23	47	70	94	17	40	64	87	1.0111	33
28	26	53	79	0.4505	32	58	84	0.9010	37	32
29	29	58	87	16	46	75	0.7904	33	62	31
30	32	64	96	28	60	92	24	56	88	30
31	35	70	0.3405	40	75	0.6809	44	79	1.0214	29
32	38	76	13	51	89	27	65	0.9102	40	28
33	41	81	22	63	0.5704	44	85	26	66	27
34	44	87	31	74	18	62	0.8005	69	92	26
35	47	93	40	86	32	79	26	92	1.0319	25
36	49	99	48	98	47	96	46	95	45	24
37	52	0.2305	57	0.4609	62	0.6914	66	0.9218	71	23
38	55	10	66	21	76	31	86	42	97	22
39	58	16	74	32	90	48	0.8106	64	1.0422	22
40	61	22	83	44	0.5805	65	26	87	48	20
41	64	28	91	55	19	83	47	0.9310	71	19
42	67	33	0.3500	7	34	0.7000	67	34	1.0500	18
43	70	39	09	78	48	18	87	57	36	17
44	73	45	18	90	63	35	0.8208	80	53	16
45	75	51	26	0.4702	77	52	28	0.9403	79	15
46	78	57	35	13	92	70	48	26	1.0605	14
47	81	62	44	25	0.5906	87	68	50	31	13
48	84	68	52	36	20	0.7104	88	72	56	12
49	87	74	61	48	35	21	0.8308	95	82	11
50	90	80	69	59	49	39	29	0.9518	1.0708	10
51	93	85	78	71	64	56	49	42	34	9
52	96	91	87	82	78	74	69	65	60	8
53	99	97	96	94	93	91	90	88	87	7
54	0.1201	0.2403	0.3604	0.4806	0.6007	0.7208	0.8410	0.9611	1.0813	6
55	04	09	13	17	22	26	30	34	39	5
56	07	14	21	28	36	43	50	57	64	4
57	10	20	30	40	50	60	70	80	90	3
58	13	25	39	52	65	77	90	0.9708	1.0916	2
59	16	31	47	63	79	95	21	26	42	1
60	0.1219	0.2437	0.3656	0.4875	0.6094	0.7312	0.8531	0.9750	1.0968	0
1	2	3	4	5	6	7	8	9	10	11

LATITUDE 83 DEGREES.

LATITUDE 7 DEGREES.									
88	1	2	3	4	5	6	7	8	9
0	0.9926	1.9851	2.9777	3.9702	4.9628	5.9553	6.9479	7.9404	8.9330
1	25	50	75	00	26	51	76	01	26
2	25	50	74	3.9699	24	49	74	7.9398	23
3	24	49	73	97	22	46	71	95	19
4	24	48	72	96	20	44	68	92	16
5	23	48	70	94	18	42	65	89	13
6	23	47	70	93	17	40	63	86	10
7	23	46	69	91	15	38	60	83	06
8	23	45	68	90	13	36	58	81	03
9	22	44	67	89	11	33	55	78	00
10	22	44	66	88	10	31	53	75	8.9297
11	21	43	64	86	08	29	50	72	93
12	21	42	63	84	06	27	48	69	90
13	20	41	62	83	03	24	45	66	87
14	20	41	61	82	02	22	43	63	84
15	20	40	60	80	00	20	40	61	80
16	20	39	59	79	4.9599	18	38	58	77
17	19	39	58	77	97	15	35	64	73
18	19	38	57	76	95	13	32	51	70
19	18	37	56	74	93	11	29	48	66
20	18	36	55	73	91	09	27	46	64
21	18	35	54	71	89	07	25	43	61
22	18	35	53	70	88	05	23	40	58
23	17	34	51	68	86	02	20	37	54
24	17	33	50	67	84	00	17	34	50
25	16	33	49	65	82	5.9498	14	31	47
26	16	32	48	64	80	96	12	28	44
27	15	30	47	62	78	93	09	25	40
28	15	30	46	61	76	91	06	22	37
29	14	29	44	59	74	88	03	18	33
30	14	29	43	58	72	86	01	15	30
31	14	28	42	56	70	84	6.9398	12	26
32	14	27	41	55	69	82	96	10	23
33	13	26	40	53	67	80	93	06	19
34	13	26	39	52	65	78	90	03	16
35	12	24	38	50	63	75	87	00	13
36	12	24	37	49	61	73	85	7.9298	10
37	11	23	35	47	59	70	82	94	06
38	11	23	34	46	57	68	80	91	03
39	11	22	33	44	55	66	77	88	8.9199
40	11	21	32	42	53	64	74	85	95
41	10	20	30	40	51	61	71	81	91
42	10	20	29	39	49	59	69	78	88
43	09	19	28	37	48	57	66	75	85
44	09	18	27	36	46	55	64	73	82
45	08	17	26	34	44	52	61	69	78
46	08	17	25	33	42	50	59	66	75
47	08	16	24	31	40	47	56	63	71
48	08	15	23	30	38	45	53	60	68
49	07	14	21	28	36	42	50	57	64
50	07	13	20	27	34	40	47	54	60
51	06	12	19	25	32	37	44	50	56
52	06	12	18	24	30	35	41	47	53
53	05	11	16	22	28	33	38	44	49
54	05	10	15	20	26	31	36	41	46
55	04	09	14	18	24	28	33	37	42
56	04	09	13	17	22	26	30	34	39
57	04	08	12	15	20	23	27	31	35
58	04	07	11	14	18	21	25	28	32
59	03	06	09	12	16	18	22	25	28
60	0.9908	1.9805	2.9708	3.9611	4.9514	5.9416	6.9319	7.9222	8.9124
	1	2	3	4	5	6	7	8	9
DEPARTURE 82 DEGREES.									

DEPARTURE 7 DEGREES.										89
1	2	3	4	5	6	7	8	9	10	
0	0.1219	0.2437	0.3656	0.4875	0.6094	0.7312	0.8531	0.9750	1.0968	60
1	22	43	65	86	0.6108	29	51	73	94	59
2	25	49	74	98	23	47	72	96	1.1021	58
3	27	55	82	0.4910	87	64	92	0.9819	47	57
4	30	60	91	21	51	81	0.8611	42	72	56
5	33	66	99	32	66	99	32	65	98	55
6	36	72	0.3708	44	80	0.7416	52	88	1.1124	54
7	39	78	17	56	95	33	72	0.9911	50	53
8	42	84	25	67	0.6209	51	93	34	76	52
9	45	89	34	79	24	68	0.8713	58	1.1202	51
10	48	95	43	90	38	86	33	81	28	50
11	50	0.2501	51	0.5002	52	0.7502	53	1.0003	54	49
12	53	07	60	13	67	20	73	26	80	48
13	56	12	69	25	81	37	93	50	1.1806	47
14	59	18	77	36	96	55	0.8814	73	32	46
15	62	24	86	48	0.6310	72	34	96	58	45
16	65	30	95	60	25	89	54	1.0119	84	44
17	68	36	0.3803	71	39	0.7607	75	42	1.1410	43
18	71	41	12	82	53	24	94	65	35	42
19	74	47	21	94	68	41	0.8915	88	62	41
20	76	53	29	0.5106	82	58	35	1.0211	88	40
21	79	59	38	17	97	76	55	34	1.1614	39
22	82	64	47	29	0.6411	93	75	58	40	38
23	85	70	55	30	26	0.7711	96	81	66	37
24	88	76	64	52	40	28	0.9016	1.0304	92	36
25	91	82	72	63	54	45	36	26	1.1617	35
26	94	87	81	75	69	62	56	50	43	34
27	97	93	90	86	83	80	76	73	69	33
28	0.1300	98	99	98	98	97	96	96	96	32
29	02	0.2605	0.3907	0.5210	0.6512	0.7814	0.9117	1.0419	1.1722	31
30	05	11	16	21	27	32	37	42	48	30
31	08	16	24	32	41	49	57	65	73	29
32	11	22	33	44	55	66	77	88	99	28
33	14	28	42	56	70	83	97	1.0511	1.1825	27
34	17	34	50	67	84	0.7901	0.9218	34	51	26
35	20	39	59	79	99	18	38	58	77	25
36	23	45	68	90	0.6613	36	58	81	1.1903	24
37	25	51	76	0.5302	27	52	78	1.0603	29	23
38	28	57	85	13	42	70	98	26	55	22
39	31	62	94	25	56	87	0.9318	50	81	21
40	34	68	0.4002	36	71	0.8005	39	73	1.2007	20
41	37	74	11	48	85	22	59	96	33	19
42	40	80	20	60	0.6700	39	79	1.0719	59	18
43	43	85	28	71	14	56	99	42	84	17
44	46	91	37	82	28	74	0.9419	65	1.2110	16
45	49	97	46	94	43	91	40	88	37	15
46	51	0.2703	54	0.5406	57	0.8108	60	1.0811	63	14
47	54	09	63	17	72	26	80	34	89	13
48	57	14	72	29	86	43	0.9500	58	1.2215	12
49	60	20	80	40	0.6800	60	20	80	40	11
50	63	26	89	52	15	77	40	1.0903	66	10
51	66	32	97	63	29	95	61	26	92	9
52	69	37	0.4106	75	44	0.8212	81	50	1.2318	8
53	72	43	15	86	58	30	0.9601	73	44	7
54	74	49	23	98	72	46	21	95	70	6
55	77	55	32	0.5509	87	64	41	1.1018	96	5
56	80	60	41	21	0.6901	81	61	42	1.2422	4
57	83	66	49	32	16	99	82	65	48	3
58	86	72	58	44	30	0.8316	0.9702	88	74	2
59	89	78	67	56	45	83	22	1.1111	1.2500	1
60	0.1392	0.2783	0.4175	0.5567	0.6959	0.8350	0.9742	1.1134	1.2525	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 82 DEGREES.										

LATITUDE 8 DEGREES.										
90	1	2	3	4	5	6	7	8	9	n
0	0.9908	1.9805	2.9708	3.9611	4.9514	5.9416	6.9319	7.9222	8.9124	60
1	02	05	07	09	12	14	16	18	21	59
2	02	04	06	08	09	11	13	15	17	58
3	01	03	05	06	08	09	11	12	14	57
4	01	02	03	04	06	07	08	09	10	56
5	00	01	02	02	03	04	04	05	05	55
6	00	00	01	01	01	01	01	02	02	54
7	0.9899	00	2.9699	3.9599	4.9499	5.9399	6.9299	7.9198	8.9098	53
8	99	1.9799	98	98	97	96	96	95	95	52
9	99	98	97	96	95	94	93	92	91	51
10	99	97	96	94	93	92	90	89	87	50
11	98	96	95	93	91	89	87	86	84	49
12	98	96	93	91	89	87	85	82	80	48
13	97	95	92	89	87	84	81	78	77	47
14	97	94	91	88	85	81	78	75	72	46
15	97	93	90	86	83	79	75	72	69	45
16	96	92	88	84	81	77	73	69	65	44
17	96	91	87	83	79	74	70	66	61	43
18	95	91	86	81	76	72	67	62	58	42
19	95	90	84	79	74	69	64	58	53	41
20	94	89	83	78	72	66	61	55	50	40
21	94	88	82	76	70	64	58	52	46	39
22	94	87	81	74	68	62	55	49	42	38
23	93	86	79	72	66	59	52	45	38	37
24	93	85	78	71	64	56	49	42	34	36
25	92	85	77	69	62	54	46	38	31	35
26	92	84	76	68	60	51	43	35	27	34
27	91	83	74	67	57	48	40	31	23	33
28	91	82	73	64	55	46	37	28	19	32
29	91	81	72	66	53	44	34	25	15	31
30	90	80	71	61	51	41	31	22	12	30
31	90	79	69	59	49	38	28	18	08	29
32	89	79	68	57	47	36	25	14	04	28
33	89	78	67	56	44	33	22	10	00	27
34	88	77	65	54	42	30	19	07	8.8996	26
35	88	76	64	52	40	28	16	04	92	25
36	88	75	63	50	38	26	13	01	88	24
37	87	74	61	48	36	23	10	7.9094	84	23
38	87	73	60	47	34	20	07	94	80	22
39	86	73	59	45	32	17	04	90	77	21
40	86	72	57	43	29	15	01	86	72	20
41	85	71	56	42	27	12	6.9198	83	69	19
42	85	70	55	40	25	09	94	79	64	18
43	84	69	54	38	23	07	92	76	61	17
44	84	68	52	36	21	05	89	73	57	16
45	83	67	51	34	18	02	85	69	53	15
46	83	66	50	33	16	5.9299	82	66	49	14
47	82	65	48	31	14	96	79	62	44	13
48	82	65	47	29	12	94	76	58	41	12
49	82	64	45	27	09	91	73	54	36	11
50	81	63	44	26	07	88	70	51	33	10
51	81	62	43	24	05	85	66	47	28	9
52	81	61	42	22	03	83	64	44	25	8
53	80	60	40	20	00	80	60	40	20	7
54	80	59	39	18	4.9398	78	57	37	16	6
55	79	58	37	16	96	75	54	33	12	5
56	79	57	36	15	94	72	51	30	08	4
57	78	56	35	13	91	69	47	26	04	3
58	78	56	33	11	89	67	45	22	00	2
59	77	55	32	09	87	64	41	18	8.8896	1
60	0.9877	1.9754	2.9631	3.9508	4.9385	5.9261	6.9138	7.9015	8.8892	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 81 DEGREES.										

DEPARTURE 8 DEGREES.										91
	1	2	3	4	5	6	7	8	9	
0	0.1392	0.2783	0.4175	0.5567	0.6959	0.8350	0.9742	1.1134	1.2526	60
1	95	89	84	78	73	68	62	57	51	59
2	98	95	94	90	88	85	83	80	78	58
3	0.1400	0.2801	91	0.5602	0.7002	0.8402	0.9803	1.1203	1.2604	57
4	03	07	0.4210	13	17	20	23	26	31	56
5	06	12	18	24	31	37	43	49	55	55
6	09	18	27	36	45	54	63	72	81	54
7	12	24	36	48	60	71	83	95	1.2707	53
8	15	30	44	59	74	89	0.9904	1.1318	33	52
9	18	35	53	71	89	0.8506	24	42	59	51
10	21	41	62	82	0.7103	23	44	64	85	50
11	23	47	70	94	17	40	64	87	1.2811	49
12	26	53	79	0.5705	32	58	84	1.1410	37	48
13	29	58	88	17	46	75	1.0004	34	63	47
14	32	64	96	28	60	92	24	56	88	46
15	35	70	0.4305	40	75	0.8609	44	79	1.2914	45
16	38	76	13	51	89	27	65	1.1502	40	44
17	41	81	22	63	0.7204	44	85	26	66	43
18	44	87	31	74	18	62	1.0105	49	92	42
19	46	93	39	86	32	78	25	71	1.3018	41
20	49	99	48	97	47	96	45	94	44	40
21	52	0.2904	57	0.5809	61	0.8713	65	1.1618	70	39
22	55	10	65	20	76	31	86	41	96	38
23	58	16	74	32	90	48	1.0206	64	1.3122	37
24	61	22	82	43	0.7304	65	26	86	47	36
25	64	27	91	55	19	82	46	1.1770	73	35
26	67	33	0.4400	66	33	0.8800	66	33	99	34
27	70	39	09	78	48	17	86	56	1.3226	33
28	72	45	17	89	62	34	1.0306	78	51	32
29	75	50	26	0.5901	76	51	26	1.1802	77	31
30	78	56	34	12	91	69	47	25	1.3303	30
31	81	62	43	24	0.7405	86	67	48	29	29
32	84	68	51	35	19	0.8903	87	70	54	28
33	87	73	60	47	34	20	1.0405	94	80	27
34	90	79	69	58	48	38	27	1.1917	1.3406	26
35	93	85	78	70	63	55	48	40	33	25
36	95	91	86	82	77	72	68	63	59	24
37	98	96	95	93	91	89	87	86	84	23
38	0.1501	0.3002	0.4503	0.6004	0.7506	0.9007	1.0508	1.2009	1.3510	22
39	04	08	12	16	20	24	28	32	36	21
40	07	13	21	28	35	41	48	55	62	20
41	10	19	39	39	49	58	68	78	87	19
42	13	25	38	50	63	76	88	1.2101	1.3613	18
43	16	31	47	62	78	93	1.0609	24	40	17
44	18	37	55	74	92	0.9110	29	47	66	16
45	21	42	64	85	0.7606	27	48	70	91	15
46	24	48	72	96	21	45	69	93	1.3717	14
47	27	54	81	0.6108	35	62	89	1.2216	48	13
48	30	60	90	20	50	79	1.0709	39	69	12
49	38	65	98	31	64	96	29	62	94	11
50	36	71	0.4607	42	78	0.9214	49	85	1.3820	10
51	39	77	16	54	93	31	70	1.2308	47	9
52	41	83	24	66	0.7707	48	90	21	73	8
53	44	88	33	77	21	65	1.0809	44	98	7
54	47	94	41	88	36	83	30	77	1.3924	6
55	50	0.3100	50	0.6200	50	0.9300	50	1.2400	50	5
56	53	06	59	12	65	17	70	23	76	4
57	56	11	67	23	79	34	90	46	1.4001	3
58	59	17	76	34	93	52	1.0910	69	27	2
59	62	23	85	46	0.7808	69	31	92	54	1
60	0.1564	0.3129	0.4693	0.6257	0.7822	0.9386	1.0950	1.2514	1.4079	0
1	2	3	4	5	6	7	8	9		
LATITUDE 81 DEGREES.										

92									
LATITUDE 9 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.9877	1.9754	2.9631	3.9508	4.9385	5.9261	6.9138	7.9015	8.8892
1	76	58	29	06	82	58	35	11	88
2	76	52	28	04	80	56	32	08	84
3	75	51	27	02	78	53	29	04	80
4	75	50	25	00	76	51	26	01	76
5	74	49	24	3.9498	73	48	22	7.8997	71
6	74	48	22	96	71	45	19	93	67
7	73	47	21	95	69	42	15	90	63
8	73	46	20	93	66	39	12	86	59
9	72	46	18	91	64	37	00	82	55
10	72	45	17	89	62	34	06	78	51
11	71	44	15	87	59	31	03	74	46
12	71	43	14	86	57	28	00	71	43
13	70	42	13	84	55	25	6.9096	67	38
14	70	41	11	82	52	22	93	63	34
15	70	40	10	79	50	20	90	60	30
16	70	39	09	78	48	17	87	56	26
17	69	38	07	76	45	14	83	52	21
18	69	37	06	74	43	12	80	49	17
19	68	36	04	72	41	09	76	45	13
20	68	35	03	70	38	06	73	41	08
21	67	34	01	68	36	03	70	37	04
22	67	33	00	67	34	00	67	34	00
23	66	32	1.9599	65	31	5.9197	63	30	8.8796
24	66	31	97	63	29	94	60	26	91
25	65	30	96	61	26	91	57	22	87
26	65	30	94	59	24	89	54	18	83
27	64	29	93	57	22	86	50	14	78
28	64	28	91	55	19	83	47	10	74
29	63	27	90	53	17	80	43	06	70
30	63	26	89	52	15	77	40	03	66
31	62	25	87	50	12	74	36	7.8899	62
32	62	24	86	48	10	71	33	95	57
33	61	23	84	46	07	68	29	91	53
34	61	22	83	44	05	65	26	87	48
35	60	21	81	42	02	62	23	83	44
36	60	20	80	40	00	60	20	80	40
37	59	19	78	38	4.9298	57	16	76	35
38	59	18	77	36	95	54	13	72	31
39	58	17	76	34	93	51	09	68	27
40	58	16	74	32	90	48	06	64	22
41	57	15	73	30	88	45	02	60	18
42	57	14	71	28	85	42	6.8999	56	13
43	56	13	70	26	83	39	96	52	09
44	56	12	68	24	81	37	93	49	05
45	55	11	66	22	78	34	89	45	00
46	55	10	65	20	76	31	86	41	8.8696
47	54	09	63	18	73	28	82	37	91
48	54	08	62	16	71	25	79	33	87
49	53	07	61	14	68	22	75	29	82
50	53	06	59	12	66	19	72	25	78
51	52	05	58	10	63	16	68	21	73
52	52	04	56	08	61	13	65	17	69
53	51	03	55	06	58	10	61	13	64
54	51	02	53	04	56	07	58	09	60
55	50	01	52	02	53	04	54	05	55
56	50	00	50	00	51	01	51	01	51
57	49	1.9699	49	3.9398	48	5.9098	47	7.8797	46
58	49	98	47	96	46	95	44	93	42
59	48	97	46	94	48	92	40	89	37
60	0.9848	1.9696	2.9544	3.9392	4.9241	5.9089	6.8937	7.8785	8.8633
	1	2	3	4	5	6	7	8	9
DEPARTURE 80 DEGREES.									

DEPARTURE 9 DEGREES.										98
1	2	3	4	5	6	7	8	9	10	
0	0.1564	0.3129	0.4693	0.6257	0.7822	0.9386	1.0950	1.2514	1.4079	60
1	67	34	0.4702	69	36	0.9408	70	38	1.4105	59
2	70	40	10	80	51	21	91	61	31	58
3	73	46	19	92	65	38	1.1011	84	57	57
4	76	52	27	0.6303	79	55	31	1.2606	82	56
5	79	57	36	15	94	72	49	30	1.4208	55
6	82	63	45	26	0.7908	90	71	53	34	54
7	85	69	54	38	23	0.9507	92	76	61	53
8	87	75	62	49	37	24	1.1111	98	86	52
9	90	80	71	61	51	41	31	1.2722	1.4312	51
10	93	86	79	72	66	59	52	45	38	50
11	96	92	88	84	80	75	71	67	63	49
12	99	98	96	95	94	93	92	90	89	48
13	0.1602	0.3203	0.4805	0.6407	0.8009	0.9610	1.1212	1.2814	1.4415	47
14	05	09	14	18	23	28	32	37	41	46
15	07	15	22	30	37	44	52	59	67	45
16	10	21	31	41	52	62	72	82	93	44
17	13	26	40	53	66	79	92	1.2906	1.4519	43
18	16	32	48	64	81	97	1.1313	29	45	42
19	19	38	57	76	95	0.9713	32	51	70	41
20	22	44	65	87	0.8109	31	53	74	96	40
21	25	49	77	98	23	48	73	97	1.4620	39
22	28	55	83	0.6510	38	65	93	1.3020	48	38
23	30	61	91	22	52	82	1.1413	43	74	37
24	33	67	0.4900	33	67	0.9800	33	66	1.4700	36
25	36	72	08	44	81	17	53	89	25	35
26	39	78	17	56	95	34	73	1.3112	51	34
27	42	84	26	68	0.8210	51	93	35	77	33
28	45	89	34	79	24	68	1.1513	58	1.4802	32
29	48	95	43	90	38	86	33	81	28	31
30	51	0.3301	52	0.6602	53	0.9903	54	1.3204	55	30
31	53	07	60	13	67	20	73	26	80	29
32	56	12	69	25	81	37	93	50	1.4906	28
33	59	18	77	36	96	55	1.1614	73	32	27
34	62	24	86	48	0.8810	72	34	96	58	26
35	65	30	94	59	24	89	54	1.3318	83	25
36	68	35	0.5003	71	39	1.0006	74	42	1.5009	24
37	70	41	12	82	53	24	94	65	35	23
38	73	47	20	94	67	40	1.1714	87	61	22
39	76	53	29	0.6705	82	58	34	1.3410	87	22
40	79	58	38	17	96	75	54	34	1.5113	20
41	82	64	46	28	0.8410	92	74	56	38	19
42	85	70	55	40	25	1.0109	94	79	64	18
43	88	76	63	51	39	27	1.1815	1.3502	90	17
44	91	81	72	62	53	44	34	25	1.5215	16
45	94	87	81	74	68	61	55	48	42	15
46	96	93	89	86	82	78	75	72	68	14
47	99	98	98	97	96	95	94	94	93	13
48	0.1702	0.3404	0.5106	0.6808	0.8511	1.0213	1.1915	1.3617	1.5319	12
49	05	10	15	20	25	30	35	40	45	11
50	08	16	23	31	39	47	55	62	70	10
51	11	21	32	43	54	64	75	86	96	9
52	14	27	41	54	68	82	95	1.3709	1.5422	8
53	16	33	49	66	82	98	1.2015	81	48	7
54	19	39	58	77	97	1.0316	85	54	74	6
55	22	44	67	89	0.8611	33	55	78	1.5500	5
56	25	50	75	0.6900	25	50	75	1.3800	25	4
57	28	56	84	12	40	67	95	23	51	3
58	31	62	92	23	54	85	1.2116	46	77	2
59	34	67	0.5201	34	68	1.0402	85	68	1.5602	1
60	0.1736	0.3473	0.5210	0.6946	0.8683	1.0419	1.2156	1.3892	1.5629	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 80 DEGREES.										

LATITUDE 10 DEGREES.										
94	1	2	3	4	5	6	7	8	9	7
0	0.9848	1.9696	2.9544	3.9392	4.9241	5.9089	6.8937	7.8785	8.8633	06
1	48	95	43	90	38	86	33	81	28	59
2	47	94	41	88	36	83	30	77	24	58
3	47	93	39	86	33	80	26	78	19	57
4	46	92	38	84	31	77	23	69	15	56
5	46	91	37	82	28	73	19	64	10	55
6	45	90	35	80	25	70	15	60	05	54
7	45	89	34	78	23	67	12	56	01	53
8	44	88	32	76	20	64	08	52	8.8596	52
9	44	87	31	74	18	61	05	48	92	51
10	43	86	29	72	15	58	01	44	87	50
11	43	85	28	70	13	55	6.8898	40	83	49
12	42	84	26	68	10	52	94	36	78	48
13	41	83	24	66	07	48	90	31	73	47
14	41	82	22	64	05	45	86	27	68	46
15	40	81	21	62	02	42	83	23	64	45
16	40	80	20	60	00	39	79	19	59	44
17	39	79	18	58	1.9197	36	76	15	55	43
18	39	78	17	56	95	33	72	11	50	42
19	38	77	15	53	92	30	68	06	45	41
20	38	76	13	51	89	27	65	02	40	40
21	37	75	11	49	87	24	61	7.8698	35	39
22	37	74	10	47	84	21	58	94	31	38
23	36	72	09	45	81	17	54	90	26	37
24	36	71	07	43	79	14	50	86	21	36
25	35	70	06	41	76	11	46	82	17	35
26	35	69	04	39	74	08	43	78	12	34
27	34	68	02	36	71	05	39	73	07	33
28	34	67	01	34	68	02	35	69	02	32
29	33	66	2.9499	32	66	5.8999	32	65	8.8498	31
30	33	65	98	30	63	95	28	60	93	30
31	32	64	96	28	60	92	24	56	88	29
32	32	63	95	26	58	89	21	52	84	28
33	31	62	93	24	55	86	17	48	79	27
34	30	61	91	22	52	82	13	43	74	26
35	30	60	90	20	50	79	09	39	69	25
36	29	59	88	18	47	76	06	35	65	24
37	29	58	86	15	44	73	02	30	60	23
38	28	57	85	13	42	70	6.8798	26	55	22
39	28	55	83	11	39	66	94	22	50	21
40	27	54	82	09	36	63	90	18	45	20
41	27	53	80	06	35	60	87	14	40	19
42	26	52	78	04	31	57	83	09	35	18
43	26	51	77	02	28	54	79	05	30	17
44	25	50	75	00	25	50	75	00	25	16
45	25	49	74	3.9298	23	47	72	7.8596	20	15
46	24	48	72	96	20	44	68	92	16	14
47	23	47	70	94	17	40	64	87	10	13
48	23	46	69	92	15	37	60	83	06	12
49	22	45	67	89	12	34	56	78	01	11
50	22	44	65	87	09	31	53	74	8.8496	10
51	21	42	64	85	06	27	48	70	91	9
52	21	41	62	83	04	24	45	66	86	8
53	20	40	60	80	01	21	41	61	81	7
54	20	39	59	78	4.9098	18	37	57	76	6
55	19	38	57	76	95	14	33	52	71	5
56	19	37	56	74	93	11	30	48	67	4
57	18	36	54	72	90	07	26	43	61	3
58	17	35	52	70	87	04	22	39	57	2
59	17	34	50	67	84	01	18	34	51	1
60	0.9816	1.9633	2.9449	3.9275	4.9082	5.8898	6.8714	7.8530	8.8346	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 79 DEGREES.										

DEPARTURE 10 DEGREES.										96
	1	2	3	4	5	6	7	8	9	0
0	0.1737	0.3473	0.5210	0.6946	0.8683	1.0419	1.2156	1.3892	1.5629	60
1	39	89	18	57	97	36	75	1.3914	54	59
2	42	84	27	69	0.8711	53	95	38	80	58
3	45	90	35	80	25	71	1.2216	61	1.5706	57
4	48	96	44	92	40	87	35	83	31	56
5	51	0.3502	52	0.7003	64	1.0505	56	1.4006	57	55
6	54	07	61	15	69	22	76	30	83	54
7	57	13	70	26	83	39	96	52	1.5809	53
8	59	19	78	38	97	56	1.2316	75	35	52
9	62	25	87	49	0.8812	74	36	98	61	51
10	65	30	95	60	26	91	56	1.4121	86	50
11	68	36	0.5304	72	40	1.0608	76	54	1.5912	49
12	71	42	12	83	54	27	96	66	37	48
13	74	47	21	95	69	42	1.2416	90	63	47
14	77	53	30	0.7106	83	60	36	1.4213	89	46
15	79	59	38	18	97	76	56	35	1.6015	45
16	82	65	47	29	0.8912	94	76	58	41	44
17	85	70	56	41	26	1.0711	96	82	67	43
18	88	76	64	52	40	28	1.2516	1.4304	92	42
19	91	82	72	63	55	45	36	27	1.6118	41
20	94	87	81	76	69	62	56	50	43	40
21	97	93	90	87	83	80	76	73	69	39
22	0.1800	99	99	98	98	97	97	96	96	38
23	02	0.3605	0.5407	0.7209	0.9012	1.0814	1.2616	1.4418	1.6221	37
24	05	10	16	21	26	31	36	42	47	36
25	08	16	24	32	41	49	57	65	73	35
26	11	22	33	44	55	65	76	87	98	34
27	14	28	41	55	69	83	97	1.4510	1.6325	33
28	17	33	50	66	83	1.0900	1.2716	33	49	32
29	20	39	59	78	98	17	37	56	76	31
30	22	45	67	90	0.9112	34	57	79	1.6402	30
31	25	50	76	0.7301	26	51	76	1.4602	27	29
32	28	56	84	12	41	69	97	25	53	28
33	31	62	92	24	55	85	1.2816	47	78	27
34	34	68	0.5501	35	69	1.1003	37	70	1.6504	26
35	37	73	10	47	84	20	57	94	30	25
36	40	79	19	38	98	37	77	1.4716	56	24
37	42	85	27	70	0.9212	54	97	39	82	23
38	45	90	35	81	26	71	1.2916	62	1.6607	22
39	48	96	44	92	41	89	37	85	33	21
40	51	0.3702	53	0.7404	55	1.1105	56	1.4807	58	20
41	54	08	61	15	69	23	77	30	84	19
42	57	13	70	27	84	40	97	54	1.6710	18
43	60	19	78	38	98	57	1.3017	76	86	17
44	62	25	87	50	0.9312	74	37	99	62	16
45	65	30	96	61	26	91	56	1.4922	87	15
46	68	36	0.5604	72	41	1.1209	77	45	1.6813	14
47	71	42	13	84	55	26	97	68	39	13
48	74	48	21	95	69	43	1.3117	90	64	12
49	77	53	30	0.7507	84	60	37	1.5014	90	11
50	80	59	39	18	98	77	57	36	1.6916	10
51	82	65	47	30	0.9412	94	77	59	42	9
52	85	70	56	41	26	1.1311	96	82	67	8
53	88	76	64	52	41	29	1.3217	1.5105	93	7
54	91	82	73	64	55	46	37	28	1.7019	6
55	94	88	81	75	69	63	57	50	44	5
56	97	93	90	87	84	80	77	74	70	4
57	0.1900	99	99	98	98	97	97	96	96	3
58	02	0.3805	0.5707	10	0.9512	1.1414	1.3317	1.5219	1.7122	2
59	05	10	15	21	26	31	36	42	47	1
60	0.1908	0.3816	0.5724	0.7632	0.9541	1.1449	1.3357	1.5265	1.7173	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 79 DEGREES.										

96									
LATITUDE 11 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.9816	1.9633	2.9449	3.9265	4.9082	5.8898	6.8714	7.8530	8.8347
1	16	31	47	63	89	94	10	26	41
2	15	30	46	61	76	91	06	22	37
3	15	29	44	58	73	88	02	27	31
4	14	28	42	56	70	84	6.8698	12	26
5	14	27	41	54	88	81	95	08	22
6	13	26	39	52	65	77	90	.03	16
7	12	25	37	50	62	74	87	7.8599	12
8	12	24	35	47	59	71	83	94	06
9	11	22	34	45	56	67	78	90	01
10	11	21	32	43	54	64	75	86	8.8296
11	10	20	30	40	51	61	71	81	91
12	10	19	29	38	48	58	67	77	86
13	09	18	27	36	45	54	63	72	81
14	08	17	25	34	42	50	59	67	76
15	08	16	26	32	40	47	55	63	71
16	07	15	22	29	37	44	51	58	66
17	07	13	20	27	34	40	57	54	60
18	06	12	18	24	31	37	43	49	55
19	06	11	17	22	28	34	39	45	50
20	05	10	15	20	25	30	35	40	45
21	04	09	13	18	22	26	33	35	40
22	04	08	12	16	20	23	27	31	35
23	03	07	10	13	17	20	23	26	30
24	03	05	08	11	14	16	15	22	24
25	02	04	06	08	11	13	18	17	20
26	02	03	05	06	08	10	11	13	14
27	01	02	03	04	05	06	07	08	09
28	00	01	01	01	02	02	03	03	04
29	00	00	2.9399	3.9199	4.8999	5.8799	6.8599	7.8398	8.8198
30	0.9799	1.9598	2.9399	3.9199	4.8999	5.8799	6.8599	7.8398	8.8198
31	99	97	96	95	94	92	91	90	88
32	98	96	94	92	91	89	87	85	83
33	98	95	92	90	88	85	83	80	78
34	97	94	91	88	85	81	78	75	72
35	96	93	89	85	82	78	74	70	67
36	96	92	87	83	79	75	71	66	62
37	95	90	86	81	76	71	66	62	57
38	95	89	84	78	73	68	62	57	51
39	94	88	82	76	70	64	58	52	46
40	93	87	80	74	67	60	54	47	41
41	93	86	78	71	64	57	50	42	35
42	92	84	77	69	61	53	45	38	30
43	92	83	75	66	58	50	41	33	24
44	91	82	73	64	55	46	37	28	19
45	91	81	72	62	53	43	34	24	15
46	90	80	70	60	50	39	29	19	09
47	89	79	68	57	47	36	25	14	04
48	89	77	66	55	44	32	21	10	8.8098
49	88	76	64	52	41	39	17	05	93
50	88	75	63	50	38	25	13	00	88
51	87	74	61	48	35	22	08	7.8295	82
52	86	73	59	45	32	18	04	90	77
53	86	71	57	43	29	14	00	86	71
54	85	70	55	40	26	11	6.8496	81	66
55	85	69	54	38	23	07	92	76	61
56	84	68	52	36	20	03	87	71	55
57	83	67	50	33	17	00	83	66	50
58	83	65	48	31	14	5.8696	79	62	54
59	82	64	46	28	11	83	75	57	39
60	0.9782	1.9568	2.9345	3.9126	4.8908	5.8689	6.8471	7.8252	8.8034
1	2	3	4	5	6	7	8	9	10
DEPARTURE 78 DEGREES.									

DEPARTURE 11 DEGREES.										97
1	2	3	4	5	6	7	8	9	10	
0	0.1908	0.3816	0.5724	0.7632	0.9541	1.1449	1.3357	1.5265	1.7173	60
1	11	22	33	44	55	66	76	88	98	59
2	14	28	41	55	69	83	97	1.5310	1.7224	58
3	17	33	50	67	84	1.1500	1.3417	34	50	57
4	20	39	59	78	98	17	37	56	76	56
5	22	45	67	90	0.9612	34	57	79	7.7302	55
6	25	50	76	0.7701	26	51	76	1.5402	27	54
7	28	56	84	12	41	69	97	25	53	53
8	31	62	93	24	55	85	1.3516	47	78	52
9	34	68	0.5801	35	69	1.1603	37	70	1.7404	51
10	37	73	10	46	83	20	56	93	29	50
11	40	79	19	58	98	37	77	1.5516	56	49
12	42	85	27	69	0.9712	54	96	38	81	48
13	45	90	36	81	26	71	1.3616	62	1.7507	47
14	48	96	44	92	41	89	37	85	33	46
15	51	0.3902	53	0.7804	55	1.1705	57	1.5607	58	45
16	54	08	61	15	69	23	77	30	84	44
17	57	13	70	26	83	40	96	53	1.7609	43
18	60	19	79	38	98	57	1.3717	76	36	42
19	62	25	87	49	0.9812	74	39	98	61	41
20	65	30	96	61	26	91	56	1.5722	87	40
21	68	36	0.5904	72	40	1.1808	76	44	1.7712	39
22	71	42	13	84	55	25	96	67	38	38
23	74	47	21	95	69	42	1.3816	90	63	37
24	77	53	30	0.7906	83	60	36	1.5813	89	36
25	79	59	38	18	97	76	56	35	1.7815	35
26	82	65	47	29	0.9912	94	76	58	41	34
27	85	70	55	40	26	1.1911	96	81	67	33
28	88	76	64	52	40	28	1.3916	1.5904	92	32
29	91	82	72	63	54	45	36	26	1.7917	31
30	94	87	81	75	69	62	56	50	43	30
31	97	93	90	86	83	79	76	72	69	29
32	99	99	98	98	97	96	96	95	95	28
33	0.2002	0.4004	0.6007	0.8009	1.0011	1.2013	1.4016	1.6018	1.8020	27
34	05	10	15	20	26	31	36	41	46	26
35	08	16	24	32	40	47	56	63	71	25
36	11	22	32	43	54	65	76	86	97	24
37	14	27	41	54	68	82	96	1.6109	1.8122	23
38	17	33	50	66	83	99	1.4116	32	49	22
39	19	39	58	77	97	1.2116	35	55	74	21
40	22	44	67	89	1.0111	33	55	78	1.8200	20
41	25	50	75	90	25	50	75	90	26	19
42	28	56	84	0.8112	40	67	95	1.6223	51	18
43	31	61	92	23	54	84	1.4215	46	76	17
44	34	67	0.6101	34	17	1.2202	35	69	1.8302	16
45	36	73	09	46	32	18	55	92	28	15
46	39	79	18	57	97	36	75	1.6314	54	14
47	42	84	26	68	1.0211	53	95	37	79	13
48	45	90	35	80	25	70	1.4315	60	1.8405	12
49	48	96	43	91	39	87	35	82	30	11
50	51	0.4101	52	0.8203	54	1.2304	55	1.6406	56	10
51	54	07	61	14	68	21	75	28	82	9
52	56	13	69	25	82	38	94	50	1.8507	8
53	59	18	78	37	96	55	1.4414	74	33	7
54	62	24	86	48	1.0310	72	34	96	58	6
55	65	30	95	60	25	89	54	1.6519	84	5
56	68	35	0.6203	71	34	1.2406	74	42	1.8609	4
57	71	41	12	82	53	24	94	65	35	3
58	73	47	20	94	37	40	1.4514	87	61	2
59	76	53	29	0.8305	52	58	34	1.6610	87	1
60	0.2079	0.4158	0.6237	0.8316	1.0396	1.2475	1.4554	1.6633	1.8712	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 78 DEGREES.										

LATITUDE 12 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.9782	0.9563	2.9345	3.9126	4.8908	5.8689	6.8471	7.8252	8.8034	60
1	81	62	43	24	05	85	66	47	28	59
2	80	61	41	21	02	82	62	42	23	58
3	80	59	39	19	4.8899	78	58	38	17	57
4	79	58	37	16	95	75	54	33	12	56
5	78	57	35	14	92	71	49	27	06	55
6	78	56	33	11	89	67	45	22	00	54
7	77	54	32	09	86	63	40	18	8.7995	53
8	77	53	30	06	83	60	36	13	89	52
9	76	52	28	04	80	56	32	08	84	51
10	75	51	26	02	77	52	28	03	79	50
11	75	50	24	3.9099	74	49	24	7.8199	73	49
12	74	48	23	97	71	45	19	94	68	48
13	74	47	21	94	68	41	15	88	62	47
14	73	46	19	92	65	37	10	83	56	46
15	72	45	17	89	62	34	06	79	51	45
16	72	43	15	87	59	30	02	74	45	44
17	71	42	13	84	56	26	6.8398	69	40	43
18	71	41	12	82	53	23	94	64	35	42
19	70	40	10	79	49	19	89	59	28	41
20	69	38	08	77	46	15	84	54	23	40
21	69	37	06	74	43	11	80	49	17	39
22	68	36	04	72	40	08	76	44	12	38
23	67	35	02	69	37	04	71	38	06	37
24	67	33	00	67	34	00	67	34	00	36
25	66	32	2.9298	64	31	5.8597	63	29	8.7895	35
26	66	31	97	62	28	93	59	24	90	34
27	65	30	94	59	24	89	54	18	83	33
28	64	28	93	57	21	85	49	14	78	32
29	64	27	91	54	18	82	45	09	72	31
30	63	26	89	52	15	78	41	04	67	30
31	62	25	87	49	12	74	36	7.8098	61	29
32	62	23	85	47	09	70	32	94	55	28
33	61	22	83	44	05	67	28	89	50	27
34	60	21	81	42	02	62	23	83	44	26
35	60	20	79	39	4.8799	58	18	78	38	25
36	59	18	78	37	96	55	14	74	33	24
37	59	17	76	34	93	51	09	68	27	23
38	58	16	74	32	90	47	05	63	21	22
39	57	15	72	29	86	44	00	58	16	21
40	57	13	70	26	83	40	6.8296	53	09	20
41	56	12	68	24	80	36	92	47	04	19
42	55	11	66	21	77	32	87	42	8.7798	18
43	55	09	64	18	74	28	83	38	92	17
44	54	08	62	16	71	25	79	33	87	16
45	53	07	60	14	67	21	74	27	81	15
46	53	06	58	11	64	17	70	22	75	14
47	52	04	56	08	61	13	65	17	69	13
48	52	03	55	06	58	09	61	12	64	12
49	51	02	52	03	54	05	56	07	58	11
50	50	00	51	01	51	01	51	02	52	10
51	49	0.9499	49	3.8998	48	5.8498	47	7.7996	46	9
52	49	98	47	96	45	93	42	91	40	8
53	48	96	45	93	42	90	38	86	34	7
54	48	95	43	90	38	86	33	81	28	6
55	47	94	41	88	35	82	28	75	23	5
56	46	93	39	85	32	78	24	70	17	4
57	46	91	37	83	28	74	20	65	11	3
58	45	90	35	80	25	70	15	60	05	2
59	44	88	33	77	22	66	11	55	8.7699	1
60	0.9744	1.9487	2.9231	3.8975	4.8719	5.8462	6.8206	7.7950	8.7693	0
1	2	3	4	5	6	7	8	9	10	11
DEPARTURE 77 DEGREES.										

DEPARTURE 12 DEGREES.										99
	1	2	3	4	5	6	7	8	9	
0	0.2079	0.4158	0.6237	0.8316	1.0396	1.2475	1.4554	1.6633	1.8712	60
1	82	64	46	28	1.0410	92	74	56	38	59
2	85	70	54	39	24	1.2509	94	78	63	58
3	88	75	63	51	39	26	1.4614	1.6702	89	57
4	91	81	72	62	53	43	34	24	1.8815	56
5	93	87	80	73	67	60	53	46	40	55
6	96	92	89	85	81	77	73	70	66	54
7	99	98	97	96	95	94	93	92	91	53
8	0.2102	0.4204	0.6306	0.8408	1.0510	1.2611	1.4713	1.6815	1.8917	52
9	05	09	14	19	24	28	33	38	42	51
10	08	15	23	30	38	46	53	61	68	50
11	10	21	31	42	54	62	73	83	94	49
12	13	26	40	53	66	79	92	1.6906	1.9019	48
13	16	32	48	64	81	97	1.4813	29	45	47
14	19	38	57	76	95	1.2713	32	51	70	46
15	22	44	65	87	1.0609	31	53	74	96	45
16	25	49	74	98	23	48	72	97	1.9121	44
17	28	55	83	0.8510	38	65	92	1.7020	48	43
18	30	61	91	21	52	82	1.4912	42	73	42
19	33	66	99	32	66	99	32	65	98	41
20	36	72	0.6408	44	80	1.2816	52	88	1.9224	40
21	39	78	16	55	94	33	72	1.7110	49	39
22	42	83	25	67	1.0709	50	92	34	75	38
23	45	89	34	78	23	67	1.5012	56	1.9801	37
24	47	95	42	90	37	84	32	79	27	36
25	50	0.4300	51	0.8601	51	1.2901	51	1.7202	52	35
26	53	06	59	12	65	18	71	24	77	34
27	56	12	68	24	80	35	91	47	93	33
28	59	17	76	35	94	52	1.5111	70	1.9428	32
29	62	23	85	46	1.0808	70	31	93	54	31
30	64	29	93	58	22	86	51	1.7315	80	30
31	67	34	0.6502	69	36	1.3003	70	38	1.9505	29
32	70	40	10	80	51	21	91	61	31	28
33	73	46	19	92	65	37	10	83	56	27
34	76	52	29	0.8703	79	55	31	1.7406	82	26
35	79	57	36	14	93	72	50	29	1.9607	25
36	81	63	44	26	1.0907	88	70	51	33	24
37	84	69	53	37	23	1.3106	90	74	59	23
38	87	74	61	48	36	23	1.5310	97	84	22
39	90	80	70	60	50	39	29	1.7519	1.9709	21
40	93	86	78	71	64	57	50	42	35	20
41	96	91	87	82	78	74	69	65	60	19
42	99	97	96	94	93	91	90	88	87	18
43	0.2201	0.4403	0.6604	0.8805	1.1007	1.3208	1.5409	1.7610	1.9812	17
44	04	08	12	16	21	25	29	33	37	16
45	07	14	21	28	35	42	49	56	63	15
46	10	20	29	39	49	59	69	78	88	14
47	13	25	38	40	63	76	88	1.7701	1.9918	13
48	16	31	47	62	78	93	1.5509	24	40	12
49	18	37	55	73	92	1.3310	28	46	65	11
50	21	42	64	85	1.1106	27	48	70	91	10
51	24	48	72	96	20	44	68	92	2.0016	9
52	27	54	80	07	34	61	88	1.7814	41	8
53	30	59	89	0.8919	49	78	1.5608	38	67	7
54	33	65	98	30	63	95	28	60	93	6
55	35	71	0.6706	41	77	1.3412	47	82	2.0118	5
56	38	76	15	53	91	29	67	1.7906	44	4
57	41	82	23	64	1.1205	46	87	28	69	3
58	44	88	31	75	19	63	1.5707	50	94	2
59	47	93	40	87	34	80	27	74	2.0220	1
60	0.2250	0.4489	0.6749	0.8998	1.1248	1.3497	1.5747	1.7996	2.0246	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 77 DEGREES.										

100									
LATITUDE 13 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.9744	1.9487	2.9231	3.8975	4.8719	5.8462	6.8206	7.7950	8.7693
1	40	86	29	72	15	58	01	44	87
2	42	85	27	70	12	54	6.8197	39	82
3	42	83	25	67	09	50	92	34	75
4	41	82	23	64	06	47	88	29	70
5	40	81	21	62	02	42	83	23	64
6	40	80	19	59	4.8699	39	79	18	58
7	39	78	17	56	96	35	74	13	52
8	38	77	15	54	92	30	69	07	46
9	38	76	13	51	89	27	65	02	40
10	37	74	11	48	86	23	60	7.7897	34
11	36	73	09	46	82	18	55	91	28
12	36	72	07	43	79	15	51	86	22
13	35	70	05	40	76	11	46	81	16
14	35	69	04	38	73	07	42	76	11
15	34	68	01	35	69	03	37	70	04
16	33	66	2.9191	32	66	5.8399	32	65	08
17	33	65	98	30	63	95	28	60	8.7594
18	32	64	95	27	59	91	23	54	86
19	31	62	93	24	56	87	18	49	80
20	30	61	91	22	52	82	13	43	74
21	30	60	89	19	49	79	09	38	68
22	29	58	87	16	46	75	04	33	62
23	28	57	85	14	42	70	6.8099	27	56
24	28	56	83	11	39	67	95	22	50
25	27	54	81	08	36	63	90	17	44
26	26	53	78	06	32	58	85	11	38
27	26	51	77	03	29	54	80	06	32
28	25	50	75	00	26	51	76	01	26
29	24	49	73	3.8898	22	46	71	7.7795	19
30	24	47	71	94	19	42	66	90	13
31	23	46	69	92	15	38	61	84	07
32	22	45	67	89	12	34	56	78	01
33	22	42	65	87	09	30	52	74	8.7495
34	21	42	63	84	05	26	47	68	89
35	20	41	61	81	02	22	42	62	83
36	20	39	59	78	4.8598	18	37	57	76
37	19	38	57	76	95	13	32	51	70
38	18	36	55	73	91	09	27	46	64
39	18	35	53	70	88	06	23	41	58
40	17	34	51	68	85	01	18	35	52
41	16	32	49	65	81	5.8297	13	30	46
42	16	31	47	62	78	93	09	24	40
43	15	30	44	59	74	89	04	18	33
44	14	28	42	56	71	85	6.7999	13	27
45	13	27	40	54	67	80	94	07	21
46	13	25	38	51	64	76	89	02	14
47	12	24	36	48	60	72	84	7.7696	08
48	11	23	34	45	57	68	79	90	02
49	11	21	32	42	53	64	74	85	8.7395
50	10	20	30	40	50	60	70	80	90
51	09	19	28	37	46	56	65	74	84
52	09	17	26	34	43	52	60	69	77
53	08	16	24	32	40	47	55	63	71
54	07	15	22	29	36	43	50	58	65
55	07	13	20	26	33	39	46	52	59
56	06	12	19	23	29	35	41	46	52
57	05	10	15	20	26	31	36	41	46
58	04	09	13	18	22	26	31	35	40
59	04	07	11	15	19	22	26	30	33
60	0.9703	1.9406	2.9109	3.8812	4.8515	5.8218	6.7921	7.7624	8.7327
1	2	3	4	5	6	7	8	9	
DEPARTURE 76 DEGREES.									

DEPARTURE 13 DEGREES.									
									101
1.	1	2	3	4	5	6	7	8	9
0	0.2250	0.4499	0.6749	0.8998	1.1248	1.3497	1.5747	1.7996	2.0246
1	52	0.4505	57	0.9009	62	1.3514	66	1.8018	71
2	55	10	66	21	76	31	86	42	97
3	58	16	74	32	94	48	1.5806	64	2.0322
4	61	22	82	43	1.1304	65	26	86	47
5	64	27	91	55	19	82	46	1.8110	73
6	67	33	0.6800	66	33	99	66	32	99
7	69	39	08	77	46	1.3616	85	54	2.0424
8	72	44	17	89	61	33	1.5905	78	50
9	75	50	25	0.9100	75	50	25	1.8200	75
10	78	56	33	11	89	67	45	22	2.0500
11	81	61	42	23	1.1404	85	65	46	26
12	84	67	52	34	18	1.3701	85	67	52
13	86	73	69	45	32	18	1.6004	90	77
14	89	79	68	57	46	35	24	1.8314	2.0603
15	92	84	76	68	60	52	44	36	28
16	95	90	84	79	74	69	64	58	53
17	98	95	93	91	89	86	84	82	79
18	0.2301	0.4601	0.6902	0.9202	1.1503	1.3803	1.6104	1.8404	2.0705
19	03	07	10	13	17	20	23	26	30
20	06	12	19	25	31	37	43	50	56
21	09	18	27	36	45	54	63	72	81
22	12	24	35	47	59	71	83	94	2.0806
23	15	29	44	58	73	88	1.6206	1.8517	31
24	18	35	53	70	88	1.3905	23	40	58
25	20	41	61	81	1.1602	22	42	62	83
26	23	46	69	92	16	39	62	85	2.0908
27	26	52	78	0.9304	30	56	82	1.8608	34
28	29	58	86	15	44	73	1.6302	30	60
29	31	63	95	26	58	90	21	53	84
30	35	69	0.7004	38	73	1.4007	42	1.8676	2.1011
31	37	75	12	49	87	24	61	98	35
32	40	80	20	60	1.1701	41	81	1.8721	61
33	43	86	29	72	15	57	1.6400	43	86
34	46	92	37	83	29	75	21	66	2.1112
35	49	97	46	94	43	92	40	89	37
36	51	0.4703	54	0.9406	57	1.4108	60	1.8811	63
37	54	08	63	17	71	25	79	34	88
38	57	14	71	28	86	43	1.6500	57	2.1214
39	60	20	80	40	1.1800	59	19	79	39
40	63	25	88	51	14	76	39	1.8902	64
41	66	31	97	62	28	94	59	25	90
42	68	37	0.7105	74	42	1.4210	79	47	2.1316
43	71	42	14	85	56	27	98	70	41
44	74	48	22	96	70	44	1.6618	92	66
45	77	54	31	0.9508	85	61	38	1.9015	92
46	80	59	39	19	99	78	58	38	2.1417
47	83	65	48	30	1.1913	95	78	60	43
48	85	71	56	41	27	1.4312	97	82	68
49	88	76	65	53	41	29	1.6717	1.9106	94
50	91	82	73	64	55	46	37	28	2.1519
51	94	88	81	75	69	63	57	50	44
52	97	93	90	86	83	80	76	73	69
53	0.2400	99	99	98	1.2000	97	97	96	96
54	02	0.4805	0.7207	0.9609	12	1.4414	1.6816	1.9218	2.1621
55	05	10	15	20	26	31	36	41	46
56	08	16	24	32	40	47	55	63	71
57	11	22	32	43	54	65	76	86	97
58	14	27	41	54	68	82	95	1.9309	2.1722
59	16	33	49	65	82	98	1.6915	31	48
60	0.2419	0.4838	0.7258	0.9677	1.2096	1.4515	1.6934	1.9354	2.1773
	1	2	3	4	5	6	7	8	9
LATITUDE 76 DEGREES.									

102									
LATITUDE 14 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.9708	1.9406	2.9109	3.8812	4.8515	5.8218	6.7921	7.7624	8.7327
1	02	05	07	09	12	14	16	18	21
2	02	03	05	06	08	09	11	12	14
3	01	02	02	03	04	05	06	06	07
4	00	00	00	00	01	01	01	01	01
5	0.9699	1.9399	2.9098	3.8798	4.8497	5.8196	6.7896	7.7595	8.7295
6	99	97	96	95	94	92	91	90	88
7	98	96	94	92	90	88	86	84	82
8	97	95	92	89	87	84	81	78	76
9	97	93	90	86	83	80	76	73	69
10	96	92	88	84	80	75	71	67	63
11	95	90	86	81	76	71	66	62	57
12	95	89	84	78	73	67	62	56	51
13	94	87	81	75	69	62	56	50	43
14	93	86	79	72	65	58	51	44	37
15	92	85	77	69	62	54	46	39	31
16	92	83	75	66	58	50	41	33	24
17	91	82	73	64	55	45	36	27	18
18	90	80	71	61	51	41	31	22	12
19	89	79	68	58	47	36	26	15	05
20	89	77	66	55	44	32	21	10	8.7198
21	88	76	64	52	40	28	16	04	92
22	87	75	62	49	37	24	11	7.7498	86
23	87	73	60	46	33	20	06	93	79
24	86	72	57	43	29	15	01	86	72
25	85	70	55	41	26	11	6.7796	81	66
26	84	69	53	38	22	06	91	75	60
27	84	67	51	35	18	02	86	70	53
28	83	66	49	32	15	5.8097	80	63	46
29	82	64	47	29	11	93	75	58	40
30	82	63	45	26	08	89	71	52	34
31	81	61	42	23	04	84	65	46	27
32	80	60	40	20	00	80	60	40	20
33	79	59	38	17	4.8897	76	55	34	14
34	79	57	36	14	93	72	50	29	07
35	78	56	33	11	89	67	45	22	00
36	77	54	31	08	86	63	40	17	8.7094
37	76	53	29	06	82	58	35	11	88
38	76	51	27	02	78	54	29	05	80
39	75	50	25	3.8699	75	49	24	7.7399	74
40	74	48	23	97	71	45	19	94	68
41	73	47	20	95	67	40	14	87	61
42	73	45	18	91	64	36	09	82	54
43	72	44	16	88	60	31	03	75	47
44	71	42	14	85	56	27	6.7698	70	41
45	71	41	11	82	53	23	94	64	35
46	70	39	09	79	49	18	88	58	27
47	69	38	07	76	45	14	83	52	21
48	68	36	05	73	41	09	77	46	14
49	68	35	03	70	38	05	73	40	08
50	67	33	00	67	34	00	67	34	00
51	66	32	2.8998	64	30	5.7996	62	28	8.6994
52	65	31	96	61	27	92	57	22	88
53	65	39	94	58	24	87	52	16	81
54	64	28	91	55	19	83	47	10	74
55	63	26	89	52	15	78	41	04	67
56	62	25	87	49	12	74	36	7.7298	61
57	62	23	85	46	08	69	31	92	54
58	61	22	82	43	05	65	26	86	47
59	60	20	80	40	00	60	20	80	40
60	0.9659	1.9319	2.8978	3.8637	4.8297	5.7956	6.7615	7.7274	8.6934
1	2	3	4	5	6	7	8	9	
DEPARTURE 75 DEGREES.									

DEPARTURE 14 DEGREES.										108
1	2	3	4	5	6	7	8	9	10	
0	0.2419	0.4838	0.7258	0.9677	1.2096	1.4515	1.6934	1.9354	2.1773	60
1	22	44	66	88	1.2110	32	54	76	98	59
2	25	50	75	0.9700	25	49	74	99	2.1824	58
3	28	55	83	11	39	66	94	1.9422	49	57
4	31	61	92	22	53	83	1.7014	44	75	56
5	33	67	0.7300	33	67	1.4600	33	66	2.1900	55
6	36	72	09	45	81	17	53	90	26	54
7	39	78	17	56	95	34	73	1.9512	51	53
8	42	84	25	67	1.2209	51	93	34	76	52
9	45	89	34	78	33	68	1.7112	57	2.2001	51
10	47	95	42	90	37	84	32	79	27	50
11	50	0.4901	51	0.9801	52	7.4702	52	1.9602	53	49
12	53	06	59	12	66	19	72	25	78	48
13	56	12	68	24	80	35	91	47	2.2103	47
14	59	17	76	35	94	52	1.7211	70	28	46
15	62	23	85	46	1.2308	69	31	92	54	45
16	64	29	93	58	22	86	51	1.9715	80	44
17	67	34	0.7402	69	36	1.4803	70	38	2.2205	43
18	70	40	10	80	50	20	90	60	30	42
19	73	46	18	91	64	37	1.7310	82	55	41
20	76	51	27	0.9902	78	54	29	1.9805	80	40
21	78	57	35	14	92	70	49	27	2.2306	39
22	81	63	44	25	1.2407	88	69	50	32	38
23	84	68	52	36	21	1.4905	89	73	57	37
24	87	74	61	47	35	21	1.7408	95	82	36
25	90	79	69	59	49	38	29	1.9918	2.2407	35
26	93	85	78	70	63	55	48	40	33	34
27	95	91	86	81	77	72	67	62	58	33
28	98	96	95	93	91	89	87	86	84	32
29	0.2501	0.5002	0.7503	1.0004	1.2505	1.5006	1.7507	2.0008	2.2509	31
30	04	08	11	15	19	23	27	30	34	30
31	07	13	20	26	33	40	46	53	59	29
32	09	19	28	38	47	56	66	75	85	28
33	12	24	37	49	61	73	85	98	2.2610	27
34	15	30	45	60	76	91	1.7606	2.0121	36	26
35	18	36	54	72	90	1.5107	25	43	61	25
36	21	41	62	83	1.2604	24	45	66	86	24
37	24	47	71	94	18	41	65	88	2.2712	23
38	26	53	79	1.0105	32	58	84	2.0210	37	22
39	29	58	87	16	46	75	1.7704	33	62	22
40	32	64	96	28	60	92	24	56	88	20
41	35	70	0.7604	39	74	1.5209	44	78	2.2813	19
42	38	75	13	50	88	26	63	2.0301	38	18
43	40	81	21	62	1.2702	42	83	23	64	17
44	43	86	30	73	16	59	1.7802	46	89	16
45	46	92	38	84	30	76	22	68	2.2914	15
46	49	98	46	95	41	93	42	90	39	14
47	52	0.5103	55	1.0206	58	1.5310	61	2.0413	64	13
48	55	09	64	18	73	27	82	36	91	12
49	57	15	72	29	87	44	1.7901	58	2.3016	11
50	60	20	80	40	1.2801	61	21	81	41	10
51	63	26	89	52	15	77	40	2.0503	66	9
52	66	31	97	63	29	94	60	26	91	8
53	69	37	0.7706	74	43	1.5411	80	48	2.3117	7
54	71	43	14	85	57	28	99	70	42	6
55	74	48	22	96	71	45	1.8019	93	67	5
56	77	54	31	1.0308	85	61	38	2.0615	92	4
57	80	60	39	19	99	79	59	38	2.3218	3
58	83	65	48	30	1.2913	96	78	61	43	2
59	85	71	56	42	27	1.5512	98	83	69	1
60	0.2588	0.5176	0.7765	1.0353	1.2941	1.5529	1.8117	2.0706	2.3294	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 75 DEGREES.										

104 LATITUDE 15 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.9659	1.9319	2.8978	3.8637	4.8297	5.7956	6.7615	7.7274	8.6934	60
1	59	17	86	34	93	51	10	68	27	59
2	58	16	73	31	89	47	05	62	20	58
3	57	14	71	28	85	42	6.7599	56	13	57
4	56	12	69	25	81	37	93	50	06	56
5	56	11	67	22	76	33	88	44	00	55
6	55	08	64	19	74	28	83	38	8.6892	54
7	54	07	62	16	70	24	78	32	86	53
8	53	06	60	13	66	19	72	26	79	52
9	52	05	57	10	62	14	67	19	72	51
10	52	03	55	07	59	10	62	14	65	50
11	51	02	53	04	55	05	56	07	58	49
12	50	00	51	01	51	01	51	02	52	48
13	49	1.9399	48	3.8598	47	5.7896	46	7.7195	45	47
14	49	97	46	94	43	92	40	89	37	46
15	48	96	44	92	40	87	35	83	31	45
16	47	94	41	88	36	83	30	77	24	44
17	46	93	39	85	32	78	24	70	17	43
18	46	91	37	82	28	74	19	65	10	42
19	45	90	34	79	24	69	14	58	03	41
20	44	88	32	76	20	64	08	52	8.6796	40
21	43	87	30	73	17	60	03	46	90	39
22	43	85	28	70	13	55	6.7498	40	83	38
23	42	83	25	67	09	50	92	34	75	37
24	41	82	23	64	05	46	87	28	69	36
25	40	80	21	61	01	41	81	22	62	35
26	39	79	18	58	4.8197	36	76	15	55	34
27	39	77	16	54	93	32	70	09	47	33
28	38	76	14	52	90	27	65	03	41	32
29	37	74	11	48	86	23	60	7.7096	34	31
30	36	73	09	45	82	18	54	90	27	30
31	36	71	06	42	78	13	48	84	20	29
32	35	69	04	39	74	08	43	78	12	28
33	34	68	02	36	70	04	38	72	06	27
34	33	66	00	33	66	5.7799	32	66	8.6699	26
35	32	65	2.8897	30	62	94	27	59	92	25
36	32	63	95	26	58	90	21	53	84	24
37	31	62	92	23	54	85	16	46	78	23
38	30	60	90	20	51	81	11	41	71	22
39	29	59	88	17	47	76	05	34	64	21
40	29	57	86	14	43	71	00	28	57	20
41	28	55	83	11	39	66	6.7394	22	49	19
42	27	54	81	08	35	61	88	15	42	18
43	26	52	78	04	31	57	83	09	35	17
44	25	51	76	01	27	52	77	02	28	16
45	25	49	73	3.8498	23	48	72	7.6997	21	15
46	24	48	71	95	19	43	67	90	14	14
47	23	46	69	92	15	38	61	84	07	13
48	22	44	67	89	11	33	55	78	00	12
49	21	43	64	86	07	28	50	71	8.6593	11
50	21	41	62	82	03	24	44	65	85	10
51	20	40	59	79	4.8099	19	39	58	78	9
52	19	38	57	76	95	14	33	52	71	8
53	18	36	55	73	91	09	27	46	64	7
54	17	35	52	70	87	04	22	39	57	6
55	16	33	50	66	83	00	16	33	49	5
56	16	32	47	63	79	5.7695	11	26	42	4
57	15	30	45	60	75	90	05	20	35	3
58	14	28	43	57	71	85	6.7299	14	28	2
59	13	27	40	53	67	80	94	07	21	1
60	0.9613	1.9225	2.8838	3.8450	4.8063	5.7676	6.7288	7.6901	8.6513	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 74 DEGREES.										

DEPARTURE 15 DEGREES.										105
1	2	3	4	5	6	7	8	9	10	
0	0.2588	0.5176	0.7765	1.0353	1.2941	1.5529	1.8117	2.0706	2.3294	60
1	91	82	73	64	55	46	37	28	2.3319	59
2	94	88	81	75	69	63	57	50	44	58
3	97	93	90	86	83	80	76	73	69	57
4	99	99	98	98	97	96	96	95	95	56
5	0.2602	0.5204	0.7807	1.0409	1.3011	1.5613	1.8215	2.0818	2.3420	55
6	05	10	15	20	25	30	35	40	45	54
7	08	16	24	32	40	47	55	63	71	53
8	11	21	32	43	54	64	75	86	96	52
9	14	27	42	54	68	81	95	2.0908	2.3522	51
10	16	33	49	65	82	98	1.8314	30	47	50
11	19	38	57	76	95	1.5715	34	53	72	49
12	22	44	66	88	1.3110	31	53	75	97	48
13	25	49	74	99	24	48	73	98	2.3622	47
14	28	55	83	1.0510	38	65	93	2.1020	48	46
15	30	61	91	21	52	82	1.8412	42	73	45
16	33	66	99	32	66	99	32	65	98	44
17	36	72	0.7908	44	80	1.5815	51	87	2.3723	43
18	39	77	16	55	94	32	71	2.1110	48	42
19	42	83	25	66	1.3208	49	91	32	74	41
20	44	89	33	77	22	66	1.8510	54	99	40
21	47	94	41	88	36	83	30	77	2.3824	39
22	50	0.5300	50	1.0600	50	1.5900	50	2.1200	50	38
23	53	07	58	11	64	17	70	22	75	37
24	55	11	67	22	78	34	89	45	2.3900	36
25	58	18	75	34	92	50	1.8609	67	26	35
26	61	22	84	45	1.3306	67	28	90	51	34
27	66	28	92	56	20	84	48	2.1312	76	33
28	67	34	0.8000	67	34	1.6001	68	34	2.4001	32
29	70	39	09	78	48	18	87	57	26	31
30	72	45	17	90	62	34	1.8707	79	52	30
31	75	50	26	1.0701	76	51	26	2.1402	77	29
32	78	56	34	12	90	68	46	24	2.4102	28
33	81	62	42	23	1.3404	85	66	46	27	27
34	84	67	51	34	18	1.6102	85	69	52	26
35	86	73	59	46	32	18	1.8805	91	78	25
36	89	78	68	57	46	35	24	2.1514	2.4203	24
37	92	84	76	68	60	52	44	36	28	23
38	95	90	84	79	74	69	64	58	53	22
39	98	95	93	90	78	86	83	81	78	21
40	0.2700	0.5401	0.8101	1.0802	1.3502	1.6202	1.8903	2.1603	2.4304	20
41	03	06	10	13	16	19	22	26	29	19
42	06	12	18	24	30	36	42	48	54	18
43	09	18	26	35	44	53	62	70	79	17
44	12	23	35	46	58	70	81	93	2.4404	16
45	14	29	43	58	72	86	1.9001	2.1715	30	15
46	17	34	52	69	86	1.6303	20	38	55	14
47	20	40	60	80	1.3600	20	40	60	80	13
48	23	46	68	91	14	37	60	82	2.4505	12
49	26	51	77	1.0902	28	54	79	2.1805	30	11
50	28	57	85	14	42	70	99	27	56	10
51	31	62	94	25	56	87	1.9118	50	81	9
52	34	68	0.8202	36	70	1.6404	38	72	2.4606	8
53	37	74	10	47	84	21	58	94	31	7
54	40	79	19	58	98	38	77	2.1917	56	6
55	42	85	27	70	1.3712	54	97	39	82	5
56	45	90	36	81	26	71	1.9216	62	2.4707	4
57	48	96	44	92	44	88	36	84	32	3
58	51	0.5502	52	1.1003	54	1.6505	56	2.2006	57	2
59	54	07	61	14	68	22	75	29	82	1
60	0.2756	0.5513	0.8269	1.1026	1.3782	1.6538	1.9295	2.2051	2.4808	0
1	2	3	4	5	6	7	8	9		
LATITUDE 74 DEGREES.										

106 LATITUDE 16 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.9613	1.9225	2.8838	3.8450	4.8063	5.7676	6.7288	7.6901	8.6513	60
1	12	24	35	47	59	71	83	7.6894	06	59
2	11	22	33	44	55	66	77	89	8.6499	58
3	10	20	31	41	51	61	71	82	92	57
4	09	19	28	38	47	56	66	75	85	56
5	09	17	26	34	43	52	60	69	77	55
6	08	16	23	31	39	47	55	62	70	54
7	07	14	21	28	35	42	49	56	63	53
8	06	12	19	25	31	37	43	50	56	52
9	05	11	16	22	27	32	38	43	49	51
10	05	09	14	18	23	28	32	37	41	50
11	04	07	11	15	19	22	26	30	33	49
12	03	06	09	12	15	17	20	23	26	48
13	02	04	06	08	11	13	15	17	19	47
14	01	03	04	05	07	08	09	10	12	46
15	00	01	02	02	03	03	04	04	05	45
16	00	1.9199	2.8799	3.8399	4.7999	5.7598	6.7198	7.6798	8.6397	44
17	0.9599	98	97	96	95	93	92	91	90	43
18	98	96	94	92	91	89	87	85	83	42
19	97	94	92	89	86	83	80	78	75	41
20	96	93	89	87	82	78	75	71	68	40
21	96	91	87	83	78	74	69	65	60	39
22	95	90	84	79	74	69	64	58	53	38
23	94	88	82	76	70	64	58	52	46	37
24	93	86	79	72	66	59	52	45	38	36
25	92	85	77	69	62	54	46	38	31	35
26	92	83	75	66	58	49	41	32	24	34
27	91	81	72	62	54	44	35	26	16	33
28	90	80	69	59	49	39	29	18	08	32
29	89	78	67	56	45	34	23	12	01	31
30	88	76	65	53	41	29	17	06	8.6294	30
31	87	75	62	50	37	24	11	7.6699	87	29
32	87	73	60	46	33	19	06	92	79	28
33	86	71	57	43	29	14	00	86	71	27
34	85	70	55	40	25	09	6.7094	79	64	26
35	84	68	52	36	20	05	89	73	57	25
36	83	66	50	33	16	5.7499	82	66	49	24
37	82	65	47	30	12	94	77	59	41	23
38	82	63	45	26	08	90	71	53	34	22
39	81	61	42	23	04	84	65	46	26	21
40	80	60	40	20	00	79	59	39	19	20
41	79	58	37	16	4.7895	75	54	33	11	19
42	78	56	35	13	91	69	47	26	04	18
43	77	55	32	10	87	64	42	19	8.6197	17
44	76	53	30	06	83	60	36	13	89	16
45	76	51	27	03	79	54	30	06	81	15
46	75	50	25	00	75	49	24	7.6599	74	14
47	74	48	22	3.8296	70	44	18	92	66	13
48	73	46	20	99	66	39	12	86	59	12
49	72	45	17	90	62	34	07	79	52	11
50	72	43	15	86	58	29	01	72	44	10
51	71	41	12	83	54	24	6.6995	66	36	9
52	70	40	09	79	49	19	89	58	28	8
53	69	38	07	76	45	14	83	52	21	7
54	68	36	04	72	41	09	77	45	13	6
55	67	34	02	69	36	03	70	38	05	5
56	66	33	2.8699	66	32	5.7398	65	31	8.6098	4
57	65	31	97	62	28	93	59	24	90	3
58	65	29	94	59	24	88	53	18	82	2
59	64	28	91	55	19	83	47	10	74	1
60	0.9563	1.9126	2.8689	3.8252	4.7815	5.7378	6.6941	7.6504	8.6067	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 73 DEGREES.										

DEPARTURE 16 DEGREES.										107
1	2	3	4	5	6	7	8	9	10	
0	0.2756	0.5513	0.8269	1.1026	1.3782	1.6538	1.9295	2.2051	2.4808	60
1	59	18	78	37	96	55	1.9314	74	33	59
2	62	24	86	48	1.3810	72	34	96	58	58
3	65	30	94	59	24	89	54	2.2118	38	57
4	68	35	0.8303	70	38	1.6606	73	41	2.4908	56
5	70	41	11	82	52	22	93	63	34	55
6	73	46	19	92	66	39	1.9412	85	58	54
7	76	52	28	1.1104	80	55	31	2.2207	83	53
8	79	57	36	15	94	72	51	30	2.5008	52
9	82	63	45	26	1.8908	89	71	52	34	51
10	84	69	53	37	22	1.6706	90	74	59	50
11	87	74	61	48	36	23	1.9510	97	84	49
12	90	80	70	60	50	39	29	2.2319	2.5109	48
13	93	85	78	71	64	56	49	32	34	47
14	96	91	87	82	78	73	69	64	60	46
15	98	97	95	92	94	88	86	84	82	45
16	0.2801	0.5602	0.8403	1.1204	1.4006	1.6807	1.9608	2.2409	2.5210	44
17	04	08	12	16	30	23	27	31	35	43
18	07	13	20	27	34	40	47	54	60	42
19	10	19	29	38	48	57	67	76	86	41
20	12	25	37	49	62	74	86	98	2.5311	40
21	15	30	45	60	75	90	1.9705	2.2510	35	39
22	18	36	53	71	89	1.6907	25	42	60	38
23	21	41	62	82	1.4103	24	44	65	86	37
24	23	47	70	94	17	40	64	87	2.5411	36
25	26	52	79	1.1305	81	57	83	2.2600	36	35
26	29	58	87	16	45	74	1.9803	32	61	34
27	32	64	95	27	59	91	23	54	86	33
28	35	69	0.8504	38	73	1.7008	42	77	2.5511	32
29	37	75	12	50	87	25	62	99	37	31
30	40	80	21	61	1.4201	41	81	2.2722	62	30
31	43	86	29	72	15	58	1.9900	43	86	29
32	46	91	37	83	29	75	20	66	2.5611	28
33	49	97	46	94	43	91	40	88	87	27
34	51	0.5703	54	1.1405	57	1.7108	59	2.2810	62	26
35	54	08	62	16	71	25	79	33	87	25
36	57	14	71	28	85	41	98	55	2.5712	24
37	60	19	79	39	99	58	2.0018	78	37	23
38	63	25	88	50	1.4313	75	38	2.2900	63	22
39	65	31	96	61	26	91	56	22	87	22
40	68	36	0.8604	72	40	1.7208	76	44	2.5812	20
41	71	42	12	83	54	25	96	66	37	19
42	74	47	21	94	68	42	2.0115	89	62	18
43	77	53	29	1.1508	82	61	38	2.3015	82	17
44	79	58	38	17	96	75	54	34	2.5913	16
45	82	64	46	28	1.4420	92	74	56	88	15
46	85	69	54	39	24	1.7308	93	78	62	14
47	88	75	63	50	38	25	2.0213	2.3100	88	13
48	90	81	71	61	52	42	32	22	2.6013	12
49	93	86	79	72	66	59	52	45	38	11
50	96	92	87	84	80	75	71	67	63	10
51	99	97	96	95	94	92	91	90	88	9
52	0.2902	0.5803	0.8705	1.1606	1.4508	1.7409	2.0311	2.3212	2.6114	8
53	04	09	13	17	21	25	29	34	38	7
54	07	14	21	28	35	42	49	56	63	6
55	10	20	29	39	49	59	69	78	88	5
56	13	25	38	50	63	76	88	2.3301	2.6213	4
57	15	31	46	62	77	92	2.0408	23	39	3
58	18	36	55	73	91	1.7509	27	46	64	2
59	21	42	63	84	1.4605	25	46	67	88	1
60	0.2924	0.5847	0.8771	1.1695	1.4619	1.7542	2.0466	2.3390	2.6318	0
1	2	3	4	5	6	7	8	9		
LATITUDE 73 DEGREES.										

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LATITUDE 17 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.9563	1.9126	2.8689	3.8652	4.7815	5.7378	6.6941	7.6504	8.6067
1	62	24	87	49	11	73	35	7.6498	60 59
2	61	23	84	45	07	68	29	90	52 58
3	61	21	82	41	03	63	24	84	45 57
4	60	19	79	38	4.7798	58	17	77	36 56
5	59	18	76	35	94	53	12	70	29 55
6	58	16	74	32	90	47	05	63	21 54
7	57	14	71	28	86	43	00	57	14 53
8	56	12	69	25	81	36	6.6893	49	06 52
9	55	11	66	22	77	32	88	43	8.5999
10	55	09	64	18	73	27	82	36	91 50
11	54	07	61	14	68	22	75	29	82 49
12	53	06	58	11	64	17	70	22	75 48
13	52	04	56	08	60	11	63	15	67 47
14	51	02	53	04	56	07	58	09	60 46
15	50	00	51	01	51	01	51	01	52 45
16	49	1.9099	48	3.8197	47	5.7296	45	7.6394	44 44
17	49	97	46	94	43	91	40	88	37 43
18	48	95	43	90	38	86	33	81	28 42
19	47	93	40	87	34	80	27	74	20 41
20	46	92	38	84	30	75	21	67	13 40
21	45	90	35	80	25	70	15	60	05 39
22	44	88	32	76	21	65	09	53	8.5897
23	43	86	30	73	17	60	03	46	90 37
24	42	85	27	70	12	54	6.6797	39	82 36
25	42	83	25	66	08	49	91	32	74 35
26	41	81	22	63	04	44	85	25	66 34
27	40	79	19	59	4.7699	39	79	18	58 33
28	39	78	17	56	95	33	72	11	50 32
29	38	76	14	52	90	28	66	04	42 31
30	37	74	12	49	86	23	60	7.6297	35 30
31	36	73	09	45	82	18	54	90	27 29
32	35	71	06	42	77	12	48	83	19 28
33	35	69	04	38	73	07	42	76	11 27
34	34	67	01	35	69	02	36	69	03 26
35	33	66	2.8598	31	64	5.7197	30	62	8.5795
36	32	64	96	28	60	91	23	55	87 24
37	31	62	93	24	55	86	17	48	79 23
38	30	60	90	20	51	81	11	41	71 22
39	29	59	88	17	47	76	05	34	64 21
40	28	57	85	14	42	70	6.6699	27	56 20
41	28	55	83	10	38	65	93	20	48 19
42	27	53	80	06	33	60	86	13	39 18
43	26	51	77	03	29	54	80	05	31 17
44	25	50	74	3.8099	24	49	74	7.6198	23 16
45	24	48	72	96	20	44	68	92	16 15
46	23	46	69	92	16	39	62	85	08 14
47	22	44	67	89	11	32	55	77	00 13
48	21	43	64	85	07	28	49	70	8.5692
49	20	41	61	82	02	22	43	63	84 11
50	20	39	59	78	4.7598	17	37	56	76 10
51	19	37	56	74	93	12	30	49	67 9
52	18	35	53	71	89	06	24	41	59 8
53	17	32	50	67	84	01	18	34	51 7
54	16	34	48	64	80	5.7095	11	27	43 6
55	15	30	45	60	75	90	05	20	35 5
56	14	28	43	57	71	85	6.6599	13	28 4
57	13	27	40	53	67	80	93	06	20 3
58	12	25	37	50	62	74	87	7.6099	12 2
59	12	23	35	46	58	69	81	92	04 1
60	0.9511	1.9021	2.8532	3.8042	4.7553	5.7064	6.6574	7.6085	8.5595
	1	2	3	4	5	6	7	8	9
DEPARTURE 72 DEGREES.									

DEPARTURE 17 DEGREES.										109
1	2	3	4	5	6	7	8	9	10	
0	0.2924	0.5847	0.8771	1.1695	1.4619	1.7542	2.0466	2.3390	2.6313	06
1	27	53	80	1.1706	33	59	86	2.3412	39	59
2	29	59	88	17	47	76	2.0505	84	64	58
3	32	64	96	28	61	93	25	57	89	57
4	35	70	0.8804	39	74	1.7609	44	78	2.6413	56
5	38	75	13	50	88	26	63	2.3501	38	55
6	40	81	21	62	1.4702	42	83	23	64	54
7	43	86	30	73	16	59	2.0602	46	89	53
8	46	92	38	84	30	76	22	68	2.6514	52
9	49	97	46	95	44	92	41	90	38	51
10	52	0.5903	55	1.1806	58	1.7709	61	2.3612	64	50
11	54	09	63	17	72	26	80	34	2.6609	49
12	57	14	71	28	86	43	2.0700	57	14	48
13	60	20	80	40	1.4800	59	19	79	39	47
14	63	25	88	50	13	76	38	2.3701	63	46
15	65	31	96	62	27	92	58	23	89	45
18	68	36	0.8905	73	41	1.7809	77	46	2.6714	44
17	71	42	13	84	55	26	97	68	39	43
18	74	47	21	95	69	42	2.0816	90	63	42
19	77	53	30	1.1906	83	59	36	2.3812	89	41
20	79	59	38	17	97	76	55	34	2.6814	40
21	82	64	46	28	1.4911	93	75	57	39	39
22	85	70	55	40	25	1.7909	94	79	64	38
23	88	75	63	50	38	26	2.0913	2.3901	88	37
24	90	81	71	62	52	40	33	23	2.6914	36
25	93	86	80	73	66	59	52	46	39	35
26	96	92	88	84	80	76	72	68	64	34
27	99	97	96	95	94	92	91	90	88	33
28	0.3002	0.6003	0.9005	1.2006	1.5008	1.8009	2.1011	2.4012	2.7014	32
29	04	09	13	17	22	26	30	34	39	31
30	07	14	21	28	34	43	50	57	64	30
31	10	20	29	39	49	59	69	78	88	29
32	13	25	38	50	63	76	88	2.4101	2.7113	28
33	15	31	46	68	77	92	2.1108	23	39	27
34	18	36	55	73	91	1.8109	27	46	64	26
35	21	42	63	84	1.5105	25	46	67	88	25
36	24	47	71	95	19	42	66	90	2.7213	24
37	27	53	80	1.2106	33	59	86	2.4212	39	23
38	29	58	88	17	46	75	2.1204	34	63	22
39	32	64	96	28	60	92	24	56	88	21
40	35	70	0.9104	39	74	1.8209	44	78	2.7313	20
41	38	75	13	50	88	26	63	91	38	19
42	40	81	21	61	1.5202	42	82	2.4322	63	18
43	43	86	29	72	16	59	2.1302	45	88	17
44	46	92	38	84	30	75	21	67	2.7413	16
45	49	97	46	94	43	92	40	89	37	15
46	51	0.6103	54	1.2206	57	1.8308	60	2.4411	64	14
47	54	08	63	17	71	25	79	34	88	13
48	57	14	71	28	85	42	99	56	2.7513	12
49	60	19	79	39	99	58	2.1418	78	37	11
50	63	25	88	50	1.5313	75	38	2.4500	63	10
51	65	31	96	61	27	92	57	22	87	9
52	68	36	0.9204	72	40	1.8408	76	44	2.7612	8
53	71	42	12	83	54	25	96	66	87	7
54	74	47	21	94	68	42	2.1515	89	62	6
55	76	53	29	1.2305	82	58	34	2.4610	87	5
56	79	58	37	16	96	75	54	13	2.7712	4
57	82	64	46	28	1.5410	91	73	35	37	3
58	85	69	54	38	23	1.8508	92	77	61	2
59	87	75	62	50	47	24	2.1612	99	87	1
60	0.3090	0.6180	0.9271	1.2361	1.5451	1.8541	2.1631	1.4722	2.7812	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 72 DEGREES.										

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LATITUDE 18 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9511	1.9021	2.8532	3.8042	4.7553	5.7064	6.6574	7.6085	8.5595	60
1	10	19	29	39	49	58	68	78	87	59
2	09	18	26	35	44	53	62	70	79	58
3	08	16	24	32	40	47	55	63	71	57
4	07	14	21	28	35	42	49	56	63	56
5	06	12	18	24	31	37	43	49	55	55
6	05	10	16	21	26	31	36	42	47	54
7	04	09	13	17	22	26	30	34	39	53
8	03	07	10	13	17	20	23	26	30	52
9	02	05	07	10	12	14	17	19	22	51
10	02	03	05	06	08	09	11	12	14	50
11	01	01	02	02	03	04	04	05	05	49
12	00	1.3999	2.8499	3.7999	4.7499	5.6998	6.6498	7.5998	8.5497	48
13	0.9499	98	96	95	94	93	92	90	89	47
14	98	96	94	92	90	87	86	83	81	46
15	97	94	91	88	85	82	79	76	73	45
16	96	92	88	84	81	77	73	69	65	44
17	95	90	86	81	76	71	66	62	57	43
18	94	89	83	77	72	66	60	54	49	42
19	93	87	80	73	67	60	53	46	40	41
20	92	85	77	70	62	56	47	39	32	40
21	92	83	75	66	58	49	41	32	23	39
22	91	81	72	62	53	44	34	25	15	38
23	90	79	69	59	49	38	28	18	07	37
24	89	78	66	55	44	33	22	10	8.5399	36
25	88	76	63	51	39	27	15	02	90	35
26	87	74	61	48	35	21	08	7.5895	82	34
27	86	72	58	44	30	16	02	88	74	33
28	85	70	55	40	26	11	6.6396	81	66	32
29	84	68	53	37	21	05	89	74	58	31
30	83	66	50	33	16	5.6899	82	66	49	30
31	82	65	47	29	12	94	76	58	41	29
32	81	63	44	26	07	88	70	51	33	28
33	81	61	42	22	03	83	64	44	25	27
34	80	59	39	18	4.7398	77	57	36	16	26
35	79	57	36	14	93	72	50	29	07	25
36	78	55	33	11	89	66	44	22	8.5299	24
37	77	54	30	07	84	61	38	14	91	23
38	76	52	27	03	79	55	31	06	82	22
39	75	50	25	00	75	49	24	7.5799	74	21
40	74	48	22	3.7896	70	44	18	92	66	20
41	73	46	19	92	65	38	11	84	57	19
42	72	44	16	88	61	33	05	77	49	18
43	71	42	14	85	56	27	98	70	41	17
44	70	40	11	81	51	21	6.6291	62	32	16
45	69	39	08	77	47	16	85	54	24	15
46	68	37	05	74	42	10	79	47	16	14
47	67	35	02	70	37	04	72	39	07	13
48	67	33	00	66	33	5.6799	66	32	8.5199	12
49	66	31	2.8397	62	28	94	59	25	90	11
50	65	29	94	58	23	88	52	17	81	10
51	64	27	91	55	19	82	46	10	73	9
52	63	25	88	51	14	76	39	02	64	8
53	62	24	85	47	09	71	33	7.5694	56	7
54	61	22	83	44	05	65	26	87	48	6
55	60	20	80	40	00	59	19	79	39	5
56	59	18	77	36	4.7295	54	13	72	31	4
57	58	16	74	32	90	48	06	64	22	3
58	57	14	71	28	86	43	00	57	14	2
59	56	12	68	24	81	37	6.6193	49	05	1
60	0.9455	1.8910	2.8366	3.7821	4.7276	5.6731	6.6186	7.5642	8.5097	0

DEPARTURE 71 DEGREES.

DEPARTURE 18 DEGREES.										111
	1	2	3	4	5	6	7	8	9	
0	0.8090	0.6180	0.9271	1.2381	1.5451	1.8541	2.1631	2.4722	2.7812	60
1	93	86	79	72	65	57	50	43	36	59
2	96	91	87	83	79	74	70	66	61	58
3	99	97	96	94	93	91	90	88	87	57
4	0.8101	0.6202	0.9304	1.2405	1.5506	1.8607	2.1708	2.4810	2.7911	56
5	04	08	12	16	20	24	28	32	36	55
6	07	14	20	27	34	41	48	54	61	54
7	10	19	29	38	48	57	67	76	86	53
8	12	25	37	49	62	74	86	98	2.8011	52
9	15	30	45	60	76	91	2.1806	2.4921	36	51
10	18	36	53	71	89	1.8707	24	42	60	50
11	21	41	62	82	1.5603	24	44	65	85	49
12	23	47	70	93	17	40	63	86	2.8110	48
13	26	52	78	1.2504	31	57	83	2.5009	35	47
14	29	58	87	16	45	73	2.1902	31	60	46
15	32	63	95	26	58	90	21	53	84	45
16	34	69	0.9403	38	72	1.8806	41	75	2.8210	44
17	37	74	12	49	86	23	60	98	35	43
18	40	80	20	60	1.5700	39	79	2.5119	59	42
19	43	85	28	71	14	56	99	42	84	41
20	45	91	36	82	27	72	2.2018	63	2.8309	40
21	48	96	45	93	41	89	37	86	34	39
22	51	0.6302	53	1.2604	55	1.8906	57	2.5208	59	38
23	54	07	61	15	69	22	76	30	83	37
24	57	13	70	26	83	39	96	52	2.8409	36
25	59	19	78	37	97	56	2.2115	74	34	35
26	62	24	86	48	1.5810	72	34	96	59	34
27	65	30	94	59	24	89	54	2.5318	83	33
28	68	35	0.9503	70	38	1.9005	73	40	2.8508	32
29	70	41	11	81	52	22	92	62	33	31
30	73	46	19	92	65	38	2.2211	84	57	30
31	76	52	27	1.2703	79	55	31	2.5406	82	29
32	79	57	36	14	93	72	50	29	2.8607	28
33	81	63	44	25	1.5907	88	69	50	32	27
34	84	68	52	36	21	1.9105	89	73	56	26
35	87	74	60	47	34	21	2.2308	94	81	25
36	90	79	69	58	48	38	27	2.5517	2.8706	24
37	92	85	77	69	62	54	46	38	31	23
38	95	90	85	80	76	71	66	61	56	22
39	98	96	94	92	90	87	85	83	81	21
40	0.8201	0.6401	0.9602	1.2802	1.6003	1.9204	2.2404	2.5605	2.8805	20
41	03	07	10	14	17	20	24	27	31	19
42	06	12	18	24	31	37	43	49	55	18
43	09	18	27	36	45	53	62	71	80	17
44	12	23	35	46	58	70	81	93	2.8904	16
45	14	29	43	58	72	86	2.2501	2.5715	30	15
46	17	34	51	68	86	1.9303	20	37	54	14
47	20	40	60	80	1.6105	19	39	59	79	13
48	23	45	68	91	14	36	59	82	2.9004	12
49	25	51	76	1.2902	27	52	78	2.5803	29	11
50	28	58	85	13	41	69	97	26	54	10
51	31	62	93	24	55	85	2.2616	47	78	9
52	34	67	0.9701	35	69	1.9402	36	70	2.9103	8
53	36	73	09	46	82	18	55	91	28	7
54	39	78	18	57	96	35	74	2.5904	53	6
55	42	84	26	68	1.6210	51	93	35	77	5
56	45	89	34	79	24	68	2.2718	57	2.9202	4
57	47	95	42	90	37	84	32	79	27	3
58	50	0.6500	51	1.8001	51	1.9501	51	2.6002	52	2
59	53	06	59	12	65	17	70	23	76	1
60	0.8256	0.6511	0.9767	1.8023	1.6279	1.9534	2.2790	2.6046	2.9301	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 71 DEGREES.										

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LATITUDE 19 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.9455	1.8910	2.8366	3.7821	4.7276	5.6731	6.6186	7.5642	8.5097	60
1	54	08	63	17	71	25	79	34	89	59
2	53	07	60	13	67	20	73	26	80	58
3	52	05	57	09	62	14	66	18	71	57
4	51	03	54	06	57	08	60	11	63	56
5	50	01	51	02	52	02	53	03	54	55
6	50	1.8899	49	3.7798	48	5.6697	47	7.5596	46	54
7	49	97	46	94	43	91	40	88	37	53
8	48	95	43	90	38	86	33	81	28	52
9	47	93	40	86	33	80	26	73	19	51
10	46	91	37	83	29	74	20	66	11	50
11	45	89	34	79	24	68	13	58	03	49
12	44	88	31	75	19	63	07	50	8.4994	48
13	43	86	28	71	14	57	00	42	85	47
14	42	84	25	67	09	51	6.6093	34	76	46
15	41	82	23	64	05	45	86	27	68	45
16	40	80	20	60	00	39	79	19	59	44
17	39	78	17	56	4.7193	34	73	12	51	43
18	38	76	14	52	90	28	66	04	42	42
19	37	74	11	48	85	22	59	7.5496	33	41
20	36	72	08	44	81	17	53	89	25	40
21	35	70	05	40	76	11	46	81	16	39
22	34	68	03	37	71	05	39	74	08	38
23	33	66	00	33	66	5.6599	32	66	8.4899	37
24	32	64	2.8297	29	61	93	25	58	90	36
25	31	63	94	25	57	88	19	50	82	35
26	30	61	91	21	52	82	12	42	73	34
27	29	59	88	18	47	76	05	34	64	33
28	28	57	85	14	42	70	3.5999	27	56	32
29	27	55	82	10	37	64	92	19	47	31
30	26	53	79	06	32	58	85	11	38	30
31	25	51	76	02	27	52	78	03	39	29
32	25	49	74	3.7698	23	47	72	7.5396	21	28
33	24	47	71	94	18	41	65	88	12	27
34	23	45	68	90	13	35	58	80	03	26
35	22	43	65	86	08	29	51	72	8.4794	25
36	21	41	62	82	03	24	44	65	85	24
37	20	39	59	78	4.7098	18	37	57	76	23
38	19	37	56	74	93	12	30	49	67	22
39	18	35	53	70	88	06	23	41	58	21
40	17	33	50	67	84	00	17	34	50	20
41	16	31	47	63	79	5.6494	10	26	41	19
42	15	29	44	59	74	88	03	18	32	18
43	14	27	41	55	69	82	6.5896	10	23	17
44	13	25	38	51	64	76	89	02	14	16
45	12	24	35	47	59	71	83	7.5294	06	15
46	11	22	32	43	54	65	76	86	8.4697	14
47	10	20	29	39	49	59	69	78	88	13
48	09	18	26	35	44	53	62	70	79	12
49	08	16	23	31	39	47	55	62	70	11
50	07	14	20	27	34	41	48	54	61	10
51	06	12	17	23	29	35	41	46	52	9
52	05	10	15	20	25	29	34	39	44	8
53	04	08	12	16	20	23	27	31	35	7
54	03	06	09	12	15	17	20	23	26	6
55	02	04	06	08	10	11	13	15	17	5
56	01	02	03	04	05	05	06	07	08	4
57	00	00	00	00	00	5.6299	6.5799	7.5199	8.4599	3
58	0.9399	1.8798	2.8197	3.7596	4.6995	93	92	91	90	2
59	98	96	94	92	90	87	85	83	81	1
60	0.9397	1.8794	2.8191	3.7588	4.6985	5.6381	6.5778	7.5175	8.4572	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 70 DEGREES.										

DEPARTURE 19 DEGREES.										113
1	2	3	4	5	6	7	8	9	10	
0	0.3256	0.6511	0.9767	1.3023	1.6278	1.9534	2.2790	2.6046	2.9801	60
1	58	17	75	84	92	50	2.2809	67	26	59
2	61	22	84	45	1.6306	67	28	90	51	58
3	64	28	92	56	20	83	47	2.6111	75	57
4	67	33	0.9800	67	34	1.9600	67	34	2.9400	56
5	69	39	08	78	47	16	86	55	25	55
6	72	44	17	89	61	33	2.2905	78	50	54
7	75	50	25	1.3100	75	50	24	99	74	53
8	78	55	33	11	89	66	44	2.6222	99	52
9	80	60	41	22	1.6402	82	63	43	2.9524	51
10	83	66	50	33	16	99	82	66	49	50
11	86	72	58	44	30	1.9715	2.3001	87	73	49
12	89	77	66	55	44	32	21	2.6310	98	48
13	91	83	74	66	57	48	40	31	2.9623	47
14	94	88	83	77	71	65	59	54	48	46
15	97	94	91	88	85	81	78	75	72	45
16	0.3300	99	99	99	99	98	98	98	97	44
17	02	0.6605	0.9907	1.3210	1.6512	1.9814	2.3197	2.6419	2.6722	43
18	05	10	15	20	26	31	36	41	46	42
19	08	16	24	32	40	47	55	63	71	41
20	11	21	32	42	53	64	74	85	95	40
21	13	27	40	56	67	80	94	2.6507	2.9821	39
22	16	32	48	64	81	97	2.3213	29	45	38
23	19	38	57	76	95	1.9913	32	51	70	37
24	22	43	65	86	1.6608	30	51	73	94	36
25	24	49	73	98	22	46	71	95	2.9920	35
26	27	54	81	1.3308	36	63	90	2.6617	44	34
27	30	60	89	19	49	79	2.3309	38	68	33
28	33	65	98	30	63	96	28	61	93	32
29	35	71	1.0006	41	77	2.0012	47	82	3.0018	31
30	38	76	14	52	91	29	67	2.6705	43	30
31	41	82	22	63	1.6704	45	86	26	67	29
32	44	87	31	74	18	62	2.3405	49	92	28
33	46	93	39	85	32	78	24	70	3.0117	27
34	49	98	47	96	45	94	43	92	41	26
35	52	0.6704	55	1.3407	59	2.0111	63	2.6814	66	25
36	55	09	64	18	73	27	82	36	91	24
37	57	15	72	29	87	44	2.3501	58	3.0216	23
38	60	20	80	40	1.6800	60	20	80	40	22
39	63	25	88	51	14	76	39	2.6902	64	21
40	66	31	97	62	28	93	59	24	90	20
41	68	36	1.0105	73	41	2.0209	77	46	3.0314	19
42	71	42	13	84	55	26	97	68	39	18
43	74	47	21	95	69	42	2.3616	90	63	17
44	76	53	29	1.3506	82	58	35	2.7011	88	16
45	79	59	38	17	96	75	54	34	3.0413	15
46	82	64	46	28	1.6910	91	73	55	37	14
47	85	69	54	38	23	2.0308	92	77	61	13
48	87	75	62	50	37	24	2.3712	99	87	12
49	90	80	70	60	51	41	31	2.7121	3.0511	11
50	93	86	79	72	65	57	50	43	36	10
51	96	91	87	82	78	74	69	65	60	9
52	98	97	95	93	92	90	88	86	85	8
53	0.3401	0.6802	1.0203	1.3604	1.7006	2.0407	1.3808	2.7209	3.0610	7
54	04	08	11	15	19	23	27	30	34	6
55	07	13	20	26	33	39	46	52	59	5
56	09	19	28	37	47	56	65	74	84	4
57	12	24	36	48	60	72	84	96	3.0708	3
58	15	29	44	59	74	88	2.3903	2.7318	32	2
59	18	35	53	70	88	2.0505	23	40	58	1
60	0.3420	0.6840	1.2661	1.3681	1.7101	2.0521	2.3941	2.7362	3.0782	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 70 DEGREES.										

	1	2	3	4	5	6	7	8	9	10
0	0.9397	1.8794	2.8191	3.7588	4.6985	5.6381	6.5778	7.5175	8.4572	60
1	96	92	88	84	80	75	71	67	63	59
2	95	90	85	80	75	69	64	59	54	58
3	94	88	82	76	70	63	57	51	45	57
4	93	86	79	72	65	57	50	43	36	56
5	92	84	76	68	60	51	43	35	27	55
6	91	82	73	64	55	45	36	27	18	54
7	90	80	70	60	50	39	29	19	09	53
8	89	78	67	56	45	33	22	11	00	52
9	88	76	64	52	40	37	15	03	8.4491	51
10	87	74	* 61	48	35	21	08	7.5095	82	50
11	86	72	58	44	30	15	01	87	73	49
12	85	70	55	40	25	09	6.5694	79	64	48
13	84	68	52	36	20	03	87	71	55	47
14	83	66	49	32	15	5.6297	80	63	46	46
15	82	64	46	28	10	91	73	55	37	45
16	81	62	43	24	05	85	66	47	28	44
17	80	60	40	20	00	79	59	39	19	43
18	79	58	37	16	4.6895	73	52	31	10	42
19	78	56	34	12	90	67	45	23	01	41
20	77	54	31	08	85	61	38	15	8.4392	40
21	76	52	28	04	79	55	32	07	83	39
22	75	50	24	3.7499	74	49	24	7.4998	73	38
23	74	48	21	95	69	43	17	90	64	37
24	73	46	18	91	64	37	10	82	55	36
25	72	44	15	87	59	31	02	74	46	35
26	71	42	12	83	54	25	6.5596	66	37	34
27	70	40	09	79	49	19	89	58	28	33
28	69	38	06	75	44	13	82	50	19	32
29	68	36	03	71	39	07	75	42	10	31
30	67	33	00	67	34	00	67	34	00	30
31	66	31	2.8097	63	29	5.6194	60	26	8.4291	29
32	65	29	94	59	24	88	53	18	82	28
33	64	27	91	55	19	82	45	09	72	27
34	63	25	88	50	13	76	38	01	63	26
35	62	23	85	46	08	70	31	7.4893	54	25
36	61	21	82	42	03	64	24	85	45	24
37	60	19	79	38	4.6798	58	17	77	36	23
38	59	17	76	34	93	51	10	68	27	22
39	58	15	73	30	89	45	03	60	18	21
40	57	13	70	26	83	39	6.5496	52	09	20
41	56	11	66	22	78	33	89	44	00	19
42	54	09	63	18	72	26	81	35	8.4190	18
43	53	07	60	14	67	20	74	27	81	17
44	52	05	57	10	62	14	67	19	72	16
45	51	03	54	06	57	08	60	11	63	15
46	50	01	51	01	52	02	52	02	53	14
47	49	1.8699	48	3.7397	47	5.6096	45	7.4794	44	13
48	48	97	45	93	42	90	38	86	35	12
49	47	94	42	89	36	83	30	78	25	11
50	46	92	39	85	31	77	23	70	16	10
51	45	90	36	81	26	71	16	62	07	9
52	44	88	32	76	21	65	09	53	8.4097	8
53	43	86	29	72	16	59	02	45	88	7
54	42	84	26	68	10	52	6.5394	36	78	6
55	41	82	23	64	05	46	87	28	69	5
56	40	80	20	60	00	40	80	20	60	4
57	39	78	18	56	4.6695	33	72	11	50	3
58	38	76	14	52	90	27	65	03	41	2
59	37	74	10	47	84	21	58	7.4694	31	1
60	0.9336	1.8672	2.8007	3.7343	4.6679	5.6015	6.5351	7.4686	8.4022	0

DEPARTURE 69 DEGREES.

DEPARTURE 20 DEGREES.										115
1	2	3	4	5	6	7	8	9	10	
0	0.3420	0.6840	1.0261	1.3681	1.7101	2.0521	2.3941	2.7362	3.0782	06
1	23	46	69	92	15	37	60	83	3.0806	59
2	26	51	77	1.3708	29	54	80	2.7404	31	58
3	28	57	85	14	42	70	99	27	56	57
4	31	62	93	24	56	87	2.4018	49	80	56
5	34	68	1.0302	36	70	2.0603	37	71	3.0905	55
6	37	73	10	46	83	20	56	93	29	54
7	39	79	18	57	97	33	75	2.7514	54	53
8	42	84	26	68	1.7211	53	95	37	79	52
9	45	90	34	79	24	69	2.4114	58	3.1003	51
10	48	95	43	90	38	85	33	80	28	50
11	50	0.6901	51	1.3801	52	2.0702	52	2.7602	53	49
12	53	06	59	12	65	18	71	24	77	48
13	56	11	67	23	79	34	90	46	3.1101	47
14	58	17	75	34	92	50	2.4209	67	26	46
15	61	22	84	45	1.7306	67	28	90	51	45
16	64	28	92	56	20	83	47	2.7711	75	44
17	67	33	1.0400	66	33	2.0800	66	33	99	43
18	69	39	08	78	47	16	86	53	3.1225	42
19	72	44	16	88	61	33	2.4305	77	49	41
20	75	50	24	99	74	49	24	98	73	40
21	78	55	33	1.3910	88	65	43	2.7820	98	39
22	80	61	41	21	1.7402	82	62	42	3.1323	38
23	83	66	49	32	15	98	81	64	47	37
24	86	71	57	43	29	2.0914	2.4400	86	71	36
25	88	77	65	54	42	30	19	2.7907	96	35
26	91	82	74	65	56	47	38	30	3.1421	34
27	94	88	82	76	70	63	57	51	45	33
28	97	93	90	86	83	80	76	73	69	32
29	99	99	98	97	97	96	95	94	94	31
30	0.3502	0.7004	1.0506	1.4008	1.7511	2.1013	2.4515	2.8017	3.1519	30
31	05	10	14	19	24	29	34	38	43	29
32	08	15	23	30	38	45	53	60	68	28
33	10	20	31	41	51	61	71	82	92	27
34	13	26	39	52	65	78	91	2.8104	3.1617	26
35	16	31	47	63	79	94	2.4610	26	41	25
36	18	37	55	74	92	2.1110	29	47	66	24
37	21	42	62	84	1.7606	27	48	69	90	23
38	24	48	72	96	20	43	67	91	3.1715	22
39	27	53	80	1.4106	33	60	86	2.8218	39	21
40	29	59	88	17	47	76	2.4705	34	64	20
41	32	64	96	28	60	92	24	56	88	19
42	35	69	1.0604	39	74	2.1208	43	78	3.1812	18
43	38	75	13	50	87	25	63	2.8300	38	17
44	40	80	21	61	1.7701	41	81	22	62	16
45	43	86	29	72	15	57	2.4800	43	86	15
46	46	91	37	82	28	74	19	65	3.1910	14
47	48	97	45	94	42	90	39	87	86	13
48	51	0.7102	53	1.4204	56	2.1307	58	2.8409	60	12
49	54	08	61	15	69	23	77	80	84	11
50	57	13	70	26	83	39	96	52	3.2009	10
51	59	18	78	37	96	55	2.4914	74	33	9
52	62	24	86	48	1.7810	71	83	95	57	8
53	65	29	94	59	24	88	53	2.8518	82	7
54	67	35	1.0702	70	37	2.1404	72	39	3.2107	6
55	70	40	10	80	51	21	91	61	81	5
56	73	46	18	91	64	37	2.5010	82	55	4
57	76	51	27	1.4302	78	53	29	2.8604	80	3
58	78	56	35	13	91	69	47	26	3.2204	2
59	81	62	43	24	1.7905	86	67	48	29	1
60	0.3584	0.7167	1.0761	1.4385	1.7919	2.1502	2.5086	2.8670	3.2258	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 69 DEGREES.										

116										
LATITUDE 21 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.9336	1.8672	2.8007	3.7343	4.6679	5.6015	6.5351	7.4686	8.4022	60
1	35	70	04	39	74	09	44	78	13	59
2	34	67	01	35	69	02	36	70	03	58
3	33	65	2.7998	31	64	5.5996	29	61	8.3994	57
4	32	63	95	26	58	90	21	53	84	56
5	31	61	92	22	53	84	14	44	75	55
6	30	59	89	18	48	77	07	36	66	54
7	29	57	86	14	43	71	6.5299	28	57	53
8	27	55	82	10	37	64	92	19	47	52
9	26	53	79	06	32	58	84	11	38	51
10	25	51	76	01	27	52	77	02	28	50
11	24	49	73	3.7297	22	46	69	7.4594	19	49
12	23	46	70	93	16	39	62	86	09	48
13	22	44	67	89	11	33	55	78	00	47
14	21	42	63	84	06	27	48	69	8.3890	46
15	20	40	60	80	01	21	41	61	81	45
16	19	38	57	76	4.6595	14	33	52	71	44
17	18	36	54	72	90	08	26	44	62	43
18	17	34	51	68	85	01	18	35	52	42
19	16	32	48	64	80	5.5895	11	26	43	41
20	15	30	44	59	74	89	04	18	33	40
21	14	27	41	55	69	82	6.5196	10	23	39
22	13	25	38	51	64	76	89	02	14	38
23	12	23	35	46	58	70	81	7.4493	04	37
24	11	21	32	42	53	64	74	85	8.3795	36
25	10	19	29	38	48	57	67	76	86	35
26	08	17	25	34	42	50	59	67	76	34
27	07	15	22	30	37	44	52	59	67	33
28	06	13	19	25	32	38	44	50	57	32
29	05	10	16	21	26	31	36	42	47	31
30	04	08	13	17	21	25	29	34	38	30
31	03	06	09	12	16	19	22	25	28	29
32	02	04	06	08	10	12	14	16	18	28
33	01	02	03	04	05	05	07	08	09	27
34	00	00	00	00	00	5.5799	6.5099	7.4399	8.3699	26
35	0.9299	1.8598	2.7896	3.7195	4.6494	93	92	90	89	25
36	98	96	93	91	89	87	85	82	80	24
37	97	93	90	87	84	80	77	74	70	23
38	96	91	87	82	78	74	69	65	60	22
39	95	89	84	78	73	67	62	56	51	21
40	94	87	81	74	68	61	55	48	42	20
41	92	85	77	70	62	54	47	39	32	19
42	91	83	74	65	57	48	39	30	22	18
43	90	80	71	61	51	41	31	22	12	17
44	89	78	68	57	46	35	24	14	03	16
45	88	76	64	52	40	29	17	05	8.3593	15
46	87	74	61	48	35	22	09	7.4296	83	14
47	86	72	58	44	30	15	01	87	73	13
48	85	70	55	40	25	09	6.4994	79	64	12
49	84	68	51	35	19	03	87	70	54	11
50	83	65	48	31	14	5.5696	79	62	44	10
51	82	63	45	26	08	90	71	53	34	9
52	81	61	42	22	03	83	64	44	25	8
53	79	59	38	18	4.6397	76	56	35	16	7
54	78	57	35	14	92	70	49	27	06	6
55	77	55	32	09	87	64	41	18	8.3496	5
56	76	52	29	05	81	57	33	10	86	4
57	75	50	25	00	76	51	26	02	76	3
58	74	48	22	3.7096	70	44	18	7.4192	66	2
59	73	46	19	91	65	87	10	88	56	1
60	0.9272	1.8544	2.7815	3.7087	4.6359	5.5631	6.4903	7.4174	8.3446	0
1	2	3	4	5	6	7	8	9		
DEPARTURE 68 DEGREES.										

DEPARTURE 21 DEGREES.										117
1	2	3	4	5	6	7	8	9	10	
0	0.3584	0.7167	1.0751	1.4335	1.7919	2.1502	2.5086	2.8670	3.2253	60
1	86	73	59	46	32	18	2.5105	91	78	59
2	89	78	67	56	46	35	24	2.8713	3.2202	58
3	92	84	75	67	59	51	43	34	26	57
4	95	89	84	78	73	67	62	56	51	56
5	97	95	92	89	87	84	81	78	76	55
6	0.3600	0.7200	1.0800	1.4400	1.8000	2.1600	2.5200	2.8800	3.2300	54
7	03	05	08	11	14	16	19	22	24	53
8	05	11	16	21	27	32	38	43	49	52
9	08	16	24	32	41	49	57	65	73	51
10	11	22	32	43	54	65	76	86	97	50
11	14	27	41	54	68	81	95	2.8908	3.2422	49
12	16	32	49	65	81	97	2.5313	30	46	48
13	19	38	57	76	95	2.1714	33	52	71	47
14	22	43	65	87	1.8109	30	52	74	95	46
15	24	49	73	98	22	46	71	95	3.2520	45
16	27	54	81	1.4508	36	63	90	2.9017	44	44
17	30	60	89	19	49	79	2.5409	38	68	43
18	33	65	98	30	63	95	28	60	93	42
19	35	70	1.0906	41	76	2.1811	46	82	3.2617	41
20	38	76	14	52	90	27	65	2.9103	41	40
21	41	81	22	62	1.8203	44	84	25	65	39
22	43	87	30	74	17	60	2.5504	57	91	38
23	46	92	38	84	31	77	23	69	3.2715	37
24	49	98	46	95	44	93	42	90	39	36
25	52	0.7303	55	1.4606	58	2.1909	61	2.9212	64	35
26	54	08	63	17	71	25	79	34	88	34
27	57	14	71	28	85	41	98	55	3.2812	33
28	60	19	79	38	98	58	2.5617	77	36	32
29	62	25	87	49	1.8312	74	36	98	61	31
30	65	30	95	60	25	90	55	2.9320	3.2985	30
31	68	35	1.1003	71	39	2.2006	74	42	3.3019	29
32	70	41	10	82	52	22	93	63	34	28
33	73	46	19	92	66	39	2.5712	85	58	27
34	76	52	27	1.4703	79	55	31	2.9406	82	26
35	79	57	36	14	93	71	50	28	3.3107	25
36	81	62	43	25	1.8406	87	68	50	31	24
37	84	68	52	36	20	2.2103	87	71	55	23
38	87	73	60	47	34	20	2.5807	94	80	22
39	89	79	68	58	47	36	26	2.9515	3.3205	21
40	92	84	76	68	61	53	45	37	29	20
41	95	90	84	79	74	69	64	58	53	19
42	98	95	93	90	88	85	83	80	78	18
43	0.3700	0.7400	1.1101	1.4801	1.8501	2.2201	2.5901	2.9602	3.3302	17
44	03	06	09	12	15	17	20	23	26	16
45	06	11	17	22	28	34	39	45	50	15
46	08	17	25	33	42	50	58	66	75	14
47	11	22	33	44	55	66	77	88	99	13
48	14	27	41	55	69	82	96	2.9710	3.3423	12
49	16	33	49	66	82	98	2.6015	31	48	11
50	19	38	57	76	96	2.2315	34	53	72	10
51	22	44	65	87	1.8609	31	53	74	96	9
52	25	49	74	98	23	47	72	96	3.3521	8
53	27	54	82	1.4909	36	63	90	2.9818	45	7
54	30	60	90	20	50	79	2.6109	39	69	6
55	33	65	98	30	63	96	28	61	93	5
56	35	71	1.1206	41	77	2.2412	47	82	3.3618	4
57	38	76	14	52	90	28	66	2.9904	42	3
58	41	81	22	63	1.8704	44	85	26	66	2
59	43	87	30	74	17	60	2.6204	47	91	1
60	0.3746	0.7492	1.1238	1.4984	1.8731	2.2477	2.6223	2.9967	3.3715	0
1	2	3	4	5	6	7	8	9		
LATITUDE 68 DEGREES.										

LATITUDE 22 DEGREES.									
118	1	2	3	4	5	6	7	8	9
0	0.9272	1.8544	2.7815	3.7087	4.6359	5.5631	6.4903	7.4174	8.3446
1	71	41	12	83	54	24	6.4895	66	36 59
2	70	39	09	79	49	18	88	58	27 58
3	69	37	06	74	43	12	80	49	17 57
4	68	35	03	70	38	05	73	40	08 56
5	66	33	2.7799	66	32	5.5598	65	31	8.3398 55
6	65	31	96	61	27	92	57	22	88 54
7	64	28	93	56	21	85	49	14	78 53
8	63	26	89	52	16	79	42	05	68 52
9	62	24	86	48	10	72	34	7.4096	58 51
10	61	22	83	44	05	65	26	87	48 50
11	60	20	79	39	00	59	19	78	38 49
12	59	17	76	35	4.6294	52	11	70	28 48
13	58	15	73	30	88	46	03	61	18 47
14	57	13	70	26	83	39	6.4796	52	09 46
15	55	11	66	22	77	33	88	43	8.3299 45
16	54	09	63	17	72	26	80	34	89 44
17	53	06	60	12	66	19	72	25	79 43
18	52	04	56	08	61	13	65	17	69 42
19	51	02	53	04	55	06	57	08	59 41
20	50	00	50	00	50	5.5499	49	7.3999	49 40
21	49	1.8498	46	3.6995	46	93	42	90	39 39
22	48	95	43	91	39	86	34	82	29 38
23	47	93	40	86	33	80	27	73	19 37
24	46	91	37	82	28	73	19	64	10 36
25	44	89	33	78	22	66	11	55	00 35
26	43	86	30	73	16	59	02	46	8.3189 34
27	42	84	26	68	11	53	6.4695	37	79 33
28	41	82	23	64	05	46	87	28	69 32
29	40	80	20	60	00	39	79	19	59 31
30	39	78	16	55	4.6194	33	72	10	49 30
31	38	75	13	51	89	26	64	02	39 29
32	37	73	10	46	83	20	56	7.3893	29 28
33	36	71	07	42	78	13	49	84	20 27
34	34	69	03	37	72	06	40	74	09 26
35	33	66	00	32	66	5.5399	32	66	8.3099 25
36	32	64	2.7696	28	61	93	25	57	89 24
37	31	62	93	24	55	86	17	48	79 23
38	30	60	90	20	50	79	09	39	69 22
39	29	57	86	15	44	72	01	30	58 21
40	28	55	83	10	38	66	6.4593	21	48 20
41	27	53	80	06	33	59	86	12	39 19
42	25	51	76	02	27	52	78	03	29 18
43	24	49	73	3.6897	22	46	70	7.3794	19 17
44	23	46	69	92	16	39	62	85	08 16
45	22	44	66	88	10	32	54	76	8.2998 15
46	21	42	63	84	05	25	46	67	88 14
47	20	40	59	79	4.6099	19	39	58	78 13
48	19	37	56	74	93	12	30	49	67 12
49	18	35	53	70	88	05	23	40	58 11
50	16	33	49	66	82	5.5298	15	31	48 10
51	15	30	46	61	76	91	06	22	37 9
52	14	28	42	56	71	85	6.4499	13	27 8
53	13	26	39	52	65	78	91	06	17 7
54	12	24	36	48	60	71	83	7.3695	07 6
55	11	21	32	43	54	64	75	86	8.2896 5
56	10	19	29	38	48	58	67	77	86 4
57	09	17	26	34	43	51	60	68	77 3
58	07	15	22	29	37	44	51	58	66 2
59	06	12	19	25	31	37	43	50	56 1
60	0.9205	1.8410	2.7615	3.6820	4.6025	5.5230	6.4435	7.3640	8.2845 0
	1	2	3	4	5	6	7	8	9
DEPARTURE 67 DEGREES.									

DEPARTURE 22 DEGREES.										119
	1	2	3	4	5	6	7	8	9	
0	0.8746	0.7492	1.1238	1.4984	1.8781	2.2477	2.6223	2.9969	3.3715	60
1	49	98	46	95	44	93	42	90	39	59
2	52	0.7503	55	1.5006	58	2.2509	61	3.0012	64	58
3	54	08	63	17	71	25	79	34	88	57
4	57	14	71	28	85	41	98	55	3.3812	56
5	60	19	79	38	98	57	2.6317	76	36	55
6	62	24	87	49	1.8811	73	35	98	60	54
7	65	30	95	60	25	89	54	3.0119	84	53
8	68	35	1.1303	70	38	2.2606	73	41	3.3908	52
9	70	41	11	81	52	22	92	62	33	51
10	73	46	19	92	65	38	2.6411	84	57	50
11	76	51	27	1.5103	79	54	30	3.0206	81	49
12	78	57	35	14	92	70	49	27	3.4006	48
13	81	62	43	24	1.8906	87	68	49	30	47
14	84	68	51	35	19	2.2703	87	70	54	46
15	87	73	60	46	33	19	2.6506	92	79	45
16	89	78	68	57	46	35	24	3.0314	3.4103	44
17	92	84	76	68	60	51	43	85	27	43
18	95	89	84	78	73	68	62	57	51	42
19	97	95	92	89	87	84	81	78	76	41
20	0.3800	0.7600	1.1400	1.5200	1.9000	99	99	99	99	40
21	03	05	08	10	13	2.2816	2.6618	3.0421	3.4223	39
22	05	11	16	21	27	32	37	42	48	38
23	08	16	24	32	40	48	56	64	72	37
24	11	21	32	43	54	64	75	86	96	36
25	13	27	40	54	67	80	94	3.0507	3.4321	35
26	16	32	48	64	81	97	2.6713	29	45	34
27	19	38	56	75	94	2.2913	32	50	69	33
28	22	43	65	86	1.9108	29	51	72	94	32
29	24	48	72	96	21	45	69	93	3.4417	31
30	27	54	80	1.5307	34	61	88	3.0614	41	30
31	30	59	89	18	48	77	2.6807	36	66	29
32	32	64	97	29	61	93	25	58	90	28
33	35	70	1.1505	40	75	2.3009	44	79	3.4514	27
34	38	75	13	50	88	26	63	3.0701	38	26
35	40	81	21	61	1.9202	42	82	22	63	25
36	43	86	29	72	15	58	2.6901	44	87	24
37	46	91	37	82	28	74	19	65	3.4610	23
38	48	97	45	93	42	90	38	86	35	22
39	51	0.7702	53	1.5404	55	2.3106	57	3.0808	59	22
40	54	07	61	15	69	22	76	30	83	20
41	56	13	69	26	82	38	95	51	3.4708	19
42	59	18	77	36	96	55	2.7014	72	32	18
43	62	23	85	47	1.9309	70	32	94	55	17
44	64	29	93	58	22	86	51	3.0916	80	16
45	67	34	1.1601	68	36	2.3203	70	37	3.4804	15
46	70	40	09	79	49	19	89	58	28	14
47	73	45	18	90	63	35	2.7108	80	53	13
48	75	50	26	1.5501	76	57	26	3.1002	77	12
49	78	56	33	11	89	67	45	22	3.4900	11
50	81	61	42	22	1.9403	83	64	44	25	10
51	83	66	50	33	16	99	82	66	49	9
52	86	72	58	44	30	2.3315	2.7201	87	73	8
53	89	77	66	54	43	32	20	3.1109	97	7
54	91	82	74	65	56	47	38	30	3.5021	6
55	94	88	82	76	70	63	57	51	45	5
56	97	93	90	86	83	80	76	73	69	4
57	99	99	98	97	97	96	95	94	94	3
58	0.3902	0.7804	1.1706	1.5608	1.9510	2.3412	2.7314	3.1216	3.5118	2
59	05	09	14	18	23	28	32	37	41	1
60	0.3907	0.7815	1.1722	1.5629	1.9537	2.3444	2.7351	3.1258	3.5166	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 67 DEGREES.										

120										
LATITUDE 23 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.9205	1.8410	2.7615	3.6820	4.6025	5.5230	6.4435	7.3640	8.2845	60
1	04	08	12	16	20	23	27	31	35	59
2	03	06	08	11	14	17	20	22	25	58
3	02	03	05	06	08	10	11	13	14	57
4	01	01	02	02	03	03	04	04	05	56
5	0.9199	00	2.7598	3.6798	4.5997	5.5196	6.4396	7.3595	8.2795	55
6	98	1.8396	95	93	91	89	87	86	84	54
7	97	94	91	88	86	83	80	77	74	53
8	96	92	88	84	80	75	71	67	63	52
9	95	90	84	79	74	69	64	58	53	51
10	94	87	81	74	68	62	55	49	42	50
11	93	85	78	70	63	55	48	40	33	49
12	91	83	74	66	57	48	40	31	23	48
13	90	80	71	61	51	41	31	22	12	47
14	89	78	67	56	46	35	24	13	02	46
15	88	76	64	52	40	28	15	03	8.2691	45
16	87	74	60	47	34	21	08	7.3494	81	44
17	86	71	57	42	28	14	00	85	70	43
18	85	69	54	38	23	07	6.4292	76	61	42
19	83	67	50	33	17	00	83	67	50	41
20	82	64	47	29	11	5.5093	75	58	40	40
21	81	62	43	24	05	86	67	48	29	39
22	80	60	40	20	00	79	59	39	19	38
23	79	57	36	15	4.5894	72	51	30	08	37
24	78	55	33	10	88	65	43	20	8.2598	36
25	76	53	29	06	82	58	35	11	88	35
26	75	50	26	01	76	51	26	02	77	34
27	74	48	22	3.6695	71	45	19	7.3393	67	33
28	73	46	19	92	65	37	10	83	56	32
29	72	44	15	87	59	31	03	74	46	31
30	71	41	12	82	53	24	6.4194	65	35	30
31	69	39	08	78	47	16	86	55	25	29
32	68	37	05	73	42	10	78	46	15	28
33	67	34	01	68	36	03	70	37	04	27
34	66	32	2.7498	64	30	5.4996	62	28	8.2494	26
35	65	30	94	59	24	89	54	18	83	25
36	64	27	91	54	18	82	45	09	72	24
37	63	25	88	50	13	75	38	00	63	23
38	61	23	84	45	07	68	29	7.3290	52	22
39	60	20	80	40	01	61	21	81	41	21
40	59	18	77	36	4.5795	54	13	72	31	20
41	58	16	73	31	89	47	05	62	20	19
42	57	13	70	26	83	40	6.4096	53	09	18
43	56	11	67	22	78	33	89	44	00	17
44	54	09	63	17	72	26	80	34	8.2389	16
45	53	06	59	12	66	19	72	25	78	15
46	52	04	56	08	60	11	63	15	67	14
47	51	02	52	03	54	05	56	06	57	13
48	50	1.8299	49	3.6598	48	5.4898	47	7.3197	46	12
49	48	97	45	94	42	90	39	87	36	11
50	47	94	42	89	36	83	30	78	25	10
51	46	92	38	84	31	77	23	69	15	9
52	45	90	35	80	25	69	14	59	04	8
53	44	87	31	75	19	62	06	50	8.2298	7
54	43	85	28	70	13	55	6.3998	40	83	6
55	41	83	24	65	07	48	90	31	73	5
56	40	80	21	61	01	41	81	22	62	4
57	39	78	17	56	4.5695	34	73	12	51	3
58	38	76	13	51	89	27	65	02	40	2
59	37	73	10	46	83	20	56	7.3098	29	1
60	0.9136	1.8271	2.7407	3.6542	4.5678	5.4813	6.3949	7.3084	8.2220	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 66 DEGREES.										

DEPARTURE 23 DEGREES.										121
1	2	3	4	5	6	7	8	9	10	
0	0.3907	0.7815	1.1722	1.5629	1.9537	2.3444	2.7351	3.1258	3.5166	60
1	10	20	30	40	50	60	70	80	90	59
2	13	25	38	51	63	76	89	3.1302	3.5214	58
3	15	31	46	61	77	92	2.7407	22	38	57
4	18	36	54	72	90	2.3508	26	44	62	56
5	21	41	62	83	1.9604	24	45	66	86	55
6	23	47	70	94	17	40	64	87	3.5311	54
7	26	52	78	1.5704	30	56	82	3.1408	34	53
8	29	57	86	15	44	72	2.7501	30	58	52
9	31	63	94	26	57	88	20	51	83	51
10	34	68	1.1802	36	71	2.3605	39	73	3.5407	50
11	37	73	10	47	84	20	57	94	30	49
12	39	79	18	58	97	36	76	3.1515	55	48
13	42	84	26	68	1.9711	52	95	87	79	47
14	45	90	34	79	24	69	2.7614	58	3.5503	46
15	47	95	42	90	37	84	32	79	27	45
16	50	0.7900	50	1.5800	51	2.3701	51	3.1601	51	44
17	53	06	58	11	64	17	70	22	75	43
18	56	11	67	22	78	33	89	44	3.5600	42
19	58	16	74	32	91	49	2.7707	65	23	41
20	61	22	82	43	1.9804	65	26	86	47	40
21	64	27	91	54	18	81	45	3.1708	72	39
22	66	32	98	64	31	97	63	29	95	38
23	69	38	1.1906	75	44	2.3813	82	50	3.5719	37
24	72	43	15	86	58	29	2.7801	72	44	36
25	74	48	22	96	71	45	19	93	67	35
26	77	54	30	1.5907	84	61	38	3.1814	91	34
27	80	59	39	18	98	77	57	36	3.5816	33
28	82	64	47	29	1.9911	93	75	59	40	32
29	85	70	54	39	24	2.3909	94	78	63	31
30	88	75	63	50	38	25	2.7913	3.1900	88	30
31	90	80	71	61	51	41	31	22	3.5912	29
32	93	86	78	71	64	57	50	42	35	28
33	96	91	87	82	78	73	69	64	60	27
34	98	96	95	93	91	89	87	86	84	26
35	0.4001	0.8002	1.2002	1.6003	2.0004	2.4005	2.8006	3.2006	3.6007	25
36	04	07	11	14	18	21	25	28	32	24
37	06	12	19	25	31	37	43	50	56	23
38	09	18	26	35	44	53	62	70	79	22
39	12	23	35	46	58	69	81	92	3.6104	21
40	14	28	42	56	71	85	99	3.2113	27	20
41	17	34	50	67	84	2.4101	2.8118	34	51	19
42	20	39	59	78	98	17	37	56	76	18
43	22	44	66	88	2.0111	33	55	77	99	17
44	25	50	74	99	24	49	74	98	3.6223	16
45	28	55	83	1.6110	38	65	98	3.2220	48	15
46	30	60	90	20	51	81	2.8211	41	71	14
47	33	66	98	31	64	97	30	62	95	13
48	36	71	1.2107	42	78	2.4213	49	84	3.6320	12
49	38	76	14	52	91	29	67	3.2305	43	11
50	41	82	22	63	2.0204	45	86	26	67	10
51	43	87	30	74	17	61	2.8304	47	91	9
52	46	92	38	84	31	77	23	69	3.6414	8
53	49	98	46	95	44	93	42	90	39	7
54	51	0.8103	54	1.6206	57	2.4308	60	3.2411	63	6
55	54	08	62	16	71	25	79	83	87	5
56	57	13	70	27	84	40	97	54	3.6510	4
57	59	19	78	38	97	56	2.8416	75	85	3
58	62	24	86	48	2.0311	73	85	97	59	2
59	65	29	94	59	24	88	53	3.2518	82	1
60	0.4067	0.8135	1.2202	1.6270	2.0337	2.4404	2.8472	3.2539	3.6607	0
1	2	3	4	5	6	7	8	9		
LATITUDE 66 DEGREES.										

G

122 ° LATITUDE 24 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.9136	1.8271	2.7407	3.6542	4.5678	5.4813	6.3949	7.3084	8.2220	60
1	34	69	03	37	72	06	40	74	09	59
2	38	66	2.7399	32	66	5.4799	32	65	8.2198	58
3	32	64	96	28	60	91	23	55	87	57
4	31	61	92	28	54	84	15	46	76	56
5	80	59	89	18	48	77	07	36	66	55
6	28	57	85	13	42	70	6.3898	26	55	54
7	27	54	82	09	36	63	90	18	45	53
8	26	52	78	04	30	56	82	08	34	52
9	25	50	74	3.6499	24	49	74	7.2998	23	51
10	24	47	71	94	18	42	65	89	12	50
11	22	45	67	90	12	34	57	79	02	49
12	21	42	64	85	06	27	48	70	8.2091	48
13	20	40	60	80	00	20	40	60	80	47
14	19	38	56	75	4.5594	13	32	50	69	46
15	18	35	53	70	88	06	23	41	58	45
16	16	33	49	66	82	5.4698	15	31	48	44
17	15	30	46	61	76	91	06	22	37	43
18	14	28	42	56	70	84	6.3798	12	26	42
19	13	26	38	51	64	77	90	02	15	41
20	12	23	35	46	58	70	81	7.2893	04	40
21	10	21	31	42	52	62	73	83	8.1994	39
22	09	18	28	37	46	55	64	74	83	38
23	08	16	24	32	40	48	56	64	72	37
24	07	14	20	27	34	41	48	54	61	36
25	06	11	17	22	28	34	39	45	51	35
26	04	09	13	18	22	26	31	35	40	34
27	03	06	10	13	16	19	22	26	29	33
28	02	04	06	08	10	12	14	16	18	32
29	01	02	03	03	04	05	06	06	07	31
30	00	1.8199	2.7299	3.6398	4.5498	5.4598	6.3697	7.2797	8.1896	30
31	0.9098	97	95	94	92	90	89	87	86	29
32	97	94	92	89	86	83	80	78	75	28
33	96	92	88	84	80	76	72	68	64	27
34	95	90	84	79	74	69	64	58	53	26
35	94	87	81	74	68	62	55	49	42	25
36	92	85	77	70	62	54	47	39	32	24
37	91	82	73	64	56	47	38	29	20	23
38	90	80	70	60	50	39	29	19	09	22
39	89	77	66	55	44	32	21	10	8.1798	21
40	88	75	63	50	38	25	13	00	88	20
41	86	73	59	45	32	18	04	7.2690	77	19
42	85	70	55	40	26	11	6.3596	81	66	18
43	84	67	52	36	20	03	87	71	55	17
44	83	65	48	30	13	5.4496	78	61	43	16
45	81	63	44	26	07	88	70	51	33	15
46	80	60	41	21	01	81	61	42	22	14
47	79	58	37	16	4.5395	74	53	32	11	13
48	78	56	33	11	89	67	45	22	00	12
49	77	53	30	06	83	60	36	13	8.1689	11
50	75	51	26	01	77	52	27	02	78	10
51	74	48	22	3.6296	71	45	19	7.2593	67	9
52	73	46	19	92	65	37	10	83	56	8
53	72	43	15	87	59	30	02	74	45	7
54	70	41	11	82	52	22	6.3498	63	34	6
55	69	38	08	77	46	15	84	54	23	5
56	68	36	04	72	40	08	76	44	12	4
57	67	34	00	67	34	01	68	34	01	3
58	66	31	2.7197	62	28	5.4393	59	24	8.1590	2
59	64	29	93	57	22	86	50	14	79	1
60	0.9063	1.8126	2.7189	3.6252	4.5316	5.4379	6.3442	7.2505	8.1568	0
1	2	3	4	5	6	7	8	9	10	11
DEPARTURE 65 DEGREES.										

DEPARTURE 24 DEGREES.										123
1	2	3	4	5	6	7	8	9	10	
0	0.4067	0.8135	1.2202	1.6270	2.0337	2.4404	2.8472	3.2539	3.6607	60
1	70	40	10	80	50	20	90	60	30	59
2	73	45	18	91	64	36	2.8507	82	54	58
3	75	51	26	1.6301	77	52	27	3.2602	78	57
4	78	56	34	12	90	68	46	24	3.6702	56
5	81	61	42	22	2.0403	84	64	45	25	55
6	83	67	50	33	17	2.4500	83	66	51	54
7	86	72	58	44	30	16	2.8602	88	74	53
8	89	77	66	54	43	32	20	3.2709	97	52
9	91	83	74	65	57	48	39	30	3.6822	51
10	94	88	92	76	70	63	57	51	45	50
11	97	93	90	86	83	80	76	73	69	49
12	99	98	98	97	97	95	94	94	93	48
13	0.4102	0.8204	1.2306	1.6408	2.0510	2.4611	2.8713	3.2815	3.6917	47
14	05	09	14	18	23	27	32	36	41	46
15	07	14	22	29	36	43	50	58	65	45
16	10	20	29	39	49	59	69	78	88	44
17	13	25	38	50	63	75	88	3.2900	3.7018	43
18	15	30	45	60	76	91	2.8806	21	36	42
19	18	36	53	71	89	2.4707	25	42	60	41
20	20	41	61	82	2.0602	22	43	63	84	40
21	23	46	69	92	16	39	62	85	3.7108	39
22	26	51	77	1.8503	29	54	80	3.8006	31	38
23	28	57	85	14	42	70	99	27	56	37
24	31	62	93	24	55	86	2.8917	48	79	36
25	34	67	1.2401	35	69	2.4802	36	70	3.7203	35
26	36	73	09	45	82	18	54	90	27	34
27	39	78	17	56	95	34	73	3.8112	51	33
28	42	83	25	66	2.0708	50	91	33	74	32
29	44	89	33	77	22	66	2.9010	54	99	31
30	47	94	41	88	35	81	28	75	3.7322	30
31	50	99	49	98	48	98	47	97	46	29
32	52	0.8304	57	1.6609	61	2.4913	65	3.3218	70	28
33	55	10	65	20	75	29	84	39	94	27
34	58	15	73	30	88	45	2.9103	60	3.7418	26
35	60	20	81	41	2.0801	61	21	82	42	25
36	63	21	88	51	14	77	40	3.8302	65	24
37	66	31	97	62	28	93	59	24	90	23
38	68	36	1.2504	72	41	2.5009	77	45	3.7513	22
39	71	41	12	83	54	24	95	66	36	22
40	73	47	20	94	67	40	2.9214	87	61	20
41	76	52	28	1.6704	80	56	32	3.8408	84	19
42	79	57	36	15	94	72	51	30	3.7608	18
43	81	63	44	25	2.0907	88	69	50	32	17
44	84	68	52	36	20	2.5104	88	72	56	16
45	87	73	60	46	33	20	2.9306	93	79	15
46	89	78	68	57	46	35	24	3.8514	3.7703	14
47	92	84	76	68	60	51	43	35	27	13
48	95	89	84	78	73	67	62	56	51	12
49	97	94	92	89	86	83	80	78	75	11
50	0.4200	0.8400	99	99	99	99	99	98	98	10
51	02	05	1.2607	1.6810	2.1012	2.5214	2.9417	3.8619	3.7822	9
52	05	10	15	20	26	31	36	41	46	8
53	08	15	23	31	39	46	54	62	69	7
54	10	21	31	42	52	62	73	83	94	6
55	13	26	39	52	65	78	91	3.8704	3.7917	5
56	16	31	47	62	78	94	2.9509	25	40	4
57	18	37	55	73	92	2.5310	28	46	65	3
58	21	42	63	84	2.1105	25	46	67	88	2
59	24	47	71	94	18	41	65	88	3.8012	1
60	0.4226	0.8452	1.2679	1.6905	2.1131	2.5357	2.9583	3.8310	3.8036	0
1	2	3	4	5	6	7	8	9		
LATITUDE 65 DEGREES.										

124 LATITUDE 25 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.9063	1.8126	2.7189	3.6252	4.5316	5.4379	6.3442	7.2505	8.1568	9.0631
1	62	24	85	47	09	71	33	7.2495	56	59
2	61	21	82	42	08	64	24	85	45	58
3	59	19	78	38	4.5297	56	16	75	35	57
4	58	16	75	33	91	49	07	66	24	56
5	57	14	71	28	85	41	3.3398	55	12	55
6	56	11	67	23	79	34	90	46	01	54
7	55	09	64	18	73	27	82	36	8.1491	53
8	53	06	60	13	66	19	72	26	79	52
9	52	04	56	08	60	12	64	16	68	51
10	51	01	52	03	54	04	55	06	56	50
11	50	1.8099	49	3.6198	48	5.4297	47	7.2396	46	49
12	48	97	45	93	42	90	38	86	35	48
13	47	94	41	88	35	83	29	76	23	47
14	46	92	37	83	29	75	21	66	12	46
15	45	89	34	78	23	68	12	57	01	45
16	43	87	30	73	17	60	03	46	8.1390	44
17	42	84	26	68	11	53	6.3295	37	79	43
18	41	82	22	63	04	45	86	26	67	42
19	40	79	19	58	4.5198	38	77	17	56	41
20	38	77	15	53	92	30	68	06	45	40
21	37	74	11	48	86	23	60	7.2297	34	39
22	36	72	07	43	79	15	51	86	22	38
23	35	69	04	38	73	08	43	77	11	37
24	33	67	00	34	67	00	34	67	01	36
25	32	64	1.7096	28	61	5.4193	25	57	8.1289	35
26	31	62	93	24	55	85	16	47	78	34
27	30	59	89	18	48	78	07	37	66	33
28	28	57	85	14	42	70	6.3199	27	56	32
29	27	54	81	08	36	63	90	17	44	31
30	26	52	78	04	30	55	81	07	33	30
31	25	49	74	3.6098	23	48	72	7.2197	21	29
32	23	47	70	93	17	40	63	86	10	28
33	22	44	66	88	10	33	55	77	8.1199	27
34	21	42	62	83	04	25	46	66	87	26
35	20	39	59	78	4.5098	18	37	56	76	25
36	18	37	55	73	92	10	28	46	65	24
37	17	34	51	68	86	03	20	36	54	23
38	16	32	47	63	79	5.4095	11	26	42	22
39	15	30	44	58	73	88	02	17	31	21
40	13	27	40	53	67	80	6.3093	06	20	20
41	12	24	36	48	60	72	84	7.2096	08	19
42	11	22	32	43	54	65	76	86	8.1097	18
43	10	19	29	38	48	57	67	76	86	17
44	08	16	25	33	41	49	57	66	74	16
45	07	14	21	28	35	42	49	56	63	15
46	06	11	17	23	29	34	40	46	51	14
47	05	09	14	18	23	27	31	36	41	13
48	03	06	10	13	16	19	22	26	29	12
49	02	04	06	08	10	11	13	15	17	11
50	01	01	02	03	04	04	05	06	06	10
51	0.8999	1.7999	2.6998	3.5998	4.4997	5.3997	6.2996	7.1995	8.0995	9
52	98	96	94	92	91	89	87	85	83	8
53	97	94	90	87	84	82	78	75	71	7
54	96	91	87	82	78	74	69	65	60	6
55	94	89	83	77	72	66	60	54	49	5
56	93	86	79	72	65	58	51	44	37	4
57	92	84	75	67	59	51	43	34	26	3
58	91	81	72	62	53	43	34	24	15	2
59	89	78	68	57	46	35	24	14	03	1
60	0.8988	1.7976	2.6964	3.5952	4.4940	5.3927	6.2915	7.1903	8.0891	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 64 DEGREES.

DEPARTURE 25 DEGREES.										125
1	2	3	4	5	6	7	8	9	10	1
0	4.2226	0.8452	1.2679	1.6905	2.1131	2.5357	2.9583	3.3810	3.8036	06
1	29	58	86	15	44	73	2.9602	30	59	59
2	32	63	95	26	58	89	21	52	84	58
3	34	68	1.2702	36	71	2.5405	39	73	3.8107	57
4	37	73	10	47	84	20	57	94	30	56
5	39	79	18	58	97	36	76	3.3915	55	55
6	42	84	26	68	2.1211	52	94	36	78	54
7	45	89	34	78	23	68	2.9712	57	3.8201	53
8	47	95	42	89	37	84	31	78	26	52
9	50	0.8500	50	1.7000	50	99	49	99	49	51
10	53	05	58	10	63	2.5515	68	3.4020	73	50
11	55	10	66	21	76	31	86	42	97	49
12	58	16	73	31	89	47	2.9805	62	3.8320	48
13	60	21	81	42	2.1302	62	23	83	44	47
14	63	26	89	52	16	79	42	3.4105	68	46
15	66	31	97	63	29	94	60	26	91	45
18	68	37	1.2805	73	42	2.5610	78	46	3.8415	44
17	71	42	13	84	55	25	.96	67	38	43
18	74	47	21	94	68	42	2.9915	89	62	42
19	76	52	29	1.7105	81	57	33	3.4210	86	41
20	79	58	36	15	94	73	52	30	3.8599	40
21	82	63	45	26	2.1408	89	71	52	34	39
22	84	68	52	36	21	2.5705	89	73	57	38
23	87	73	60	47	34	20	3.0007	94	80	37
24	89	79	68	58	47	36	26	3.4315	3.8605	36
25	92	84	76	68	60	52	44	36	28	35
26	95	89	84	78	73	68	62	57	51	34
27	97	94	92	89	86	83	80	78	75	33
28	0.4300	0.8600	1.2900	1.7200	2.1500	99	99	99	99	32
29	03	06	08	10	13	2.5815	3.0118	3.4420	3.8723	31
30	05	10	15	20	26	31	36	41	46	30
31	08	15	23	31	39	46	54	62	69	29
32	10	21	31	42	52	62	73	83	94	28
33	13	26	39	52	65	78	91	3.4504	3.8817	27
34	16	31	47	62	78	94	3.0209	25	40	26
35	18	36	55	73	91	2.5909	27	46	64	25
36	21	42	63	84	2.1605	25	46	67	88	24
37	24	47	71	94	18	41	65	88	3.8912	23
38	26	52	78	1.7304	31	57	83	3.4609	35	22
39	29	57	86	15	44	72	3.0301	30	58	21
40	31	63	94	25	57	88	19	50	82	20
41	34	68	1.3002	36	70	2.6004	38	72	3.9006	19
42	37	73	10	46	83	20	56	93	29	18
43	39	78	18	57	96	35	74	3.4714	53	17
44	42	84	25	67	2.1709	51	93	34	76	16
45	45	89	34	78	23	67	3.0412	56	3.9101	15
46	47	94	41	88	36	83	30	77	88	14
47	50	99	49	99	49	98	48	98	47	13
48	52	0.8705	57	1.7409	62	2.6114	66	3.4819	71	12
49	55	10	65	20	75	29	84	39	94	11
50	58	15	73	30	88	45	3.0503	60	3.9218	10
51	60	20	81	41	2.1801	61	21	82	42	9
52	63	26	88	51	14	77	40	3.4902	65	8
53	65	31	96	62	27	92	58	23	89	7
54	68	36	1.3104	72	40	2.6208	76	44	3.9312	6
55	71	41	12	82	53	24	94	65	35	5
55	73	47	20	93	87	40	3.0213	86	60	4
57	76	52	28	1.7504	80	55	31	3.5007	88	3
58	79	57	36	14	93	71	50	28	3.9407	2
59	81	62	43	24	2.1906	87	68	49	30	1
60	0.4384	0.8767	1.3151	1.7535	2.1919	2.6302	3.0686	3.5070	3.9453	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 64 DEGREES.										

126										
LATITUDE 26 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.8988	1.7976	2.6964	3.5952	4.4940	5.3927	6.2915	7.1903	8.0891	60
1	87	73	60	47	33	20	70	7.1894	80	59
2	85	71	56	42	27	12	6.2898	83	69	58
3	83	68	52	36	21	05	89	73	57	57
4	82	66	48	31	14	5.3897	80	62	45	56
5	81	63	45	26	08	90	71	53	34	55
6	80	61	41	21	02	82	62	42	23	54
7	79	58	37	16	4.4895	74	53	32	11	53
8	78	55	33	11	89	66	44	22	8.0799	52
9	76	53	29	06	82	58	35	11	88	51
10	75	50	26	01	76	51	26	02	77	50
11	74	48	22	3.5896	70	43	17	7.1791	65	49
12	73	45	18	90	63	36	08	81	53	48
13	71	43	14	85	57	28	6.2799	70	42	47
14	70	40	10	80	50	20	90	60	30	46
15	69	37	06	75	44	12	81	50	18	45
16	67	35	02	70	37	04	72	39	07	44
17	66	32	2.6899	65	31	5.3797	63	30	8.0696	43
18	65	30	95	60	25	89	54	19	84	42
19	64	27	91	54	18	82	45	09	72	41
20	62	25	87	49	12	74	36	7.1698	61	40
21	61	22	83	44	05	66	27	88	49	39
22	60	19	79	39	4.4799	58	18	78	37	38
23	58	17	75	34	92	50	09	67	25	37
24	57	14	71	28	86	43	00	57	14	36
25	56	12	68	23	79	35	6.2691	46	02	35
26	55	09	64	18	73	27	82	36	8.0591	34
27	53	06	60	13	66	19	72	26	79	33
28	52	04	56	08	60	11	63	15	67	32
29	51	01	52	02	53	04	54	05	55	31
30	49	1.7899	48	3.5797	47	5.3696	45	7.1594	44	30
31	48	96	44	92	40	88	36	84	32	29
32	47	93	40	87	34	80	27	74	20	28
33	45	91	36	82	27	72	18	63	09	27
34	44	88	32	76	21	65	09	53	97	26
35	43	86	28	71	14	57	00	42	8.0485	25
36	42	83	25	66	08	49	6.2591	32	74	24
37	40	80	21	61	01	41	81	22	62	23
38	39	78	17	56	4.4695	33	72	11	50	22
39	38	75	13	50	88	26	63	01	38	21
40	36	73	09	45	82	18	54	7.1490	27	20
41	35	70	05	40	75	10	45	80	15	19
42	34	67	01	35	69	02	36	70	03	18
43	32	65	2.6797	30	62	5.3594	27	59	8.0392	17
44	31	62	93	24	56	87	18	49	80	16
45	30	60	89	19	49	79	09	38	68	15
46	29	57	86	14	43	71	00	28	57	14
47	27	54	82	09	36	63	6.2490	18	45	13
48	26	52	78	04	30	55	81	07	33	12
49	25	49	74	3.5698	23	47	71	7.1396	21	11
50	23	46	70	93	16	39	62	86	09	10
51	22	44	66	88	10	32	53	75	8.0297	9
52	21	41	62	82	03	24	44	65	85	8
53	19	39	58	77	4.4597	16	35	54	74	7
54	18	36	54	72	90	08	26	44	62	6
55	17	33	50	67	84	00	17	34	50	5
56	15	31	46	61	77	5.3492	07	22	38	4
57	14	28	42	56	70	84	6.2398	12	26	3
58	13	25	38	51	64	76	89	02	14	2
59	11	23	34	46	57	68	80	7.1291	03	1
60	0.8910	1.7820	2.6780	3.5640	4.4551	5.3461	6.2371	7.1281	8.0191	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 68 DEGREES.										

DEPARTURE 26 DEGREES.										127
1	2	3	4	5	6	7	8	9	10	11
0	0.4384	0.8767	1.3151	1.7535	2.1919	2.6302	3.0686	3.5070	3.9453	60
1	86	73	59	45	32	18	3.0704	90	77	59
2	89	78	67	56	45	33	22	3.5111	3.9500	58
3	92	83	75	66	58	50	41	33	24	57
4	94	88	83	77	71	65	59	54	48	56
5	97	94	90	87	84	81	80	74	71	55
6	99	99	98	98	97	96	96	95	95	54
7	0.4402	0.8804	1.3206	1.7608	2.2010	2.6412	3.0814	3.5216	3.9618	53
8	05	09	14	18	23	28	32	37	41	52
9	07	14	22	29	36	43	50	58	65	51
10	10	20	29	39	49	59	69	78	88	50
11	12	25	37	50	62	74	87	99	3.9712	49
12	15	30	45	60	76	91	3.0906	3.5321	36	48
13	18	35	53	71	89	2.6506	24	42	57	47
14	20	41	61	81	2.2102	22	42	62	83	46
15	23	46	69	92	15	37	60	83	3.9806	45
16	26	51	77	1.7702	28	53	79	3.5404	30	44
17	28	56	84	12	41	69	97	25	53	43
18	31	61	92	23	54	84	3.1015	46	76	42
19	33	67	1.3300	33	67	2.6600	33	66	3.9900	41
20	36	72	08	44	80	15	51	87	23	40
21	39	77	16	54	93	31	70	3.5508	47	39
22	41	82	23	64	2.2206	47	88	29	70	38
23	44	87	31	75	19	62	3.1106	50	93	37
24	46	93	39	86	32	78	25	71	4.0018	36
25	49	98	47	96	45	94	43	92	41	35
26	52	0.8903	55	1.7806	58	2.6710	61	3.5613	64	34
27	54	08	63	17	71	25	79	34	88	33
28	57	14	70	27	84	41	98	54	4.0111	32
29	59	19	78	38	97	56	3.1216	75	35	31
30	62	24	86	48	2.2310	72	34	96	58	30
31	65	29	94	58	23	88	52	3.5717	81	29
32	67	34	1.3402	69	36	2.6803	70	38	4.0205	28
33	70	40	09	79	49	19	09	58	28	27
34	72	45	17	90	62	34	3.1307	79	52	26
35	75	50	25	1.7900	75	50	25	3.5800	75	25
36	78	55	33	10	88	66	43	18	98	24
37	80	60	41	21	2.2401	81	61	42	4.0322	23
38	83	66	48	31	14	97	80	62	45	22
39	85	71	56	42	27	2.6912	98	83	69	21
40	88	76	64	52	40	28	3.1416	3.5904	92	20
41	91	81	72	62	53	44	34	25	4.0415	19
42	93	86	80	73	66	59	52	46	39	18
43	96	92	87	83	79	75	71	66	62	17
44	98	97	95	94	92	90	89	87	86	16
45	0.4501	0.9002	1.3503	1.8004	2.2505	2.7006	3.1507	3.6008	4.0509	15
46	04	07	11	14	18	22	25	29	32	14
47	06	12	19	25	31	37	43	50	56	13
48	09	18	26	35	44	53	62	70	79	12
49	11	23	34	46	57	68	80	91	4.0608	11
50	14	28	42	56	70	84	98	3.6112	26	10
51	17	33	50	66	83	2.7100	3.1616	33	49	9
52	19	38	58	77	96	15	34	54	73	8
53	22	44	65	87	2.2609	31	53	74	96	7
54	24	49	73	97	22	46	70	94	4.0719	6
55	27	54	81	1.8108	35	61	88	3.6215	42	5
56	30	59	89	18	48	77	3.1707	26	66	4
57	32	64	96	28	61	93	25	57	89	3
58	35	69	1.3604	39	74	2.7208	43	78	4.0812	2
59	37	75	12	49	87	24	61	98	86	1
60	0.4540	0.9080	1.3620	1.8160	2.2700	2.7239	3.1779	3.6319	4.0859	0
1	2	3	4	5	6	7	8	9	10	11
LATITUDE 68 DEGREES.										

128										
LATITUDE 27 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.8910	1.7820	2.6730	3.5640	4.4551	5.3461	6.2371	7.1281	8.0191	60
1	09	17	26	35	44	52	61	70	78	59
2	07	15	22	30	37	44	52	59	67	58
3	06	12	18	24	31	37	43	49	55	57
4	05	10	14	19	24	29	34	38	43	56
5	04	07	10	14	18	21	25	28	32	55
6	02	04	06	08	11	13	15	17	19	54
7	01	02	2.6602	03	04	05	06	06	07	53
8	00	1.7799	99	3.5598	4.4498	5.3397	6.2297	7.1196	8.0096	52
9	0.8898	96	94	92	91	89	87	85	83	51
10	97	94	90	87	84	81	78	74	71	50
11	96	91	87	82	78	73	69	64	60	49
12	94	88	83	77	71	65	60	54	48	48
13	93	86	78	71	64	57	50	42	35	47
14	92	83	75	66	58	49	41	32	24	46
15	90	80	71	61	51	41	31	22	12	45
16	89	78	66	55	44	33	22	10	7.9999	44
17	88	75	63	50	38	25	13	01	88	43
18	86	72	59	45	31	17	03	7.1090	76	42
19	85	70	54	39	24	09	6.2194	78	63	41
20	84	67	51	34	18	01	85	68	52	40
21	82	64	47	29	11	5.3293	75	58	40	39
22	81	62	42	23	04	85	66	46	27	38
23	80	59	39	18	4.4398	77	57	36	16	37
24	78	56	35	13	91	69	47	26	04	36
25	77	54	30	07	84	61	38	14	7.9891	35
26	76	51	27	02	78	53	29	04	80	34
27	74	48	22	3.5496	71	45	19	7.0993	67	33
28	73	46	18	91	64	37	10	82	55	32
29	72	43	15	86	58	29	01	72	44	31
30	70	40	10	80	51	21	6.2091	61	31	30
31	69	38	06	75	44	13	82	50	19	29
32	67	35	02	70	37	04	72	39	07	28
33	66	32	2.6598	64	31	5.3197	63	29	7.9795	27
34	65	29	94	59	24	88	53	18	82	26
35	63	27	90	54	17	80	44	07	71	25
36	62	24	86	48	10	72	34	7.0896	58	24
37	61	21	82	43	04	61	25	86	46	23
38	59	19	78	37	4.4297	56	15	74	34	22
39	58	16	74	32	90	48	06	64	22	21
40	57	13	70	26	83	40	6.1996	58	09	20
41	55	11	66	21	77	32	87	42	7.9698	19
42	54	08	62	16	70	23	77	31	85	18
43	53	05	58	10	63	16	68	21	73	17
44	51	02	54	05	56	07	58	10	61	16
45	50	00	50	00	50	5.3099	49	7.0799	49	15
46	49	1.7697	46	3.5394	43	91	40	88	37	14
47	47	94	42	89	36	83	30	78	25	13
48	46	92	37	83	29	75	21	66	12	12
49	45	89	34	78	23	67	12	56	01	11
50	43	86	29	72	16	59	02	45	7.9588	10
51	42	83	25	67	09	50	6.1892	34	75	9
52	40	81	21	62	02	42	83	23	64	8
53	39	78	17	56	4.4195	34	73	12	51	7
54	38	75	13	51	89	26	64	02	39	6
55	36	73	09	45	82	18	54	7.0690	27	5
56	35	70	05	40	75	09	44	79	14	4
57	34	67	01	34	68	02	35	69	02	3
58	32	64	2.6497	29	61	5.2993	25	58	7.9490	2
59	31	62	92	23	54	85	16	46	77	1
60	0.8830	1.7659	2.6489	3.5318	4.4148	5.2977	6.1807	7.0636	7.9466	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 62 DEGREES.										

DEPARTURE 27 DEGREES.									
									129
1	2	3	4	5	6	7	8	9	10
0	0.4540	0.9080	1.3620	1.8160	2.2700	2.7239	3.1779	3.6319	4.0859
1	43	85	28	70	113	155	198	240	283
2	45	90	35	80	126	171	216	261	306
3	48	95	43	91	139	186	234	282	330
4	50	0.9101	51	1.8201	52	2.7302	52	3.6402	53
5	53	06	59	12	65	17	70	23	76
6	55	11	66	22	77	32	88	43	99
7	58	16	74	32	90	48	3.1906	64	4.1022
8	61	21	82	42	2.2803	64	24	85	45
9	63	26	90	53	16	79	42	3.6506	69
10	66	32	97	63	29	95	61	26	92
11	68	37	1.3705	74	42	2.7410	79	47	4.1116
12	71	42	13	84	55	26	97	68	39
13	74	47	21	94	68	42	3.2015	89	62
14	76	52	29	1.8305	81	57	33	3.6610	86
15	79	57	36	15	94	72	51	30	4.1208
16	81	63	44	25	2.2907	88	69	50	32
17	84	68	52	36	20	2.7503	87	71	55
18	87	73	60	46	33	19	3.2105	92	79
19	89	78	67	56	46	35	24	3.6713	4.1302
20	92	83	75	67	59	50	42	34	25
21	94	88	83	77	71	65	59	54	48
22	97	94	90	87	84	81	78	74	71
23	99	99	98	98	97	96	96	95	95
24	0.4602	0.9204	1.3806	1.8408	2.3010	2.7612	3.2214	3.6816	4.1418
25	05	09	14	18	23	28	32	37	41
26	07	14	22	29	36	43	50	58	65
27	10	19	29	39	49	58	68	78	87
28	12	25	37	49	62	74	86	98	4.1511
29	15	30	45	60	75	89	3.2304	3.6919	34
30	18	35	53	70	88	2.7705	23	40	58
31	20	40	60	80	2.3101	21	41	61	81
32	23	45	68	90	13	36	58	81	4.1603
33	25	50	76	1.8501	26	51	76	3.7002	27
34	28	56	83	11	39	67	95	22	50
35	30	61	91	22	52	82	3.2413	43	74
36	33	66	99	32	65	98	31	64	97
37	36	71	1.3907	42	78	2.7813	49	84	4.1720
38	38	76	14	52	91	29	67	3.7105	43
39	41	81	22	63	2.3204	44	85	26	66
40	43	87	30	73	17	60	3.2503	46	91
41	46	92	37	83	29	75	21	66	4.1812
42	48	97	45	94	42	90	39	87	36
43	51	0.9302	53	1.8604	55	2.7906	57	3.7208	59
44	54	07	61	14	68	22	75	29	82
45	56	12	68	24	81	37	93	49	4.1905
46	59	17	76	35	94	52	3.2611	70	28
47	61	23	84	45	2.3307	68	29	90	52
48	64	28	92	56	20	83	47	3.7311	75
49	66	33	99	66	32	98	65	31	98
50	69	38	1.4007	76	45	2.8014	83	52	4.2021
51	72	43	15	86	58	30	3.2701	73	44
52	74	48	23	97	71	45	19	94	68
53	77	53	30	1.8707	84	60	37	3.7414	90
54	79	59	38	17	97	76	55	34	4.2114
55	82	64	46	28	2.3410	91	73	55	37
56	84	69	53	38	22	2.8106	91	75	60
57	87	74	61	48	35	22	3.2809	96	83
58	90	79	70	58	48	38	27	3.7517	4.2206
59	92	84	76	68	61	53	45	37	29
60	0.4695	0.9389	1.4084	1.8779	2.3474	2.8168	3.2863	3.7558	4.2252
1	2	3	4	5	6	7	8	9	

LATITUDE 62 DEGREES.

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1	2	3	4	5	6	7	8	9	10
0	0.8680	1.7659	2.6489	3.5318	4.4148	5.2977	6.1807	7.0636	7.9466
1	28	56	84	12	41	69	6.1797	25	53
2	27	53	80	07	34	60	87	14	40
3	25	51	76	02	27	52	78	03	29
4	24	48	72	3.5296	20	44	68	7.0592	16
5	23	45	68	90	13	36	58	81	03
6	21	43	64	85	07	28	49	70	7.9391
7	20	40	60	80	00	19	39	59	79
8	19	37	56	74	4.4093	11	30	48	67
9	17	34	52	69	86	03	21	38	55
10	16	32	47	63	79	5.2895	11	26	42
11	14	29	43	58	72	86	01	15	30
12	13	26	39	52	65	78	6.1691	04	17
13	12	23	35	47	59	70	82	7.0494	05
14	10	21	31	41	52	62	72	82	7.9293
15	09	18	27	36	45	53	62	71	80
16	08	15	23	30	38	45	53	60	68
17	06	12	19	25	31	37	43	50	56
18	05	10	14	19	24	29	34	38	43
19	03	07	10	14	17	20	24	27	31
20	02	04	06	08	10	12	14	16	18
21	01	01	02	02	03	04	04	05	05
22	0.8799	1.7599	2.6398	3.5197	4.3997	5.2796	6.1595	7.0394	7.9194
23	98	96	94	92	90	87	85	83	81
24	97	93	90	86	83	79	76	72	69
25	95	90	85	80	76	70	66	61	56
26	94	87	81	75	69	62	56	50	43
27	92	85	77	69	63	53	46	38	31
28	91	82	73	64	55	45	36	27	18
29	90	79	69	58	48	38	27	17	06
30	88	76	65	53	41	29	17	06	7.9094
31	87	74	60	47	34	21	08	7.0294	81
32	85	71	56	42	27	12	6.1498	83	69
33	84	68	52	36	20	04	88	72	56
34	83	65	48	30	13	5.2696	78	61	43
35	81	62	44	25	06	87	68	50	31
36	80	60	39	19	00	79	59	38	18
37	78	57	35	14	4.3892	70	49	27	06
38	77	54	31	08	85	62	39	16	7.8993
39	76	52	27	02	78	54	29	05	80
40	74	49	23	3.5097	72	46	20	7.0194	69
41	73	46	19	92	65	37	10	83	56
42	72	43	15	86	58	29	01	72	44
43	70	40	10	80	51	21	6.1391	61	31
44	69	37	06	75	44	12	81	50	18
45	67	35	02	69	37	04	71	38	06
46	66	32	2.6298	64	30	5.2595	61	27	7.8893
47	65	29	94	58	23	87	52	16	81
48	63	26	89	52	16	79	41	05	68
49	62	23	85	47	09	70	32	7.0094	55
50	60	21	81	41	02	62	22	82	43
51	59	18	77	36	4.3795	53	12	71	30
52	58	15	73	30	88	45	03	60	18
53	56	12	68	24	81	37	6.1293	49	05
54	55	09	64	18	73	28	82	37	7.8791
55	53	06	60	13	66	19	72	26	79
56	52	04	55	07	59	11	63	14	66
57	50	01	51	02	52	02	53	03	54
58	49	98	47	3.4996	45	5.2494	43	6.9992	41
59	48	1.7495	43	90	38	86	33	81	28
60	0.8746	1.7492	2.6239	3.4985	4.3731	5.2477	6.1223	6.9970	7.8716
	1	2	3	4	5	6	7	8	9

DEPARTURE 61 DEGREES.

DEPARTURE 28 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.4695	0.9389	1.4084	1.8779	2.3474	2.8168	3.2863	3.7558	4.2252
1	97	95	92	89	87	84	81	78	76
2	0.4700	0.9400	1.4100	1.8800	2.3500	2.8214	3.2917	3.7629	4.2322
3	02	05	07	10	12	15	18	21	24
4	05	10	15	20	25	30	35	40	45
5	08	15	23	30	38	46	53	61	68
6	10	20	30	40	51	61	71	81	91
7	13	25	38	51	64	76	89	3.7702	4.2414
8	15	31	46	61	77	92	3.3007	22	38
9	18	36	53	71	89	2.8307	25	42	60
10	20	41	61	82	2.3602	22	43	63	84
11	23	46	69	92	15	37	60	83	4.2506
12	26	51	77	1.8902	28	53	79	3.7804	30
13	28	56	84	12	41	69	97	25	53
14	31	61	92	22	53	84	3.3114	45	75
15	33	66	1.4200	33	66	99	32	66	99
16	36	72	07	43	79	2.8415	51	86	4.2622
17	38	77	15	53	92	30	68	3.7906	45
18	41	82	23	64	2.3705	45	86	27	68
19	43	87	30	74	17	60	3.3204	47	91
20	46	92	38	84	30	76	22	68	4.2714
21	49	97	46	94	43	92	40	89	37
22	51	0.9502	53	1.9004	56	2.8507	58	3.8009	60
23	54	07	61	15	69	22	76	30	83
24	56	12	69	25	81	37	93	50	4.2806
25	59	18	76	35	94	53	3.3312	70	29
26	61	23	84	46	2.3807	68	30	91	53
27	64	28	92	56	20	83	47	3.8111	75
28	67	33	1.4300	66	33	99	66	32	99
29	69	38	07	76	45	2.8614	83	52	4.2921
30	72	43	15	86	58	30	3.3401	73	44
31	74	48	22	96	71	45	19	93	67
32	77	53	30	1.9107	84	60	37	3.8214	90
33	79	59	38	17	97	76	55	34	4.3014
34	82	64	45	27	2.3909	91	73	54	36
35	84	69	53	38	22	2.8706	91	75	60
36	87	74	61	48	35	21	3.3508	95	82
37	90	79	69	58	48	37	27	3.8316	4.3106
38	92	84	76	68	60	52	44	36	28
39	95	89	84	78	73	68	62	57	51
40	97	94	91	88	86	83	80	77	74
41	0.4800	99	99	99	99	98	98	98	97
42	02	0.9604	1.4407	1.9209	2.4011	2.8813	3.3615	3.8418	4.3220
43	05	10	14	19	24	29	34	38	43
44	07	15	22	29	37	44	51	58	66
45	10	20	30	40	50	59	69	79	89
46	12	25	37	50	62	74	87	99	4.3312
47	15	30	45	60	75	90	3.3705	3.8520	35
48	18	35	53	70	88	2.8905	23	40	58
49	20	40	60	80	2.4101	21	41	61	81
50	22	45	68	90	13	36	58	81	4.3403
51	25	50	76	1.9301	26	51	76	3.8602	27
52	28	55	83	11	39	66	94	22	49
53	30	61	91	21	52	82	3.3812	42	73
54	33	66	98	31	64	97	30	62	95
55	35	71	1.4506	42	77	2.9012	48	83	4.3519
56	38	76	14	52	90	27	65	3.8703	41
57	41	81	22	62	2.4203	43	84	24	65
58	43	86	29	72	15	58	3.3901	44	87
59	46	91	37	82	28	74	19	65	4.3610
60	0.4848	0.9696	1.4544	1.9392	2.4241	2.9089	3.3937	3.8785	4.3633
1	2	3	4	5	6	7	8	9	10
LATITUDE 61 DEGREES.									

LATITUDE 29 DEGREES.										
132	1	2	3	4	5	6	7	8	9	'
0	0.8746	1.7492	2.6239	3.4985	4.3731	5.2477	6.1223	6.9970	7.8716	60
1	45	90	34	79	24	69	14	58	03	59
2	43	87	30	74	17	60	04	47	7.8691	58
3	42	84	26	68	10	52	6.1194	36	78	57
4	41	81	22	62	03	44	84	25	65	56
5	39	78	17	56	4.3696	35	74	13	52	55
6	38	75	13	51	89	26	64	02	39	54
7	36	73	09	45	82	18	54	6.9890	27	53
8	35	70	05	40	75	09	44	79	14	52
9	33	67	01	34	68	01	35	68	02	51
10	32	64	2.6196	28	61	5.2393	25	57	7.8589	50
11	31	61	92	22	53	84	14	45	75	49
12	29	58	88	17	46	75	04	34	63	48
13	28	56	83	11	39	67	6.1095	22	50	47
14	26	53	79	06	32	58	85	11	38	46
15	25	50	75	00	25	50	75	6.9700	25	45
16	24	47	71	3.4894	18	41	65	88	12	44
17	22	44	66	88	11	33	55	77	7.8499	43
18	21	41	62	83	04	24	45	66	86	42
19	19	39	58	77	4.3597	16	35	54	74	41
20	18	36	53	71	89	07	25	42	60	40
21	16	33	49	66	82	5.2298	15	31	48	39
22	15	30	45	60	75	90	05	20	35	38
23	14	27	41	54	68	82	6.0995	09	22	37
24	12	24	36	48	61	73	85	6.9697	09	36
25	11	21	32	43	54	64	75	86	7.8396	35
26	09	19	28	37	47	56	65	74	84	34
27	08	16	24	32	40	47	55	63	71	33
28	06	13	19	26	32	38	45	51	58	32
29	05	10	15	20	25	30	35	40	45	31
30	04	07	11	14	18	22	25	29	32	30
31	02	04	06	08	11	13	15	17	19	29
32	01	01	02	03	04	04	05	06	06	28
33	0.8699	1.7399	2.6098	3.4797	4.3497	5.2196	6.0895	6.9594	7.8294	27
34	98	96	93	91	89	87	85	82	80	26
35	96	93	89	86	82	78	75	71	68	25
36	95	90	85	80	75	69	64	59	54	24
37	94	87	81	74	68	61	55	48	42	23
38	92	84	76	68	61	53	45	37	29	22
39	91	81	72	62	53	44	34	25	15	21
40	89	78	68	57	46	35	24	14	03	20
41	88	76	63	51	39	27	15	02	7.8190	19
42	86	73	59	45	32	18	04	6.9490	77	18
43	85	70	55	40	25	09	6.0794	79	64	17
44	83	67	50	34	17	00	84	67	51	16
45	82	64	46	28	10	5.2092	74	56	38	15
46	81	61	42	22	03	83	64	44	25	14
47	79	58	37	16	4.3396	75	54	33	12	13
48	78	55	33	11	89	66	44	22	7.8099	12
49	76	52	29	05	81	57	33	20	86	11
50	75	50	24	3.4699	74	49	24	6.9398	73	10
51	73	47	20	93	67	40	13	86	60	9
52	72	44	16	88	60	31	03	75	47	8
53	70	41	11	82	52	22	6.0693	63	34	7
54	69	38	07	76	45	14	83	52	21	6
55	68	35	03	70	38	05	73	40	08	5
56	66	32	2.5998	64	31	5.1997	63	29	7.7995	4
57	65	29	94	58	23	88	52	17	81	3
58	63	26	90	53	16	79	42	06	69	2
59	62	23	85	47	09	70	32	94	55	1
60	0.8660	1.7321	2.5981	3.4641	4.3302	5.1962	6.0622	6.9282	7.7948	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 60 DEGREES.										

DEPARTURE 29 DEGREES.										133
1	2	3	4	5	6	7	8	9	10	
0	0.4848	0.9696	1.4544	1.9392	2.4241	2.9089	3.3937	3.8785	4.3633	60
1	51	0.9701	52	1.9402	53	2.9104	54	3.8805	55	59
2	53	06	60	13	66	19	72	26	79	58
3	56	11	67	23	79	84	90	46	4.8701	57
4	58	17	75	33	92	50	3.4008	66	25	56
5	61	22	82	43	2.4304	65	26	86	47	55
6	63	27	90	54	17	80	44	3.8907	71	54
7	66	32	98	64	30	95	61	27	93	53
8	68	37	1.4605	74	42	2.9210	79	47	4.3816	52
9	71	42	13	84	55	26	97	68	39	51
10	74	47	21	94	68	41	3.4115	88	62	50
11	76	52	28	1.9504	81	57	33	3.9009	85	49
12	79	57	37	14	93	72	50	29	4.3907	48
13	81	62	43	24	2.4406	87	68	49	30	47
14	84	67	51	35	19	2.9302	86	70	53	46
15	86	72	59	45	31	17	3.4203	90	76	45
16	89	78	66	55	44	33	22	3.9110	99	44
17	91	83	74	65	57	48	39	30	4.4022	43
18	94	88	81	75	69	63	57	50	44	42
19	96	93	89	86	82	78	75	71	68	41
20	99	98	97	96	95	93	92	91	90	40
21	0.4901	0.9803	1.4704	1.9606	2.4507	2.9408	3.4310	3.9211	4.4113	39
22	04	08	12	16	20	24	28	32	36	38
23	07	13	20	26	33	39	46	52	59	37
24	09	18	27	36	45	54	63	72	81	36
25	12	23	35	46	58	70	81	93	4.4204	35
26	14	28	42	56	71	85	99	3.9313	27	34
27	17	33	50	66	83	80	3.4416	33	49	33
28	19	38	58	77	96	2.9515	34	54	73	32
29	22	43	65	87	2.4609	30	52	74	95	31
30	24	48	73	97	21	45	69	94	4.4318	30
31	27	54	80	1.9707	34	61	88	3.9414	41	29
32	29	59	88	17	47	76	3.4505	34	64	28
33	32	64	95	27	59	91	23	54	86	27
34	34	69	1.4803	38	72	2.9606	41	75	4.4410	26
35	37	74	11	48	85	21	58	95	32	25
36	39	79	18	58	97	36	76	3.9515	55	24
37	42	84	26	68	2.4710	51	93	35	77	23
38	45	89	34	78	23	67	3.4612	56	4.4501	22
39	47	94	41	88	35	82	29	76	23	22
40	50	99	49	98	48	97	47	96	46	20
41	52	0.9904	56	1.9808	61	2.9713	65	3.9617	69	19
42	55	09	64	18	73	28	82	37	91	18
43	57	14	71	28	86	43	3.4700	57	4.4614	17
44	60	19	79	38	98	58	17	77	36	16
45	62	24	87	49	2.4811	73	35	98	60	15
46	65	29	94	59	24	88	53	3.9718	82	14
47	67	34	1.4902	69	36	2.9803	70	38	4.4705	13
48	70	39	09	79	49	18	88	58	27	12
49	72	45	17	89	62	34	3.4806	78	51	11
50	75	50	24	99	74	49	24	98	73	10
51	77	55	32	1.9909	87	64	41	3.9818	96	9
52	80	60	39	19	99	79	59	38	4.4818	8
53	82	65	47	30	2.4912	94	77	59	42	7
54	85	70	55	40	25	2.9909	94	79	64	6
55	87	75	62	50	37	24	3.4912	99	87	5
56	90	80	70	60	50	39	29	3.9919	4.4909	4
57	92	85	77	70	62	54	47	39	32	3
58	95	90	85	80	75	70	65	60	55	2
59	98	95	98	90	88	85	83	80	78	1
60	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 60 DEGREES.										

134										
LATITUDE 30 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.8660	1.7321	2.5981	3.4641	4.3302	5.1962	6.0622	6.9282	6.7943	60
1	59	18	76	35	4.3294	53	12	70	29	59
2	57	15	72	29	87	44	01	58	16	58
3	56	12	68	24	80	35	6.0591	47	03	57
4	54	09	63	18	72	26	81	35	7.7890	56
5	53	06	59	12	65	18	71	24	77	55
6	52	03	55	06	58	09	61	12	64	54
7	50	00	50	00	51	01	51	01	51	53
8	49	1.7297	46	3.4594	43	5.1892	40	6.9189	37	52
9	47	94	41	88	36	83	30	77	24	51
10	46	91	37	83	29	74	20	66	11	50
11	44	88	33	77	21	65	09	54	7.7798	49
12	43	85	28	71	14	56	6.0499	42	84	48
13	41	83	24	65	07	48	89	30	72	47
14	40	80	19	59	4.3199	39	79	18	58	46
15	38	77	15	54	92	30	69	07	46	45
16	37	74	11	48	85	21	58	6.9095	32	44
17	35	71	06	42	77	12	48	83	19	43
18	34	68	02	36	70	04	38	72	06	42
19	33	65	2.5898	30	63	5.1795	28	60	7.7693	41
20	31	62	93	24	55	86	17	48	79	40
21	30	59	89	18	48	77	07	36	66	39
22	28	56	84	12	41	69	6.0397	25	53	38
23	27	53	80	06	33	60	86	13	39	37
24	25	50	75	00	26	51	76	01	26	36
25	24	47	71	3.4495	19	42	66	6.8990	13	35
26	22	44	67	89	11	33	55	78	00	34
27	21	41	62	83	04	24	45	66	7.7586	33
28	19	38	58	77	4.3096	15	34	54	73	32
29	18	36	53	71	89	07	25	42	60	31
30	16	33	49	65	82	5.1698	14	30	47	30
31	15	30	44	59	74	89	04	18	33	29
32	13	27	40	53	67	80	6.0293	06	20	28
33	12	24	36	48	60	71	83	6.8895	07	27
34	10	21	31	42	52	62	73	83	7.7494	26
35	09	18	27	36	45	53	62	71	80	25
36	07	15	22	30	37	44	52	59	67	24
37	06	12	18	24	30	35	41	47	53	23
38	05	09	14	18	23	27	32	36	41	22
39	03	06	09	12	15	18	21	24	27	21
40	02	03	05	06	08	09	11	12	14	20
41	00	00	00	00	00	00	00	00	00	19
42	0.8599	1.7197	2.5796	3.4394	4.2993	5.1591	6.0190	6.8788	7.7387	18
43	97	94	91	88	85	82	79	76	73	17
44	96	91	87	82	78	74	69	65	60	16
45	94	88	82	76	71	65	59	53	47	15
46	93	85	78	70	63	56	48	41	33	14
47	91	82	73	64	56	47	37	29	20	13
48	90	79	69	58	48	38	27	17	06	12
49	88	76	64	52	41	29	17	05	7.7293	11
50	87	73	60	46	33	20	06	6.8693	79	10
51	85	70	55	40	25	11	6.0096	81	66	9
52	84	67	51	34	18	02	85	69	52	8
53	82	64	46	28	10	5.1493	75	57	39	7
54	81	61	42	22	03	84	64	45	25	6
55	79	58	38	17	4.2895	75	54	34	18	5
56	78	55	33	11	88	66	44	22	7.7199	4
57	76	52	29	05	81	57	33	10	86	3
58	75	49	24	3.4299	74	48	23	98	72	2
59	73	46	20	93	66	39	12	6.8586	59	1
60	0.8572	1.7143	2.5715	3.4287	4.2859	5.1430	6.0002	6.8574	7.7145	0
1	2	3	4	5	6	7	8	9		
DEPARTURE 59 DEGREES.										

DEPARTURE 30 DEGREES.										135
1	2	3	4	5	6	7	8	9	10	
0	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000	60
1	03	05	08	10	13	15	18	20	23	59
2	05	11	15	21	25	30	35	40	45	58
3	08	15	23	30	38	46	53	61	68	57
4	10	20	30	40	51	61	71	81	91	56
5	13	25	38	50	63	76	88	4.0101	4.5113	55
6	15	30	45	60	76	91	3.5106	21	36	54
7	18	35	53	70	88	3.0106	23	41	58	53
8	20	40	60	80	2.5101	21	41	61	81	52
9	23	45	68	91	14	36	56	82	4.5204	51
10	25	50	76	2.0101	26	51	76	4.0202	27	50
11	28	55	83	11	39	66	94	22	49	49
12	30	60	91	21	51	81	3.5211	42	72	48
13	33	65	98	31	64	96	29	62	94	47
14	35	70	1.5106	41	76	3.0211	46	82	4.5317	46
15	38	75	13	51	89	26	64	4.0302	39	45
16	40	81	21	61	2.5202	42	82	22	63	44
17	43	86	28	71	14	57	00	42	85	43
18	45	91	36	81	27	72	3.5317	62	4.5408	42
19	48	96	43	91	39	87	35	82	30	41
20	50	1.0101	51	2.0201	52	3.0303	52	4.0402	53	40
21	53	06	58	11	64	17	70	22	75	39
22	55	11	66	21	77	32	87	42	98	38
23	58	16	73	31	89	47	3.5407	62	4.5520	37
24	60	21	81	41	2.5302	62	22	82	43	36
25	63	26	88	51	14	75	40	4.0502	65	35
26	65	31	96	62	27	92	58	23	89	34
27	68	36	1.5204	72	40	3.0408	75	43	4.5611	33
28	70	41	11	82	52	22	93	63	34	32
29	73	46	19	92	65	37	3.5503	83	56	31
30	75	51	26	2.0302	77	52	28	4.0603	79	30
31	78	56	34	12	90	67	45	23	4.5701	29
32	80	61	41	22	2.5402	82	63	43	24	28
33	83	66	49	32	15	97	80	63	46	27
34	85	71	56	42	27	3.0512	98	83	69	26
35	88	76	64	52	40	27	3.5615	4.0703	91	25
36	90	81	71	62	52	42	33	23	4.5814	24
37	93	86	79	72	65	57	50	43	36	23
38	95	91	86	82	77	72	68	63	59	22
39	98	96	94	92	90	87	85	83	81	21
40	0.5100	1.0201	1.5301	2.0402	2.5502	3.0602	3.5703	4.0803	4.5904	20
41	03	06	09	12	15	17	20	23	26	19
42	05	11	16	22	27	32	38	43	49	18
43	08	16	24	32	40	47	55	63	71	17
44	10	21	31	42	52	62	73	83	94	16
45	13	26	39	52	65	77	90	4.0903	4.6016	15
46	15	31	46	62	77	92	3.5808	23	39	14
47	18	36	54	72	90	3.0707	25	43	61	13
48	20	41	61	82	2.5602	22	43	63	84	12
49	23	46	69	92	15	37	60	83	4.6106	11
50	25	51	76	2.0502	27	52	78	4.1003	29	10
51	28	56	84	12	40	67	95	23	51	9
52	30	61	91	22	52	82	3.5913	43	74	8
53	33	66	99	32	65	97	30	63	96	7
54	35	71	1.5406	42	77	3.0812	48	83	4.6219	6
55	38	76	14	52	90	87	65	4.1103	41	5
56	40	81	21	62	2.5702	42	83	23	64	4
57	43	86	29	72	15	57	3.6000	43	86	3
58	45	91	36	82	27	72	18	63	4.6309	2
59	48	96	44	92	40	87	35	83	31	1
60	0.5150	1.0301	1.5451	2.0602	2.5752	3.0902	3.6053	4.1203	4.6354	0
1	2	3	4	5	6	7	8	9		

LATITUDE 59 DEGREES.

186 LATITUDE 31 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.8572	1.7143	2.5715	3.4287	4.2859	5.1430	6.0002	6.8574	7.7145
1	70	40	11	81	51	21	5.9991	62	82
2	69	37	06	75	44	12	81	50	18
3	67	34	02	69	36	03	70	38	05
4	66	31	2.5697	63	29	5.1894	60	26	7.7091
5	64	28	93	57	21	85	49	14	78
6	63	25	88	51	14	76	39	02	64
7	61	22	84	45	06	67	28	6.8490	51
8	60	19	79	39	4.2799	58	18	78	37
9	58	16	75	33	91	49	07	66	24
10	57	13	70	27	84	40	5.9897	54	10
11	55	10	65	20	76	31	86	41	7.6996
12	54	07	61	14	68	22	75	29	82
13	52	04	56	08	61	13	65	17	69
14	51	01	52	02	53	04	54	05	55
15	49	1.7098	47	3.4196	46	5.1295	44	6.8393	42
16	48	95	43	90	38	86	33	81	28
17	46	92	38	84	31	77	23	69	15
18	45	89	34	78	23	68	12	57	01
19	43	86	29	72	16	59	02	45	7.6888
20	42	83	25	66	08	50	5.9791	33	74
21	40	80	20	60	01	41	81	21	61
22	39	77	16	54	4.2693	31	70	08	47
23	37	74	11	48	85	22	59	6.8296	33
24	36	71	07	42	78	13	49	84	20
25	34	68	02	36	70	04	38	72	06
26	33	65	2.5598	30	63	5.1195	28	60	7.6793
27	31	62	93	24	55	86	17	48	79
28	29	59	88	18	47	76	06	35	65
29	28	56	84	12	40	67	5.9695	23	51
30	26	53	79	06	32	58	85	11	38
31	25	50	75	00	25	49	74	6.8199	24
32	23	47	70	3.4094	17	40	64	87	11
33	22	44	65	87	09	31	53	74	7.6696
34	20	41	61	81	02	22	42	62	83
35	19	38	56	75	4.2594	13	32	50	69
36	17	35	52	69	87	04	21	38	56
37	16	31	47	63	79	5.1094	10	26	41
38	14	28	43	57	71	85	5.9599	14	28
39	13	25	38	51	64	76	89	02	14
40	11	22	34	45	56	67	78	6.8090	01
41	10	19	29	38	48	58	67	77	7.6586
42	08	16	24	32	41	49	57	65	73
43	07	13	20	26	33	40	46	53	59
44	05	10	15	20	26	31	36	41	46
45	04	07	11	14	18	21	25	28	32
46	02	04	06	08	10	12	14	16	18
47	01	01	02	02	03	03	04	04	05
48	0.8499	1.6998	2.5497	3.3996	4.2495	5.0993	5.9492	6.7991	7.6490
49	97	95	92	90	87	84	82	79	77
50	96	92	88	84	80	75	71	67	63
51	94	89	83	77	72	66	60	54	49
52	93	86	78	71	64	57	50	42	35
53	91	83	74	65	57	48	39	30	22
54	90	79	69	59	49	38	28	18	07
55	88	76	65	53	41	29	17	06	7.6394
56	87	73	60	46	33	20	06	6.7893	79
57	85	70	55	40	26	11	5.9396	81	66
58	84	67	51	34	18	02	85	69	52
59	82	64	46	28	10	5.0892	74	56	38
60	0.8481	1.6961	2.5442	3.3922	4.2403	5.0883	5.9364	6.7844	7.6325
1	2	3	4	5	6	7	8	9	10

DEPARTURE 58 DEGREES.

DEPARTURE 81 DEGREES.										187
	1	2	3	4	5	6	7	8	9	
0	0.5150	1.0301	1.5451	2.0602	2.5752	3.0902	3.6053	4.1203	4.6354	60
1	53	06	59	12	65	17	70	23	76	59
2	55	11	66	22	77	32	88	43	99	58
3	58	16	74	32	90	47	3.6105	63	4.6421	57
4	60	21	81	42	2.5802	62	23	83	44	56
5	63	26	88	51	14	77	40	4.1302	65	55
6	65	31	96	61	27	92	57	22	88	54
7	68	36	1.5503	71	39	3.1007	74	42	4.6510	53
8	70	41	11	81	52	22	92	62	33	52
9	73	46	18	91	64	37	3.6209	82	55	51
10	75	51	26	2.0701	77	52	27	4.1402	78	50
11	78	56	33	11	89	67	44	22	9.6600	49
12	80	61	41	21	2.5902	82	62	42	23	48
13	83	66	43	31	14	97	80	62	45	47
14	85	70	56	41	26	3.1111	96	82	67	46
15	88	75	63	51	39	26	3.6314	4.1502	89	45
16	90	80	71	61	51	41	31	22	4.6712	44
17	93	85	78	71	64	56	49	42	34	43
18	95	90	86	81	76	71	66	62	57	42
19	98	95	93	91	89	86	84	82	79	41
20	0.5200	00	1.5601	2.0801	2.6001	3.1201	3.6401	4.1602	4.6802	40
21	03	1.0405	08	10	13	16	18	21	23	39
22	05	10	15	20	26	31	36	41	46	38
23	08	15	23	30	38	46	53	61	68	37
24	10	20	30	40	51	61	71	81	91	86
25	13	25	08	50	63	76	88	4.1701	4.6913	85
26	15	30	45	60	76	91	3.6506	21	36	84
27	18	35	53	70	88	3.1305	23	40	58	83
28	20	40	60	80	2.6100	20	40	60	80	82
29	23	45	60	90	13	35	58	80	4.7003	81
30	25	50	75	2.0900	25	50	75	4.1800	25	80
31	28	55	83	10	38	65	93	20	47	29
32	30	60	90	20	50	79	3.6609	39	69	28
33	32	65	97	30	63	94	27	59	92	27
34	35	70	1.5705	40	75	3.1409	44	79	4.7114	26
35	37	75	12	50	87	24	62	99	37	25
36	40	80	20	60	2.6200	39	79	4.1919	59	24
37	43	85	27	70	12	53	96	38	81	23
38	45	90	34	79	24	69	2.6714	58	4.7203	22
39	47	95	42	89	37	84	31	78	26	21
40	50	1.0500	49	99	49	99	49	98	48	20
41	52	04	57	2.1010	61	3.1514	65	4.2018	70	19
42	55	09	64	19	74	28	83	38	92	18
43	57	14	72	29	86	43	2.6800	58	4.7315	17
44	60	19	79	29	99	58	18	78	37	16
45	62	24	86	49	2.6311	73	35	98	59	15
46	65	29	94	48	28	88	52	4.2117	81	14
47	67	34	1.5801	68	36	3.1603	69	37	4.7404	13
48	70	39	09	78	48	18	87	57	26	12
49	72	44	16	88	60	32	2.6904	76	48	11
50	75	49	24	98	73	47	22	96	71	10
51	77	54	31	2.1108	85	62	39	4.2216	93	9
52	79	59	38	18	97	76	56	35	4.7515	8
53	82	64	46	28	2.6410	91	73	55	87	7
54	84	69	53	38	22	3.1706	91	75	60	6
55	87	74	61	48	35	21	2.7008	95	82	5
56	89	79	68	37	47	36	25	4.2314	4.7604	4
57	92	84	75	67	59	51	43	34	26	3
58	94	89	83	77	72	66	60	54	49	2
59	97	92	90	87	84	80	77	74	70	1
60	0.5299	1.0598	1.5898	2.1197	2.6496	3.1795	3.7094	4.2394	4.7698	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 58 DEGREES.										

188									
LATITUDE 32 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.8481	1.6961	2.5442	3.3922	4.2403	5.0883	5.9364	6.7844	7.6325
1	79	58	37	16	4.2395	73	52	31	10 59
2	77	55	32	10	87	64	42	19	7.6297 58
3	76	52	28	04	80	55	31	07	83 57
4	74	49	23	3.8897	72	46	20	6.7794	69 56
5	73	46	18	91	64	37	10	82	55 55
6	71	42	14	85	56	27	5.9298	70	41 54
7	70	39	09	79	49	18	88	58	27 53
8	68	36	04	72	41	09	77	45	13 52
9	67	33	00	66	33	00	66	33	7.6199 51
10	65	20	2.5395	60	25	5.0790	55	20	85 50
11	64	27	91	54	18	81	45	08	72 49
12	62	24	86	48	10	71	33	6.7695	57 48
13	60	21	81	42	02	62	23	83	44 47
14	59	18	76	35	4.2294	53	12	70	29 46
15	57	15	72	29	87	44	01	58	16 45
16	56	11	67	23	79	34	5.9190	46	01 44
17	54	08	63	17	71	25	79	34	7.6088 43
18	53	05	58	10	63	16	68	21	73 42
19	51	02	53	04	56	06	58	09	60 41
20	50	1.6899	49	3.3798	48	5.0697	47	6.7596	46 40
21	48	96	44	92	40	88	36	84	32 39
22	46	93	39	86	32	78	25	71	18 38
23	45	90	34	79	24	69	14	58	03 37
24	43	87	30	73	17	60	03	46	7.5990 36
25	42	83	25	67	09	50	5.9092	34	75 35
26	40	80	21	61	01	41	81	22	62 34
27	39	77	16	54	4.2193	32	70	09	47 33
28	37	74	11	48	85	22	59	6.7496	33 32
29	36	71	07	42	78	13	49	84	20 31
30	34	68	02	36	70	03	37	71	05 30
31	32	65	2.5297	30	62	5.0594	27	59	7.5892 29
32	31	62	92	23	54	85	16	46	77 28
33	29	58	88	17	46	75	04	34	63 27
34	28	55	83	11	39	66	5.8994	22	49 26
35	26	52	78	04	31	57	83	09	35 25
36	25	49	74	3.3698	23	47	72	6.7396	21 24
37	23	46	69	92	15	38	61	84	07 23
38	21	43	64	86	07	28	50	71	7.5793 22
39	20	40	59	79	4.2099	19	39	58	78 21
40	18	36	55	73	91	09	27	46	64 20
41	17	33	50	67	84	00	17	34	50 19
42	15	30	45	60	76	5.0491	06	21	36 18
43	14	27	41	54	68	81	5.8895	08	22 17
44	12	24	36	48	60	72	84	6.7296	08 16
45	10	21	31	42	52	62	73	83	7.5694 15
46	09	18	26	35	44	53	62	70	79 14
47	07	14	22	29	36	43	50	58	65 13
48	06	11	17	23	29	34	40	46	51 12
49	04	08	12	16	21	25	29	33	37 11
50	03	05	08	10	13	15	18	20	23 10
51	01	02	03	04	05	06	07	08	09
52	0.8399	1.6799	2.5198	3.3598	4.1997	5.0396	5.8796	2.7195	7.5595 8
53	98	96	93	91	89	87	85	82	80 7
54	96	92	89	85	81	77	73	70	66 6
55	95	89	84	78	73	68	62	57	51 5
56	93	86	79	72	65	58	51	44	37 4
57	92	83	75	66	58	49	41	32	24 3
58	90	80	70	60	50	39	29	19	09 2
59	88	77	65	53	42	30	18	06	7.5495 1
60	0.8387	1.6773	2.5160	3.3547	4.1934	5.0320	5.8707	6.7094	7.5480 0
	1	2	3	4	5	6	7	8	9
DEPARTURE 57 DEGREES.									

DEPARTURE 32 DEGREES.										189
1	2	3	4	5	6	7	8	9		
0	0.5299	1.0598	1.5898	2.1197	2.6496	3.1795	3.7094	4.2394	4.7693	60
1	0.5302	1.0603	1.5905	2.1207	2.6509	3.1810	3.7112	4.2414	4.7715	59
2	04	08	12	16	21	25	29	33	37	58
3	07	13	20	26	33	40	46	53	59	57
4	09	18	27	36	46	55	64	73	82	56
5	12	23	35	46	58	69	81	92	4.7804	55
6	14	28	42	56	70	84	98	4.2512	26	54
7	16	33	49	66	82	98	3.7215	31	48	53
8	19	38	57	76	95	3.1913	32	51	70	52
9	21	43	64	86	2.6607	28	50	71	93	51
10	24	48	71	95	19	43	67	90	4.7614	50
11	26	53	79	2.1305	32	58	84	4.2610	37	49
12	29	58	86	15	44	73	3.7302	30	59	48
13	31	62	94	25	56	87	18	50	81	47
14	34	67	1.6001	35	69	3.2002	36	70	4.8004	46
15	36	72	08	44	81	17	53	89	24	45
16	39	77	16	54	93	32	70	4.2709	47	44
17	41	82	23	64	2.6706	47	88	29	70	43
18	44	87	31	74	18	61	3.7405	48	92	42
19	46	92	38	84	30	76	22	68	4.8114	41
20	48	97	45	94	42	90	39	87	36	40
21	51	1.0702	53	5.1404	55	3.2105	56	4.2807	58	39
22	53	07	60	14	67	20	74	27	81	38
23	56	12	67	24	79	35	91	46	4.8202	37
24	58	17	75	33	92	50	3.7508	66	25	36
25	61	21	82	43	2.6804	64	25	86	46	35
26	63	26	90	53	16	79	42	4.2906	69	34
27	66	31	97	62	28	94	59	25	90	33
28	68	36	1.6104	72	41	3.2209	77	45	4.8313	32
29	71	41	12	82	53	23	94	64	35	31
30	73	46	19	92	65	38	3.7611	84	57	30
31	75	51	26	2.1502	87	52	28	4.3003	79	29
32	78	56	34	12	90	67	45	23	4.8401	28
33	80	61	41	22	2.6902	82	63	43	24	27
34	83	66	48	31	14	97	80	62	45	26
35	85	71	56	41	27	3.2312	97	82	68	25
36	88	75	63	51	39	26	3.7714	4.3102	89	24
37	90	80	71	61	51	41	31	22	4.8512	23
38	93	85	78	70	63	56	48	41	33	22
39	95	90	85	80	76	71	66	61	56	21
40	98	95	93	90	88	85	83	80	78	20
41	0.5400	1.0800	1.6200	2.1600	2.7000	3.2400	3.7800	4.3200	4.8600	19
42	02	05	07	10	12	14	17	19	22	18
43	05	10	15	20	25	29	34	39	44	17
44	07	15	22	29	37	44	51	58	66	16
45	10	19	29	39	49	58	68	78	87	15
46	12	24	37	49	61	73	85	98	4.8710	14
47	15	29	44	58	73	88	3.7902	4.3317	31	18
48	17	34	51	68	86	3.2503	20	37	54	12
49	20	39	59	78	98	17	37	56	76	11
50	22	44	66	88	2.7110	32	54	76	98	10
51	24	49	73	98	22	46	71	95	4.8820	9
52	27	54	81	2.1708	35	61	88	4.3415	42	8
53	29	59	88	17	47	76	3.8005	34	64	7
54	32	63	95	27	59	90	22	54	85	6
55	34	68	1.6303	37	71	3.2605	39	74	4.8908	5
56	37	72	10	46	83	20	56	93	29	4
57	39	78	17	56	96	35	74	4.3513	52	3
58	42	83	25	66	2.7208	49	91	32	74	2
59	44	88	32	76	22	64	3.1808	52	96	1
60	0.5446	1.0893	1.6339	2.1786	2.7232	3.2678	3.8125	4.3571	1.9018	0
1	2	3	4	5	6	7	8	9		
LATITUDE 57 DEGREES.										

140 LATITUDE 38 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.8387	1.8773	2.5160	3.3547	4.1934	5.0320	5.8707	6.7094	7.5480	60
1	85	70	55	40	26	11	5.8696	81	66	59
2	84	67	51	34	18	01	85	68	52	58
3	82	64	46	28	10	5.0291	73	55	37	57
4	80	61	41	22	02	82	63	43	24	56
5	79	58	36	15	4.1994	73	52	30	09	55
6	77	54	32	09	86	63	40	18	7.5395	54
7	76	51	27	02	78	55	29	05	80	53
8	74	48	22	3.3496	70	44	18	6.6992	66	52
9	72	45	17	90	62	34	07	79	52	51
10	71	42	12	83	54	25	5.8596	66	37	50
11	69	38	08	77	46	15	84	54	23	49
12	68	35	03	70	38	06	73	41	08	48
13	66	32	2.5098	64	30	5.0196	62	28	7.5294	47
14	65	29	94	58	23	87	51	16	81	46
15	63	26	89	52	15	77	40	03	66	45
16	61	23	84	45	07	68	28	6.6890	52	44
17	60	19	79	39	4.1899	58	17	78	37	43
18	58	16	74	32	91	49	06	65	23	42
19	57	13	70	26	83	39	5.8495	52	09	41
20	55	10	65	20	75	29	84	39	7.5194	40
21	53	07	60	13	67	20	73	26	80	39
22	52	03	55	07	59	10	66	14	65	38
23	50	00	50	00	51	01	51	01	51	37
24	49	1.6697	46	3.3394	43	5.0091	40	6.6788	37	36
25	47	94	41	88	35	87	28	75	22	35
26	45	91	37	81	27	72	17	62	08	34
27	44	87	31	75	19	62	06	50	7.5093	33
28	42	34	26	68	11	53	5.8395	37	79	32
29	41	31	22	62	03	43	84	24	65	31
30	39	28	17	56	4.1795	33	72	11	50	30
31	37	25	12	49	87	24	61	6.6698	36	29
32	36	21	07	42	78	14	49	85	20	28
33	34	18	02	36	70	04	38	72	06	27
34	32	15	2.4997	30	62	4.9994	27	59	7.4992	26
35	31	12	92	23	54	85	16	46	77	25
36	29	09	88	17	46	75	04	34	63	24
37	28	05	83	10	38	66	5.8293	21	48	23
38	26	02	78	04	30	56	82	08	34	22
39	24	00	73	3.8298	22	46	71	6.6595	20	21
40	23	00	68	91	14	37	60	82	05	20
41	21	00	64	85	06	27	48	70	7.4891	19
42	20	00	59	78	4.1698	17	37	56	76	18
43	18	00	54	72	90	07	25	43	61	17
44	16	00	49	65	82	4.9898	14	30	47	16
45	15	00	44	59	74	88	03	18	32	15
46	13	00	39	52	66	79	5.8192	05	18	14
47	12	00	35	46	58	69	81	6.6492	04	13
48	10	00	29	39	49	59	69	78	7.4788	12
49	08	00	25	33	41	49	57	66	74	11
50	07	00	20	26	33	40	46	53	59	10
51	05	00	15	20	25	30	35	40	45	9
52	03	00	10	14	17	20	24	27	31	8
53	02	00	05	07	09	10	12	14	15	7
54	00	00	00	00	01	01	01	01	01	6
55	0.8299	1.6597	3.4896	3.3194	4.1593	4.9791	5.8090	6.6388	7.4687	5
56	97	94	91	88	85	81	78	75	72	4
57	96	91	86	80	77	72	67	62	58	3
58	94	87	81	74	68	62	55	49	42	2
59	92	84	76	68	60	52	44	36	28	1
60	0.8290	1.6581	3.4871	3.3162	4.1452	4.9742	5.8033	6.6323	7.4614	0
1	2	3	4	5	6	7	8	9	10	11
DEPARTURE 56 DEGREES.										

DEPARTURE 38 DEGREES.										141
1	2	3	4	5	6	7	8	9	10	
0	0.5446	1.0898	1.6839	2.1786	2.7282	3.2678	3.8125	4.3571	4.9018	60
1	49	98	46	95	44	93	42	90	39	59
2	51	1.0903	54	2.1805	57	3.2708	59	4.3610	62	58
3	54	07	61	15	69	22	76	30	88	57
4	56	12	68	24	81	37	93	49	4.9105	56
5	59	17	76	34	93	52	3.8210	69	27	55
6	61	22	83	44	2.7305	66	27	88	49	54
7	64	27	91	54	18	81	45	4.3708	72	53
8	66	32	98	64	30	95	61	27	98	52
9	68	37	1.6405	73	42	3.2810	78	46	4.9215	51
10	71	42	12	83	54	25	96	66	37	50
11	73	46	20	93	66	39	3.8312	86	59	49
12	76	51	27	2.1902	78	54	29	4.3805	80	48
13	78	56	34	12	91	69	47	25	4.9303	47
14	81	61	42	22	2.7403	83	64	44	25	46
15	83	66	49	32	15	97	80	63	46	45
16	85	71	56	42	27	3.2912	98	83	69	44
17	88	76	63	51	39	27	3.8415	4.3902	90	43
18	90	80	71	61	51	41	31	22	4.9412	42
19	93	85	78	72	64	56	49	44	34	41
20	95	90	85	80	76	71	66	61	56	40
21	98	95	93	90	88	85	83	80	78	39
22	0.5500	1.1000	00	2 2000	2.7500	99	99	99	99	38
23	02	05	1.6507	10	12	3.3014	3.8517	4.4019	4.9522	37
24	05	10	14	19	24	29	34	38	43	36
25	07	14	22	29	36	43	50	58	65	35
26	10	19	29	39	49	58	68	78	87	34
27	12	24	36	48	61	73	85	97	4.9609	33
28	15	29	44	58	73	87	3.8602	4.4116	31	32
29	17	34	51	68	85	3.3101	18	35	52	31
30	19	39	58	78	97	16	36	55	75	30
31	22	44	65	87	2.7609	31	53	74	96	29
32	24	48	73	97	21	45	69	94	4.9718	28
33	27	53	80	2.2106	33	60	86	4.4213	39	27
34	29	58	87	16	46	75	3.8704	33	62	26
35	31	63	95	26	58	89	21	52	84	25
36	34	68	1.6602	36	70	3.3203	37	71	4.9805	24
37	36	73	09	45	82	18	54	90	27	23
38	39	78	16	55	94	33	72	4.4310	49	22
39	41	82	24	65	2.7706	47	88	30	71	21
40	44	87	31	74	18	62	3.8805	49	92	20
41	46	92	38	84	30	76	22	68	4.9914	19
42	48	97	45	94	42	90	39	87	36	18
43	51	1.1102	53	2.2204	55	3.3305	56	4.4407	58	17
44	53	07	60	13	67	20	73	26	80	16
45	56	11	67	23	79	34	90	46	5.0001	15
46	58	16	74	32	91	49	3.8907	65	23	14
47	61	21	82	42	2.7803	63	24	84	45	13
48	63	26	89	52	15	78	41	4.4504	67	12
49	65	31	96	62	27	92	58	23	89	11
50	68	36	1.6703	71	39	3.3407	75	42	5.0110	10
51	70	40	11	81	51	21	91	62	32	9
52	73	45	18	90	63	36	3.9008	81	53	8
53	75	50	25	2.2300	75	50	25	4.4600	75	7
54	78	55	33	10	88	65	43	20	98	6
55	80	60	40	20	2.7900	79	59	39	5.0219	5
56	82	65	47	29	12	94	76	58	41	4
57	85	69	54	39	24	3.8508	93	78	62	3
58	87	74	61	48	36	23	3.9110	97	84	2
59	90	79	69	58	48	37	27	4.4716	5.0306	1
60	0.5592	1.1184	1.6776	2.2368	2.7960	3.8551	3.9143	4.4735	5.0827	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 56 DEGREES.										

142									
LATITUDE 84 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.8290	1.6581	2.4871	3.3162	4.1452	4.9742	5.8033	6.6323	7.4614
1	89	77	66	55	44	32	21	10	7.4598
2	87	74	61	48	36	23	10	6.6297	84 58
3	86	71	57	42	28	13	5.7999	84	70 57
4	84	68	52	36	20	03	87	71	55 56
5	82	64	47	29	11	4.9693	75	58	40 55
6	81	61	42	22	03	84	64	45	25 54
7	79	58	37	16	4.1395	74	53	32	11 53
8	77	55	32	09	87	64	41	18	7.4496
9	76	51	27	03	79	54	30	06	81 51
10	74	48	22	3.3096	71	45	19	5.6193	67 50
11	72	45	17	90	62	34	07	79	52 49
12	71	42	12	83	54	25	5.7896	66	37 48
13	69	08	08	77	46	15	84	54	23 47
14	68	35	03	70	38	05	73	40	08 46
15	66	32	2.4798	64	30	4.9595	61	27	7.4394
16	64	29	93	57	22	86	50	14	79 44
17	63	25	88	50	13	76	38	01	64 43
18	61	22	83	44	05	66	27	6.6088	49 42
19	59	19	78	37	4.1297	56	15	74	34 41
20	58	15	73	31	89	46	04	62	19 40
21	56	12	68	24	81	37	2.7793	49	05 39
22	54	09	63	18	72	26	81	35	7.4290
23	53	06	58	11	64	17	70	22	75 37
24	51	02	53	04	56	07	50	09	60 36
25	50	1.6499	49	3.2998	48	4.9497	47	6.5996	46 35
26	48	96	43	91	39	87	35	82	30 34
27	46	92	39	85	31	77	23	70	16 33
28	45	89	34	78	23	68	12	57	01 32
29	43	86	29	72	15	58	00	43	7.4186
30	41	83	24	65	07	48	5.7639	30	72 30
31	40	79	19	58	4.1198	38	77	17	56 29
32	38	76	14	52	90	28	66	04	42 28
33	36	73	09	45	82	18	54	6.5890	27 27
34	35	69	04	39	74	08	43	78	12 26
35	33	66	2.4699	32	65	4.9398	31	64	7.4097
36	31	63	94	26	57	88	20	51	83 24
37	30	59	89	19	49	78	08	38	67 23
38	28	56	84	12	41	69	5.7597	25	53 22
39	26	53	79	06	32	58	85	11	38 21
40	25	50	74	3.2899	24	49	74	6.5798	23 20
41	23	46	69	92	16	39	62	85	08 19
42	21	43	64	86	07	28	50	71	7.3993
43	20	40	59	79	4.1099	19	39	58	78 17
44	18	36	54	72	91	09	27	45	63 16
45	17	33	50	66	83	4.9299	16	32	49 15
46	15	30	44	59	74	89	04	18	33 14
47	13	26	40	53	66	79	5.7492	06	19 13
48	12	23	35	46	58	69	81	6.5692	04 12
49	10	20	29	39	49	59	69	78	7.3888
50	08	16	25	33	41	49	57	66	74 10
51	07	13	20	26	32	39	46	52	59 9
52	05	10	14	19	24	29	34	38	43 8
53	03	06	10	13	16	19	22	26	29 7
54	02	03	05	06	08	09	11	12	14 6
55	00	00	00	00	00	4.9199	5.7399	6.5599	7.3799
56	0.8198	1.6395	2.4595	3.2793	4.0991	89	87	86	84 4
57	97	93	90	86	83	79	76	72	69 3
58	95	90	85	80	75	69	64	59	54 2
59	93	86	80	73	66	59	52	46	39 1
60	0.8192	1.6383	2.4575	3.2766	4.0958	4.9149	5.7341	6.5532	7.3724
	1	2	3	4	5	6	7	8	9
DEPARTURE 55 DEGREES.									

DEPARTURE 84 DEGREES.										148
1	2	3	4	5	6	7	8	9	10	
0	0.5592	1.1184	1.6776	2.2368	2.7960	3.3551	3.9143	4.4735	5.0327	60
1	94	89	83	77	72	66	60	54	49	59
2	97	94	90	87	84	81	78	74	71	58
3	99	98	98	97	96	95	94	94	93	57
4	0.5602	1.1203	1.6805	2.2406	1.8008	3.3610	3.9211	4.4813	5.0414	56
5	04	08	12	16	20	24	28	32	36	55
6	06	13	19	26	32	38	45	51	58	54
7	09	18	26	35	44	53	62	70	79	53
8	11	22	34	45	56	67	78	90	5.0501	52
9	14	27	41	54	68	82	95	4.4909	22	51
10	16	32	48	64	80	96	3.9812	28	44	50
11	18	37	55	74	92	3.3710	29	47	66	49
12	21	42	62	83	2.8104	25	46	66	87	48
13	23	46	70	93	16	39	62	86	5.0609	47
14	26	51	77	2.2502	28	54	79	4.5005	30	46
15	28	56	84	12	40	68	96	24	52	45
16	31	61	92	22	53	83	3.9414	44	75	44
17	33	66	99	32	65	97	30	63	96	43
18	35	71	1.6906	41	77	3.3812	47	82	5.0718	42
19	38	75	13	51	89	26	64	4.5102	39	41
20	40	80	20	60	2.8201	41	81	21	61	40
21	43	85	28	70	13	55	98	40	83	39
22	45	90	35	80	25	69	3.9514	59	5.0804	38
23	47	95	42	89	37	84	31	78	26	37
24	50	99	49	99	49	98	48	98	47	36
25	52	1.1304	56	2.2608	61	3.3913	65	4.5217	69	35
26	55	09	64	18	73	27	82	36	91	34
27	57	14	71	28	85	41	98	55	5.0912	33
28	59	19	78	37	97	56	3.9615	74	34	32
29	62	23	85	47	2.8309	70	32	94	55	31
30	64	28	92	56	21	85	49	4.5313	77	30
31	67	33	1.7000	66	33	99	66	32	99	29
32	69	38	07	76	45	3.4013	82	51	5.1020	28
33	71	43	14	85	57	28	99	70	42	27
34	74	47	21	94	68	42	3.9715	89	62	26
35	76	52	28	2.2704	80	56	32	4.5408	84	25
36	78	57	35	14	92	70	49	27	5.1106	24
37	81	62	42	23	2.8404	85	66	46	27	23
38	83	66	50	33	16	99	82	66	49	22
39	86	71	57	42	28	3.4114	99	85	70	21
40	88	76	64	52	40	28	3.9816	4.5504	92	20
41	90	81	71	62	52	42	33	23	3.1214	19
42	93	86	78	71	64	57	50	42	35	18
43	95	90	86	81	76	71	66	62	57	17
44	98	95	93	90	88	86	83	81	78	16
45	0.5700	1.1400	1.7100	2.2800	2.8500	3.4200	3.9900	4.5600	5.1300	15
46	02	05	07	10	12	14	17	19	22	14
47	05	09	14	19	24	28	33	38	42	13
48	07	14	21	28	36	43	50	57	64	12
49	10	19	29	38	48	57	67	76	86	11
50	12	24	36	48	60	71	83	95	5.1407	10
51	14	29	43	57	72	86	4.0000	4.5714	29	9
52	17	33	50	67	84	3.4300	17	34	50	8
53	20	38	57	76	96	15	34	53	72	7
54	22	43	65	86	2.8608	29	51	72	94	6
55	24	48	71	95	19	43	67	90	5.1514	5
56	26	52	79	2.2905	31	57	83	4.5810	36	4
57	29	57	86	14	43	72	4.0100	29	57	3
58	31	62	93	24	55	86	17	48	79	2
59	33	67	1.7200	34	67	3.4400	34	67	5.1601	1
60	0.5736	1.1472	1.7207	2.2943	2.8679	3.4415	4.0151	4.5886	5.1622	0
1	2	3	4	5	6	7	8	9		
LATITUDE 55 DEGREES.										

144									
LATITUDE 35 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.8192	1.6383	2.4575	3.2766	4.0958	4.9149	5.7341	6.5532	7.3724
1	90	80	70	60	50	39	29	19	09
2	88	76	65	53	41	29	17	06	7.3694
3	87	73	59	46	33	19	06	6.5492	78
4	85	70	54	39	24	09	5.7294	78	63
5	83	66	50	33	16	4.9099	82	66	49
6	82	63	45	26	08	89	71	52	34
7	80	60	39	19	4.0899	79	59	38	18
8	78	56	34	12	91	69	47	25	03
9	77	53	30	06	83	59	36	12	7.3689
10	75	50	24	2.2699	74	49	24	6.5398	73
11	73	46	19	92	66	39	12	85	58
12	71	43	14	86	57	82	00	71	43
13	70	40	09	79	49	19	5.7189	58	28
14	68	36	04	72	41	09	77	45	13
15	66	33	2.4499	66	32	4.8994	65	31	7.3497
16	65	29	94	59	24	88	53	18	82
17	63	26	89	52	16	79	42	05	68
18	61	23	84	46	07	68	30	6.5291	53
19	60	19	79	39	4.0799	58	18	77	37
20	58	16	74	32	90	48	06	64	22
21	56	13	69	25	82	38	5.7094	50	07
22	55	09	64	18	73	28	82	37	7.3391
23	53	06	59	12	65	18	71	43	77
24	51	03	54	05	97	08	59	10	62
25	50	1.6299	49	2.2598	48	4.8898	47	6.5196	46
26	48	96	44	92	40	87	35	83	31
27	46	92	39	85	31	77	23	69	16
28	45	89	34	78	23	67	12	56	01
29	43	86	28	71	14	57	00	43	7.3285
30	41	82	24	65	06	47	5.6988	30	71
31	40	79	19	58	4.0698	27	77	16	56
32	38	76	13	51	89	27	65	02	40
33	36	72	08	44	81	17	53	6.5089	25
34	34	69	03	38	72	06	41	75	10
35	33	65	2.4398	31	64	4.8796	29	62	7.3194
36	31	62	93	24	55	86	17	48	79
37	29	59	88	17	47	76	05	34	64
38	28	55	83	10	38	66	5.6893	21	48
39	26	52	78	04	30	55	81	07	33
40	24	48	73	2.2497	21	45	69	6.4994	18
41	23	45	68	90	13	35	58	80	03
42	21	42	62	83	04	25	46	66	7.3087
43	19	38	57	76	4.0596	15	34	53	72
44	17	35	52	70	87	04	22	39	57
45	16	31	47	63	79	4.8694	10	26	41
46	14	28	42	56	70	84	5.6798	12	26
47	14	25	37	49	62	74	86	6.4898	11
48	11	21	32	42	53	64	74	85	7.2995
49	09	17	26	34	43	51	62	68	77
50	07	14	22	29	36	43	50	58	65
51	06	11	17	22	28	33	39	44	50
52	04	08	11	15	19	23	27	30	34
53	02	04	06	08	11	13	15	17	19
54	00	01	01	02	02	02	03	03	04
55	0.8099	1.6297	2.4296	2.2395	4.0494	4.8592	5.6691	6.4790	7.2888
56	97	94	91	88	85	82	79	76	73
57	95	91	86	81	77	72	67	62	58
58	94	87	81	74	68	62	55	49	42
59	92	84	76	68	60	51	43	35	27
60	0.8090	1.6180	2.4271	3.2361	4.0451	4.8541	5.6631	6.4722	7.2812
1	2	3	4	5	6	7	8	9	10
DEPARTURE 54 DEGREES.									

DEPARTURE 35 DEGREES.										145
1	2	3	4	5	6	7	8	9	10	
0	0.5786	1.1472	1.7207	2.2943	2.8679	3.4415	4.0151	4.5886	5.1622	60
1	38	76	14	52	91	29	67	4.5905	48	59
2	41	81	22	62	2.8703	43	84	24	65	58
3	43	86	29	72	15	57	4.0200	43	86	57
4	45	91	36	81	27	72	17	62	5.1708	56
5	48	95	43	91	39	86	34	82	29	55
6	50	1.1500	50	2.3000	51	3.4501	51	4.6001	51	54
7	52	05	57	10	62	14	67	19	72	53
8	55	10	64	19	74	29	84	38	93	52
9	57	14	72	29	86	43	4.0300	57	5.1815	51
10	60	19	79	38	98	58	17	77	36	50
11	62	24	86	48	2.8810	71	33	95	57	49
12	64	29	93	57	22	86	50	4.6114	79	48
13	67	33	1.7300	67	34	3.4600	67	34	5.1900	47
14	69	38	07	76	46	15	84	53	22	46
15	72	43	15	86	58	39	4.0401	72	44	45
16	74	48	21	95	69	43	17	90	64	44
17	76	52	29	2.3105	81	57	33	4.6210	86	43
18	79	57	36	14	93	72	50	29	5.2007	42
19	81	62	43	24	2.8905	85	66	47	28	41
20	83	67	50	33	17	3.4700	83	66	50	40
21	86	71	57	43	29	14	4.0500	86	71	39
22	88	76	64	52	41	29	17	4.6305	93	38
23	90	81	71	62	52	42	33	23	5.2114	37
24	93	86	78	71	64	57	50	42	35	36
25	95	90	86	81	76	71	66	62	57	35
26	98	95	93	90	88	86	83	81	78	34
27	0.5800	1.1600	1.7400	2.3200	2.9000	99	99	99	99	83
28	02	05	07	09	12	3.4814	4.0616	4.6418	5.2221	82
29	05	09	14	19	24	28	33	38	42	81
30	07	14	21	28	35	42	49	56	63	80
31	09	19	28	38	47	56	66	75	85	29
32	12	24	35	47	59	71	83	94	5.2306	28
33	14	28	42	56	71	85	99	4.6513	27	27
34	17	33	50	66	83	99	4.0716	32	49	26
35	19	38	57	76	95	3.4913	32	51	70	25
36	21	42	64	85	2.9106	27	48	70	91	24
37	24	47	71	94	18	42	65	89	5.2412	23
38	26	52	78	2.3304	30	56	82	4.6608	34	22
39	28	57	85	13	42	70	98	26	55	21
40	31	61	92	23	54	84	4.0815	46	76	20
41	33	66	99	32	65	98	31	64	97	19
42	35	71	1.7506	42	77	3.5012	48	83	5.2519	18
43	38	76	13	51	89	27	65	4.6702	40	17
44	40	80	20	60	2.9201	41	82	21	61	16
45	43	85	28	70	13	55	98	40	83	15
46	45	90	35	80	25	69	4.0914	59	5.2604	14
47	47	94	42	89	36	83	30	78	25	13
48	50	99	49	98	48	98	57	97	46	12
49	52	1.1704	56	2.3408	60	3.5111	63	4.6815	67	11
50	54	09	63	17	72	26	80	34	89	10
51	57	13	70	27	84	40	97	54	5.2710	9
52	59	18	77	36	95	54	4.1013	72	31	8
53	61	23	84	46	2.9307	68	80	91	53	7
54	64	27	91	55	19	82	46	4.6910	78	6
55	66	32	98	64	31	97	63	29	95	5
56	68	37	05	74	42	3.5210	79	47	5.2816	4
57	71	42	12	83	54	25	96	66	37	3
58	73	46	19	92	66	39	4.1112	85	58	2
59	76	51	27	2.3502	78	53	29	4.7004	80	1
60	0.5878	1.1756	1.7634	2.3512	2.9390	3.5267	4.1145	4.7023	5.2901	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 54 DEGREES.										

J

146									
LATITUDE 36 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.8090	1.6180	2.4271	3.2361	4.0451	4.8541	5.6631	6.4722	7.2812
1	89	77	66	54	43	31	19	08	7.2797
2	87	73	60	47	34	20	07	6.4694	80 58
3	85	70	55	40	25	10	5.6595	80	65 57
4	83	67	50	33	17	00	83	66	50 56
5	82	63	45	26	08	4.8490	71	53	34 55
6	80	60	40	22	00	79	59	39	19 54
7	78	56	35	13	4.0391	69	47	26	04 53
8	77	53	30	06	83	59	36	12	7.2689
9	75	50	24	3.2299	74	49	24	6.4598	73 51
10	73	46	19	92	65	38	11	84	57 50
11	71	43	14	85	57	28	5.6499	70	42 49
12	70	39	09	78	48	18	87	57	26 48
13	68	36	04	72	40	07	75	43	11 47
14	66	32	2.4199	65	31	4.8397	63	30	7.2596
15	64	29	93	58	22	86	51	15	80 45
16	63	25	88	51	14	76	39	02	64 44
17	61	22	83	44	06	66	27	6.4488	49 43
18	59	19	78	37	4.0297	56	15	74	34 42
19	58	16	73	30	88	46	03	61	18 41
20	56	12	67	23	79	35	5.6391	46	02 40
21	54	08	62	16	71	25	79	33	7.2487
22	52	05	57	10	62	14	67	19	72 38
23	51	01	52	03	54	04	55	06	56 37
24	49	1.6098	47	3.2196	45	4.8293	42	6.4391	40 36
25	47	94	42	89	37	83	30	78	25 35
26	46	91	37	82	28	73	19	64	10 34
27	44	88	31	75	19	63	07	50	7.2394
28	42	84	26	68	10	52	5.6294	36	78 32
29	40	81	21	61	02	42	82	22	63 31
30	39	77	16	54	4.0193	32	70	09	47 30
31	37	74	10	47	84	21	58	6.4294	31 29
32	35	70	05	40	76	11	46	81	16 28
33	33	67	00	34	67	00	34	67	01 27
34	32	63	2.4095	26	58	4.8190	21	53	7.2284
35	30	60	90	20	50	79	09	39	69 25
36	28	56	85	13	41	69	5.6197	26	54 24
37	26	53	79	06	32	58	85	11	38 23
38	25	49	74	3.2099	24	48	73	6.4198	22 22
39	23	46	69	92	15	38	61	84	07 21
40	21	42	64	85	06	27	48	70	7.2191
41	20	39	59	78	4.0098	17	37	56	76 19
42	18	36	53	71	89	07	25	42	60 18
43	16	32	48	64	81	4.8096	12	28	44 17
44	14	29	43	57	72	86	00	14	29 16
45	13	25	38	50	63	75	5.6088	00	13 15
46	11	22	32	43	54	65	76	6.4086	7.2097
47	09	18	27	36	46	55	64	73	82 13
48	07	15	22	29	37	44	51	58	66 12
49	06	11	17	22	28	34	39	45	50 11
50	04	08	11	15	19	23	27	30	34 10
51	02	04	06	08	11	13	15	17	19 9
52	00	01	01	01	02	02	02	02	03 8
53	0.7999	1.5997	2.3996	3.1994	3.9993	4.7992	5.5990	6.3989	7.1987
54	97	94	90	87	84	81	78	74	71 6
55	95	90	85	81	76	71	66	61	56 5
56	93	87	80	74	67	60	54	47	41 4
57	92	83	75	67	58	50	41	33	24 3
58	90	80	70	60	50	39	29	19	09 2
59	88	76	64	52	41	29	17	05	7.1893
60	0.7986	1.5973	2.3959	3.1946	3.9932	4.7918	5.5905	6.3891	7.1878
	1	2	3	4	5	6	7	8	9
DEPARTURE 53 DEGREES.									

DEPARTURE 36 DEGREES.										147
1	2	3	4	5	6	7	8	9	10	1
0	0.5878	1.1756	1.7634	2.3512	2.9390	3.5267	4.1145	4.7023	5.2901	60
1	80	60	41	21	2.9401	81	61	42	22	59
2	83	65	48	30	13	96	78	61	43	58
3	85	70	55	40	25	3.5309	94	79	64	57
4	87	75	62	49	37	24	4.1211	98	86	56
5	90	79	69	58	48	38	27	4.7117	5.3006	55
6	92	84	76	68	60	52	44	36	28	54
7	94	89	83	77	72	66	60	54	49	53
8	97	93	90	87	84	80	77	74	70	52
9	99	98	97	96	95	94	93	92	91	51
10	0.5901	1.1803	1.7704	2.3606	2.9507	3.5408	4.1310	4.7211	4.3113	50
11	04	07	11	15	19	22	26	30	33	49
12	06	12	18	24	31	37	43	49	55	48
13	08	17	25	34	42	50	59	67	76	47
14	11	21	32	43	54	64	75	86	96	46
15	13	26	39	52	66	79	92	4.7305	4.3218	45
16	15	31	46	62	77	92	4.1408	23	39	44
17	18	36	53	71	89	3.5507	25	42	59	43
18	20	40	60	80	2.9601	21	41	61	81	42
19	23	45	68	90	13	35	58	80	4.3303	41
20	25	50	74	99	24	49	74	98	23	40
21	27	54	82	2.3709	36	63	90	4.7418	45	39
22	30	59	89	18	48	77	4.1507	36	66	38
23	32	64	95	27	59	91	23	54	86	37
24	34	68	1.7803	37	71	3.5605	39	74	4.3408	36
25	37	73	10	46	83	19	56	92	29	35
26	39	78	17	56	95	33	72	4.7511	50	34
27	41	82	24	65	2.9706	47	88	30	71	33
28	44	87	31	74	18	61	4.1605	48	92	32
29	46	92	38	84	30	75	21	67	4.3513	31
30	48	96	45	93	41	89	37	86	34	30
31	51	01	52	2.3802	53	3.5704	54	4.7605	55	29
32	53	1.1906	59	12	65	17	70	23	76	28
33	55	10	66	21	76	31	86	42	97	27
34	58	15	73	30	88	46	4.1703	61	4.3618	26
35	60	20	80	39	2.9800	59	19	80	39	25
36	62	24	87	49	11	73	35	98	60	24
37	64	29	94	58	23	88	52	4.7717	81	23
38	67	34	1.7901	68	35	3.5801	68	35	4.3702	22
39	69	39	08	77	47	16	85	54	24	21
40	72	43	15	86	58	30	4.1801	73	44	20
41	74	48	22	96	70	43	17	91	65	19
42	76	53	29	2.3905	82	58	34	4.7810	87	18
43	79	58	37	16	95	73	52	31	4.3810	17
44	81	62	43	24	2.9905	85	66	47	28	16
45	83	66	50	33	16	99	82	66	49	15
46	86	71	57	42	28	3.5914	99	85	70	14
47	88	76	64	52	40	27	4.1915	4.7903	91	13
48	90	80	71	61	51	41	31	22	5.3912	12
49	93	85	78	70	63	56	48	41	33	11
50	95	90	85	80	75	69	64	59	54	10
51	97	94	92	89	86	83	80	78	75	9
52	0.6000	99	99	98	98	97	97	96	96	8
53	02	1.2004	1.8006	2.4008	3.0010	3.6011	42.013	4.8015	5.4017	7
54	04	08	13	17	21	25	29	34	38	6
55	07	13	20	26	33	39	46	52	59	5
56	09	18	27	36	45	53	62	71	80	4
57	11	22	34	45	56	67	78	90	01	3
58	14	27	41	54	68	81	4.2195	4.8108	5.4122	2
59	16	32	47	63	79	95	11	26	42	1
60	0.6018	1.2036	1.8054	2.4072	3.0091	3.6106	4.2127	4.8145	5.4163	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 53 DEGREES.										

148									
LATITUDE 37 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.7986	1.5973	2.3959	3.1946	3.9932	4.7918	5.5905	6.3891	7.1878
1	85	69	54	38	23	08	5.5892	77	61
2	83	66	49	32	15	4.7897	80	63	46
3	81	62	43	24	06	87	68	49	30
4	79	59	38	17	3.9897	76	55	34	14
5	78	55	33	10	88	66	43	21	7.1798
6	76	52	27	03	79	55	31	06	82
7	74	48	22	3.1896	71	45	19	5.3793	67
8	72	45	17	89	62	34	06	78	51
9	71	41	12	82	53	24	5.5794	65	35
10	69	38	06	75	44	13	82	50	19
11	67	34	01	68	36	03	70	37	04
12	65	31	2.3896	61	27	4.7792	57	22	7.1688
13	64	27	91	54	18	82	45	08	72
14	62	24	85	47	09	71	33	6.3694	56
15	60	20	80	48	00	60	20	80	40
16	58	17	75	33	3.9792	50	08	66	25
17	57	13	70	26	83	39	2.5696	52	09
18	55	09	64	19	74	28	83	38	7.1592
19	53	06	59	12	65	18	71	24	77
20	51	02	54	05	56	07	58	10	61
21	49	1.5899	48	3.1798	47	4.7696	46	6.3595	45
22	48	95	43	91	39	86	34	82	29
23	46	92	38	84	30	75	21	67	13
24	44	88	32	76	21	65	09	53	7.1497
25	42	85	27	70	12	54	5.5597	39	82
26	41	81	22	62	03	44	94	25	65
27	39	78	16	55	3.9694	33	72	10	49
28	37	74	11	48	86	23	60	6.3497	34
29	35	71	06	41	77	12	47	82	18
30	34	67	01	34	68	01	35	68	02
31	32	64	2.3795	27	59	4.7591	23	54	7.1386
32	30	60	90	20	50	80	10	40	70
33	28	56	85	13	41	69	5.5497	26	54
34	26	53	79	06	32	58	85	11	38
35	25	49	74	3.1699	24	48	73	6.3398	22
36	23	46	69	92	15	37	60	83	06
37	21	42	63	84	06	27	48	69	7.1290
38	19	39	58	77	3.9597	16	35	54	74
39	18	35	53	70	88	06	23	41	58
40	16	32	47	63	79	4.7495	11	26	42
41	14	28	42	56	70	84	5.5398	12	26
42	12	24	37	49	61	73	85	6.3298	10
43	11	21	32	42	53	63	74	84	7.1195
44	09	17	26	35	44	52	61	70	78
45	07	14	21	28	35	41	48	55	62
46	05	10	15	20	26	31	36	41	46
47	03	07	10	13	17	20	23	26	30
48	02	03	05	06	08	09	11	12	14
49	00	00	2.8699	3.1599	3.9499	4.7399	5.5299	6.3198	7.1098
50	0.7898	1.5796	94	92	90	88	86	84	82
51	96	92	89	85	81	77	73	70	66
52	94	89	83	78	72	66	51	55	50
53	93	85	78	70	63	56	48	41	33
54	91	82	72	63	54	45	36	26	17
55	89	78	67	56	46	35	24	13	02
56	87	75	62	49	37	24	11	6.3098	7.0986
57	86	71	57	42	28	18	5.5199	84	70
58	84	67	51	35	19	02	86	70	53
59	82	64	45	28	10	4.7291	78	55	47
60	0.7880	1.5760	2.8640	3.1520	3.9401	4.7281	5.5161	6.3041	7.0921
	1	2	3	4	5	6	7	8	9
DEPARTURE 52 DEGREES.									

DEPARTURE 37 DEGREES.										149
1	2	3	4	5	6	7	8	9	10	
0	0.6018	1.2036	1.8054	2.4072	3.0091	3.6109	4.2127	4.8145	5.4163	60
1	21	41	62	81	3.0103	23	44	64	85	59
2	23	46	68	91	14	37	60	82	5.4205	58
3	25	50	75	2.4100	26	51	76	4.8201	26	57
4	27	55	82	10	37	64	92	19	47	56
5	30	60	89	19	49	79	4.2209	38	68	55
6	32	64	96	28	61	93	25	57	89	54
7	34	69	1.8103	38	72	3.6206	41	75	5.4310	53
8	37	73	10	47	84	21	57	94	30	52
9	39	78	17	56	95	34	73	4.8313	51	51
10	41	83	24	66	3.0207	48	90	31	73	50
11	44	87	31	75	19	62	4.2306	50	93	49
12	46	92	38	84	30	76	22	68	5.4414	48
13	48	97	45	93	42	90	38	86	35	47
14	51	1.2101	52	2.4202	53	3.6304	54	4.8405	55	46
15	53	06	59	12	65	17	70	23	76	45
16	55	11	66	21	77	32	87	42	98	44
17	58	15	73	30	88	46	4.2403	61	5.4518	43
18	60	20	80	40	3.0300	59	19	79	39	42
19	62	25	87	49	11	73	35	98	60	41
20	65	29	94	58	23	87	52	4.8516	81	40
21	67	34	1.8201	67	34	3.6401	68	85	5.4601	39
22	69	38	07	76	46	15	84	53	22	38
23	71	43	14	86	57	28	4.2500	72	43	37
24	74	48	21	95	69	43	17	90	64	36
25	76	52	28	5 4804	81	57	33	4.8609	85	35
26	78	57	35	14	92	70	49	27	5.4706	34
27	81	61	42	23	3.0404	84	65	46	27	33
28	83	66	49	32	15	98	81	64	47	32
29	85	71	56	41	27	3.6512	97	83	68	31
30	88	79	63	50	38	26	4.2613	4.8701	79	30
31	90	80	70	60	50	39	29	20	5.4809	29
32	92	85	77	69	61	53	45	38	30	28
33	95	89	84	78	73	67	62	56	51	27
34	97	94	90	87	84	81	78	74	71	26
35	99	98	97	96	96	95	94	93	92	25
36	0.6102	1.2203	1.8305	2.4406	3.0508	3.6609	4.2711	4.8812	5.4914	24
37	04	08	11	15	19	23	27	31	34	23
38	06	12	18	24	31	37	43	49	55	22
39	08	17	25	34	42	50	59	68	76	21
40	11	21	32	43	54	64	75	86	96	20
41	13	26	39	52	65	78	91	4.8905	5.5017	19
42	15	31	46	61	77	92	4.2807	22	38	18
43	18	35	53	70	88	3.6706	23	41	58	17
44	20	40	60	80	3.0600	19	39	59	79	16
45	22	45	67	89	11	33	55	78	5.5100	15
46	25	49	74	98	23	47	72	96	21	14
47	27	54	80	2.4507	34	61	88	4.9014	41	13
48	29	58	87	16	45	75	4.2904	33	62	12
49	31	63	94	26	57	88	20	51	83	11
50	34	67	1.8401	35	69	3.6802	36	70	5.5203	10
51	36	72	08	44	80	16	52	88	24	9
52	38	77	15	53	92	30	68	4.9106	45	8
53	41	81	22	62	3.0703	44	84	25	65	7
54	43	86	29	72	15	57	4.3000	43	86	6
55	45	90	35	80	26	71	16	62	5.5306	5
56	47	95	42	90	37	84	32	79	37	4
57	50	99	49	99	49	98	48	98	58	3
58	52	1.2304	56	2.4608	60	3.6912	64	4.9216	65	2
59	54	09	63	17	72	26	80	35	86	1
60	1.0157	1.2313	1.8470	2.4626	3 0783	3.6940	4.3096	4.9252	5.5409	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 52 DEGREES.										

150										
LATITUDE 38 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.7880	1.5760	2.3640	3.1520	3.9401	4.7281	5.5161	6.3041	7.0921	60
1	78	57	35	13	3.9392	70	48	26	05	59
2	77	53	30	06	83	59	36	12	7.0889	58
3	75	50	24	99	74	48	23	6.2998	72	57
4	73	46	19	3.1492	65	37	10	83	56	56
5	71	42	13	84	56	27	5.5098	69	40	55
6	69	39	08	77	47	16	85	54	24	54
7	68	35	03	70	38	06	73	41	08	53
8	66	32	2.3597	63	29	4.7195	61	26	7.0792	52
9	64	28	92	56	20	84	48	12	76	51
10	62	24	87	49	11	73	35	6.2798	60	50
11	60	21	81	42	02	62	23	83	44	49
12	59	17	76	34	3.9293	52	10	69	27	48
13	57	14	70	27	84	41	5.4998	54	11	47
14	55	10	65	20	75	30	85	40	7.0695	46
15	53	06	60	13	66	19	72	26	79	45
16	51	03	54	06	57	08	60	11	63	44
17	50	00	49	3.1398	48	4.7098	47	6.2697	46	43
18	48	1.5696	43	91	39	87	35	82	30	42
19	46	92	38	84	30	76	22	68	14	41
20	44	88	33	77	21	65	09	54	7.0598	40
21	42	85	27	70	12	54	5.4897	39	82	39
22	41	81	22	62	3.9103	43	84	24	65	38
23	39	77	16	55	94	32	71	10	48	37
24	37	74	11	48	85	21	58	6.2595	32	36
25	35	71	05	40	76	11	46	81	16	35
26	33	67	00	33	67	00	33	66	00	34
27	32	63	2.3495	26	58	4.6989	21	52	7.0484	33
28	30	59	89	19	49	78	08	38	67	32
29	28	56	84	12	40	67	5.4795	23	51	31
30	26	52	78	04	31	57	83	09	35	30
31	24	49	73	3.1297	22	46	70	6.2494	19	29
32	23	45	68	90	13	35	58	80	03	28
33	21	41	62	82	03	24	44	65	7.0385	27
34	19	38	56	75	3.9094	13	32	50	69	26
35	17	34	51	68	85	02	19	36	53	25
36	15	30	46	61	76	4.6891	06	22	37	24
37	13	27	40	54	67	80	5.4694	07	21	23
38	12	23	35	46	58	70	81	6.2393	04	22
39	10	20	29	39	49	59	69	78	7.0288	21
40	08	16	24	32	40	47	55	63	71	20
41	06	12	18	24	31	37	43	49	55	19
42	04	09	13	17	22	26	30	34	39	18
43	03	05	08	10	13	15	18	20	23	17
44	01	01	02	03	04	04	05	06	06	16
45	0.7799	1.5598	2.3396	3.1195	3.8994	4.6793	5.4592	6.2290	7.0189	15
46	97	94	91	88	85	82	79	76	73	14
47	95	91	86	81	76	71	66	62	57	13
48	93	87	80	74	67	60	54	47	41	12
49	92	83	75	66	58	50	41	03	24	11
50	90	79	69	59	49	38	28	18	07	10
51	88	76	64	52	40	27	15	03	7.0091	9
52	86	72	58	44	31	17	03	6.2189	75	8
53	84	69	53	37	22	06	5.4490	74	59	7
54	82	65	47	30	12	4.6694	77	59	42	6
55	81	61	41	22	03	84	64	45	25	5
56	79	58	36	15	94	73	52	30	09	4
57	77	54	31	08	85	61	38	15	6.9992	3
58	75	50	25	00	76	51	26	01	76	2
59	73	47	20	3.1093	3.8867	40	13	86	60	1
60	0.7772	1.5543	2.3315	3.1086	3.8858	4.6629	5.4401	6.2172	6.9944	0
1	2	3	4	5	6	7	8	9		
DEPARTURE 51 DEGREES.										

DEPARTURE 38 DEGREES.										151
	1	2	3	4	5	6	7	8	9	
0	0.6157	1.2313	1.8470	2.4626	3.0783	3.6940	4.3096	4.9253	5.5409	60
1	59	18	77	36	95	53	4.3112	71	30	59
2	61	22	84	45	3.0806	67	28	90	51	58
3	64	27	91	54	18	81	45	4.9308	72	57
4	66	32	97	63	29	95	61	26	92	56
5	68	36	1.8504	72	41	3.7009	77	45	5.5513	55
6	70	41	11	82	52	22	93	63	34	54
7	73	45	18	90	63	36	4.3208	81	53	53
8	75	50	25	2.4700	75	49	24	4.9400	74	52
9	77	54	32	09	86	63	40	18	95	51
10	80	59	39	18	98	77	57	36	5.5616	50
11	82	64	45	27	3.0909	91	73	54	36	49
12	84	68	52	36	21	3.7105	89	73	57	48
13	86	73	59	46	32	18	4.3305	91	78	47
14	89	77	66	55	44	32	21	4.9510	98	46
15	91	82	73	64	56	45	36	27	5.5718	45
16	93	86	80	73	66	59	52	46	39	44
17	96	91	87	82	78	73	69	64	60	43
18	98	96	93	91	89	87	85	82	80	42
19	0.6200	1.2400	1.8600	2.4800	3.1001	3.7201	4.3401	4.9601	5.5801	41
20	02	05	07	10	12	14	17	19	22	40
21	05	09	14	18	23	28	32	37	41	39
22	07	14	21	28	35	41	48	55	62	38
23	09	18	28	37	46	55	64	74	83	37
24	12	23	35	46	58	69	81	92	5.5904	36
25	14	28	41	55	69	83	97	4.9710	24	35
26	16	32	48	64	80	96	4.3512	28	44	34
27	18	37	55	73	92	3.7310	28	46	65	33
28	21	41	62	82	3.1103	24	44	65	85	32
29	23	46	69	92	15	37	60	83	5.6006	31
30	25	50	75	2.4900	26	51	76	4.9801	26	30
31	27	55	82	10	37	64	92	19	47	29
32	30	59	89	19	49	78	4.3608	38	67	28
33	32	64	96	28	60	92	24	56	88	27
34	34	68	1.8703	37	71	3.7405	39	74	5.6108	26
35	37	73	10	46	83	19	56	92	29	25
36	39	78	16	55	94	33	72	4.9910	49	24
37	41	82	23	64	3.1206	47	88	29	70	23
38	43	87	30	73	17	60	4.3703	46	90	22
39	46	91	37	82	28	74	19	65	5.6210	21
40	48	96	44	92	40	87	35	83	31	20
41	50	1.2500	51	2.5001	51	3.7501	51	5.0002	52	19
42	52	05	57	10	62	14	67	19	72	18
43	55	09	64	19	74	28	83	38	92	17
44	57	14	71	28	85	42	99	56	5.6313	16
45	59	18	78	37	96	55	4.3814	74	33	15
46	62	23	85	46	3.1308	69	31	92	54	14
47	64	28	91	55	19	83	47	5.0110	74	13
48	66	32	98	64	30	96	62	28	94	12
49	68	37	05	73	42	3.7610	78	46	5.6415	11
50	71	41	1.8812	82	53	24	94	65	35	10
51	73	46	18	91	65	37	4.3910	82	55	9
52	75	50	25	2.5100	76	51	26	5.0201	76	8
53	77	55	32	10	87	64	42	19	97	7
54	80	59	39	18	98	78	57	37	5.6516	6
55	82	64	46	28	3.1410	91	73	55	37	5
56	84	68	53	37	21	3.7705	89	74	58	4
57	86	73	59	46	32	18	4.4005	91	78	3
58	89	77	66	55	44	32	21	5.0810	98	2
59	91	82	73	64	55	45	36	27	5.6618	1
60	0.6293	1.2586	1.8880	2.5173	3.1446	3.7759	4.4052	5.0346	5.6639	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 51 DEGREES.										

152 LATITUDE 39 DEGREES.										
1	2	3	4	5	6	7	8	9	10	
0	0.7772	1.5543	2.3315	3.1086	3.8858	4.6629	5.4401	6.2172	6.9944	60
1	70	39	09	78	48	18	87	57	26	59
2	68	36	03	71	39	07	5.4375	42	10	58
3	66	32	2.3298	64	30	4.6596	62	28	6.9894	57
4	64	28	92	56	21	85	59	13	77	56
5	62	25	87	49	12	74	36	6.2098	61	55
6	61	21	82	42	03	63	24	84	45	54
7	59	17	76	34	3.8793	52	10	69	27	53
8	57	14	70	27	84	41	5.4298	54	11	52
9	55	10	65	20	75	30	85	40	6.9795	51
10	53	06	59	12	66	19	72	25	78	50
11	51	03	54	05	77	08	59	10	62	49
12	49	1.5499	48	3.0998	47	4.6496	46	6.1995	45	48
13	48	95	43	90	38	86	33	81	28	47
14	46	92	37	83	29	75	21	66	12	46
15	44	88	32	76	20	63	07	51	6.9695	45
16	42	84	26	68	11	53	5.4195	37	79	44
17	40	80	21	61	01	41	81	22	62	43
18	38	77	15	54	3.8692	30	69	07	46	42
19	37	73	10	46	83	20	56	6.1893	29	41
20	35	69	04	39	74	08	43	78	12	40
21	33	66	2.3199	32	65	4.6397	30	63	6.9596	39
22	31	62	93	24	55	86	17	48	79	38
23	29	59	88	17	46	75	04	34	63	37
24	27	55	82	09	37	64	5.4091	18	46	36
25	26	51	77	02	28	53	79	04	30	35
26	24	47	71	3.0894	18	42	65	6.1789	12	34
27	22	44	65	87	09	31	53	74	6.9496	33
28	20	40	60	80	00	19	39	59	79	32
29	18	36	54	72	3.8591	09	27	45	63	31
30	16	32	49	65	81	4.6297	13	30	46	30
31	14	29	43	58	72	86	01	15	30	29
32	13	25	38	50	63	75	5.3983	00	13	28
33	11	21	32	43	54	64	75	6.1685	6.9396	27
34	09	18	26	35	44	53	62	70	79	26
35	07	14	21	28	35	42	49	56	63	25
36	05	10	15	20	26	31	36	41	46	24
37	03	07	10	13	16	20	23	26	30	23
38	01	03	04	06	07	08	10	11	13	22
39	00	1.5399	2.3099	3.0798	3.8498	4.6198	5.3897	6.1597	6.9266	21
40	0.7698	95	93	91	89	86	84	82	79	20
41	96	92	88	84	80	75	71	67	63	19
42	94	88	82	76	70	64	58	52	46	18
43	92	84	76	68	61	53	45	37	29	17
44	90	81	71	61	52	42	32	22	13	16
45	88	77	65	54	42	30	19	07	6.9196	15
46	87	73	60	46	33	20	06	6.1493	79	14
47	85	69	54	39	24	08	5.3793	78	62	13
48	83	66	48	31	14	4.6097	80	62	45	12
49	81	62	43	24	05	86	67	48	29	11
50	79	58	37	16	96	75	54	33	12	10
51	77	54	32	09	3.8386	63	41	18	6.9095	9
52	75	51	26	02	77	52	28	03	79	8
53	74	47	21	2.0694	68	41	15	6.1388	62	7
54	72	43	15	87	59	30	02	74	45	6
55	70	40	09	79	49	19	5.3689	58	28	5
56	68	36	04	72	40	07	75	43	11	4
57	66	32	98	64	31	4.5997	63	29	6.8995	3
58	64	28	93	57	21	85	49	14	78	2
59	62	25	87	49	12	74	36	6.1298	61	1
60	0.7660	1.5321	2.2981	3.0642	3.8302	4.5962	5.3623	6.1283	6.8944	0
1	2	3	4	5	6	7	8	9		
DEPARTURE 50 DEGREES.										

DEPARTURE 89 DEGREES.										158
1	2	3	4	5	6	7	8	9	10	
0	0.6293	1.2586	1.8880	2.5173	3.1466	3.7759	4.4052	5.0346	5.6639	60
1	96	91	87	82	78	73	69	64	59	59
2	98	95	93	91	89	86	84	82	79	58
3	0.6300	1.2600	1.8900	2.5200	3.1500	3.7800	4.4100	5.0400	5.6700	57
4	02	04	07	09	11	13	15	18	20	56
5	05	09	14	18	23	27	32	36	41	55
6	07	14	20	27	34	41	48	54	61	54
7	09	18	27	36	45	54	63	72	81	53
8	11	23	34	45	57	68	79	90	5.6802	52
9	14	27	41	54	68	81	95	5.0508	22	51
10	16	32	47	63	79	95	4.4211	26	42	50
11	18	36	54	72	90	3.7908	26	44	62	49
12	20	41	61	81	3.1602	22	42	62	83	48
13	23	45	68	90	13	35	58	80	5.6903	47
14	25	50	74	99	24	49	74	98	23	46
15	27	54	81	2.5308	36	63	90	5.0617	44	45
16	29	60	88	17	47	76	4.4305	34	64	44
17	32	68	95	26	58	90	21	53	84	43
18	34	68	1.9001	35	69	3.8003	37	70	5.7004	42
19	36	72	08	44	81	17	53	89	25	41
20	38	77	15	53	92	30	68	5.0706	45	40
21	41	81	22	62	3.1703	44	84	25	65	39
22	43	86	28	71	14	57	4.4400	42	85	38
23	45	90	35	80	26	71	16	61	5.7106	37
24	47	95	42	89	37	84	31	78	26	36
25	50	99	49	98	48	98	47	97	46	35
26	52	1.2704	55	2.5407	59	3.8111	63	5.0814	66	34
27	54	08	62	16	70	24	78	32	86	33
28	56	13	69	25	82	38	94	50	5.7207	32
29	59	17	76	34	93	51	4.4510	68	27	31
30	61	22	82	43	3.1804	65	26	86	47	30
31	63	26	89	52	15	78	41	5.0904	67	29
32	65	31	96	61	27	92	57	22	88	28
33	68	35	1.9103	70	38	3.8205	73	40	5.7308	27
34	70	40	09	79	49	19	89	58	28	26
35	72	44	16	88	60	32	4.4604	76	48	25
36	74	48	23	97	71	45	19	94	68	24
37	77	53	30	2.5506	83	59	36	5.1012	89	23
38	79	57	36	15	94	72	51	30	5.7408	22
39	81	62	43	24	3.1905	86	67	48	29	21
40	83	66	50	33	16	99	82	66	49	20
41	85	71	56	42	27	3.8312	98	83	69	19
42	88	75	63	51	39	26	4.4714	5.1102	89	18
43	90	80	70	60	50	39	29	19	8.7509	17
44	92	84	77	69	61	53	45	38	30	16
45	94	89	83	78	72	66	61	55	50	15
46	97	93	90	86	83	80	76	73	69	14
47	99	98	97	96	95	93	92	91	90	13
48	0.6401	1.2802	1.9203	2.5604	3.2006	3.8407	4.4808	5.1209	5.7610	12
49	03	07	10	13	17	20	23	26	30	11
50	06	11	17	22	28	34	39	45	50	10
51	08	16	23	31	39	47	55	62	70	9
52	10	20	30	40	50	60	70	80	90	8
53	12	25	37	49	62	74	86	98	5.7711	7
54	15	29	44	58	73	87	4.4902	5.1316	31	6
55	17	33	50	67	84	3.8500	17	34	50	5
56	19	38	57	76	95	14	33	52	71	4
57	21	42	64	85	3.2006	27	48	70	91	3
58	23	47	70	94	17	40	64	87	5.7811	2
59	26	51	77	2.5702	28	54	79	5.1405	80	1
60	0.6428	1.2856	1.9284	2.5712	3.2140	3.8567	4.4995	5.1423	5.7851	0
1	2	3	4	5	6	7	8	9		
LATITUDE 50 DEGREES.										

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154										
LATITUDE 40 DEGREES.										
	1	2	3	4	5	6	7	8	9	
0	0.7660	1.5321	2.2981	3.0642	3.8302	4.5962	5.3623	6.1283	6.8944	60
1	59	17	76	34	3.8293	52	10	69	27	59
2	57	13	70	27	84	40	5.3597	54	10	58
3	55	10	64	19	74	29	84	38	6.8893	57
4	53	06	59	12	65	18	71	24	77	56
5	51	02	53	04	56	07	58	09	60	55
6	49	1.5298	48	3.0597	46	4.5895	44	6.1194	43	54
7	47	95	42	89	37	84	31	78	26	53
8	46	91	37	82	28	73	19	64	10	52
9	44	87	31	74	18	62	05	49	6.8792	51
10	42	83	25	67	09	50	5.3492	34	75	50
11	40	80	19	59	3.8199	39	79	18	58	49
12	38	76	14	52	90	28	66	04	42	48
13	36	72	08	44	81	17	53	6.1089	25	47
14	34	68	03	37	71	05	39	74	08	46
15	32	65	2.2897	29	62	4.5794	26	58	6.8691	45
16	30	61	91	22	52	82	13	43	74	44
17	29	57	86	14	43	72	00	29	57	43
18	27	53	80	07	34	60	5.3387	14	40	42
19	25	50	74	3.0499	24	49	74	6.0998	23	41
20	23	46	69	92	15	37	60	83	06	40
21	21	42	63	84	05	26	47	68	6.8590	39
22	19	38	58	77	3.8096	15	34	54	73	38
23	17	35	52	69	87	04	21	38	56	37
24	15	31	46	62	77	4.5692	08	23	39	36
25	14	27	41	54	68	81	5.3295	08	22	35
26	12	23	35	46	58	70	81	6.0893	04	34
27	10	19	29	39	49	58	68	78	6.8487	33
28	08	16	23	31	39	47	55	62	70	32
29	06	12	18	24	30	35	41	47	53	31
30	04	08	12	16	21	25	29	33	37	30
31	02	04	07	09	11	13	15	18	20	29
32	00	01	01	01	02	02	02	02	03	28
33	0.7598	1.5197	2.2795	3.0394	3.7992	4.5590	5.3189	6.0787	6.8386	27
34	97	93	90	86	83	79	76	72	69	26
35	95	89	84	78	73	68	62	57	51	25
36	93	85	78	71	64	56	49	42	34	24
37	91	82	72	93	54	45	36	26	17	23
38	89	78	67	56	45	33	22	11	00	22
39	87	74	61	48	35	22	09	7.0696	6.8283	21
40	85	70	55	40	26	11	5.3096	81	66	20
41	83	66	50	33	16	4.5499	82	66	49	19
42	81	63	44	25	07	88	69	50	32	18
43	79	59	38	18	3.7897	76	56	35	15	17
44	78	55	33	10	88	65	43	20	6.8198	16
45	76	51	27	02	78	54	29	05	80	15
46	74	48	21	3.0295	69	43	17	6.0590	64	14
47	72	44	16	88	60	31	03	75	47	13
48	70	40	10	80	50	19	5.2989	59	29	12
49	68	36	04	72	40	08	76	44	12	11
50	66	32	2.2698	64	31	4.5397	63	29	6.8096	10
51	64	28	93	57	21	85	49	14	78	9
52	62	25	87	49	12	74	36	6.0498	61	8
53	60	21	81	42	02	62	23	83	44	7
54	59	17	76	34	3.7793	51	10	68	27	6
55	57	13	70	26	83	40	5.2896	53	09	5
56	55	09	64	19	74	28	83	38	6.7992	4
57	53	06	58	11	64	17	70	22	75	3
58	51	02	53	04	55	05	56	07	58	2
59	49	1.5098	47	8.0196	45	4.5294	43	6.0392	41	1
60	0.7547	1.5094	2.2641	3.0188	3.7786	4.5283	5.2830	6.0377	6.7924	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 49 DEGREES.										

DEPARTURE 40 DEGREES.										155
1	2	3	4	5	6	7	8	9	10	
0	0.6428	1.2856	2.9284	2.5712	3.2140	3.8567	4.4995	5.1423	5.7851	60
1	30	60	90	20	51	81	4.5011	41	71	59
2	32	65	97	29	62	94	26	58	91	58
3	35	69	1.9304	38	73	3.8608	42	77	5.7911	57
4	37	74	10	47	84	21	58	94	31	56
5	39	78	17	56	95	34	73	5.1512	51	55
6	41	82	24	65	3.2206	47	88	30	71	54
7	44	87	31	74	18	61	4.5105	48	92	53
8	46	91	37	83	29	74	20	66	5.8011	52
9	48	96	44	92	40	87	35	83	31	51
10	50	1.2900	50	2.5800	51	3.8701	51	5.1601	51	50
11	52	05	57	10	62	14	67	19	72	49
12	55	09	64	18	73	28	82	37	91	48
13	57	14	70	27	84	41	98	54	5.8111	47
14	59	18	77	36	95	54	4.5213	72	31	46
15	61	22	84	45	3.2306	67	28	90	51	45
16	64	27	91	54	18	81	45	5.1708	72	44
17	66	31	97	63	29	94	60	26	91	43
18	68	36	1.9404	72	40	3.8807	75	43	5.8211	42
19	70	40	10	80	51	21	91	61	31	41
20	72	45	17	89	62	34	4.5306	78	51	40
21	75	49	24	98	73	48	22	97	71	39
22	77	54	30	2.5907	84	61	38	5.1814	91	38
23	79	58	37	16	95	74	53	32	5.8311	37
24	81	62	44	25	3.2406	87	68	50	31	36
25	83	67	51	34	17	3.8900	84	67	51	35
26	86	71	57	42	28	14	99	85	70	34
27	88	76	63	51	39	28	4.5415	5.1902	90	33
28	90	80	70	60	51	41	31	21	5.8411	32
29	92	85	77	69	62	54	46	38	31	31
30	95	89	84	78	73	67	62	56	51	30
31	97	93	90	87	84	81	77	74	70	29
32	99	98	97	96	95	93	92	91	90	28
33	0.6501	1.3002	1.9503	2.6004	3.2506	3.9007	4.5508	5.2009	5.8510	27
34	03	07	10	13	17	20	23	26	30	26
35	06	11	17	22	28	33	39	44	50	25
36	08	15	23	31	39	46	54	62	69	24
37	10	20	30	40	50	59	69	79	89	23
38	12	24	37	49	61	73	85	98	5.8610	22
39	14	29	43	58	72	86	4.5601	5.2115	30	21
40	17	33	50	66	83	3.9100	16	33	49	20
41	19	38	56	75	94	13	32	51	69	19
42	21	42	63	84	3.2605	26	47	68	89	18
43	23	46	70	93	16	39	62	86	5.8709	17
44	25	51	76	2.6102	27	52	78	5.2203	29	16
45	28	55	83	10	38	66	94	21	48	15
46	30	60	89	19	49	79	4.5709	38	68	14
47	32	64	96	28	60	92	24	56	88	13
48	34	68	1.9603	37	71	3.9205	39	74	5.8808	12
49	36	73	09	46	82	18	55	91	28	11
50	39	77	16	54	93	32	70	5.2309	47	10
51	41	82	22	63	3.2704	45	86	26	67	9
52	43	86	29	72	15	58	4.5801	44	87	8
53	45	90	36	81	26	71	16	62	5.8907	7
54	47	95	42	90	37	84	32	79	27	6
55	50	99	49	98	48	98	47	97	47	5
56	52	1.8104	55	2.6207	59	3.9311	63	5.2414	66	4
57	54	08	62	16	70	24	78	32	86	3
58	56	12	69	25	81	37	93	50	5.9006	2
59	58	17	75	34	92	50	4.5909	67	26	1
60	0.6561	1.8121	1.9682	2.6242	3.2808	3.9364	4.5924	5.2485	5.9045	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 49 DEGREES.										

156 LATITUDE 41 DEGREES.										
	1	2	3	4	5	6	7	8	9	!
0	0.7547	1.5094	2.2641	3.0188	3.7786	4.5283	5.2880	6.0377	6.7924	60
1	45	90	36	81	26	71	16	62	07	59
2	48	87	30	73	17	60	03	46	6.7890	58
3	41	83	24	66	08	48	5.2790	31	73	57
4	40	79	19	58	3.7698	37	77	16	56	56
5	88	75	13	50	88	25	63	00	38	55
6	36	71	07	42	78	14	49	6.0285	20	54
7	34	67	01	35	69	02	36	70	03	53
8	32	64	2.2595	27	59	4.5191	23	54	6.7786	52
9	30	60	90	20	50	79	09	39	69	51
10	28	56	84	12	40	68	5.2696	24	52	50
11	26	52	78	04	31	57	83	09	35	49
12	24	48	72	3.0096	21	45	69	6.0103	17	48
13	22	44	67	89	11	33	55	78	00	47
14	20	41	61	81	02	22	42	62	6.7683	46
15	18	37	55	74	3.7592	10	29	47	66	45
16	17	33	50	66	88	4.5099	16	32	49	44
17	15	29	44	58	73	88	02	17	31	43
18	13	25	38	50	63	76	5.2588	01	13	42
19	11	21	32	43	54	64	75	6.0086	6.7596	41
20	09	18	26	35	44	53	62	70	79	40
21	07	14	21	28	35	41	48	55	62	39
22	05	10	15	20	25	30	35	40	45	38
23	03	06	09	12	15	18	21	24	27	37
24	01	02	03	04	06	07	88	09	10	36
25	0.7499	1.4998	2.2498	2.9997	3.7496	4.4995	5.2494	5.9994	6.7493	35
26	97	95	92	89	87	84	81	78	76	34
27	95	91	86	81	77	72	67	62	58	33
28	93	87	80	74	67	60	54	47	41	32
29	92	83	75	66	58	49	41	32	24	31
30	90	79	69	58	48	38	27	17	06	80
31	88	75	63	50	38	26	13	01	6.7388	29
32	86	71	57	43	29	14	00	5.9886	71	28
33	84	68	51	35	19	03	5.2387	70	54	27
34	82	64	45	27	09	91	73	54	36	26
35	80	60	40	20	00	4.4879	59	39	19	25
36	78	56	34	12	3.7390	68	46	24	02	24
37	76	52	28	04	80	56	32	08	6.7284	23
38	74	48	22	2.9896	71	45	19	5.9793	67	22
39	72	44	17	89	61	33	05	78	50	21
40	70	41	11	81	52	22	5.2292	62	33	20
41	68	37	05	73	42	10	78	46	15	19
42	66	33	2.2399	66	32	4.4798	65	31	6.7198	18
43	64	29	93	58	24	86	51	15	80	17
44	63	25	88	50	13	75	38	00	63	16
45	61	21	82	42	03	64	24	5.9685	45	15
46	59	17	76	34	3.7293	52	10	69	27	14
47	57	13	70	27	84	40	5.2197	54	10	13
48	55	10	64	19	74	29	84	38	6.7093	12
49	53	06	58	11	64	17	70	22	75	11
50	51	02	53	04	55	05	56	07	58	10
51	49	1.4898	47	2.9796	45	4.4693	42	5.9591	40	9
52	47	94	41	88	35	82	29	76	23	8
53	45	90	35	80	26	71	16	61	06	7
54	43	86	29	72	16	59	02	45	6.6988	6
55	41	82	24	65	06	47	5.2088	30	71	5
56	39	78	18	57	3.7196	35	74	14	53	4
57	37	75	12	49	87	24	61	5.9498	36	3
58	35	71	06	41	77	12	47	82	18	2
59	33	67	00	34	67	00	84	67	01	1
60	0.7431	1.4863	2.2394	2.9726	3.7157	4.4588	5.2020	5.9451	6.6883	0
	1	2	3	4	5	6	7	8	9	!
DEPARTURE 48 DEGREES.										

DEPARTURE 41 DEGREES.										157
	1	2	3	4	5	6	7	8	9	
0	0.6561	1.3121	1.9682	2.6242	3.2803	3.9364	4.5924	5.2485	5.9045	60
1	63	26	88	51	14	77	40	5.2502	65	59
2	65	30	95	60	25	90	55	20	85	58
3	67	34	1.9702	69	36	3.9403	70	38	5.9105	57
4	69	39	08	78	47	16	86	55	25	56
5	72	43	15	86	58	30	4.6001	73	44	55
6	74	48	21	95	69	43	17	90	64	54
7	76	52	28	2.6304	80	55	31	5.2607	83	53
8	78	56	34	12	91	68	47	25	5.9203	52
9	80	61	41	21	3.2902	82	62	42	23	51
10	83	65	48	30	13	95	78	60	43	50
11	85	69	54	39	24	3.9508	93	78	62	49
12	87	74	61	48	35	21	4.6108	95	82	48
13	89	78	67	56	46	35	24	5.2713	5.9302	47
14	91	83	74	65	57	48	39	30	22	46
15	94	87	81	74	68	61	55	48	42	45
16	96	91	87	82	78	74	69	65	60	44
17	98	96	93	91	89	87	85	82	80	43
18	0.6600	1.3200	1.9800	2.6400	3.3000	3.9600	4.6200	5.2800	5.9400	42
19	02	04	07	09	11	13	15	18	20	41
20	04	09	13	18	22	26	31	35	40	40
21	07	13	20	26	33	40	46	53	59	39
22	09	18	26	35	44	53	62	70	79	38
23	11	22	33	44	55	65	76	87	98	37
24	13	26	39	52	66	79	92	5.2905	5.9518	36
25	15	31	46	61	77	92	4.6307	22	38	35
26	18	35	53	70	88	3.9705	23	40	58	34
27	20	39	59	79	99	18	38	58	77	33
28	22	44	65	87	3.3109	31	53	74	96	32
29	24	48	72	96	20	44	68	92	5.9616	31
30	26	52	79	2.6505	31	57	83	5.3090	36	30
31	28	57	85	14	42	70	99	27	56	29
32	31	61	92	22	53	84	4.6414	45	75	28
33	33	65	98	31	64	96	29	62	94	27
34	35	70	1.9905	40	75	3.9809	44	80	5.9714	26
35	37	74	11	48	86	23	60	97	34	25
36	39	79	18	57	97	36	75	5.3114	54	24
37	41	83	24	66	3.3207	48	90	31	73	23
38	44	87	31	74	18	62	4.6505	49	92	22
39	46	92	37	83	29	75	21	66	5.9812	21
40	48	96	44	92	40	88	36	84	32	20
41	50	1.3300	50	2.6600	51	3.9901	51	5.3201	51	19
42	52	05	57	09	62	14	66	18	71	18
43	54	09	64	18	73	27	82	36	91	17
44	57	13	70	26	83	40	96	53	5.9909	16
45	59	18	76	35	94	53	4.6612	70	29	15
46	61	22	83	44	3.3305	66	27	88	49	14
47	63	26	90	53	16	79	42	5.3306	69	13
48	65	31	96	61	27	92	57	22	88	12
49	68	35	2.0003	70	38	4.0005	73	40	6.0008	11
50	70	39	05	79	49	18	88	58	27	10
51	72	44	16	87	59	31	4.6703	74	46	9
52	74	48	22	96	70	44	18	92	66	8
53	76	52	29	2.6705	81	57	33	5.3410	86	7
54	78	57	35	13	92	70	48	26	6.0105	6
55	81	61	42	22	3.3403	83	64	44	25	5
56	83	65	48	31	14	96	79	62	44	4
57	85	70	54	39	24	4.0109	94	78	63	3
58	87	74	61	48	35	22	4.6809	96	83	2
59	89	78	67	56	46	35	24	5.3513	6.0202	1
60	0.6691	1.3383	2.0074	2.6765	3.3457	4.0148	4.6839	5.3530	6.0222	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 48 DEGREES.										

158									
LATITUDE 42 DEGREES.									
	1	2	3	4	5	6	7	8	9
0	0.7431	1.4863	2.2294	2.9726	3.7157	4.4588	5.2020	5.9451	6.6883
1	20	59	89	18	48	77	07	36	66
2	28	55	83	10	38	66	5.1993	21	48
3	26	51	77	02	28	54	79	05	30
4	24	47	71	2.9695	19	42	66	5.9390	13
5	22	43	65	87	09	30	52	74	6.6795
6	20	40	59	79	3.7099	20	39	58	78
7	18	36	53	71	89	07	25	42	60
8	16	32	48	64	80	4.4495	11	27	43
9	14	28	42	56	70	83	5.1897	11	25
10	12	24	36	48	60	72	84	5.9296	08
11	10	20	30	40	50	60	70	80	6.6690
12	08	16	24	32	40	48	56	64	72
13	06	12	18	24	30	37	43	49	55
14	04	08	12	16	21	25	29	33	37
15	02	04	07	09	11	13	15	18	20
16	00	00	01	01	01	01	01	02	02
17	6.7398	1.4797	2.2195	2.9593	3.6992	4.4390	5.1788	5.9186	6.6585
18	96	93	89	85	82	78	74	70	67
19	94	89	83	78	72	66	61	55	50
20	92	85	77	70	62	54	47	39	32
21	90	81	71	62	52	42	33	23	14
22	89	77	66	54	43	31	20	08	6.6497
23	87	73	60	46	33	19	06	5.9092	79
24	85	69	54	38	23	08	5.1692	77	61
25	83	65	48	30	13	4.4296	78	61	43
26	81	61	42	22	03	84	64	45	25
27	79	57	36	15	3.6894	72	51	30	08
28	77	53	20	07	84	60	37	14	6.6390
29	75	49	24	2.9499	74	48	23	5.8998	72
30	73	46	18	91	64	37	10	82	55
31	71	42	12	83	54	25	5.1596	66	37
32	69	38	06	75	44	13	82	50	19
33	67	34	01	68	35	01	68	35	02
34	65	30	2.2095	60	25	4.4189	54	19	6.6284
35	63	26	89	52	15	77	40	03	66
36	61	22	83	44	05	66	27	5.8888	49
37	59	18	77	36	3.6795	54	13	72	31
38	57	14	71	28	85	42	5.1499	56	13
39	55	10	65	30	76	31	86	41	6.6196
40	53	06	59	12	66	19	72	25	78
41	51	02	53	04	56	07	58	09	59
42	49	1.4698	47	2.9896	46	95	44	5.8793	42
43	47	94	42	89	36	83	30	78	25
44	45	90	36	81	26	71	16	62	07
45	43	86	20	73	16	59	02	42	6.6089
46	41	82	24	65	06	47	5.1388	30	71
47	39	79	18	57	2.6697	36	75	14	54
48	37	75	12	49	87	24	61	5.8698	36
49	35	71	06	41	77	12	47	82	18
50	33	67	00	33	67	00	33	66	00
51	31	63	2.1994	26	57	4.3988	20	51	6.5983
52	29	59	88	18	47	76	06	85	65
53	27	55	82	10	37	64	5.1292	19	47
54	25	51	76	02	27	52	78	03	29
55	23	47	70	2.9294	17	40	64	5.8587	11
56	22	43	65	86	08	29	51	72	6.5894
57	20	39	59	78	2.6598	17	37	56	76
58	18	35	53	70	88	05	23	40	58
59	16	31	47	62	78	4.3893	09	24	40
60	0.7814	1.4627	2.1941	2.9254	3.6568	4.3881	5.1195	5.8508	6.5822
	1	2	3	4	5	6	7	8	9
DEPARTURE 47 DEGREES.									

DEPARTURE 42 DEGREES.										159
1	2	3	4	5	6	7	8	9	10	
0	0.6691	1.3883	2.0074	2.6765	3.3457	4.0148	4.6839	5.3530	6.0222	60
1	94	87	81	74	68	61	55	48	42	59
2	96	91	87	82	78	74	69	65	60	58
3	98	96	93	91	89	87	85	82	80	57
4	0.6700	1.3400	2.0100	00	3.3500	99	99	99	99	56
5	02	04	06	2.6808	11	4.0213	4.6915	5.3617	6.0309	55
6	04	09	13	17	22	26	30	34	39	54
7	06	13	19	26	32	38	45	51	58	53
8	09	17	26	34	43	52	60	69	77	52
9	11	21	32	43	54	64	75	86	96	51
10	13	26	39	52	65	77	90	5.3703	6.0416	50
11	15	30	45	60	76	91	4.7006	21	36	49
12	17	34	52	69	86	4.0303	20	38	55	48
13	19	39	58	78	97	16	36	55	75	47
14	22	43	65	86	3.3608	29	51	72	94	46
15	24	47	71	95	19	42	66	90	6.0513	45
16	26	52	77	2.6903	29	55	81	5.3806	32	44
17	28	56	84	12	40	68	96	24	52	43
18	30	60	90	20	51	81	4.7111	41	71	42
19	32	65	97	29	62	94	26	58	91	41
20	34	69	03	38	72	4.0406	41	75	6.0610	40
21	37	73	2.0210	46	83	20	56	93	29	39
22	39	77	16	55	94	32	71	5.3910	48	38
23	41	82	22	64	3.3705	45	86	27	68	37
24	43	86	29	72	15	58	4.7201	44	87	36
25	45	90	36	81	26	71	16	62	6.0707	35
26	47	95	42	89	37	84	31	78	26	34
27	50	99	49	98	48	97	47	96	46	33
28	52	1.3503	55	2.7006	58	4.0510	61	5.4013	64	32
29	54	08	61	15	69	23	77	30	84	31
30	56	12	68	24	80	35	91	47	6.0803	30
31	58	16	74	32	90	48	4.7306	64	22	29
32	60	20	81	41	3.3801	61	21	82	42	28
33	62	25	87	49	12	74	36	98	61	27
34	65	29	94	58	23	87	52	5.4116	81	26
35	67	33	2.0300	66	33	4.0600	66	33	99	25
36	69	38	06	75	44	13	82	50	6.0919	24
37	71	42	13	84	55	25	96	67	38	23
38	73	46	19	92	65	38	4.7411	84	57	22
39	75	50	26	2.7101	76	51	26	5.4202	77	21
40	77	55	32	09	87	64	41	18	96	20
41	80	59	39	18	98	77	57	36	6.1016	19
42	82	63	45	26	3.3908	90	71	53	34	18
43	84	67	51	35	19	4.0702	86	70	53	17
44	86	72	58	44	30	15	4.7501	87	78	16
45	88	76	64	52	40	28	16	5.4304	92	15
46	90	80	70	60	51	41	31	21	6.1111	14
47	92	85	77	69	62	53	46	38	31	13
48	94	89	83	78	72	66	61	55	50	12
49	97	93	90	86	83	79	76	72	69	11
50	99	97	96	95	94	92	91	90	88	10
51	0.6801	1.3602	2.0402	2.7203	3.4004	4.0805	4.7606	5.4406	6.1207	9
52	03	06	09	12	15	17	20	23	26	8
53	05	10	15	20	26	31	36	41	46	7
54	07	14	22	29	36	43	50	58	65	6
55	09	19	28	37	47	56	65	74	84	5
56	12	23	35	46	58	69	81	92	9.1304	4
57	14	27	41	54	68	82	95	5.4509	22	3
58	16	31	47	63	79	94	4.7710	26	41	2
59	18	36	54	72	90	4.0907	25	43	61	1
60	0.6820	1.3640	2.0460	2.7280	3.4100	4.0920	4.7740	5.4560	6.1380	0
1	2	3	4	5	6	7	8	9		
LATITUDE 47 DEGREES.										

160										
LATITUDE 48 DEGREES.										
	1	2	3	4	5	6	7	8	9	10
0	0.7814	1.4627	2.1941	2.9254	3.6568	4.3881	5.1195	5.8508	6.5822	60
1	12	23	35	46	58	70	81	5.8498	04	59
2	10	19	29	38	48	58	67	77	6.5786	58
3	08	15	23	30	38	46	53	41	68	57
4	06	11	17	22	28	34	39	45	50	56
5	04	07	11	14	18	22	25	29	32	55
6	02	03	05	06	08	10	11	13	14	54
7	00	1.4599	2.1899	2.9198	3.6498	4.3798	5.1097	5.8397	06	53
8	0.7298	95	93	90	88	86	83	81	6.5678	52
9	96	91	87	83	79	74	70	66	61	51
10	94	87	81	75	69	62	56	50	43	50
11	92	83	75	67	59	50	42	34	25	49
12	90	79	69	59	49	38	28	18	07	48
13	88	75	63	51	39	26	14	01	6.5589	47
14	86	71	57	43	29	14	00	5.8286	71	46
15	84	67	51	35	19	02	5.0986	70	52	45
16	82	63	45	27	09	4.3690	72	54	35	44
17	80	59	39	19	3.6399	78	58	38	17	43
18	78	55	33	11	89	66	44	22	6.5499	42
19	76	51	27	03	79	54	30	06	81	41
20	74	47	21	2.9095	69	42	16	5.8190	63	40
21	72	43	15	87	59	30	02	74	45	39
22	70	39	09	79	49	18	5.0888	58	27	38
23	68	35	03	71	39	06	74	42	09	37
24	66	31	2.1797	63	29	4.3594	60	26	6.5391	36
25	64	27	91	55	19	82	46	10	73	35
26	62	23	85	47	09	70	32	5.8094	55	34
27	60	19	79	39	3.6299	58	18	78	37	33
28	58	15	73	31	89	46	04	62	19	32
29	56	11	67	23	79	34	5.0790	46	01	31
30	54	07	61	15	69	22	76	30	6.5283	30
31	52	03	55	07	59	10	62	14	65	29
32	50	1.4499	49	2.8999	49	4.3498	48	5.7998	47	28
33	48	95	43	91	39	86	34	82	29	27
34	46	91	37	83	29	74	20	66	11	26
35	44	87	31	75	19	62	06	50	6.5193	25
36	42	83	25	67	09	50	5.0692	34	75	24
37	40	79	19	59	3.6199	38	78	18	57	23
38	38	75	13	51	89	26	64	02	39	22
39	36	71	07	43	79	14	60	5.7886	21	21
40	34	67	01	35	69	02	36	70	03	20
41	32	63	2.1795	27	59	4.3390	22	54	6.5085	19
42	30	59	89	19	49	78	08	38	67	18
43	28	55	83	11	39	66	5.0594	22	49	17
44	26	51	77	03	29	54	80	06	31	16
45	24	47	71	2.8894	18	42	65	5.7789	12	15
46	22	43	65	86	08	30	51	73	6.4994	14
47	20	39	59	78	3.6098	18	37	57	76	13
48	18	35	53	70	88	06	23	41	58	12
49	16	31	47	62	78	4.3294	09	25	40	11
50	14	27	41	54	68	82	5.0495	09	22	10
51	12	23	35	46	58	70	81	5.7693	04	9
52	10	19	29	38	48	57	67	76	6.4886	8
53	08	15	23	30	38	45	53	60	68	7
54	06	11	17	22	28	33	39	44	50	6
55	04	07	11	14	18	21	25	28	32	5
56	02	03	05	06	08	09	11	12	14	4
57	00	1.4399	2.1699	2.8798	3.5997	4.3197	5.0397	5.7596	6.4796	3
58	0.7197	95	92	90	87	84	82	79	77	2
59	95	91	86	82	77	72	68	63	59	1
60	0.7193	1.4387	2.1580	2.8774	3.5967	4.3160	5.0354	5.7547	6.4741	0
	1	2	3	4	5	6	7	8	9	
DEPARTURE 46 DEGREES.										

DEPARTURE 43 DEGREES.										161
1	2	3	4	5	6	7	8	9	10	
0	0.6820	1.3640	2.0460	2.7280	3.4100	4.0920	4.7740	5.4560	6.1380	60
1	22	44	66	88	11	33	55	77	99	59
2	24	48	73	97	21	45	69	94	6.1418	58
3	26	53	79	2.7306	32	58	85	5.4611	38	57
4	29	57	86	14	43	71	4.7800	28	57	56
5	31	61	92	22	53	84	14	45	75	55
6	33	65	98	31	64	96	29	62	94	54
7	35	70	2.0505	40	75	4.1009	44	79	6.1514	53
8	37	74	11	48	85	22	59	96	33	52
9	39	78	17	56	96	35	74	5.4713	52	51
10	41	82	24	65	3.4206	47	88	30	71	50
11	43	87	30	73	17	60	4.7903	46	90	49
12	46	91	37	82	28	73	19	64	6.1610	48
13	48	95	43	90	38	86	33	81	28	47
14	50	99	49	99	49	98	48	98	47	46
15	52	1.3704	55	2.7407	59	4.1111	63	5.4814	66	45
16	54	08	62	16	70	23	77	31	85	44
17	56	12	68	24	81	37	93	49	6.1705	43
18	58	16	75	33	91	49	4.8007	66	24	42
19	60	21	81	41	3.4302	62	22	82	43	41
20	62	25	87	50	12	74	37	99	62	40
21	65	29	94	58	23	87	52	5.4916	81	39
22	67	33	2.0600	66	33	4.1200	66	33	99	38
23	69	38	06	75	44	13	82	50	6.1819	37
24	71	42	13	84	55	25	96	67	38	36
25	73	46	19	92	65	38	4.8111	84	57	35
26	75	50	25	2.7500	76	51	26	5.5001	76	34
27	77	54	32	09	86	63	40	18	95	33
28	79	59	38	17	97	76	55	24	6.1914	32
29	81	63	44	26	3.4407	88	70	51	33	31
30	84	67	51	34	18	4.1301	85	68	52	30
31	86	71	57	43	29	14	4.8200	86	71	29
32	88	76	63	51	39	27	15	5.5102	90	28
33	90	80	70	60	50	39	29	19	6.2009	27
34	92	84	76	68	60	52	44	36	28	26
35	94	89	83	78	72	66	61	35	50	25
36	96	92	89	85	81	77	73	70	66	24
37	98	97	95	93	92	90	88	86	85	23
38	0.6900	1.3801	2.0701	2.7602	3.4502	4.1402	4.8303	5.5203	6.2104	22
39	03	05	07	10	13	15	18	20	23	21
40	05	09	14	18	23	28	32	37	41	20
41	07	13	20	27	34	40	47	54	60	19
42	09	18	26	35	44	53	62	70	79	18
43	11	22	33	44	55	65	76	87	98	17
44	13	26	39	52	65	78	91	5.5304	6.2217	16
45	15	30	45	60	76	91	4.8406	21	36	15
46	17	34	52	69	86	4.1503	20	38	55	14
47	19	38	58	77	97	16	35	54	74	13
48	21	43	64	86	3.4607	28	50	71	93	12
49	24	47	71	94	18	41	65	88	6.2312	11
50	26	51	77	2.7702	28	54	79	5.5405	30	10
51	28	55	83	11	39	66	94	22	49	9
52	30	60	89	19	49	79	4.8509	38	68	8
53	32	64	96	28	60	91	23	55	87	7
54	34	68	2.0802	36	70	4.1604	38	72	6.2406	6
55	36	72	08	44	81	17	53	89	25	5
56	38	76	15	53	91	29	67	5.5506	44	4
57	40	80	21	61	3.4702	42	82	22	63	3
58	42	85	27	70	12	54	97	39	82	2
59	44	89	34	78	23	67	4.8612	56	6.2501	1
60	0.6947	1.3893	2.0840	2.7786	3.4733	4.1680	6.8626	5.5573	6.2519	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 46 DEGREES.										

162 LATITUDE 44 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.7193	1.4887	2.1580	2.8774	3.5967	4.3160	5.0354	5.7547	6.4741	60
1	91	83	74	66	57	48	40	31	23	59
2	89	79	68	58	47	36	26	15	05	58
3	87	75	62	49	37	24	11	5.7498	6.4686	57
4	85	71	56	41	27	12	97	82	68	56
5	83	67	50	33	17	00	5.0283	66	50	55
6	81	63	44	25	07	4.3088	69	50	32	54
7	79	58	38	17	3.5896	75	54	34	13	53
8	77	54	32	09	86	63	40	18	6.4595	52
9	75	50	26	01	76	51	26	02	77	51
10	73	46	20	2.8693	66	39	12	5.7386	59	50
11	71	42	13	84	56	27	5.0198	69	40	49
12	69	38	07	76	46	15	84	53	22	48
13	67	34	01	68	36	03	70	37	04	47
14	65	20	2.1495	60	25	4.2990	55	20	6.4485	46
15	63	26	89	52	15	78	41	04	67	45
16	61	22	83	44	05	66	27	5.7288	49	44
17	59	18	77	36	3.5795	54	13	72	31	43
18	57	14	71	28	85	41	5.0098	55	12	42
19	55	10	65	20	75	29	84	39	6.4394	41
20	53	06	59	12	65	17	70	23	76	40
21	51	02	52	03	54	05	56	06	57	39
22	49	1.4298	46	2.8595	44	4.2893	42	90	39	38
23	47	94	40	87	34	81	28	5.7174	21	37
24	45	89	34	79	24	68	13	58	02	36
25	43	85	28	71	14	56	4.9999	42	6.4284	35
26	41	81	22	63	04	44	85	26	66	34
27	39	77	16	54	3.5693	32	70	09	47	33
28	37	73	10	46	83	20	56	5.7093	29	32
29	35	69	04	38	73	07	42	76	11	31
30	33	65	2.1398	30	63	4.2795	28	60	6.4193	30
31	31	61	92	22	53	83	14	43	74	29
32	28	57	85	14	42	70	4.9899	27	56	28
33	26	53	79	06	32	58	85	11	37	27
34	24	49	73	2.8497	22	46	70	5.6994	19	26
35	22	45	67	89	12	34	56	78	01	25
36	20	41	61	81	02	22	42	62	6.4083	24
37	18	36	55	73	3.5591	09	27	46	64	23
38	16	32	49	65	81	4.2697	13	30	46	22
39	14	28	42	56	71	85	4.9799	13	27	21
40	12	24	36	48	61	73	85	5.6897	09	20
41	10	20	30	40	50	60	70	80	6.3990	19
42	08	16	24	32	40	48	56	64	72	18
43	06	12	18	24	30	35	41	47	53	17
44	04	08	12	16	20	23	27	31	35	16
45	02	04	06	08	10	11	13	15	17	15
46	00	00	00	2.8399	3.5499	4.2599	4.9699	5.6798	2.3898	14
47	0.7098	1.4196	2.1293	91	89	87	85	82	80	13
48	96	91	87	83	79	74	70	66	61	12
49	94	87	81	75	69	62	56	49	43	11
50	92	83	75	66	58	50	41	33	24	10
51	90	79	69	58	48	38	27	17	06	9
52	88	75	63	50	38	25	13	00	6.3788	8
53	86	71	57	42	28	13	4.9599	5.6684	70	7
54	83	67	50	34	17	00	84	67	51	6
55	81	63	44	25	07	4.2488	69	50	32	5
56	79	59	38	17	3.5397	76	55	34	14	4
57	77	54	32	09	86	63	40	18	6.3695	3
58	75	50	26	01	76	51	26	02	77	2
59	73	46	19	2.8292	66	39	12	5.6585	58	1
60	0.7071	1.4142	2.1213	2.8284	3.5356	4.2427	4.9498	5.6569	6.3640	0
1	2	3	4	5	6	7	8	9		
DEPARTURE 45 DEGREES.										

DEPARTURE 44 DEGREES.										163
	1	2	3	4	5	6	7	8	9	
0	0.6947	1.3893	2.0840	2.7786	3.4733	4.1680	4.8626	5.5572	6.2519	60
1	49	97	46	95	44	92	41	90	38	59
2	51	1.3902	52	2.7803	54	4.1705	56	5.5606	57	58
3	53	06	59	12	65	17	70	23	76	57
4	55	10	65	20	75	29	84	39	94	56
5	57	14	71	28	85	42	99	56	6.2613	55
6	59	18	77	36	96	55	4.8714	73	82	54
7	61	22	84	45	2.4806	67	28	90	51	53
8	63	27	90	53	17	80	43	5.5706	70	52
9	65	31	96	62	27	92	58	23	89	51
10	68	35	2.0903	70	38	4.1805	73	40	6.2708	50
11	70	39	09	78	48	18	87	57	26	49
12	72	43	15	87	59	30	4.8802	74	45	48
13	74	47	21	95	69	42	16	90	63	47
14	76	52	27	2.7903	79	55	31	5.5806	82	46
15	78	56	34	12	90	67	46	23	6.2801	45
16	80	60	40	20	2.4900	80	60	40	20	44
17	82	64	46	28	11	93	75	57	39	43
18	84	68	53	37	21	4.1905	89	74	58	42
19	86	72	59	45	31	17	4.8903	90	76	41
20	88	77	65	53	42	30	18	5.5906	95	40
21	90	81	71	62	52	42	33	23	6.2914	39
22	93	85	78	70	63	55	48	40	33	38
23	95	89	84	78	73	68	62	57	51	37
24	97	93	90	86	83	80	76	73	69	36
25	99	97	96	95	94	92	91	90	88	35
26	0.7001	1.4002	2.1002	2.8003	2.5004	4.2005	4.9006	5.6006	6.3007	34
27	03	06	09	12	15	17	20	23	26	33
28	05	10	15	20	25	29	34	39	44	32
29	07	14	21	28	35	42	49	56	63	31
30	09	18	27	36	46	55	64	73	82	30
31	11	22	34	45	56	67	78	90	6.3101	29
32	13	26	40	53	66	79	92	5.6106	19	28
33	15	31	46	61	77	92	4.9107	22	38	27
34	17	35	52	70	87	4.2104	22	39	57	26
35	20	39	59	78	98	17	37	56	76	25
36	22	43	65	86	2.5108	29	51	72	94	24
37	24	47	71	94	18	42	65	89	6.3212	23
38	26	51	77	2.8103	29	54	80	5.6206	81	22
39	28	55	83	11	39	66	94	22	49	21
40	30	60	89	19	49	79	4.9209	38	68	20
41	32	64	96	28	60	92	23	55	87	19
42	34	68	2.1102	36	70	4.2203	37	71	6.3305	18
43	36	72	08	44	80	16	52	88	24	17
44	38	76	14	52	91	29	67	5.6305	43	16
45	40	80	20	60	2.5201	41	81	21	61	15
46	42	84	27	69	11	53	95	38	80	14
47	44	89	33	77	22	66	4.9310	54	99	13
48	46	93	39	85	32	78	24	70	6.3417	12
49	48	97	45	94	42	90	39	87	36	11
50	51	1.4101	52	2.8202	53	4.2303	54	5.6404	55	10
51	53	05	58	10	63	15	68	20	73	9
52	55	09	64	18	73	28	82	37	91	8
53	57	13	70	27	84	40	97	54	6.3510	7
54	59	17	76	35	94	52	4.9411	70	28	6
55	61	22	82	43	2.5304	65	26	86	47	5
56	63	26	88	51	14	77	40	5.6502	65	4
57	65	30	95	60	25	89	54	19	84	3
58	67	34	2.1201	68	35	4.2402	69	36	6.3608	2
59	69	38	07	76	45	14	83	52	21	1
60	0.7071	1.4142	2.1213	2.8284	3.5350	4.2427	4.9498	5.6569	6.3640	0
	1	2	3	4	5	6	7	8	9	
LATITUDE 45 DEGREES.										

EXPLANATION OF THE TRAVERSE TABLE.

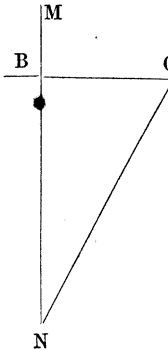
Latitude is the distance made in a north or south direction on a given meridian, by running a line at any bearing less than 90 degrees from that meridian; or it is the distance on any line parallel to a given meridian. When the given meridian is assumed at true north and south, the distance made in running on a course in a northerly direction is termed north latitude, or northing; and if ran southerly, the distance south is termed south latitude, or southing.

Departure is the distance perpendicular to the given meridian that is made by running on a given course. East departure, or easting, is when the line is run east of the meridian. West departure, or westing, is when the line is run west of the meridian.

Example. Let M N represent the meridian, such as any line (generally assumed north and south); let the point M = north, and the point N = south; let the bearing of the line N C = N. 44°, 17' E., and the distance N C = 9,74 chains = 9 chains and 74 links.

Here N B is the latitude made, and B C is the departure perpendicular to the meridian or base line N M; consequently, N B is north latitude or northing, and B C is the east departure or easting.

Or, latitude N B = cosine of the $\angle$ C N B $\times$ by the distance N C. And departure B C = sine of the $\angle$ C N B $\times$ by the distance N C.



The degrees are at the top and bottom, and the minutes in the outer columns. The distances 2, 3, etc., to 9, at top and bottom, may be used as chains, tenths of a chain, or links.

<i>Example.</i> Lat. N C for 44° 17', and distance 9 chains =	6,4431
Lat. N C for 44° 17', and distance 90 chains, remove the decimal point one place to the right =	64,431
Lat. N C for 44° 17', and distance 900 chains, remove the point two places to the right =	644,31
Lat. N C for 44° 17', and distance 90 links, or ,9 chains, remove the point one place to the left =	0,64431
Lat. N C for 44° 17', and distance 9 links, or ,09 chains, remove the point two places to the left =	0,064431

Application. Given the course N 44° 17' E., and distance N C = 97,48 chains, to find the latitude N B and departure B C.

Take a piece of card paper, two inches wide, and as long as the width of the page; have it ruled, and numbered 1, 2, 3, etc., to 9, similar to the tables. Lay this across, from 17' to 23', under latitude 44°. Lay a small weight on the guide paper; then under the edge of the paper you will have the required numbers to be taken out.

Under 9 chains we have 6,4431 .. for ,90 chains we have	64,4310
Under 7 chains we have	5,0113
Under 4 chains we have 2,8636 .. for ,4 chains we have	0,2864
Under 8 chains we have 5,7272 .. for ,08 chains we have	0,0573
	Latitude N B = 69,7860 chains.

Let the distance be 9748 links.

Under 9 we have 6,4431 .. 9000, remove the point 3 places =	6443,1
Under 7 we have 5,0113 .. 700, remove the point 2 places =	501,13
Under 4 we have 2,8636 .. 40, remove the point 1 place =	28,636
Under 8 we have 5,7272 .. 8,	5,723
	6975,589
	Latitude N C = 6978 ⁵⁹ / ₁₀₀ links.

TABLE II.—*Expansion of Solids in Direction of their Lengths, from 32° to 212° (Change of Temperature 180°).*

Name of Substance.	Authority.	Vulgar fractions.	Dec. frac. 180° ch.	con. orex. in 10° ch.
Platinum.	Troughton.	1 in 1008	0.0009918	0.0000551
do.	Dulong & Petit.	1 in 1131	0.0008242	0.0000458
do.	Borda.	1 in 1167	0.0008566	0.0000476
do.	Hasler.	1 in 1082	0.0009242	0.0000512
Mean of the four.		1 in 1094	0.0009142	0.0000508
Glass, white barometer tube.	Smeaton.	1 in 1175	0.0008510	0.0000472
" flint.	Brunner.	1 in 1248	0.0008012	0.0000445
" tube, without lead (4 sorts).	Lavoisier & Laplace.	1 in 1115	0.0008969	0.0000492
" with lead.	Brunner.	1 in 1142	0.0008757	0.0000486
Steel, not tempered.	Lavoisier & Laplace.	1 in 927	0.0010788	0.0000599
" tempered yellow at 149°.	do.	1 in 807	0.0012396	0.0000699
" rod.	Major General Roy.	1 in 847	0.0011807	0.0000656
" blistered.	Smeaton.	1 in 870	0.0011500	0.0000632
" tempered.	do.	1 in 816	0.0012583	0.0000699
Iron wire.	Troughton.	1 in 840	0.0011899	0.0000661
"	Brunner.	1 in 812	0.0012550	0.0000685
"	Smeaton.	1 in 795	0.0012583	0.0000699
" cast (prism).	Major General Roy.	1 in 901	0.0011000	0.0000617
" bar.	Smeaton.	1 in 795	0.0012583	0.0000699
"	Hasler.	1 in 797	0.0012534	0.0000696
" forged.	Lavoisier & Laplace.	1 in 819	0.0012205	0.0000678
Copper, mean of three specimens.	do.	1 in 582	0.0017122	0.0000951
"	Troughton.	1 in 521	0.0019188	0.0001066
" hammered.	Smeaton.	1 in 588	0.0017000	0.0000944
"	Brunner.	1 in 581	0.0017211	0.0000956
" eight parts, tin 1.	Smeaton.	1 in 550	0.0018167	0.0001009
Brass, cast.	do.	1 in 538	0.0018750	0.0001042
" wire.	do.	1 in 517	0.0019833	0.0001074
" Hamburgh.	Roy.	1 in 539	0.0018555	0.0001031
" English angular.	do.	1 in 528	0.0018945	0.0001052
" English round rod.	do.	1 in 528	0.0018930	0.0001052
" mean of three specimens.	Lavoisier & Laplace.	1 in 532	0.0018797	0.0001044
Antimony.	Smeaton.	1 in 928	0.0010823	0.0000602
Bismuth.	do.	1 in 719	0.0013917	0.0000772
Lead.	do.	1 in 849	0.0012867	0.0001592
Tin, fine.	do.	1 in 438	0.0022833	0.0001257
" grain.	do.	1 in 403	0.0024833	0.0001229
Zinc.	do.	1 in 340	0.0029417	0.0001634
Pine, white, Norway.	Captain Kater.		0.004083	0.0000227

Example. A surveyor had adjusted his chain at a temperature of 60°, the standard chain of 66 feet or 100 links being cut in the floor of a public hall. During the time that he measured a line of 8000 links, the mean temperature had been 105° Required the *true length of the line*, the chain being of iron wire.

From col. 10° correction for 1° = 0,00000685

45°

to be added,

,00030825

1

1 + c = 1,00030825 here = c = correction.

8000

8002,466 links = true length.

(1 + c) • L = true length, when chain or box expanded.

(1 - c) • L = true length, when chain contracted.

Here L = measured length, and c = tabular correction for change of temperature.

The above correction 2,466 links would be subtracted if the mean temperature was 15° above zero (Fahrenheit).

Note 1. If the above line had been measured by a Norway pine pole or rod, 15 feet long (*see measuring of base lines*), the correction would only be 0,82 link, nearly eight tenths of a link in a mile.

Note 2. It appears from this table that there is no sensible or practical benefit to be derived in using a steel chain, in reference to expansion or contraction. However, steel chains are to be preferred, as they are not liable to bend like the iron wire chain.

TABLE III.—To Reduce Links to Feet.

lk.	Feet	100	200	300	400	500	600	700	800	900
0	0.00	66.00	132.00	198.00	264.00	330.00	396.00	462.00	528.00	594.00
1	0.66	66.66	132.66	198.66	264.66	330.66	396.66	462.66	528.66	594.66
2	1.32	67.32	133.32	199.32	265.32	331.32	397.32	463.32	529.32	595.32
3	1.98	67.98	133.98	199.98	265.98	331.98	397.98	463.98	529.98	595.98
4	2.64	68.64	134.64	200.64	266.64	332.64	398.64	464.64	530.64	596.64
5	3.30	69.30	135.30	201.30	267.30	333.30	399.30	465.30	531.30	597.30
6	3.96	69.66	135.96	201.96	267.96	333.96	399.96	465.96	531.96	597.96
7	4.62	70.62	136.62	202.62	268.62	334.62	400.62	466.62	532.62	598.62
8	5.28	71.28	137.28	203.28	269.28	335.28	401.28	467.28	533.28	599.28
9	5.94	71.94	137.94	203.94	269.94	335.94	401.94	467.94	533.94	599.94
10	6.60	72.60	138.60	204.60	270.60	336.60	402.60	468.60	534.60	600.60
11	7.26	73.26	139.26	205.26	271.26	337.26	403.26	469.26	535.26	601.26
12	7.92	73.92	139.92	205.92	271.92	337.92	403.92	469.92	535.92	601.92
13	8.58	74.58	140.58	206.58	272.58	338.58	404.58	470.58	536.58	602.58
14	9.24	75.24	141.24	207.24	273.24	339.24	405.24	471.24	537.24	603.24
15	9.90	75.90	141.90	207.90	273.90	339.90	405.90	471.90	537.90	603.90
16	10.56	76.56	142.56	208.56	274.56	340.56	406.56	472.56	538.56	604.56
17	11.22	77.22	143.22	209.22	275.22	341.22	407.22	473.22	539.22	605.22
18	11.88	77.88	143.88	209.88	275.88	341.88	407.88	473.88	539.88	605.88
19	12.54	78.54	144.54	210.54	276.54	342.54	408.54	474.54	540.54	606.54
20	13.20	79.20	145.20	211.20	277.20	343.20	409.20	475.20	541.20	607.20
21	13.86	79.86	145.86	211.86	277.86	343.86	409.86	475.86	541.86	607.86
22	14.52	80.52	146.52	212.52	278.52	344.52	410.52	476.52	542.52	608.52
23	15.18	81.18	147.18	213.18	279.18	345.18	411.18	477.18	543.18	609.18
24	15.84	81.84	147.84	213.84	279.84	345.84	411.84	477.84	543.84	609.84
25	16.50	82.50	148.50	214.50	280.50	346.50	412.50	478.50	544.50	610.50
26	17.16	83.16	149.16	215.16	281.16	347.16	413.16	479.16	545.16	611.16
27	17.82	83.82	149.82	215.82	281.82	347.82	413.82	479.82	545.82	611.82
28	18.48	84.48	150.48	216.48	282.48	348.48	414.48	480.48	546.48	612.48
29	19.14	85.14	151.14	217.14	283.14	349.14	415.14	481.14	547.14	613.14
30	19.80	85.80	151.80	217.80	283.80	349.80	415.80	481.80	547.80	613.80
31	20.46	86.46	152.46	218.46	284.46	350.46	416.46	482.46	548.46	614.46
32	21.12	87.12	153.12	219.12	285.12	351.12	417.12	483.12	549.12	615.12
33	21.78	87.78	153.78	219.78	285.78	351.78	417.78	483.78	549.78	615.78
34	22.44	88.44	154.44	220.44	286.44	352.44	418.44	484.44	550.44	616.44
35	23.10	89.10	155.10	221.10	287.10	353.10	419.10	485.10	551.10	617.10
36	23.76	89.76	155.76	221.76	287.76	353.76	419.76	485.76	551.76	617.76
37	24.42	90.42	156.42	222.42	288.42	354.42	420.42	486.42	552.42	618.42
38	25.08	91.08	157.08	223.08	289.08	355.08	421.08	487.08	553.08	619.08
39	25.74	91.74	157.74	223.74	289.74	355.74	421.74	487.74	553.74	619.74
40	26.40	92.40	158.40	224.40	290.40	356.40	422.40	488.40	554.40	620.40
41	27.06	93.06	159.06	225.06	291.06	357.06	423.06	489.06	555.06	621.06
42	27.72	93.72	159.72	225.72	291.72	357.72	423.72	489.72	555.72	621.72
43	28.38	94.38	160.38	226.38	292.38	358.38	424.38	490.38	556.38	622.38
44	29.04	95.04	161.04	227.04	293.04	359.04	425.04	491.04	557.04	623.04
45	29.70	95.70	161.70	227.70	293.70	359.70	425.70	491.70	557.70	623.70
46	30.36	96.36	162.36	228.36	294.36	360.36	426.36	492.36	558.36	624.36
47	31.02	97.02	163.02	229.12	295.02	361.02	427.02	493.02	559.02	625.02
48	31.68	97.68	163.68	229.78	295.68	361.68	427.68	493.68	559.68	625.68
49	32.34	98.34	164.34	230.34	296.34	362.34	428.34	494.34	560.34	626.34
50	33.00	99.00	165.00	231.00	297.00	363.00	429.00	495.00	561.00	627.00
51	33.66	99.66	165.66	231.66	297.66	363.66	429.66	495.66	561.66	627.66
52	24.32	100.32	166.32	232.32	298.32	364.32	430.32	496.32	562.32	628.32
53	34.98	100.98	166.98	232.98	298.98	364.98	430.98	496.98	562.98	628.98
54	35.64	101.64	167.64	233.64	299.64	365.64	431.64	497.64	563.64	629.64
55	36.30	102.30	168.30	234.30	300.30	366.30	432.30	498.30	564.30	630.30
56	36.96	102.96	168.96	234.96	300.96	366.96	432.96	498.96	564.96	630.96
57	37.62	103.62	169.62	235.62	301.62	367.62	433.62	499.62	565.62	631.62
58	38.28	104.28	170.28	236.28	302.28	368.28	434.28	500.28	566.28	632.28
59	38.94	104.94	170.94	236.94	302.94	368.94	434.94	500.94	566.94	632.94
60	39.60	105.60	171.60	237.60	303.60	369.60	435.60	501.60	567.60	633.60

TABLE III.—To Reduce Links to Feet.

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Lk.	Feet.	100	200	300	400	500	600	700	800	900
61	40.26	106.26	172.26	238.26	304.26	370.26	436.26	502.26	568.26	634.26
62	40.92	106.92	172.92	238.92	304.92	370.92	436.92	502.92	568.92	634.92
63	41.58	107.58	173.58	239.58	305.58	371.58	437.58	503.58	569.58	635.58
64	42.24	108.24	174.24	240.24	306.24	372.24	438.24	504.24	570.24	636.24
65	42.90	108.90	174.90	240.90	306.90	372.90	438.90	504.90	570.90	636.90
66	43.56	109.56	175.56	241.56	307.56	373.56	439.56	505.56	571.56	637.56
67	44.22	110.22	176.22	242.22	308.22	374.22	440.22	506.22	572.22	638.22
68	44.88	110.88	176.88	242.88	308.88	374.88	440.88	506.88	572.88	638.88
69	45.54	111.54	177.54	243.54	309.54	375.54	441.54	507.54	573.54	639.54
70	46.20	112.20	178.20	244.20	310.20	376.20	442.20	508.20	574.20	640.20
71	46.86	112.86	178.86	244.86	310.86	376.86	442.86	508.86	574.86	640.86
72	47.52	113.52	179.52	245.52	311.52	377.52	443.52	509.52	575.52	641.52
73	48.18	114.18	180.18	246.18	312.18	378.18	444.18	510.18	576.18	642.18
74	48.84	114.84	180.84	246.84	312.84	378.84	444.84	510.84	576.84	642.80
75	49.50	115.50	181.50	247.50	313.50	379.50	445.50	511.50	577.50	643.50
76	50.16	116.16	182.16	248.16	314.16	380.16	446.16	512.16	578.16	644.16
77	50.82	116.82	183.82	248.82	314.82	380.82	446.82	512.82	578.82	644.82
78	51.48	117.48	183.48	249.48	315.48	381.48	447.48	513.48	579.48	645.48
79	52.14	118.14	184.14	250.14	316.14	382.14	448.14	514.14	580.14	646.14
80	52.80	118.80	184.80	250.80	316.80	382.80	448.80	514.80	580.80	646.80
81	53.46	119.46	185.46	251.46	317.46	383.46	449.46	515.46	581.46	647.46
82	54.12	120.12	186.12	252.12	318.12	384.12	450.12	516.12	582.12	648.12
83	54.78	120.78	186.78	252.78	318.78	384.78	450.78	516.78	582.78	648.78
84	55.44	121.44	187.44	253.44	319.44	385.44	451.44	517.44	583.44	649.44
85	56.10	122.10	188.10	254.10	320.10	386.10	452.10	518.10	584.10	650.10
86	56.76	123.76	188.76	254.76	320.76	386.76	452.76	518.76	584.76	650.76
87	57.42	123.42	189.42	255.42	321.42	387.42	453.42	519.42	585.42	651.42
88	58.08	124.08	190.08	256.08	322.08	388.08	454.08	520.08	586.08	652.08
89	58.74	124.74	190.74	256.74	322.74	388.74	454.74	520.74	586.74	653.74
90	59.40	125.40	191.40	257.40	323.40	389.40	455.40	521.40	587.40	653.40
91	60.06	126.06	192.06	258.06	324.06	390.06	456.06	522.06	588.06	654.06
92	60.72	126.72	192.72	258.72	324.72	390.72	456.72	522.72	588.72	654.72
93	61.38	127.38	193.38	259.38	325.38	391.38	457.38	523.38	589.38	655.38
94	62.04	128.04	194.04	260.04	326.04	392.04	458.04	524.04	590.04	656.04
95	62.70	128.70	194.70	260.70	326.70	392.70	458.70	524.70	590.70	656.70
96	63.36	129.36	195.36	261.36	327.36	393.36	459.36	525.36	591.36	657.36
97	64.02	130.02	196.02	262.02	328.02	394.02	460.02	526.02	592.02	658.02
98	64.68	130.68	196.68	262.68	328.68	394.68	460.68	526.68	592.68	658.68
99	65.34	131.34	197.34	263.34	329.34	395.34	461.34	527.34	593.34	659.34

Lks.	1000	2000	3000	4000	5000	6000	7000	8000	9000
000	660	1320	1980	2640	3300	3960	4620	5280	5940
100	726	1386	2046	2706	3366	4026	4686	5346	6006
200	792	1452	2112	2772	3432	4092	4752	5412	6072
300	858	1518	2178	2838	3498	4158	4818	5478	6138
400	924	1584	2244	2904	3564	4224	4884	5544	6204
500	990	1650	2310	2970	3630	4290	4950	5610	6270
600	1056	1716	2376	3036	3696	4356	5016	5676	6336
700	1122	1782	2442	3102	3762	4422	5082	5742	6402
800	1188	1848	2508	3168	3828	4488	5148	5808	6468
900	1254	1914	2574	3234	3894	4554	5214	5874	6534

Example. Reduce 9664 links to feet.

From the bottom table, under 9000 at top, and opposite 600 in the left hand column, we find 6336

Opposite 64, in upper table, and under 0 = 42.24

6378.24 feet.

TABLE IV.—To Reduce Feet to Links.

Ft.	Links.	100	200	300	400	500	600	700	800	900
0	0.00	151.52	303.03	454.55	606.06	757.58	909.09	1060.60	1212.12	1363.64
1	1.515	153.03	304.55	5.07	7.58	9.10	910.61	2.12	3.64	5.16
2	3.03	154.55	6.06	7.58	9.09	760.61	2.12	3.63	5.15	6.67
3	4.55	6.07	7.58	9.10	610.61	2.13	3.64	5.15	6.67	8.19
4	6.06	7.58	9.09	460.61	2.12	3.64	5.15	6.66	8.18	9.70
5	7.58	9.10	310.61	2.13	4.64	5.16	6.67	8.18	9.70	1371.22
6	9.09	160.61	2.12	3.64	5.15	6.67	8.18	9.69	1221.21	2.73
7	10.61	2.13	3.64	5.16	6.67	8.19	9.70	1071.21	2.73	4.25
8	12.12	3.64	5.15	6.67	8.18	9.70	921.21	2.72	4.24	6.76
9	13.64	5.16	6.67	8.19	9.70	771.22	2.73	4.24	5.76	7.28
10	15.15	6.67	8.18	9.70	621.21	2.73	4.24	5.75	7.27	8.79
11	16.67	8.19	9.70	471.22	2.73	4.25	5.76	7.27	8.79	1380.31
12	18.18	9.70	321.21	2.73	4.24	5.76	7.27	8.78	1230.30	1.82
13	19.70	171.22	2.73	4.25	5.76	7.28	8.79	1080.30	1.82	3.34
14	21.21	2.73	4.24	5.76	7.27	8.79	930.30	1.81	3.33	4.85
15	22.73	4.25	5.76	7.28	8.79	780.31	1.82	3.33	4.85	6.37
16	24.24	5.76	7.27	8.79	630.30	1.82	3.33	4.84	6.36	7.88
17	25.76	7.28	8.79	480.31	1.82	3.34	4.85	6.36	7.88	9.40
18	27.27	8.79	330.30	1.82	3.33	4.85	6.36	7.87	9.39	1390.91
19	28.79	180.31	1.82	3.34	4.85	6.37	7.88	9.39	1240.91	2.43
20	30.30	1.82	3.33	4.85	6.36	7.88	9.39	1090.90	2.42	3.94
21	31.82	3.34	4.85	6.37	7.85	9.40	940.91	2.42	4.09	5.46
22	33.33	4.85	6.36	7.88	8.39	790.91	2.42	3.93	5.45	6.97
23	34.85	6.37	7.88	9.40	640.91	2.43	3.94	5.95	6.97	8.49
24	36.36	7.88	9.39	490.91	2.42	3.94	5.45	6.96	8.48	1400.00
25	37.88	9.40	340.91	2.43	3.94	5.46	6.97	8.45	1250.00	1.52
26	39.39	190.91	2.42	3.94	5.45	6.97	8.48	9.99	1.51	3.03
27	40.91	2.43	3.94	5.56	6.97	8.49	950.00	1101.51	3.03	4.55
28	42.43	3.95	5.46	6.98	8.49	800.01	1.52	3.03	4.54	6.07
29	43.94	4.46	6.97	8.49	650.00	1.52	3.03	4.54	6.06	7.58
30	45.46	6.98	8.49	500.01	1.52	3.04	4.55	6.06	7.58	9.10
31	46.97	8.49	350.00	1.52	3.03	4.55	6.06	7.57	9.09	1410.61
32	48.48	200.00	1.51	3.03	4.54	6.06	7.57	9.08	1260.60	2.12
33	50.00	1.52	3.03	4.55	6.06	7.58	9.09	1110.60	2.12	3.64
34	51.52	3.04	4.55	6.07	7.58	9.10	960.61	2.12	3.64	5.16
35	53.03	4.55	6.06	7.58	9.09	810.61	2.12	3.63	5.15	6.67
36	54.54	6.06	7.58	9.09	660.60	2.12	3.63	5.14	6.66	8.18
37	56.06	7.58	9.09	510.61	2.12	3.64	5.15	6.66	8.18	1420.70
38	57.57	9.09	360.60	2.12	3.63	5.15	6.66	8.17	9.69	1.21
39	59.09	210.61	2.12	3.64	5.15	6.67	8.18	9.69	1271.22	2.73
40	60.60	2.12	3.63	5.15	6.66	8.18	9.69	1121.20	2.72	4.24
41	62.12	3.64	5.15	6.67	8.18	9.70	971.21	2.72	4.24	5.76
42	63.63	5.15	6.66	8.18	9.69	821.21	2.72	4.23	5.75	7.27
43	65.15	6.17	8.18	9.70	521.21	2.73	4.24	5.75	7.27	8.79
44	66.66	8.18	9.69	521.21	2.72	4.24	5.75	7.26	8.78	1430.30
45	68.18	9.70	371.21	2.73	4.24	5.76	7.27	8.78	1280.30	1.82
46	69.69	221.21	2.72	4.24	5.75	7.27	8.78	1130.29	1.81	3.33
47	71.21	2.73	4.24	5.76	7.27	8.79	980.30	1.81	3.33	4.85
48	72.73	4.25	5.76	7.28	8.79	830.31	1.82	3.33	4.85	6.37
49	74.24	5.76	7.27	8.79	680.30	1.82	3.33	4.84	6.36	7.88
50	75.77	7.29	8.80	530.32	1.83	3.35	4.86	6.37	7.89	9.41
51	77.27	8.79	380.30	1.82	3.33	4.85	6.36	7.87	9.39	1440.91
52	78.79	230.31	1.82	3.34	4.85	6.37	7.88	9.38	1290.91	2.43
53	80.30	1.82	3.33	4.85	6.36	7.88	9.39	1140.90	2.42	3.94
54	81.81	3.33	4.84	6.36	7.87	9.39	990.90	2.41	3.93	5.45
55	83.33	4.85	6.36	7.88	9.39	840.91	2.42	3.93	5.45	6.97
56	84.85	6.37	7.88	9.40	690.91	2.43	3.94	5.45	6.97	8.49
57	86.36	7.88	9.39	540.91	2.43	3.94	5.45	6.96	8.48	1450.00
58	87.88	9.40	390.91	2.43	4.94	5.46	6.97	8.48	1300.00	1.52
59	89.29	240.91	2.42	3.94	5.45	6.97	8.48	9.96	1.51	3.03
60	90.91	2.43	3.94	5.46	6.97	8.49	1000.00	1151.51	3.03	4.55

TABLE IV.—To Reduce Feet to Links.

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ft.	Links	100	200	300	400	500	600	700	800	900
61	92.42	243.94	395.45	546.97	698.48	850.00	1001.51	1153.02	1304.54	1456.06
62	93.94	5.46	6.97	8.49	10.00	1.52	3.03	4.54	6.06	7.58
63	95.05	6.97	8.48	9.99	1.51	3.02	4.53	6.04	7.55	9.06
64	96.57	248.99	400.00	551.52	703.03	854.55	1006.06	1157.57	1309.09	1460.61
65	98.08	250.00	401.51	553.03	704.55	856.07	1007.58	1159.09	1310.61	1462.13
66	100.00	251.52	403.04	554.56	706.08	857.60	1009.10	1160.61	1312.13	1463.65
67	101.52	3.04	4.55	6.07	7.58	9.09	10.61	12.12	13.64	15.16
68	103.03	4.55	6.06	7.58	9.09	10.61	12.12	13.64	15.16	16.67
69	104.55	6.07	7.58	9.10	10.61	12.12	13.64	15.16	16.67	18.19
70	106.06	7.58	9.09	10.61	12.12	13.64	15.16	16.67	18.19	19.70
71	107.58	9.10	10.61	12.13	13.64	15.16	16.67	18.19	19.70	21.22
72	109.09	10.61	12.13	13.64	15.16	16.67	18.19	19.70	21.22	22.73
73	110.61	12.13	13.64	15.16	16.67	18.19	19.70	21.22	22.73	24.25
74	112.12	13.64	15.16	16.67	18.19	19.70	21.22	22.73	24.25	25.76
75	113.64	15.16	16.67	18.19	19.70	21.22	22.73	24.25	25.76	27.28
76	115.15	16.67	18.19	19.70	21.22	22.73	24.25	25.76	27.28	28.79
77	116.67	18.19	19.70	21.22	22.73	24.25	25.76	27.28	28.79	30.31
78	118.18	19.70	21.22	22.73	24.25	25.76	27.28	28.79	30.31	31.82
79	119.70	21.22	22.73	24.25	25.76	27.28	28.79	30.31	31.82	33.33
80	121.21	22.73	24.25	25.76	27.28	28.79	30.31	31.82	33.33	34.85
81	122.73	24.25	25.76	27.28	28.79	30.31	31.82	33.33	34.85	36.37
82	124.24	25.76	27.28	28.79	30.31	31.82	33.33	34.85	36.37	37.88
83	125.76	27.28	28.79	30.31	31.82	33.33	34.85	36.37	37.88	39.40
84	127.27	28.79	30.31	31.82	33.33	34.85	36.37	37.88	39.40	40.91
85	128.79	30.31	31.82	33.33	34.85	36.37	37.88	39.40	40.91	42.43
86	130.30	31.82	33.33	34.85	36.37	37.88	39.40	40.91	42.43	43.94
87	131.82	33.33	34.85	36.37	37.88	39.40	40.91	42.43	43.94	45.46
88	133.33	34.85	36.37	37.88	39.40	40.91	42.43	43.94	45.46	46.97
89	134.85	36.37	37.88	39.40	40.91	42.43	43.94	45.46	46.97	48.49
90	136.36	37.88	39.40	40.91	42.43	43.94	45.46	46.97	48.49	50.00
91	137.88	39.40	40.91	42.43	43.94	45.46	46.97	48.49	50.00	51.52
92	139.39	40.91	42.43	43.94	45.46	46.97	48.49	50.00	51.52	53.03
93	140.91	42.43	43.94	45.46	46.97	48.49	50.00	51.52	53.03	54.55
94	142.42	43.94	45.46	46.97	48.49	50.00	51.52	53.03	54.55	56.06
95	143.94	45.46	46.97	48.49	50.00	51.52	53.03	54.55	56.06	57.58
96	145.46	46.97	48.49	50.00	51.52	53.03	54.55	56.06	57.58	59.10
97	146.97	48.49	50.00	51.52	53.03	54.55	56.06	57.58	59.10	60.61
98	148.48	50.00	51.52	53.03	54.55	56.06	57.58	59.10	60.61	62.13
99	150.00	51.52	53.03	54.55	56.06	57.58	59.10	60.61	62.13	63.65

Ft.	1000	2000	3000	4000	5000	6000	7000	8000	9000
0	1515.15	3030.30	4545.45	6060.61	7575.76	9090.91	10606.06	12121.21	13636.36
100	1666.67	3333.33	4999.99	6222.22	7727.27	9242.42	10757.58	12272.73	13787.88
200	1818.18	3636.36	5454.55	6818.18	8323.23	9838.38	11353.54	12868.69	14383.84
300	1969.70	3939.39	5909.09	7318.18	8823.23	10338.38	11853.54	13368.69	14879.01
400	2121.21	4242.42	6363.64	7818.18	9318.18	10833.33	12348.48	13863.64	15374.15
500	2272.73	4545.45	6818.18	8318.18	9818.18	11333.33	12833.33	14348.48	15869.29
600	2424.24	4848.48	7272.73	8818.18	10318.18	11833.33	13333.33	14848.48	16364.43
700	2575.76	5151.51	7727.27	9318.18	10818.18	12333.33	13833.33	15348.48	16859.57
800	2727.27	5454.55	8181.82	9818.18	11318.18	12833.33	14333.33	15848.48	17354.71
900	2878.79	5757.58	8636.36	10318.18	11818.18	13333.33	14833.33	16348.48	17849.85

inches.	Feet.	Links.	inches.	Feet.	Links.	inches.	Feet.	Links.
1	0.083	0.126	5	0.416	0.631	9	0.750	1.126
2	0.176	0.253	6	0.500	0.757	10	0.833	1.262
3	0.250	0.379	7	0.583	0.883	11	0.947	1.388
4	0.333	0.505	8	0.667	1.010	12	1.000	1.515

Example. Reduce 9874 feet to links.

From the bottom table we find 9800 feet = 14848.48 links.
 From the upper table, 74 links = 112.12
 14960.60 links.

TABLE V.—Lengths of Circular Arcs to Radius 1.

Deg.	Arc.	Deg.	Arc.	Deg.	Arc.	M.	Arc.	S.	Arc.
1	0.017453	61	1.064651	121	1.111848	1	291	1	5
2	0.034907	2	1.082104	2	1.129302	2	582	2	10
3	0.052360	3	1.099557	3	1.146750	3	873	3	15
4	0.069813	4	1.117011	4	1.164208	4	1164	4	19
5	0.087267	5	1.134464	5	1.181662	5	1454	5	24
6	0.104720	6	1.151917	6	1.199115	6	1745	6	29
7	0.122173	7	1.169371	7	1.216568	7	2036	7	34
8	0.139626	8	1.186824	8	1.234021	8	2327	8	39
9	0.157080	9	1.204277	9	1.251475	9	2618	9	44
10	0.174533	10	1.221731	10	1.268928	10	2909	10	49
11	0.191986	11	1.239184	11	1.286381	11	3120	11	52
12	0.209439	12	1.256637	12	1.303834	12	3491	12	58
13	0.226892	13	1.274090	13	1.321287	13	3782	13	63
14	0.244345	14	1.291543	14	1.338740	14	4072	14	68
15	0.261799	15	1.308997	15	1.356194	15	4363	15	73
16	0.279252	16	1.326450	16	1.373647	16	4654	16	78
17	0.296705	17	1.343903	17	1.391100	17	4945	17	83
18	0.314158	18	1.361356	18	1.408553	18	5235	18	87
19	0.331611	19	1.378809	19	1.426006	19	5526	19	92
20	0.349066	20	1.396263	20	1.443461	20	5818	20	97
21	0.366519	21	1.413716	21	1.460914	21	6109	21	102
22	0.383972	22	1.431169	22	1.478367	22	6400	22	107
23	0.401425	23	1.448622	23	1.495820	23	6690	23	112
24	0.418878	24	1.466075	24	1.513273	24	6981	24	116
25	0.436332	25	1.483529	25	1.530727	25	7272	25	121
26	0.453785	26	1.500982	26	1.548180	26	7563	26	126
27	0.471238	27	1.518435	27	1.565633	27	7854	27	131
28	0.488691	28	1.535888	28	1.583086	28	8145	28	136
29	0.506144	29	1.553341	29	1.600539	29	8436	29	141
30	0.523599	30	1.570796	30	1.617994	30	8727	30	145
31	0.541052	31	1.588249	31	1.635447	31	9018	31	150
32	0.558505	32	1.605702	32	1.652900	32	9308	32	155
33	0.575958	33	1.623155	33	1.670353	33	9599	33	160
34	0.593411	34	1.640608	34	1.687806	34	9890	34	165
35	0.610865	35	1.658062	35	1.705260	35	10181	35	170
36	0.628318	36	1.675515	36	1.722713	36	10471	36	175
37	0.645771	37	1.692968	37	1.740165	37	10762	37	179
38	0.673224	38	1.710421	38	1.757618	38	11053	38	184
39	0.690677	39	1.727874	39	1.775071	39	11344	39	189
40	0.698132	40	1.745329	40	1.792527	40	11636	40	194
41	0.715585	41	1.762782	41	1.809980	41	11926	41	199
42	0.733038	42	1.780235	42	1.827433	42	12217	42	204
43	0.750491	43	1.797688	43	1.844886	43	12508	43	208
44	0.767944	44	1.815141	44	1.862339	44	12799	44	213
45	0.785398	45	1.832595	45	1.879793	45	13090	45	218
46	0.802851	46	1.850048	46	1.897246	46	13381	46	223
47	0.820304	47	1.867501	47	1.914699	47	13672	47	228
48	0.837757	48	1.884954	48	1.932152	48	13963	48	233
49	0.855210	49	1.902407	49	1.949605	49	14254	49	238
50	0.872665	50	1.919862	50	1.967060	50	14544	50	242
51	0.890118	51	1.937315	51	1.984513	51	14835	51	247
52	0.907571	52	1.954768	52	2.001966	52	15126	52	252
53	0.925024	53	1.972221	53	2.019419	53	15417	53	257
54	0.942477	54	1.989674	54	2.036872	54	15708	54	262
55	0.959931	55	2.007128	55	2.054326	55	15999	55	267
56	0.977384	56	2.024581	56	2.071779	56	16290	56	272
57	0.994837	57	2.042034	57	2.089232	57	16581	57	277
58	1.012290	58	2.059487	58	2.106685	58	16872	58	281
59	1.029743	59	2.076940	59	2.124138	59	17162	59	286
60	1.047198	60	2.094395	60	2.141593	60	17453	60	291

Rule 1. Multiply the tabular number corresponding to the given angle by the radius, the product will be the length of the arc.

Rule 2. Multiply the length of the arc found by rule 1 by half the radius, the product will be the area of the sector of which the angle is given.

Here M = minutes, S = seconds.

TABLE VI.—Lengths of Circular Arcs obtained by having the Chord or Base, and Height or Versed Sine given.

$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.
.100	1.0265	.160	1.0669	.220	1.1245	.280	1.1974	.340	1.2848
1	70	1	78	1	56	1	89	1	58
2	75	2	86	2	66	2	1.2001	2	74
3	81	3	94	3	77	3	15	3	90
4	86	4	1.0703	4	89	4	28	4	1.2905
.105	91	.165	11	.225	1.1800	.285	42	.345	21
6	97	6	19	6	11	6	56	6	37
7	1.0803	7	28	7	22	7	70	7	52
8	08	8	37	8	33	8	83	8	68
9	14	9	45	9	44	9	97	9	84
.110	20	.170	54	.230	56	.290	1.2110	.350	1.3000
1	35	1	62	1	67	1	24	1	16
2	31	2	71	2	79	2	38	2	32
3	37	3	80	3	90	3	52	3	47
4	43	4	89	4	1.1402	4	66	4	63
.115	49	.175	98	.235	14	.295	79	.355	79
6	55	6	1.0807	6	25	6	93	6	95
7	61	7	16	7	36	7	1.2206	7	1.3112
8	67	8	25	8	48	8	20	8	28
9	73	9	34	9	60	9	35	9	44
.120	80	.180	43	.240	71	.300	50	.360	60
1	86	1	52	1	83	1	64	1	76
2	92	2	61	2	95	2	78	2	92
3	99	3	70	3	1.1507	3	92	3	1.3209
4	1.0405	4	80	4	19	4	1.2306	4	25
.125	12	.185	89	.245	31	.305	21	.365	41
6	18	6	98	6	43	6	35	6	58
7	25	7	1.0908	7	55	7	49	7	74
8	31	8	17	8	67	8	64	8	91
9	38	9	27	9	79	9	78	9	1.3307
.130	45	.190	36	.250	91	.310	98	.370	23
1	52	1	46	1	1.1603	1	1.2407	1	40
2	58	2	56	2	16	2	22	2	56
3	65	3	65	3	28	3	36	3	73
4	72	4	75	4	40	4	51	4	90
.135	79	.195	85	.255	53	.315	65	.375	1.3406
6	86	6	95	6	65	6	80	6	23
7	93	7	1.1005	7	77	7	95	7	40
8	1.0500	8	15	8	90	8	1.2510	8	56
9	08	9	25	9	1.1702	9	24	9	73
.140	15	.200	35	.260	15	.320	39	.380	90
1	22	1	45	1	28	1	54	1	1.3507
2	29	2	55	2	40	2	69	2	24
3	37	3	65	3	53	3	84	3	41
4	44	4	75	4	66	4	99	4	58
.145	52	.205	85	.265	78	.325	1.2614	.385	74
6	59	6	96	6	91	6	29	6	91
7	67	7	1.1106	7	1.1804	7	44	7	1.3608
8	74	8	17	8	16	8	59	8	25
9	82	9	27	9	29	9	74	9	43
.150	90	.210	37	.270	43	.330	89	.390	60
1	97	1	48	1	56	1	1.2704	1	77
2	1.0605	2	58	2	69	2	20	2	94
3	13	3	69	3	82	3	35	3	1.3711
4	21	4	80	4	97	4	50	4	28
.155	29	.215	90	.275	1.1908	.335	66	.395	46
6	37	6	1.1201	6	21	6	81	6	63
7	45	7	12	7	34	7	96	7	80
8	53	8	23	8	48	8	1.2812	8	97
9	61	9	33	9	61	9	27	9	15
.160	1.0669	.220	1.1245	.280	1.1974	.340	1.2848	.400	1.3882
									1.4927

TABLE VI.—Lengths of Circular Arcs obtained by having the Chord or Base, and Height or Versed Sine given.

$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.
.460	1.4927	.467	1.5061	.474	1.5196	.481	1.5332	.488	1.5470
1	46	8	80	5	1.5215	2	52	9	89
2	65	9	99	6	35	3	71	.490	1.5509
3	84	.470	1.5119	7	54	4	91	1	29
4	1.5003	1	38	8	74	.485	1.5411	2	49
.465	22	2	57	9	93	6	30	3	69
6	42	3	76	.480	1.5313	7	50	4	.500
7	1.5061	4	1.5196	1	1.5332	8	1.5470	.595	1.5608

Example. Given the chord = 12,16 feet, and the height 3,48 feet, to find the length of the arc. Here $h = 3,48$, and $b = 12,16$,

$$\text{and } \frac{h}{b} = \frac{3,48}{12,16} = \text{tabular height} = ,2862 \text{ nearly.}$$

Tabular arc corresponding to 286 = 1,2056 = 1,2056

Tabular arc corresponding to 287 = 1,2070

$$\begin{array}{r} \text{difference, 14 multiplied by ,2,} \\ \hline = 0002,8 \\ 1,2058,8 \\ \hline 12,16 \end{array}$$

Length of the curve = 14,6635 feet nearly.

Rule. To the tabular arc corresponding to the first three figures, add the product of the fourth decimal, if any, by the difference of the tabular heights, of the one less and the other greater than the given tabular number. The sum will be the required tabular length to the nearest ten thousandth part, which sum multiplied by the given chord, will give the required length.

Example 2. Let chord $b = 40,20$ feet, and height $h = 5,16$ feet.

$$\text{Here } \frac{h}{b} = \frac{5,16}{40,2} = ,1277 = \text{tabular height.}$$

$$\begin{array}{r} ,127 = 1,0425 \\ ,128 = 1,0431 \end{array} \quad = 1,0425$$

$$\begin{array}{r} \text{difference} = ,0006, \text{ multiplied by } 7 = 4,2 \\ \hline 1,0429,2 \end{array}$$

$$\text{Base or chord} = 40,2$$

The required length of the curve = 41,92 feet.

TABLE VII.—Areas of Segments of a Circle whose Diameter is Unity.

Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.
.001	.000042	.063	.020681	.125	.056668	.187	.101553	.249	.152680
2	119	4	1168	6	7326	8	2334	.250	3546
3	219	5	1659	7	7991	9	3116	1	4412
4	337	6	2154	8	8658	.190	3900	2	5280
5	470	7	2652	9	9327	1	4685	3	6149
6	618	8	3154	.130	9999	2	5472	4	7019
7	779	9	3659	1	.060672	3	6261	5	7890
8	951	.070	4168	2	1348	4	7051	6	8762
9	.001135	1	4680	3	2026	5	7842	7	9636
.010	1329	2	5195	4	2707	6	8636	8	.160510
1	1533	3	5714	5	3389	7	9430	9	1386
2	1746	4	6236	6	4074	8	.110226	.260	2263
3	1968	5	6761	7	4760	9	1024	1	3140
4	2199	6	7289	8	5449	.200	1823	2	4019
5	2438	7	7821	9	6140	1	2624	2	4899
6	2685	8	8356	.140	6833	2	3426	4	5780
7	2940	9	8894	1	7528	3	4230	5	6663
8	3202	.080	9435	2	8225	4	5035	6	7546
9	3471	1	9979	3	8924	5	5842	7	8430
.020	3748	2	.030526	4	9625	6	6650	8	9315
1	4031	3	1076	5	.070328	7	7460	9	.170202
2	4322	4	1629	6	1033	8	8271	.270	1089
3	4618	5	2180	7	1741	9	9083	1	1978
4	4921	6	2745	8	2450	.210	9897	2	2867
5	5230	7	3307	9	3161	1	.120712	3	3758
6	5546	8	3872	.150	3874	2	1529	4	4649
7	5867	9	4441	1	4589	3	2347	5	5542
8	6194	.090	5011	2	5306	4	3167	6	6435
9	6527	1	5585	3	6026	5	3988	7	7330
.030	6865	2	6162	4	6747	6	4810	8	8225
1	7209	3	6741	5	7469	7	5634	9	9122
2	7558	4	7323	6	8194	8	6459	.280	.180019
3	7913	5	7909	7	8921	9	7285	1	0918
4	8273	6	8496	8	9649	.220	8113	2	1817
5	8438	7	9087	9	.080380	1	8942	3	2718
6	9008	8	9680	.160	1112	2	9773	4	3619
7	9383	9	.040276	1	1846	3	.130605	5	4521
8	9763	.100	0875	2	2582	4	1438	6	5425
9	.010148	1	1476	3	3320	5	2272	7	6329
.040	0537	2	2080	4	4059	6	3108	8	7234
1	0931	3	2687	5	4801	7	3945	9	8140
2	1330	4	3296	6	5544	8	4784	.290	9047
3	1734	5	3908	7	6289	9	5624	1	9955
4	2142	6	5522	8	7036	.230	6465	2	.190864
5	2554	7	5139	9	7785	1	7307	3	1775
6	2971	8	5759	.170	8535	2	8150	4	2684
7	3392	9	6381	1	9287	3	8995	5	3596
8	3818	.110	7005	2	.090041	4	9841	6	4509
9	4247	1	7632	3	0797	5	.140688	7	5422
.050	4681	2	8262	4	1554	6	1537	8	6337
1	5119	3	8894	5	2313	7	2387	9	7252
2	5561	4	9528	6	3074	8	3238	.300	8168
3	6007	5	.050165	7	3836	9	4091	1	9085
4	6457	6	0804	8	4601	.240	4944	2	.200008
5	6911	7	1446	9	5366	1	5799	3	0922
6	7369	8	2090	.180	6134	2	6655	4	1841
7	7831	9	2736	1	6903	3	7512	5	2761
8	8296	.120	3385	2	7674	4	8371	6	3683
9	8766	1	4036	3	8447	5	9230	7	4605
.060	9239	2	4689	4	9221	6	.150091	8	5527
1	9716	3	5345	5	9997	7	0953	9	6451
2	.020196	4	6003	6	.100774	8	1861	.310	7376

TABLE VII.—Areas of Segments of a Circle whose Diameter is Unity.

Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	To find the Tab- ular Versed Sine.
.311	.208301	.373	.267078	.435	.327882	.497	.389699	
2	9227	4	8045	6	8874	8	.390699	
3	.210154	5	9013	7	9866	9	.391699	Rule. Divide
4	1082	6	9982	8	.330858	.500	.392699	the height of the
5	2011	7	.270951	9	1850	given segment by the diame-		
6	2940	8	1920	.440	2843	ter of the circle of which it is		
7	3871	9	2890	1	3836	a segment. The quotient will		
8	4802	.380	3861	2	4829	be the required tabular height.		
9	5733	1	4832	3	5822	And because the areas of		
.320	6666	2	5803	4	6816	circles are to one another as		
1	7599	3	6775	5	7810	the squares of their diameters,		
2	8533	4	7748	6	8804	multiply the tabular areas in		
3	9468	5	8721	7	9798	this table by the square of the		
4	.220404	6	9694	8	.340793	diameter of the circle of which		
5	1344	7	.280668	9	1787	a segment is given. The pro-		
6	2277	8	1642	.450	2782	duct will be the area of the		
7	3215	9	2617	1	3777	required segment.		
8	4154	.390	3592	2	4772	Example. (See fig. 17.) Let		
9	5093	1	4568	3	5768	the chord A B = 42, versed		
.330	6033	2	5544	4	6764	sine D C = 7.		
1	6974	3	6521	5	7759	By Euclid III, prop. 3 and		
2	7915	4	7498	6	8755	35, the diameter cuts the chord		
3	8858	5	8476	7	9752	of the arc at right angles, con-		
4	9801	6	9453	8	.350748	sequently making the rectan-		
5	.230745	7	.290432	9	1745	gle contained by the versed		
6	1689	8	1411	.460	2742	sine, and the remaining part		
7	2634	9	2390	1	3739	of the diameter, equal to the		
8	3580	.400	3369	2	4736	square of half the chord.		
9	4526	1	4349	3	5732	∴ 21 ² divided by 7 = V D		
.340	5473	2	5330	4	6736	= 63; therefore the diameter		
1	6421	3	6311	5	7727	V C = 63 + 7 = 70, and 7		
2	7369	4	7292	6	8729	divided by 70 = .100 = tab-		
3	8318	5	8273	7	9723	ular versed sine, whose corres-		
4	9268	6	9255	8	.360721	ponding area = .040875, which		
5	.240218	7	.300238	9	1719	multiplied by the square of		
6	1169	8	1220	.470	2717	the diameter = 4900, gives		
7	2121	9	2203	1	3715	the required area = 200,2875.		
8	3074	.410	3187	2	4413	Example 2. Let the tabu-		
9	4026	1	4171	3	5712	lar versed sine = .3456, which		
.350	4980	2	5155	4	6710	is not to be found in the table.		
1	5934	3	6140	5	7709	Tab. versed sine 346,		
2	6889	4	7125	6	8708	area segment = .240218		
3	7845	5	8116	7	9707	Tab. versed sine 347,		
4	8801	6	9095	8	.370706	area segment = .241169		
5	9757	7	.310081	9	1705	difference, 951		
6	.250715	8	1068	.480	2764	As 10 : 951 :: 6 : 570,6, near-		
7	1673	9	2054	1	3703	ly 571, ∴ .240218		
8	2631	.420	3041	2	4702	more 571		
9	3590	1	4029	3	5702	.240789 = the re-		
.360	4550	2	5016	4	6702	quired area of segment.		
1	5510	3	6004	5	7701			
2	6471	4	6992	6	8701			
3	7433	5	7981	7	9700			
4	8395	6	8970	8	.380700			
5	9357	7	9959	9	1699			
6	.260320	8	.320948	.490	2699			
7	1284	9	1938	1	3699			
8	2248	.430	2928	2	4699			
9	3213	1	3918	3	5699			
.370	4178	2	4909	4	6699			
1	5144	3	5900	5	7699			
2	6111	4	6892	6	8699			

Rule. Take out the areas corresponding to the nearest tabular versed sine,—one greater and the other less than the given tabular versed sine; take the difference of the area segments; multiply this difference by the fourth decimal figure of the given tabular versed sine; cut off one figure to the right, and add the remainder to the lesser area segment. The sum will be the required area segment. (See the last example.)

Note. When the tabular versed sine is greater than ,500, the segment is greater than a semicircle; in which case subtract it from 1, find the area seg. of the difference, which take from ,785398. Multiply this difference by the square of the diameter. The product will be the required area.

Example. Let tabular versed sine = ,867, and let 60 = diameter of the circle.

From	1,000	Area circle =	,785398
take tabular versed sine	0,867		
	difference, ,133	Area segment =	,062026
Correct area of segment			= ,723372
Square of 60 (the diameter)			= 3600
Required area of the segment			= 2604,039200

TABLE VIII.—To Reduce Square Feet to Acres, and Vice Versa.

Ac.	Sq. feet.	Acres.	Sq. feet.	Acres.	Sq. feet.	Acres.	Sq. feet.	Acres.	Sq. feet.
1	43560	11	479160	21	914760	31	1350360	41	1785960
2	87120	12	522720	22	958320	32	1393920	42	1829520
3	130680	13	566280	23	1001880	33	1437480	43	1873080
4	174240	14	609840	24	1045440	34	1481040	44	1916640
5	217800	15	653400	25	1089000	35	1524600	45	1960200
6	261360	16	696960	26	1132560	36	1568160	46	2003760
7	304920	17	740520	27	1176120	37	1611720	47	2047320
8	348480	18	784080	28	1219680	38	1655280	48	2090880
9	392040	19	827640	29	1263240	39	1698840	49	2134440
10	435600	20	871200	30	1306800	40	1742400	50	2178000
0.1	4356	.01	435.6	0.001	43.56	0.0001	4.36	0.00001	0.44
.2	8712	.02	871.2	.002	87.12	.0002	8.71	.00002	0.87
.3	13068	.03	1306.8	.003	130.68	.0003	13.07	.00003	1.31
.4	17424	.04	1742.4	.004	174.24	.0004	17.42	.00004	1.74
.5	21780	.05	2178.0	.005	217.80	.0005	21.78	.00005	2.18
.6	26136	.06	2613.6	.006	261.36	.0006	26.14	.00006	2.61
.7	30492	.07	3049.2	.007	304.92	.0007	30.49	.00007	3.05
.8	34848	.08	3484.8	.008	348.48	.0008	34.85	.00008	3.49
0.9	39204	0.09	3920.4	0.009	392.04	0.0009	39.20	0.00009	3.92

Example. Reduce 1283446 square feet to acres.

From the first part, 1283240 = 29

From the second part, 2026

17424 = ,4

2782

2613,6 = ,06

168,4

180,88 = ,008

87,72

84,85 = ,0008

2,87

2,61 = ,00006

0,26 = ,000005 nearly; $\therefore$ 29,46385 = Answer.

This example, being one of the most difficult that can occur, is sufficient to show the application.

TABLE VIIIa.—*Properties of Polygons whose Sides are = Unity.*

No. sides.	Name of polygon	Area of polygon.	Angle at the centre.	Angle made by two of its sides.	Radius of the inscrib'd circle.
			° ' "	° ' "	
3	Trigon.	0.4330127	120,00,00	60,00,00	0.2886751
4	Tetragon.	1.0000000	90,00,00	90,00,00	0.5000000
5	Pentagon.	1.7204774	72,00,00	108,00,00	0.6881910
6	Hexagon.	2.5980762	60,00,00	120,00,00	0.8660254
7	Heptagon.	3.6339124	51,25,42 ⁶ / ₇	128,34,17 ¹ / ₇	0.0382617
8	Octagon.	4.8284271	45,00,00	135,00,00	1.2071068
9	Nonagon.	6.1818242	40,00,00	140,00,00	1.3737387
10	Decagon.	7.6942088	36,00,00	144,00,00	1.5388418
11	Undecagon.	9.3656404	32,00,16 ⁴ / ₁₁	147,16,21 ⁹ / ₁₁	1.7028437
12	Dodecagon.	11.1961524	30,00,00	150,00,00	1.8660254

TABLE VIIIb.

Side of a polygon inscribed in a circle whose diameter = 1

1.732051
1.414214
1.175571
1.000000
0.867768
0.765367
0.684040
0.618034
0.563366
0.517638

TABLE IX.—*Properties of the Five Regular Bodies.*

No. Sides.	Name of polygon	Area of regular polygon whose side is = 1.	Solidity of regular polygon whose side is = 1.	Side of a poly. ins. in sphere whose diameter = 1.	Side of poly. circumscrib'g sphere whose diam. = 1.	Side of a poly. = a sphere whose diameter = 1.
4	Tetrahædron.	1.732051	0.117851	0.117851	0.816497	2.44948
6	Hexædron.	6.000000	1.000000	1.000000	0.577350	1.00000
8	Octædron.	3.464102	0.471405	0.471405	0.707107	1.22474
10	Dodecædron.	20.645729	7.663119	7.663119	0.525731	0.66158
22	Icosædron.	8.660254	2.181695	2.181695	0.356822	0.44903

TABLE X.—*To Reduce Square Links to Acres, Roods and Perches.*

Perch.	Sq. links.	Per.	Sq. links.	Per.	Sq. links.	Perches	Sq. links.	Perches	Sq. links.
160	100000	29	18125	15	9875	1.0	625	0.05	31.25
120	75000	28	17500	14	8750	0.1	62.5	0.06	37.50
80	50000	27	16875	13	8125	0.2	125.0	0.07	43.75
40	25000	26	16250	12	7500	0.3	187.5	0.08	50.00
39	24375	25	15625	11	6875	0.4	250.0	0.09	56.25
38	23750	24	15000	10	6250	0.5	312.5	0.001	0.63
37	23125	23	14375	9	5625	0.6	375.0	0.002	1.25
36	22500	22	13750	8	5000	0.7	437.5	0.003	1.88
35	21875	21	13125	7	4375	0.8	500.0	0.004	2.50
34	21250	20	12500	6	3750	0.9	562.5	0.005	3.13
33	20625	19	11875	5	3125	0.01	6.25	0.006	3.75
32	20000	18	11250	4	2500	0.02	12.50	0.007	4.37
31	19375	17	10625	3	1875	0.03	18.75	0.008	5.00
30	18750	16	10000	2	1250	0.04	25.00	0.009	5.63

Example 1. Reduce 47632854 links to acres, roods and perches.

47,63285 A. R.
Cut off always five places to the right. 47,50000 = 47, 2 — 21,256
21 perches = 13285 Answer.
160
0,2 perch = 125
0,05 perch = 35,
31,25
0,006 perch = 3,75
3,75

Example 2. Reduce 1758 square links to perches.

This being less than 25000, shows that there are no roods in the answer.

1758 square links.
2 perches = 1250
508
,3 perch = 500
8
005 = 3,15
Answer, 2,805 perches.

TABLE XI.—Showing the Reduction on Each Chain of 100 Links,
to Reduce Hypothenusal to Base or Horizontal Measurements.

Angle of Inclinat'n.	Red. in lks.	Angle of Inclinat'n.	Red. in lks.	Angle of Inclinat'n.	Red. in lks.	Angle of Inclinat'n.	Red. in lks.
° / ' / "		° / ' / "		° / ' / "		° / ' / "	
2 33 40	0.10	19 26 19	5.70	27 30 05	11.30	33 47 54	16.90
3 27 29		20 19 36 34	80	27 37 30	40	33 53 04	17.00
4 26 20		30 19 46 47	90	27 44 50	50	34 00 14	10
5 07 35		40 19 56 55	6.00	27 52 20	60	34 06 26	20
5 43 55		50 20 06 56	10	27 59 40	70	34 12 30	30
6 16 45		60 20 16 54	20	28 06 56	80	34 18 36	40
6 47 00		70 20 26 46	30	28 14 12	90	34 24 41	50
7 15 07		80 20 36 35	40	28 21 27	12.00	34 30 46	60
7 30 33		90 20 46 19	50	28 28 41	10	34 36 50	70
8 06 34	1.00	20 55 58	60	28 35 52	20	34 42 53	80
8 30 23		10 21 05 33	70	28 43 02	30	34 48 54	90
8 53 07		20 21 15 04	80	28 50 11	40	34 54 55	18.00
9 14 55		30 21 24 32	90	28 57 18	50	35 00 55	10
9 35 55		40 21 33 54	7.00	29 04 23	60	35 06 54	20
9 56 11		50 21 43 14	10	29 11 27	70	35 12 52	30
10 15 47		60 21 52 30	20	29 18 29	80	35 18 49	40
10 34 48		70 22 01 41	30	29 25 30	90	35 24 45	50
10 53 16		80 22 10 50	40	29 32 29	13.00	35 30 41	60
11 11 12		90 22 19 54	50	29 39 27	10	35 36 36	70
11 28 42	2.00	22 28 55	60	29 46 20	20	35 42 30	80
11 45 46		10 22 37 53	70	29 53 18	30	35 48 22	90
12 02 26		20 22 46 47	80	30 00 10	40	35 54 15	19.00
12 18 44		30 22 55 38	90	30 07 02	50	36 00 06	10
12 34 41		40 23 04 26	8.00	30 13 52	60	36 05 56	20
12 50 20		50 23 13 12	10	30 20 42	70	36 11 46	30
13 05 38		60 23 21 52	20	30 27 29	80	36 17 36	40
13 20 00		70 23 30 31	30	30 34 15	90	36 23 23	50
13 35 37		80 23 39 07	40	30 41 00	14.00	36 29 10	60
13 49 56		90 23 47 40	50	30 47 44	10	36 34 57	70
14 08 12	3.00	23 55 59	60	30 54 26	20	36 40 43	80
14 18 13		10 24 04 36	70	31 01 07	30	36 46 27	90
14 32 02		20 24 13 00	80	31 07 47	40	36 52 12	20.00
14 45 37		30 24 21 22	90	31 14 25	50	36 57 55	10
14 59 01		40 24 29 40	9.00	31 21 02	60	37 03 37	20
15 12 14		50 24 37 10	10	31 27 38	70	37 09 20	30
15 25 14		60 24 46 10	20	31 34 12	80	37 15 01	40
15 38 05		70 24 54 20	30	31 40 46	90	37 20 41	50
15 50 45		80 25 02 30	40	31 47 18	15.00	37 26 21	60
16 03 05		90 25 10 40	50	31 53 49	10	37 32 00	70
16 15 06	4.00	25 18 40	60	32 00 19	20	37 37 38	80
16 27 48		10 25 26 40	70	32 06 47	30	37 43 15	60
16 39 52		20 25 34 40	80	32 13 15	40	37 48 52	21.00
16 51 48		30 25 42 40	90	32 19 41	50	37 54 28	10
17 03 35		40 25 50 30	10.00	32 26 06	60	38 00 04	20
17 15 14		50 25 58 20	10	32 32 30	70	38 05 38	30
17 26 45		60 26 06 10	20	32 38 53	80	38 11 12	40
17 35 10		70 26 14 00	30	32 45 15	90	38 16 46	50
17 49 27		80 26 21 50	40	32 51 36	16.00	38 22 18	60
18 00 38		90 26 29 30	50	32 57 55	10	38 27 50	70
18 11 42	5.00	26 37 10	60	33 04 14	20	38 33 21	80
18 22 39		10 26 44 50	70	33 10 31	30	38 38 52	90
18 33 30		20 26 52 30	80	33 16 47	40	38 44 22	22.00
18 44 18		30 27 00 00	90	33 23 03	50	38 49 51	10
18 54 55		40 27 07 40	11.00	33 29 17	60	38 55 20	20
19 05 27		50 17 15 10	10	33 35 32	70	39 00 48	30
19 15 56		60 27 22 40	20	33 41 43	80	39 06 15	40

Example 2. Given the hypothenuse = 12 chains, and the angle of elevation = 25d. 12m., to find the base.

Tab. angle greater than the given angle = 25d. 18m. 40s., its reduction = 9.60 lks. Given angle = 25d. 12m. 00s.
Tab. angle less than the given angle = 25d. 19m. 40s., its reduction = 9.50 lks.
1m. 20s. = 0.10 lks.

As 8m. : 0.10 links :: 1m. 20s. : 0.0167, which added to 9.50, and multiplied by 12, gives the required correction = 1.142 chains.

Example 1. The angle of elevation being 23d. 4m.

26s., and the distance measured along the slope = 15 chains, to find the base. For angle 23d. 4m. 26s., the correction is 8 links, which multiplied by 15 chains gives 1.20 chains, which taken from 15, gives the base = 13.80.

TABLE XII.—To Reduce Sidereal Time to Mean Solar Time.

Minutes.		Seconds.	
sid. t.	Mean time.	sid. t.	mean t.
m.	m. s.	s.	s.
1	0 59.84	1	0.997
2	1 59.67	2	1.995
3	2 59.51	3	2.992
4	3 59.35	4	3.989
5	4 59.18	5	4.986
6	5 59.02	6	5.984
7	6 58.85	7	6.98
8	7 58.69	8	7.98
9	8 58.53	9	8.98
10	9 58.36	10	9.97
11	10 58.20	11	10.97
12	11 58.03	12	11.97
13	12 57.87	13	12.97
14	13 57.71	14	13.96
15	14 57.54	15	14.96
16	15 57.38	16	15.96
17	16 57.22	17	16.95
18	17 57.05	18	17.95
19	18 56.89	19	18.95
20	19 56.72	20	19.95
21	20 56.56	21	20.94
22	21 56.40	22	21.94
23	22 56.23	23	22.94
24	23 56.07	24	23.93
25	24 55.90	25	24.93
26	25 55.74	26	25.93
27	26 55.58	27	26.93
28	27 55.41	28	27.92
29	28 55.25	29	28.92
30	29 55.09	30	29.92
31	30 54.92	31	30.92
32	31 54.76	32	31.91
33	32 54.59	33	32.91
34	33 54.43	34	33.91
35	34 54.26	35	34.90
36	35 54.10	36	35.90
37	36 53.94	37	36.90
38	37 53.78	38	37.90
39	38 53.61	39	38.89
40	39 53.44	40	39.89
41	40 53.28	41	40.89
42	41 53.12	42	41.89
43	42 52.96	43	42.88
44	43 52.79	44	43.88
45	44 52.63	45	44.88
46	45 52.46	46	45.87
47	46 52.30	47	46.87
48	47 52.14	48	47.87
49	48 51.97	49	48.87
50	49 51.81	50	49.86
51	50 51.65	51	50.86
52	51 51.48	52	51.86
53	52 51.32	53	52.86
54	53 51.15	54	53.85
55	54 50.99	55	54.85
56	55 50.83	56	55.85
57	56 50.66	57	56.84
58	57 50.41	58	57.84
59	58 50.33	59	58.84

TABLE XIII.—To Reduce Mean Solar Time to Sidereal Time.

hours mean time.	Hours of sidereal time.	Min. mean time.	eq'val'ts in sidereal time.	Sec. mean time.
h. m. s.	h. m. s.	m. s.	m. s.	s.
1	1 0 9.86	1	1 0.16	1 1.00
2	2 0 19.71	2	2 0.33	2 2.01
3	3 0 29.57	3	3 0.49	3 3.01
4	4 0 39.43	4	4 0.66	4 4.01
5	5 0 49.28	5	5 0.82	5 5.01
6	6 0 59.14	6	6 0.99	6 6.02
7	7 1 9.00	7	7 1.15	7 7.02
8	8 1 18.85	8	8 1.31	8 8.02
9	9 1 28.71	9	9 1.48	9 9.02
10	10 1 38.56	10	10 1.64	10 10.03
11	11 1 48.42	11	11 1.81	11 11.03
12	12 1 58.28	12	12 1.97	12 12.03
13	13 2 8.13	13	13 2.14	13 13.04
14	14 2 17.99	14	14 2.30	14 14.04
15	15 2 27.85	15	15 2.46	15 15.04
16	16 2 37.70	16	16 2.63	16 16.04
17	17 2 47.56	17	17 2.79	17 17.05
18	18 2 57.42	18	18 2.96	18 18.05
19	19 3 7.27	19	19 3.12	19 19.05
20	20 3 17.13	20	20 3.29	20 20.05
21	21 3 26.99	21	21 3.45	21 21.06
22	22 3 36.84	22	22 3.61	22 22.06
23	23 3 46.70	23	23 3.78	23 23.06
24	24 3 56.56	24	24 3.94	24 24.07
		25	25 4.11	25 25.07
		26	26 4.27	26 26.07
		27	27 4.44	27 27.07
		28	28 4.60	28 28.08
		29	29 4.76	29 29.08
		30	30 4.93	30 30.08
		31	31 5.09	31 31.08
		32	32 5.26	32 32.09
		33	33 5.42	33 33.09
		34	34 5.59	34 34.09
		35	35 5.75	35 35.10
		36	36 5.91	36 36.10
		37	37 6.08	37 37.10
		38	38 6.24	38 38.10
		39	39 6.41	39 39.11
		40	40 6.57	40 40.11
		41	41 6.74	41 41.11
		42	42 6.90	42 42.12
		43	43 7.06	43 43.12
		44	44 7.23	44 44.12
		45	45 7.39	45 45.12
		46	46 7.56	46 46.13
		47	47 7.72	47 47.13
		48	48 7.89	48 48.13
		49	49 8.05	49 49.13
		50	50 8.21	50 50.14
		51	51 8.38	51 51.14
		52	52 8.54	52 52.14
		53	53 8.71	53 53.15
		54	54 8.87	54 54.15
		55	55 9.04	55 55.15
		56	56 9.20	56 56.15
		57	57 9.36	57 57.16
		58	58 9.53	58 58.16
		59	59 9.69	59 59.16

TABLE XII.
Continued.

hours of sid. time.	Hours of mean time.
h. m. s.	h. m. s.
1	0 59 50.17
2	1 59 40.34
3	2 59 30.51
4	3 59 20.68
5	4 59 10.85
6	5 59 1.02
7	6 58 51.19
8	7 58 41.36
9	8 58 31.53
10	9 58 21.70
11	10 58 11.88
12	11 58 2.05
13	12 57 52.22
14	13 57 42.39
15	14 57 32.56
16	15 57 22.73
17	16 57 12.90
18	17 57 3.07
19	18 56 53.24
20	19 56 43.41
21	20 56 33.58
22	21 56 23.75
23	22 56 13.92
24	23 56 4.09

The fractions of seconds agree to two places of decimals for mean and sidereal.

**TABLE XIV.—To Reduce
Sidereal Time to Longitude
or Degrees.**

Hours.	Degrees.	Time. Min. Sec.	Arc. Deg. Min. Sec.
1	15	1	0 15
2	30	2	0 30
3	45	3	0 45
4	60	4	1 00
5	75	5	1 15
6	90	6	1 30
7	105	7	1 45
8	120	8	2 00
9	135	9	2 15
10	150	10	2 30
11	165	11	2 45
12	180	12	3 00
13	195	13	3 15
14	210	14	3 30
15	225	15	3 45
16	240	16	4 00
17	255	17	4 15
18	270	18	4 30
19	285	19	4 45
20	300	20	5 00
21	315	21	5 15
22	330	22	5 30
23	345	23	5 45
24	360	24	6 00
		25	6 15
		26	6 30
		27	6 45
		28	7 00
		29	7 15
		30	7 30
		31	7 45
		32	8 00
		33	8 15
		34	8 30
		35	8 45
		36	9 00
		37	9 15
		38	9 30
		39	9 45
		40	10 00
		41	10 15
		42	10 30
		43	10 45
		44	11 00
		45	11 15
		46	11 30
		47	11 45
		48	12 00
		49	12 15
		50	12 30
		51	12 45
		52	13 00
		53	13 15
		54	13 30
		55	13 45
		56	14 00
		57	14 15
		58	14 30
		59	14 45
		60	15 00

**TABLE XV.
To Reduce Longi-
tude or Degrees
to Sid. Time.**

Arc. Deg. Min. Sec.	Time. Hr. Min. Min. Sec. Sec. Th.
1	0 4
2	0 8
3	0 12
4	0 16
5	0 20
6	0 24
7	0 28
8	0 32
9	0 36
10	0 40
11	0 44
12	0 48
13	0 52
14	0 56
15	1 00
16	1 04
17	1 08
18	1 12
19	1 16
20	1 20
21	1 24
22	1 28
23	1 32
24	1 36
25	1 40
26	1 44
27	1 48
28	1 52
29	1 56
30	2 00
31	2 04
32	2 08
33	2 12
34	2 16
35	2 20
36	2 24
37	2 28
38	2 32
39	2 36
40	2 40
41	2 44
42	2 48
43	2 52
44	2 56
45	3 00
46	3 04
47	3 08
48	3 12
49	3 16
50	3 20
51	3 24
52	3 28
53	3 32
54	3 36
55	3 40
56	3 44
57	3 48
58	3 52
59	3 56
60	4 00

**TABLE XVI.
Showing the Dip or Depression
of the Horizon, and the Dis-
tance at Sea in Miles, Corres-
ponding to given Heights.**

Height in feet.	Dip in arc.	Dist. seen at sea in miles.
1	0 58	1.32
2	1 22	1.87
3	1 40	2.29
4	1 55	2.65
5	2 09	2.96
6	2 22	3.24
7	2 33	3.50
8	2 44	3.74
9	2 54	3.97
10	3 03	4.18
11	3 12	4.39
12	3 21	4.58
13	3 29	4.77
14	3 37	4.95
15	3 45	5.12
16	3 53	5.29
17	4 01	5.45
18	4 08	5.61
19	4 15	5.77
20	4 22	5.92
21	4 28	6.06
22	4 34	6.21
23	4 40	6.34
24	4 46	6.48
25	4 52	6.61
26	4 58	6.75
28	5 10	7.00
30	5 21	7.25
32	5 31	7.48
34	5 40	7.71
36	5 50	7.94
38	6 00	8.16
40	6 10	8.37
42	6 19	8.57
44	6 28	8.78
46	6 37	8.97
48	6 45	9.17
50	6 53	9.35
55	7 11	9.81
60	7 29	10.25
65	7 47	10.67
70	8 05	11.07
75	8 23	11.46
80	8 44	11.83
85	8 57	12.20
90	9 14	12.55
95	9 30	12.89
100	9 46	13.23
105	10 01	13.56
110	10 16	13.88
115	10 30	14.19
120	10 43	14.49
125	10 56	14.79
130	11 09	15.08
135	11 22	15.37
140	11 35	15.65
145	11 47	15.93
150	11 59	16.20
155	12 11	16.46
160	12 23	16.73

TABLE XVII.—Correction of the Apparent Altitude for Refraction
(Subtractive).

Ap. altitude.	Mean refract.	Cor. in 1 unit.	Ap. altit'de	Mean ref.	cor. 1 un.	Ap. altit'de	Mean ref.	cor. 1 un.	Ap. altit'de	Mean ref.	cor. 1 un.	
° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	° / ' "	
4 00	11 47	19.9	12 00	4 28	6.9	20 00	2 39	4.1	40 00	1 10	1.8	
10	26	19.2	10	25	6.8	20	36	4.0	41 00	7	1.7	
20	6	18.5	20	21	6.8	40	34	3.9	42 00	5	1.7	
30	10 46	18.0	30	18	6.7	21 00	31	3.9	43 00	3	1.6	
40	28	17.5	40	14	6.6	20	29	3.8	44 00	0	1.5	
50	11	16.9	50	11	6.4	40	26	3.7	45 00	0 58	1.5	
5 00	9 54	16.4	13 00	8	6.4	22 00	24	3.7	46 00	56	1.4	
10	38	16.0	10	5	6.4	20	21	3.7	47 00	54	1.4	
20	23	15.6	20	2	6.2	40	19	3.5	48 00	53	1.4	
30	9	15.1	30 3	59	6.2	23 00	17	3.5	49 00	51	1.3	
40	8 55	14.7	40	56	6.0	20	15	3.5	50 00	49	1.2	
50	42	14.3	50	53	6.0	40	13	3.5	51 00	47	1.2	
6 00	30	13.9	14 00	50	6.0	24 00	11	3.3	52 00	46	1.1	
10	18	13.5	10	47	5.9	20	09	3.3	53 00	44	1.1	
20	7	13.3	20	45	5.8	40	07	3.3	54 00	42	1.0	
30	7 56	12.9	30	42	5.7	25 00	05	3.3	55 00	41	1.0	
40	45	12.6	40	40	5.6	20	03	2.2	56 00	39	1.0	
50	35	12.3	50	37	5.6	40	01	3.1	57 00	38	0.9	
7 00	25	12.0	15 00	35	5.5	26 00	1	59	3.1	58 00	36	.9
10	16	11.8	10	32	5.4	20	58	3.0	59 00	35	.9	
20	7	11.5	20	30	5.4	40	56	2.9	60 00	34	.8	
30	6 59	11.3	30	28	5.3	27 00	54	2.9	61 00	32	.8	
40	50	11.0	40	25	5.2	20	53	2.9	62 00	31	.8	
50	42	10.8	50	23	5.2	40	51	2.9	63 00	30	.8	
8 00	35	10.6	16 00	21	5.2	28 00	49	2.9	64 00	28	.7	
10	27	10.4	10	19	5.2	20	48	2.8	65 00	27	.6	
20	20	10.2	20	17	5.1	40	46	2.7	66 00	26	.6	
30	13	10.0	30	15	5.0	29 00	45	2.7	67 00	25	.6	
40	7	9.8	40	13	5.0	30	44	2.7	68 00	24	.6	
50	0	9.6	50	11	4.9	40	42	2.7	69 00	22	.5	
9 00	5 44	9.4	17 00	9	4.9	30 00	41	2.7	70 00	21	.5	
10	48	9.2	10	7	4.8	20	39	2.6	71 00	20	.5	
20	42	9.1	20	5	4.8	40	38	2.6	72 00	19	.4	
30	36	8.9	30	3	4.7	31 00	37	2.5	73 00	18	.4	
40	31	8.7	50	1	4.7	20	35	2.5	74 00	17	.4	
50	25	8.4	50	0	4.7	40	34	2.4	75 00	16	.4	
10 00	20	8.5	18 00	2	58	4.6	32 00	33	2.3	76 00	15	.4
10	15	8.3	10	56	4.6	20	32	2.3	77 00	13	.3	
20	10	8.1	20	55	4.5	40	31	2.2	78 00	12	.3	
30	6	8.0	30	53	4.5	33 00	30	2.2	79 00	11	.3	
40	1	8.0	40	51	4.4	20	28	2.2	80 00	10	.3	
50	4 56	7.8	50	50	4.3	40	27	2.1	91 00	9	.2	
11 00	52	7.7	19 00	48	4.3	34 00	26	2.1	82 00	8	.2	
10	48	7.6	10	47	4.3	35 00	23	2.1	83 00	7	.1	
20	44	7.5	20	45	4.2	36 00	20	2.1	84 00	6	.1	
30	40	7.4	30	44	4.2	37 00	17	2.0	86 00	4	.0	
40	36	7.3	40	42	4.2	38 00	15	1.9	88 00	2	.0	
50	32	7.1	50	41	4.1	39 00	12	1.8	90 00	0	.0	

The first column contains the apparent altitude of the object.
The second column contains the mean refraction; that is, the refraction corresponding to the state of the atmosphere, when the height of the barometer is 30 inches and that of the thermometer 50 degrees Fahrenheit. Here 30 — one-tenth of the thermometer = 30 — 5 = 25 = given standard tabular height for mean refraction.

The third column contains the correction for every unit that the tabular height is greater or less than the standard height (25). *Example.* Let the apparent altitude be 20°; the barometer 29.5 inches; the thermometer 15° above zero. Here we have 29.5 — one-tenth of 15 = 29.5 — 1.5 = 28, which is 3 units greater than 25.

Mean refraction for apparent altitude 20° = 39'.
Correction for 1 unit = 4". This multiplied by 3 = plus 12.3.
Required refraction = 2 51.3.

When the tabular height is greater than 25, add the correction to the mean refraction. The sum will be the required refraction. When the tabular height is less than 25, subtract the correction from the mean refraction. The difference will be the required refraction.

TABLE XVIII.

Sun's Parallax in Altitude.

Sun's alt. in degrees.	Parallax in seconds.
0	9
10	9
20	8
30	8
40	7
50	6
55	5
60	4
65	4
70	3
75	2
80	2
85	1
90	0

Parallax is to be added to apparent altitude, or it may be added after the corrections, for dip and refraction are taken from the ap. alt., or it may be taken from the sum of the dip and ref., and the difference taken from ap. alt.

TABLE XIX.—Parallax in Altitude of the Planets.

PLANETS' HORIZONTAL PARALLAX. (From Naut. Alm.)

Ap. Alt.	1"	3"	5"	7"	9"	11"	13"	15"	17"	19"	21"	23"	25"	27"	29"	30"
0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	30
10	1	3	5	7	9	11	13	15	17	19	21	23	25	26	28	29
15	1	3	5	7	9	10	12	14	16	18	20	22	24	25	27	28
20	1	3	5	7	9	10	12	14	15	17	19	21	23	24	26	27
25	1	3	5	6	8	10	12	14	15	17	19	21	23	24	25	26
30	1	3	4	6	8	10	11	13	15	16	18	20	22	23	25	26
33	1	2	4	6	8	9	11	13	14	16	18	19	21	23	24	25
36	1	2	4	6	7	9	11	12	14	15	17	19	20	22	23	25
39	1	2	4	5	7	9	10	12	13	15	16	18	19	21	23	24
42	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23
45	1	2	4	5	6	8	9	11	12	13	15	16	18	19	21	21
48	1	2	3	5	6	7	9	10	11	12	13	14	15	17	18	19
51	1	2	3	4	6	7	8	9	11	12	13	14	16	17	18	19
54	1	2	3	3	5	6	8	9	10	11	12	14	15	16	17	18
57	1	2	3	3	5	6	7	8	9	10	11	12	13	14	15	15
60	0	2	3	3	5	6	7	8	9	10	11	12	13	14	15	15
62	0	1	2	3	4	5	6	7	8	10	10	11	12	13	14	14
64	0	1	2	3	4	5	6	7	8	9	10	10	11	12	13	13
66	0	1	2	3	4	4	5	6	7	8	9	9	10	11	12	12
68	0	1	2	3	3	4	5	5	6	7	8	9	9	10	11	11
70	0	1	2	2	3	4	5	5	6	7	8	9	9	10	11	10
72	0	1	2	2	2	3	4	5	5	6	7	8	9	9	10	10
74	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	7
76	0	1	1	2	2	3	3	4	4	4	4	5	5	6	6	6
78	0	1	1	1	1	2	3	3	3	3	3	4	4	4	5	5
81	0	0	1	1	1	2	2	2	2	2	2	2	3	3	3	3
84	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3
87	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	2
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE XX.—Reduction of the Time of the Moon's Passage over the Meridian at Greenwich to that over any other Meridian.

DAILY VARIATION OF THE MOON'S MERIDIAN PASSAGE.

Long. in deg.	40'	42'	44'	46'	48'	50'	52'	54'	56'	58'	60'	62'	64'	66'
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	2	2	2	2	3	3	3	3	3	3	3	3	4	4
30	3	3	4	4	4	4	4	4	4	5	5	5	5	5
40	4	4	5	5	5	5	6	6	6	6	6	7	7	7
50	5	6	6	6	6	7	7	7	7	8	8	8	9	9
60	6	7	7	8	8	8	9	9	9	10	10	10	11	11
70	8	8	9	9	9	10	10	10	11	11	12	12	12	13
80	9	9	10	10	11	11	12	12	12	13	13	14	14	15
90	10	10	11	11	12	12	13	13	14	14	15	15	16	16
100	11	12	12	13	13	14	14	15	16	16	17	17	18	18
110	12	13	13	14	15	15	16	16	17	18	19	19	20	20
120	13	14	15	15	16	17	17	18	19	19	20	21	21	22
130	14	15	16	17	17	18	19	19	20	21	22	22	23	24
140	16	16	17	18	19	19	20	21	22	23	23	24	25	26
150	17	17	18	19	20	21	22	22	23	24	25	26	27	27
160	18	19	20	20	21	22	23	24	25	26	27	28	28	29
170	19	20	21	22	23	24	25	25	26	27	28	29	30	31
180	20	21	22	23	24	25	26	27	28	29	30	31	32	33

Note. In this table, longitude means the moon's longitude, or time in arc, from her meridian passage. The corrections are in minutes of a degree.

TABLE XXI.—*Best Time for Obtaining Apparent Time.* *

Lat.	Declination of the same name with the Latitude.											
	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
0	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
1	4 0	5 2	5 22	5 31	5 37	5 41	5 44	5 46	5 48	5 49	5 50	5 51
2	0 4	0 4	4 42	5 2	5 14	5 22	5 28	5 32	5 35	5 38	5 40	5 42
3	3 13	2 46	4 0	4 32	4 51	5 3	5 11	5 18	5 23	5 27	5 30	5 33
4	4 0	0 0	3 13	4 1	4 27	4 43	4 55	5 4	5 10	5 16	5 20	5 24
5	4 26	2 28	2 15	3 26	4 1	4 23	4 38	4 49	4 58	5 4	5 10	5 15
6	4 42	3 13	0 0	2 46	3 34	4 1	4 20	4 34	4 45	4 53	5 0	5 5
7	4 54	3 41	2 5	1 56	3 3	3 39	4 2	4 19	4 31	4 41	4 49	4 56
8	5 2	4 1	2 46	0 0	2 29	3 14	3 43	4 3	4 17	4 29	4 39	4 46
9	5 9	4 15	3 14	1 50	1 44	2 47	3 22	3 46	4 3	4 17	4 28	4 37
10	5 14	4 27	3 34	2 29	0 0	2 16	3 0	3 28	3 49	4 4	4 16	4 27
11	5 19	4 36	3 49	2 55	1 40	1 35	2 35	3 9	3 33	3 51	4 5	4 16
12	5 22	4 43	4 1	3 14	2 16	0 0	2 6	2 49	3 17	3 37	3 53	4 6
13	5 25	4 49	4 12	3 30	2 41	1 32	1 29	2 26	2 59	3 23	3 41	3 55
14	5 28	4 55	4 20	3 43	3 0	2 6	0 0	1 58	2 40	3 7	3 28	3 44
15	5 30	4 59	4 28	3 53	3 15	2 30	1 26	1 23	2 18	2 50	3 14	3 32
16	5 32	5 4	4 34	4 3	3 28	2 49	1 58	0 0	1 52	2 32	2 59	3 20
17	5 34	5 7	4 40	4 11	3 39	3 4	2 21	1 21	1 19	2 11	2 43	3 7
18	5 35	5 10	4 45	4 17	3 49	3 17	2 40	1 52	0 0	1 47	2 26	2 53
19	5 37	5 13	4 49	4 24	3 57	3 28	2 54	2 14	1 17	1 16	2 6	2 37
20	5 38	5 16	4 53	4 29	4 4	3 37	3 7	2 32	1 47	0 0	1 43	2 21
21	5 39	5 18	4 56	4 34	4 11	3 46	3 18	2 47	2 9	1 14	1 32	2 2
22	5 40	5 20	5 0	4 39	4 16	3 53	3 28	2 59	2 26	1 43	0 0	1 39
23	5 41	5 22	5 3	4 43	4 22	4 0	3 36	3 10	2 40	2 41	1 11	1 10
24	5 42	5 24	5 5	4 46	4 27	4 6	3 44	3 20	2 53	2 21	1 39	0 0
25	5 43	5 26	5 8	4 50	4 31	4 12	3 51	3 28	3 3	2 35	2 0	1 9
26	5 44	5 27	5 10	4 53	4 35	4 17	3 57	3 36	3 13	2 47	2 16	1 36
27	5 44	5 28	5 12	4 56	4 39	4 21	4 3	3 43	3 22	3 0	2 30	1 56
28	5 45	5 30	5 14	4 59	4 43	4 26	4 8	3 49	3 29	3 8	2 42	1 13
29	5 46	5 31	5 16	5 1	4 46	4 30	4 13	3 55	3 36	3 16	2 53	2 26
30	5 46	5 32	5 18	5 4	4 49	4 34	4 18	4 1	3 43	3 24	3 2	2 38
31	5 47	5 33	5 20	5 6	4 52	4 37	4 22	4 6	3 49	3 31	3 11	2 49
32	5 47	5 34	5 21	5 8	4 54	4 40	4 26	4 11	3 55	3 38	3 19	2 58
33	5 48	5 35	5 23	5 10	4 57	4 44	4 30	4 15	4 0	3 44	3 26	3 7
34	5 48	5 36	5 24	5 12	4 59	4 47	4 33	4 19	4 5	3 49	3 33	3 15
35	5 49	5 37	5 25	5 14	5 2	4 49	4 37	4 23	4 9	4 55	3 39	3 22
36	5 49	5 38	5 27	5 15	5 4	4 52	4 40	4 27	4 14	4 0	3 45	3 29
37	5 49	5 39	5 28	5 17	5 6	4 54	4 43	4 31	4 18	4 4	3 50	3 35
38	5 50	5 39	5 29	5 19	5 8	4 57	4 46	4 34	4 22	4 9	3 55	3 41
39	5 50	5 40	5 30	5 20	5 10	4 59	4 48	4 37	4 25	4 13	4 0	3 47
40	5 50	5 41	5 31	5 21	5 11	5 1	4 51	4 40	4 29	4 17	4 5	2 52
41	5 51	5 42	5 32	5 23	5 13	5 3	4 54	4 43	4 32	4 21	4 9	2 57
42	5 51	5 42	5 33	5 24	5 15	5 5	4 56	4 46	4 35	4 25	4 13	4 1
43	5 51	5 43	5 34	5 25	5 16	5 7	4 58	4 48	4 38	4 28	4 17	4 6
44	5 52	5 43	5 35	5 27	5 18	5 9	5 0	4 51	4 41	4 31	4 21	4 10
45	5 52	5 44	5 36	5 28	5 19	5 11	5 2	4 53	4 44	4 35	4 25	4 14
46	5 52	5 45	5 37	5 29	5 21	5 13	5 4	4 56	4 47	4 38	4 28	4 18
47	5 53	5 45	5 38	5 30	5 22	5 14	5 6	4 58	4 49	4 41	4 31	4 22
48	5 53	5 46	5 38	5 31	5 23	5 16	5 8	5 0	4 52	4 43	4 35	4 25
49	5 53	5 46	5 39	5 32	5 25	5 17	5 10	5 2	4 54	4 46	4 38	4 29
50	5 53	5 47	5 40	5 33	5 26	5 19	5 12	5 4	4 57	4 49	4 41	4 32
51	5 54	5 47	5 41	5 35	5 28	5 22	5 15	5 8	5 1	4 54	4 46	4 39
52	5 54	5 48	5 42	5 37	5 31	5 24	5 18	5 12	5 5	4 59	4 52	4 45
53	5 55	5 49	5 44	5 38	5 33	5 27	5 21	5 15	5 9	5 3	4 57	4 50
54	5 55	5 50	5 45	5 40	5 35	5 29	5 24	5 19	5 13	5 7	5 2	4 55
55	5 55	5 51	5 46	5 41	5 37	5 32	5 27	5 22	5 17	5 11	5 5	5 0
56	5 56	5 51	5 47	5 43	5 38	5 34	5 30	5 25	5 20	5 15	5 10	5 5
57	5 56	5 52	5 48	5 44	5 40	5 36	5 32	5 28	5 24	5 19	5 15	5 10
58	5 56	5 53	5 49	5 46	5 42	5 38	5 35	5 31	5 27	5 23	5 19	5 14
59	5 57	5 54	5 50	5 47	5 44	5 40	5 37	5 33	5 30	5 26	5 22	5 19
60	5 57	5 54	5 51	5 48	5 45	5 42	5 39	5 36	5 33	5 30	5 26	5 23

TABLE XXII.—Best Altitudes for Obtaining Apparent Time.

Lat.	Declination of the same name with the Latitude.											
	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	30 0	14 29	9 37	7 12	5 46	4 49	4 8	3 38	3 14	2 55	2 40	2 28
2	90 0	30 1	19 30	14 31	11 36	9 40	8 18	7 16	6 29	5 51	5 21	4 55
3	41 49	48 37	30 3	22 5	17 32	14 35	12 30	10 57	9 45	8 48	8 2	7 24
4	30 1	90 0	41 52	30 5	23 41	19 36	16 45	14 40	13 3	11 46	10 44	9 53
5	23 36	53 10	56 30	38 46	30 8	24 47	21 7	18 26	16 23	14 46	13 27	12 22
6	19 30	41 52	90 0	48 41	37 1	30 11	25 36	22 17	19 46	17 48	16 12	14 53
7	16 38	34 55	59 4	61 7	44 34	35 53	30 15	26 14	23 14	20 52	18 59	17 26
8	14 31	30 5	48 41	90 0	53 16	42 1	35 7	30 20	26 46	24 1	21 49	20 1
9	12 53	26 29	41 56	62 50	64 16	48 48	40 17	34 35	30 25	27 13	24 41	22 37
10	11 36	23 41	37 1	53 16	90 0	56 38	45 52	39 3	34 11	30 31	27 37	25 16
11	10 32	21 27	33 13	46 50	65 31	66 36	52 4	43 48	38 8	33 55	30 37	27 59
12	9 40	19 36	30 11	42 1	56 38	90 0	59 15	48 58	42 17	37 26	33 43	30 45
13	8 56	18 4	27 41	38 13	50 32	67 33	68 25	54 42	46 43	41 8	36 54	33 35
14	8 18	16 45	25 36	35 7	45 52	59 15	90 0	61 22	51 32	45 1	40 14	36 30
15	7 45	15 38	23 49	32 32	42 8	53 27	69 11	69 53	56 93	49 11	43 42	39 31
16	7 16	14 40	22 17	30 20	39 3	48 58	61 22	90 0	63 7	53 42	47 22	42 40
17	6 51	13 48	20 57	28 26	36 26	45 20	55 50	70 31	71 7	58 44	51 18	45 57
18	6 29	23 8	19 46	26 46	34 11	42 17	51 32	63 7	90 0	64 37	55 35	49 27
19	6 9	12 22	18 44	25 18	22 14	39 41	48 0	57 51	71 39	72 9	60 21	53 10
20	5 51	11 46	17 48	24 1	30 31	37 26	45 1	53 42	64 37	90 0	65 55	57 14
21	5 35	11 13	16 58	22 51	28 59	35 28	42 28	50 17	59 34	72 38	73 4	61 47
22	5 21	10 44	16 12	21 49	27 37	33 43	40 14	47 22	55 35	65 55	90 0	67 5
23	5 7	10 17	15 31	20 52	26 23	32 9	38 15	44 52	52 16	61 5	57 3	29 73
24	4 55	9 53	14 53	20 1	25 16	30 45	36 30	42 40	49 27	57 14	67 5	90 0
25	4 44	9 30	14 19	19 14	24 16	29 28	34 55	40 43	46 59	54 1	62 25	74 16
26	4 34	9 9	13 48	18 31	23 20	28 19	33 30	38 58	44 49	51 17	58 43	68 5
27	4 25	8 50	13 19	17 51	22 29	27 15	32 12	37 23	42 54	48 53	55 36	63 37
28	4 16	8 33	12 52	17 15	21 42	26 17	31 1	35 57	41 10	46 46	52 56	60 3
29	4 8	8 16	12 27	16 41	20 59	25 24	29 56	34 39	39 36	44 52	50 36	57 2
30	4 0	8 1	12 4	16 10	20 19	24 34	28 56	33 27	38 10	43 10	48 31	54 26
31	3 53	7 47	11 43	15 41	19 42	23 49	28 1	32 21	36 52	41 37	46 40	52 10
32	3 47	7 34	11 23	15 14	19 8	23 6	27 10	31 21	35 40	40 12	44 59	50 9
33	3 40	7 22	11 4	14 48	18 36	22 26	26 22	30 24	34 34	38 54	43 27	48 18
34	3 35	7 10	10 46	14 25	18 5	21 50	25 38	29 32	33 53	37 42	42 4	46 39
35	3 29	6 59	10 30	14 3	17 37	21 15	24 57	28 43	32 36	36 36	40 47	45 11
36	3 24	6 49	10 16	13 42	17 11	20 43	24 18	27 58	31 43	35 35	39 36	43 46
37	3 19	6 39	10 0	13 22	16 46	20 13	23 42	27 16	30 54	34 38	38 30	42 31
38	3 15	6 31	9 47	13 4	16 23	19 44	23 8	26 36	30 8	33 45	37 29	41 20
39	3 11	6 22	9 34	12 47	16 1	19 18	22 36	25 59	29 25	32 55	36 32	40 16
40	3 7	6 14	9 22	12 30	15 40	18 52	22 7	25 24	28 44	32 9	35 39	39 15
41	3 3	6 6	9 10	12 15	15 21	18 29	21 38	24 51	28 6	31 25	34 49	38 18
42	2 59	5 59	8 59	12 0	15 2	18 6	21 12	24 20	27 30	30 44	34 3	37 25
43	2 56	5 52	8 49	11 47	14 45	17 45	20 47	23 50	26 57	30 6	33 19	36 36
44	2 53	5 46	8 39	11 33	14 29	17 25	20 23	23 23	26 25	29 30	32 38	35 49
45	2 50	5 40	8 30	11 21	14 13	17 6	20 0	22 57	25 55	28 56	31 59	35 6
46	2 47	5 34	8 21	11 9	13 58	16 48	19 39	22 32	25 26	28 23	31 25	34 25
47	2 44	5 28	8 13	10 58	13 44	16 31	19 19	22 8	25 0	27 53	30 49	33 47
48	2 42	5 23	8 5	10 48	13 31	16 15	19 0	21 46	24 34	27 24	30 16	33 12
49	2 39	4 18	7 58	10 38	13 18	15 59	18 42	21 25	24 10	26 57	29 46	32 35
50	2 37	5 13	7 51	10 28	13 6	15 45	18 25	21 5	22 47	26 31	29 17	32 4
52	2 32	5 5	7 37	10 10	12 44	15 18	17 53	20 28	23 5	25 43	28 23	31 7
54	2 28	4 57	7 25	9 54	12 24	14 54	17 24	19 55	22 27	25 1	27 35	30 10
56	2 25	4 50	7 15	9 40	12 5	14 31	16 58	19 25	21 53	24 22	26 52	29 22
58	2 22	4 43	7 5	9 27	11 49	14 11	16 35	18 58	21 22	23 47	26 13	28 39
60	2 19	4 37	6 56	9 15	11 34	13 53	16 13	18 34	20 54	23 16	25 38	28 0
62	2 16	4 31	6 47	8 3	11 19	13 37	15 54	18 12	20 29	22 48	25 6	27 26
64	2 14	4 27	6 41	8 54	11 8	13 23	15 37	17 52	20 8	22 22	24 38	26 54
66	2 11	4 23	6 34	8 46	10 57	13 9	15 21	17 34	19 46	21 58	24 13	26 26
68	2 9	4 19	6 28	8 38	10 48	12 57	15 7	17 18	19 28	21 39	23 50	26 1
70	2 8	4 15	6 23	8 31	10 39	12 47	14 55	17 3	19 12	21 21	23 30	25 39

TABLE XXIII.—Azimuths or Beatings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

star	Sigma in Octantis.						Alpha in Ursaminoris (Polaris).														
p.d.	40'		45'		50'		55'			1°		1° 5'		1° 10'		1° 15'		1° 20'			
lat.	/'	''	/'	''	/'	''	/'	''	'''	/'	''	/'	''	'''	/'	''	/'	''	'''		
1	40	00	45	00	0	50	00	0	55	00	1	0	1	5	00	1	10	01	1	20	01
2	02	02	02	02	02	02	02	02	02	02	2	02	03	03	03	03	03	03	03	03	03
3	03	03	03	03	04	04	04	04	04	5	05	05	05	05	06	06	06	06	06	06	06
4	06	06	06	06	07	07	07	08	08	9	09	09	10	10	11	11	12	12	12	12	12
5	09	09	10	10	11	11	13	13	14	15	15	16	16	14	14	18	18	18	18	18	18
6	13	13	14	14	16	16	18	18	20	22	22	23	23	25	25	26	26	26	26	26	26
7	17	17	20	20	23	23	25	25	27	29	29	32	32	34	34	36	36	36	36	36	36
8	24	24	26	26	29	29	33	33	35	38	38	41	41	45	45	47	47	47	47	47	47
9	30	30	33	33	37	37	41	41	45	49	49	52	52	56	56	58	58	58	58	58	58
10	37	37	41	41	46	46	51	51	56	60	60	63	63	67	67	70	70	70	70	70	70
11	45	45	50	50	56	56	62	62	68	73	73	78	78	83	83	87	87	87	87	87	87
12	54	54	60	60	67	67	74	74	81	88	88	95	95	102	102	108	108	108	108	108	108
13	41	03	11	19	19	19	27	27	35	43	51	58	58	66	66	74	74	74	74	74	74
14	14	14	23	32	32	41	50	50	60	70	80	90	90	100	100	110	110	110	110	110	110
15	25	25	35	46	46	56	66	66	77	88	99	110	110	121	121	132	132	132	132	132	132
16	37	37	49	62	62	75	88	88	101	114	127	140	140	153	153	166	166	166	166	166	166
17	50	47	03	17	17	31	46	46	61	76	91	106	106	121	121	136	136	136	136	136	136
18	42	03	18	34	34	50	67	67	85	102	119	136	136	154	154	172	172	172	172	172	172
19	18	18	35	53	53	73	93	93	115	137	159	181	181	203	203	225	225	225	225	225	225
20	34	34	53	73	73	97	121	121	147	171	195	219	219	243	243	267	267	267	267	267	267
21	51	48	12	33	33	55	77	77	101	125	149	173	173	200	200	224	224	224	224	224	224
22	43	09	32	56	56	83	107	107	137	161	185	209	209	239	239	269	269	269	269	269	269
23	18	18	41	54	54	87	111	111	143	167	191	215	215	247	247	271	271	271	271	271	271
24	37	37	49	62	62	97	121	121	153	177	201	225	225	257	257	281	281	281	281	281	281
25	44	08	39	55	55	104	128	128	161	185	209	233	233	265	265	289	289	289	289	289	289
26	30	30	51	71	71	104	128	128	161	185	209	233	233	265	265	289	289	289	289	289	289
27	42	17	52	77	77	112	136	136	171	195	219	243	243	275	275	299	299	299	299	299	299
28	45	06	44	22	22	40	58	58	80	98	116	134	134	156	156	178	178	178	178	178	178
29	31	51	12	54	54	85	109	109	139	163	187	211	211	235	235	259	259	259	259	259	259
30	44	27	57	82	82	110	134	134	163	187	211	235	235	259	259	283	283	283	283	283	283
31	46	11	57	84	84	111	135	135	165	189	213	237	237	261	261	285	285	285	285	285	285
32	25	52	13	58	58	92	116	116	125	149	173	197	197	221	221	245	245	245	245	245	245
33	40	30	50	70	70	100	124	124	153	177	201	225	225	249	249	273	273	273	273	273	273
34	48	15	54	77	77	109	133	133	163	187	211	235	235	259	259	283	283	283	283	283	283
35	32	42	55	72	72	104	128	128	157	181	205	229	229	253	253	277	277	277	277	277	277
36	49	08	46	65	65	104	128	128	157	181	205	229	229	253	253	277	277	277	277	277	277
37	26	55	37	62	62	98	122	122	151	175	199	223	223	247	247	271	271	271	271	271	271
38	50	50	56	20	20	36	54	54	76	98	120	142	142	164	164	186	186	186	186	186	186
39	25	43	43	3	3	27	45	45	67	89	111	133	133	155	155	177	177	177	177	177	177
40	46	57	06	27	27	53	73	73	95	117	139	161	161	183	183	205	205	205	205	205	205
41	51	07	30	53	53	80	100	100	122	144	166	188	188	210	210	232	232	232	232	232	232
42	28	54	54	4	4	20	38	38	60	82	104	126	126	148	148	170	170	170	170	170	170
43	50	53	29	48	48	56	74	74	96	118	140	162	162	184	184	206	206	206	206	206	206
44	18	54	54	5	5	16	34	34	56	78	100	122	122	144	144	166	166	166	166	166	166
45	52	36	59	10	10	45	63	63	85	107	129	151	151	173	173	195	195	195	195	195	195

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Polaris.		δ Ursa Minoris.		ε Ursa Minoris.			
1° 25'	1° 30'	3° 20'	3° 23'	7° 45'	7° 50'	7° 55'	8° 00'
° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "
1 25 01	1 30 00	3 20 02	3 23 55	7 45 04	7 50 04	7 55 05	8 0 04
03	08	08	24 01	45 17	50 17	55 18	0 18
07	08	17	24 10	45 39	50 39	55 39	0 40
12	13	29	24 23	46 08	51 10	56 10	1 11
20	21	46	24 40	46 47	51 49	56 50	1 51
28	30	21 06	25 00	47 35	52 36	57 38	1 33
38	40	21 30	25 25	48 31	53 33	58 35	3 38
50	53	21 58	25 53	49 36	54 39	59 42	4 45
26 03	31 07	22 30	26 26	50 50	55 54	8 0 58	6 02
19	23	23 06	27 02	52 13	57 18	2 23	7 27
36	41	23 45	27 42	53 46	58 51	3 57	9 03
54	32 01	24 29	28 27	55 27	8 0 34	5 41	10 48
27 14	22	25 16	29 15	57 19	2 27	7 34	12 51
36	45	26 08	30 08	59 20	4 29	9 38	14 48
28 00	33 10	27 04	31 05	8 1 31	6 41	11 52	17 03
26	38	28 04	32 06	3 52	9 04	14 61	19 29
53	34 07	29 09	33 15	6 23	11 37	16 51	22 05
29 29	38	30 18	34 23	9 05	14 21	19 37	24 53
54	35 11	31 32	35 39	11 58	17 16	22 34	27 51
30 27	47	32 51	36 59	15 03	20 22	25 42	31 02
31 03	36 24	34 15	38 25	18 19	23 40	29 02	34 24
22	44	34 59	39 09	20 01	25 24	30 47	36 10
41	37 04	35 44	39 55	21 46	27 11	32 34	37 58
32 00	25	36 30	40 42	23 35	29 00	34 25	39 51
20	46	37 18	41 31	25 26	30 53	36 20	41 46
42	38 09	38 07	42 21	27 21	32 49	38 16	43 44
33 03	31	38 57	43 12	29 19	34 48	40 17	45 46
25	54	39 49	44 05	31 20	36 51	42 21	47 51
47	39 19	40 42	44 59	33 25	38 57	44 28	50 00
34 11	43	41 37	45 55	35 33	41 06	46 39	52 12
34	40 08	42 33	46 52	37 44	43 19	48 53	54 28
59	34	43 51	47 51	39 59	45 35	51 11	56 48
35 24	41 01	44 30	48 52	42 17	47 55	53 33	59 11
50	28	45 31	49 54	44 33	50 19	55 58	9 1 38
36 16	56	46 33	50 57	47 06	52 47	58 28	4 08
44	42 27	47 37	52 02	49 36	55 26	9 1 01	6 43
37 12	55	48 42	53 09	52 10	57 54	3 38	9 22
40	43 25	49 50	54 18	54 47	9 0 33	6 19	12 05
38 09	55	50 59	55 28	57 29	4 32	9 04	14 52
39	44 27	52 10	56 41	9 0 15	6 05	11 54	17 43
39 10	45 00	53 22	57 54	3 05	8 57	14 48	20 39
42	33	54 37	59 10	6 00	11 53	17 46	23 39
40 15	46 08	55 53	4 0 29	8 59	14 54	20 49	26 44
47	43	57 12	1 48	12 02	17 59	23 56	29 53
41 21	47 19	58 32	3 10	15 10	21 09	27 09	33 08
56	56	59 54	4 34	18 23	24 24	30 26	36 27
42 32	48 29	4 1 18	5 59	21 41	27 44	33 48	39 27
43 09	49 13	2 45	7 28	25 04	31 09	37 15	43 21
43	52	4 13	8 58	28 31	34 39	40 47	46 55
44 25	50 33	5 44	10 30	32 04	38 15	44 25	50 35
45 07	51 15	7 17	12 05	35 42	41 55	48 08	54 20
45	51 58	8 53	13 45	39 26	45 42	51 57	58 12
46 26	52 42	10 31	15 22	43 17	49 34	55 51	10 1 52
47 09	53 27	12 11	17 05	47 11	53 31	59 52	6 12
52	54 13	13 53	18 50	51 13	57 35	10 3 58	10 21
48 37	55 00	15 38	20 37	55 20	10 1 46	8 11	14 37
49 23	55 49	17 27	22 27	59 33	6 02	12 30	18 59
50 10	56 39	19 18	24 20	10 3 53	10 25	16 56	23 27
58	57 30	21 11	26 16	8 20	14 54	21 29	28 03
51 47	58 22	23 08	28 14	12 54	19 31	26 08	32 45

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

star	Sigma in Octantis.			Alpha in Ursa Minoris (Polaris).									
	40'	45'	50'	55'	1°	1° 5'	1° 10'	1° 15'	1° 20'				
p.d.	lat.	lat.	lat.	lat.	lat.	lat.	lat.	lat.	lat.	lat.	lat.	lat.	lat.
41	0	53 00	0 59 38	1 6 15	12 53	1 19 30	1 26 08	1 32 45	1 39 23	1 46 01			
1/2		53 24	1 0 43	6 46	13 26	20 07	26 48	33 28	40 09	46 50			
42		53 50	0 23	7 17	14 01	20 44	27 28	34 12	40 56	47 40			
1/2		54 15	1 02	7 49	14 36	21 23	28 10	34 57	41 44	48 31			
43		54 42	1 31	8 22	15 11	22 03	28 53	35 44	42 34	49 24			
1/2		55 09	2 02	8 56	15 50	22 43	29 37	36 31	43 24	50 18			
44		55 37	2 34	9 31	16 28	23 25	30 22	37 19	44 16	51 13			
1/2		56 05	3 05	10 06	17 07	24 08	31 09	38 09	45 11	52 10			
45		56 34	3 39	10 43	17 47	24 52	31 56	39 00	46 04	53 09			
1/2		57 04	4 12	11 20	18 28	25 36	32 44	39 53	47 01	54 09			
46		57 35	4 47	11 59	19 11	26 23	33 35	40 47	47 55	55 10			
1/2		58 07	5 23	12 38	19 53	27 10	34 26	41 42	48 58	56 14			
47		58 39	5 59	13 19	20 39	27 59	35 19	42 39	49 59	57 19			
1/2		59 12	6 37	14 01	21 25	28 49	36 13	43 28	51 01	58 26			
48		59 47	7 15	14 54	22 12	29 40	37 09	44 37	52 06	59 34			
1/2	1	0 22	7 55	15 28	23 00	30 33	38 06	45 39	53 12	2 0 45			
49		0 58	8 36	16 13	23 50	31 28	39 05	46 42	54 20	1 57			
1/2		1 36	9 17	17 00	24 42	32 24	40 06	47 28	55 30	3 12			
50		2 14	10 01	17 47	25 34	33 21	41 08	48 55	56 41	4 28			
1/2	1	2 53	1 10 45	1 18 37	1 26 28	34 20	1 42 12	1 50 04	1 57 56	5 47			
51		3 34	11 31	19 27	27 24	35 21	43 09	51 15	59 11	7 09			
1/2		4 16	12 17	20 19	28 21	36 23	44 26	52 28	2 0 30	8 32			
52		4 58	13 06	21 13	29 20	37 28	45 35	53 43	1 50	9 58			
1/2		5 43	13 55	22 08	30 21	38 34	46 47	54 00	3 11	11 26			
53		6 28	14 47	23 28	31 24	39 42	48 01	56 20	4 38	12 57			
1/2		7 15	15 39	24 05	32 28	40 53	49 17	57 42	6 06	14 31			
54		8 03	16 34	25 04	33 35	42 05	50 35	59 06	7 37	16 08			
1/2		8 53	17 30	26 06	34 36	43 20	51 57	2 0 34	9 10	17 47			
55		9 44	18 28	27 11	35 54	44 37	53 20	2 03	10 47	19 30			
1/2		10 37	19 27	28 29	1 37 20	45 56	55 02	3 53	12 45	21 16			
56		11 32	20 29	29 25	38 22	47 19	56 15	5 12	14 08	23 06			
1/2		12 28	21 32	30 36	39 39	48 43	57 57	6 51	15 54	24 58			
57		13 37	22 38	32 01	41 14	50 20	59 38	8 50	18 03	26 15			
1/2		14 27	23 46	33 04	42 23	51 41	2 0 59	10 18	19 37	28 55			
58		15 49	24 56	34 22	43 48	53 14	2 41	12 07	21 34	31 00			
1/2		16 34	26 08	35 42	45 16	54 51	4 26	14 00	23 34	33 09			
59		17 40	27 23	37 05	46 47	56 31	6 13	15 56	25 39	35 22			
1/2		18 49	28 40	38 30	48 23	58 13	8 05	17 57	27 48	37 40			
60		20 00	30 00	40 02	50 01	2 0 11	10 02	20 02	30 02	40 03			
1/2		21 14	31 23	41 33	1 51 42	1 52 2	12 01	22 11	2 32 21	42 31			
61		22 31	32 50	43 09	54 26	3 52	14 06	24 25	34 44	45 04			
1/2		23 50	34 19	44 48	55 17	5 46	16 15	26 44	37 13	47 43			
62		25 12	35 52	46 31	57 10	7 49	18 29	29 18	39 48	50 27			
1/2		26 38	37 28	48 18	59 07	9 58	20 48	31 38	42 28	53 19			
63		28 07	39 08	50 09	2 1 10	12 11	23 12	34 14	45 15	56 17			
1/2		29 39	40 51	52 04	3 17	14 30	25 42	36 56	48 08	59 22			
64		31 15	42 40	54 05	5 29	16 54	28 19	39 44	51 09	3 2 34			
1/2		32 55	44 33	56 09	7 47	19 24	31 11	42 39	54 16	5 54			
65		34 39	46 29	58 20	10 10	22 00	33 51	45 41	57 32	9 22			
1/2		36 28	48 31	2 0 35	12 39	24 43	36 47	48 51	3 0 56	13 00			
66		38 21	50 39	2 57	15 15	27 33	39 52	52 10	4 28	16 46			
1/2		40 19	52 52	5 25	17 58	30 30	43 04	55 36	8 10	20 45			
67		42 23	55 11	8 00	20 48	33 36	46 25	59 13	12 02	24 51			
1/2		44 32	57 37	10 41	23 46	36 50	49 55	3 0 00	16 04	29 09			
68		46 48	2 0 09	13 30	26 52	40 13	53 35	6 57	20 18	33 41			
1/2		49 09	2 48	16 27	30 06	43 57	57 25	11 04	24 25	38 25			
69		51 38	5 35	19 33	33 31	47 29	3 1 27	15 25	29 24	43 22			
1/2		54 14	8 32	22 48	37 06	51 25	5 41	19 59	34 17	48 35			
70	1	56 58	11 36	2 26 14	2 40 51	2 55 30	3 10 08	3 24 47	3 39 25	3 54 04			

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.

Polaris.		δ Ursa Minoris.		ε Ursa Minoris.			
1° 25'	1° 30'	3° 20'	3° 23'	7° 45'	7° 50'	7° 55'	8° 00'
1 52 38	1 59 16	4 25 07	4 30 16	10 17 08	10 24 06	10 30 55	10 37 36
53 30	2 0 11	27 09	32 21	22 22	29 06	35 49	42 33
54 23	1 07	29 00	34 29	27 17	34 04	40 51	47 38
55 18	2 05	31 15	36 14	32 20	38 26	46 01	52 51
56 14	3 04	33 24	38 55	36 38	44 25	51 18	58 12
57 12	4 05	35 52	41 13	42 50	51 18	56 44	11 3 42
58 11	5 08	38 45	44 10	48 18	55 18	11 2 19	9 20
59 11	6 12	40 30	46 01	53 54	11 0 58	8 03	15 07
2 0 13	7 18	43 00	48 30	59 40	68 20	13 57	21 04
1 17	8 25	45 31	51 04	11 5 43	12 36	19 58	27 10
2 23	9 35	48 05	53 41	11 37	18 53	26 10	33 26
3 30	10 46	50 43	56 23	17 51	25 12	32 32	39 53
4 39	11 59	53 27	59 09	24 15	31 40	39 05	46 30
5 50	13 14	56 14	5 2 00	30 50	38 19	45 48	53 18
7 00	14 31	59 06	4 55	37 36	45 09	52 44	12 0 17
8 18	15 50	5 2 03	7 55	44 28	52 11	59 50	7 28
9 35	17 12	5 05	11 01	51 42	59 24	12 7 07	14 51
10 52	18 36	8 12	14 11	59 03	12 6 51	14 29	22 27
12 16	20 02	11 24	17 27	12 6 36	14 30	22 22	30 16
13 39	21 13	14 42	20 49	14 23	22 21	30 20	38 19
15 05	23 02	18 05	24 16	22 24	30 27	38 31	48 33
16 34	24 36	21 34	27 49	30 39	38 48	46 57	55 07
18 05	26 13	25 09	31 29	39 09	47 33	55 38	13 03 54
19 39	27 52	28 51	35 18	47 53	56 03	13 4 26	12 56
21 16	29 35	32 40	39 08	56 55	13 5 21	13 46	22 16
22 56	31 20	36 35	43 08	13 6 10	14 45	23 18	31 52
24 38	33 09	40 38	47 15	15 47	24 27	33 07	41 47
26 24	35 01	44 48	51 30	25 41	34 26	43 12	52 00
28 14	36 57	49 06	55 53	35 52	44 45	53 39	14 2 33
30 06	38 56	53 32	6 0 24	46 24	55 25	14 4 24	13 20
32 02	40 59	58 06	5 05	57 16	14 6 24	15 33	24 41
34 03	43 06	6 2 50	9 54	14 8 31	17 46	27 02	36 18
36 07	45 41	7 44	14 52	20 09	29 34	33 54	48 18
38 14	47 33	12 45	19 59	32 07	41 39	51 12	15 0 44
40 27	49 53	17 58	25 20	44 33	54 13	15 3 54	13 34
42 44	52 15	23 21	30 49	57 24	15 7 13	17 03	26 52
45 05	54 48	28 56	36 30	15 10 43	20 44	30 39	40 38
47 31	57 23	34 42	42 24	24 30	34 38	44 46	54 48
50 03	3 0 04	40 41	48 29	38 50	49 05	59 22	16 9 41
52 40	2 50	46 52	54 48	53 37	16 4 05	16 14 33	25 02
55 23	5 42	53 18	7 1 21	16 6 43	19 37	27 57	40 57
58 12	8 14	59 57	8 10	29 04	38 10	46 38	55 04
3 1 08	11 47	7 6 52	15 11	43 34	55 00	17 3 37	17 14 39
4 09	15 00	14 03	22 30	58 49	17 10 03	21 17	32 31
7 18	18 20	21 30	30 10	17 16 47	28 13	39 39	51 06
10 34	21 48	30 18	38 02	35 28	47 40	58 47	18 13 04
13 59	25 24	37 19	46 15	54 57	18 6 50	18 18 43	30 38
17 32	29 10	45 43	54 51	18 15 16	27 23	39 31	51 40
21 13	33 04	54 29	8 3 45	36 27	48 50	19 49 13	19 13 37
25 04	37 09	8 3 36	13 13	58 36	19 11 14	23 53	36 56
29 05	41 24	12 08	22 46	19 21 45	34 40	47 36	20 0 33
33 17	45 51	23 05	32 54	45 59	59 04	20 12 25	25 40
37 40	50 29	33 28	43 31	20 11 17	20 24 53	38 28	51 56
42 15	55 20	44 22	54 37	37 59	51 51	21 5 41	21 19 34
47 03	4 0 25	55 46	9 6 14	21 5 56	21 20 08	34 28	48 33
52 05	5 45	9 7 42	18 25	35 20	49 53	22 4 27	22 19 02
57 08	11 20	20 15	31 13	22 6 15	22 21 11	36 09	51 07
4 2 54	17 12	33 25	44 39	38 05	54 11	23 9 34	23 24 57
8 43	23 02	47 17	58 48	23 13 16	23 29 02	44 50	24 0 40

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.

Star.	β Chamaeleontis.			β Hydri and ζ Ursa Minoris.			
P.D.	11° 30'	11° 35'	11° 40'	11° 45'	11° 50'	11° 55'	12° 00'
Lat.	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "
1	11 30 06	11 35 06	11 40 06	11 45 07	11 50 07	11 55 07	12 0 07
2	30 26	35 26	40 26	45 26	50 26	55 27	0 27
3	30 58	35 58	40 59	45 59	50 59	56 00	1 00
4	31 42	36 43	41 44	46 45	51 46	56 46	1 47
5	32 40	37 42	42 43	47 45	52 43	57 47	2 48
6	33 51	38 53	43 55	48 56	53 58	59 00	4 02
7	35 15	40 18	45 19	50 22	55 25	12 0 27	5 29
8	36 53	41 56	46 59	52 02	57 05	2 08	7 11
9	38 43	43 27	48 51	53 55	58 59	4 03	9 07
10	40 48	45 53	50 57	56 02	12 1 07	6 12	11 17
11	43 06	48 12	53 18	58 23	3 29	8 35	13 40
12	45 38	50 45	55 52	12 0 59	6 06	11 13	16 20
13	48 24	53 33	58 41	3 49	8 51	14 06	19 14
14	51 36	56 35	12 1 45	6 55	12 04	17 14	22 23
15	54 41	59 53	5 04	10 15	15 26	20 37	25 48
16	58 13	12 3 25	8 38	13 51	19 03	24 16	28 28
17	12 1 56	7 14	12 28	17 42	22 57	28 11	31 40
18	5 31	11 18	16 34	21 51	27 07	32 23	36 39
19	10 21	15 27	21 08	26 15	31 33	36 52	42 10
20	14 57	20 17	25 37	30 57	36 18	41 38	47 09
21	19 50	25 12	30 35	35 57	41 20	46 42	52 04
$\frac{1}{2}$	22 23	27 48	33 10	38 34	43 58	49 21	55 01
22	25 01	30 26	35 51	41 16	46 40	52 05	57 30
$\frac{1}{2}$	27 44	33 10	38 36	44 02	49 27	54 55	13 0 19
23	30 30	35 58	41 25	46 52	52 20	57 47	3 14
$\frac{1}{2}$	33 23	38 51	44 20	49 48	55 17	13 0 45	6 14
24	36 20	41 49	47 19	52 49	58 19	3 49	9 18
$\frac{1}{2}$	39 21	44 52	50 23	55 55	13 1 26	6 57	12 28
25	42 29	48 00	53 33	57 16	4 27	10 11	15 43
$\frac{1}{2}$	45 40	51 14	56 48	13 8 32	7 54	13 30	19 04
26	48 57	54 32	13 0 08	5 21	11 19	16 54	22 30
$\frac{1}{2}$	52 19	57 56	3 33	9 10	14 47	20 24	26 02
27	55 47	13 1 26	7 04	12 43	18 22	24 00	29 39
$\frac{1}{2}$	59 19	5 01	10 41	16 21	21 54	27 41	33 22
28	13 2 37	8 32	14 23	20 05	25 47	31 29	37 11
$\frac{1}{2}$	6 44	12 28	18 11	23 55	29 38	35 22	41 06
29	10 12	16 12	22 05	27 51	33 24	39 21	45 07
$\frac{1}{2}$	14 31	20 18	26 05	32 06	37 40	43 27	49 14
30	18 34	24 23	30 12	36 01	41 50	47 39	53 28
$\frac{1}{2}$	22 43	28 34	34 25	40 15	46 06	51 57	57 48
31	26 58	32 57	38 44	44 37	50 29	56 22	14 2 15
$\frac{1}{2}$	31 21	37 25	43 10	49 05	54 48	14 9 54	6 49
32	35 45	41 46	47 42	53 39	59 36	5 33	11 30
$\frac{1}{2}$	40 13	46 23	52 22	58 21	14 4 20	10 19	16 18
33	45 07	50 56	57 09	14 3 11	9 11	15 12	21 13
$\frac{1}{2}$	49 57	56 00	14 2 03	8 06	14 09	20 13	26 16
34	54 54	14 0 59	6 29	13 14	19 15	25 21	31 37
$\frac{1}{2}$	59 58	6 06	12 14	18 20	24 29	30 37	36 45
35	14 5 11	11 21	17 31	23 41	29 29	36 02	42 17
$\frac{1}{2}$	10 32	16 44	22 56	29 09	35 21	41 34	47 47
36	15 59	22 13	28 29	34 44	41 00	47 15	53 30
$\frac{1}{2}$	21 36	28 06	34 01	40 29	46 34	52 04	59 22
37	27 21	33 42	40 02	46 22	52 30	59 03	15 5 24
$\frac{1}{2}$	33 15	39 39	46 01	52 24	58 41	15 5 11	11 34
38	39 19	45 43	52 10	58 22	15 5 02	11 28	17 54
$\frac{1}{2}$	47 36	52 00	58 28	15 4 45	11 26	17 55	24 24
39	53 59	58 25	15 4 56	11 28	18 00	24 31	31 03
$\frac{1}{2}$	58 25	15 5 00	11 34	18 09	24 43	31 18	37 53
40	15 5 07	11 45	18 22	25 00	31 24	38 16	44 54
$\frac{1}{2}$	11 59	18 40	25 32	32 02	38 43	45 24	52 05

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.		β Chamaeleontis.			β Hydri and ζ Ursa Minoris.					
		11° 30'	11° 35'	11° 40'	11° 45'	11° 50'	11° 55'	12° 00'		
P.D.	Lat.	° / ' //	° / ' //	° / ' //	° / ' //	° / ' //	° / ' //	° / ' //	° / ' //	° / ' //
41	$\frac{1}{2}$	15 19 03	15 26 00	15 32 30	15 39 50	15 45 59	15 52 43	15 59 27		
42	$\frac{1}{2}$	26 17	33 17	39 57	46 39	53 26	16 0 14	16 7 02		
43	$\frac{1}{2}$	33 42	40 33	45 10	52 00	16 1 05	7 57	14 48		
44	$\frac{1}{2}$	41 20	48 14	55 08	16 2 03	8 57	15 50	22 46		
45	$\frac{1}{2}$	49 10	56 07	16 3 05	10 02	17 01	23 59	30 58		
46	$\frac{1}{2}$	57 11	16 4 12	11 14	18 16	25 18	32 20	39 22		
47	$\frac{1}{2}$	16 5 06	12 13	19 57	26 42	33 49	40 54	48 00		
48	$\frac{1}{2}$	13 54	21 03	28 12	35 22	42 32	49 42	56 02		
49	$\frac{1}{2}$	22 36	29 49	37 03	44 16	51 16	58 44	17 5 58		
50	$\frac{1}{2}$	31 32	38 49	46 07	53 56	17 0 43	17 8 01	15 19		
51	$\frac{1}{2}$	40 13	48 05	55 27	17 2 49	10 11	17 34	24 56		
52	$\frac{1}{2}$	50 09	17 57 00	17 5 02	12 28	19 57	27 22	34 49		
53	$\frac{1}{2}$	59 51	7 22	14 53	22 24	29 55	37 27	44 58		
54	$\frac{1}{2}$	17 9 49	17 25	25 00	32 37	40 12	47 49	55 25		
55	$\frac{1}{2}$	20 04	27 44	35 25	43 06	50 47	58 28	18 5 34		
56	$\frac{1}{2}$	30 37	38 23	46 08	53 54	18 1 40	18 9 26	17 12		
57	$\frac{1}{2}$	41 25	49 19	57 09	18 5 01	12 52	20 43	28 18		
58	$\frac{1}{2}$	52 38	18 0 35	18 8 30	16 27	24 23	32 20	40 16		
59	$\frac{1}{2}$	18 4 08	12 10	20 11	28 12	36 14	44 16	52 19		
60	$\frac{1}{2}$	15 58	24 05	32 12	40 20	48 27	56 35	19 4 43		
61	$\frac{1}{2}$	28 10	36 23	44 36	53 09	19 1 02	19 9 16	17 30		
62	$\frac{1}{2}$	40 44	49 02	57 21	19 5 41	14 00	22 20	30 22		
63	$\frac{1}{2}$	53 40	19 2 05	19 10 30	18 56	27 21	35 47	44 13		
64	$\frac{1}{2}$	19 7 01	15 19	24 05	32 36	41 08	49 40	58 13		
65	$\frac{1}{2}$	20 47	29 25	38 03	46 42	55 20	20 4 00	20 12 39		
66	$\frac{1}{2}$	34 59	43 40	52 28	20 1 14	20 10 00	19 00	27 32		
67	$\frac{1}{2}$	49 38	58 35	20 7 02	16 15	25 07	34 01	42 54		
68	$\frac{1}{2}$	20 5 52	20 13 44	22 45	31 44	40 47	49 46	58 46		
69	$\frac{1}{2}$	20 23	29 30	38 29	47 45	56 53	21 6 00	21 15 10		
70	$\frac{1}{2}$	36 31	45 47	55 02	21 3 21	21 13 39	22 52	32 07		
71	$\frac{1}{2}$	53 14	21 2 37	21 12 00	21 24	30 48	40 13	49 38		
72	$\frac{1}{2}$	21 10 29	20 01	29 33	39 05	48 38	58 11	22 7 45		
73	$\frac{1}{2}$	31 28	41 10	50 53	22 0 35	22 10 18	22 16 47	26 30		
74	$\frac{1}{2}$	46 50	56 40	22 6 30	16 20	26 24	36 13	45 55		
75	$\frac{1}{2}$	22 6 00	22 15 59	25 58	35 58	46 00	56 00	23 6 02		
76	$\frac{1}{2}$	25 51	36 00	46 10	56 19	23 6 30	23 16 41	26 53		
77	$\frac{1}{2}$	46 26	56 47	23 7 05	23 17 26	27 47	38 09	48 31		
78	$\frac{1}{2}$	23 7 47	23 18 17	28 48	39 19	49 50	24 0 24	24 10 58		
79	$\frac{1}{2}$	29 27	40 39	51 20	24 2 03	24 13 21	23 31	34 15		
80	$\frac{1}{2}$	52 59	24 3 52	24 14 46	25 41	36 37	47 33	58 27		
81	$\frac{1}{2}$	24 16 56	28 02	39 07	50 15	25 1 42	25 12 32	25 23 41		
82	$\frac{1}{2}$	41 29	53 09	25 4 28	25 15 49	27 09	38 32	49 54		
83	$\frac{1}{2}$	25 7 46	25 19 18	30 52	42 25	54 00	26 5 36	26 17 12		
84	$\frac{1}{2}$	34 47	46 34	58 21	26 10 17	26 21 58	33 48	45 40		
85	$\frac{1}{2}$	26 2 58	26 15 00	26 27 02	39 05	51 09	27 3 15	27 15 20		
86	$\frac{1}{2}$	32 22	44 40	56 55	27 9 17	27 21 37	34 00	46 21		
87	$\frac{1}{2}$	27 3 06	27 15 39	27 28 14	40 51	53 27	28 6 06	28 18 46		
88	$\frac{1}{2}$	35 04	48 05	28 0 57	28 13 51	28 26 46	39 42	43 57		
89	$\frac{1}{2}$	28 8 52	28 22 02	35 12	48 24	29 1 39	29 14 54	29 28 08		
90	$\frac{1}{2}$	43 27	57 36	29 11 06	29 24 39	38 12	51 48	30 5 22		
91	$\frac{1}{2}$	29 21 05	29 34 56	48 47	30 2 40	30 16 36	30 30 33	44 31		
92	$\frac{1}{2}$	59 56	30 14 08	30 28 22	42 38	56 56	31 11 15	31 25 37		
93	$\frac{1}{2}$	30 40 48	55 25	31 10 02	31 24 41	31 39 23	32 39 18	32 8 53		
94	$\frac{1}{2}$	31 23 42	31 38 53	53 56	32 9 01	32 24 08	33 27 02	33 42 42		
95	$\frac{1}{2}$	32 9 17	32 24 45	32 40 16	55 49	33 11 24	34 17 30	34 33 88		
96	$\frac{1}{2}$	57 17	33 13 15	33 29 14	33 45 17	34 1 22	35 11 00	35 27 43		
97	$\frac{1}{2}$	33 48 07	34 4 36	34 21 07	34 37 45	54 19	36 7 48	36 25 08		
98	$\frac{1}{2}$	34 42 02	59 16	35 16 10	35 33 19	35 50 32	37 8 16	37 26 14		
99	$\frac{1}{2}$	35 39 21	35 57 00	36 15 48	36 33 30	36 50 32				

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.	γ Cephi.						
	12° 05'	12° 40'	12° 45'	12° 50'	22° 55'	13° 00'	13° 05'
P. D.	° / ' / ''	° / ' / ''	° / ' / ''	° / ' / ''	° / ' / ''	° / ' / ''	° / ' / ''
1	12 5 07	12 40 07	12 45 07	12 50 07	12 55 07	13 0 07	13 5 7
2	5 27	40 28	45 29	50 29	55 29	0 29	5 29
3	6 01	41 36	46 04	51 05	56 05	1 05	6 06
4	6 48	41 53	46 57	51 55	56 56	1 56	6 57
5	7 49	42 57	47 58	53 00	58 01	3 02	8 03
6	9 03	44 16	49 17	54 19	59 21	4 21	9 24
7	10 32	45 48	50 51	55 53	13 0 55	5 58	11 00
8	12 14	47 36	52 39	57 42	2 45	7 48	12 51
9	14 11	49 38	54 42	59 46	4 50	9 54	14 58
10	16 21	51 55	57 00	13 2 05	7 10	12 15	17 20
11	18 48	54 28	59 34	4 40	9 46	14 52	19 58
12	21 27	57 16	13 2 23	7 31	12 28	17 52	22 52
13	24 24	13 0 15	5 29	10 37	15 45	20 33	26 02
14	27 33	3 40	8 50	14 07	19 09	24 19	29 29
15	30 59	7 17	12 10	17 39	22 49	28 02	33 13
16	34 41	11 10	16 23	21 35	26 48	32 01	37 14
17	38 40	15 21	20 35	25 50	31 04	36 19	41 03
18	42 55	19 49	25 05	30 22	35 38	40 54	46 10
19	47 28	24 35	29 54	35 12	40 30	45 48	51 07
20	52 18	29 40	35 01	40 21	45 41	51 02	56 22
21	57 27	35 04	40 25	45 49	51 12	56 35	14 1 57
22	13 0 08	37 54	43 17	48 41	54 06	59 29	4 53
23	2 54	40 48	46 13	51 38	57 03	14 2 28	7 53
24	5 45	43 48	49 14	54 40	14 0 06	5 33	10 59
25	8 41	46 53	52 20	57 48	3 15	8 42	14 10
26	11 51	50 02	55 31	14 1 00	6 29	11 58	17 20
27	14 48	53 18	58 48	4 18	9 49	15 18	20 46
28	18 00	56 39	14 2 10	7 42	13 13	18 45	24 19
29	21 15	14 0 06	5 38	11 11	16 44	22 17	27 56
30	24 38	3 37	9 12	14 46	20 21	25 55	31 30
31	28 06	7 15	12 51	18 27	24 03	29 39	35 15
32	31 50	10 59	16 37	22 14	27 52	33 29	39 07
33	35 07	14 49	20 28	26 07	31 46	37 00	43 04
34	39 02	18 45	24 25	30 07	35 47	41 58	47 03
35	42 53	22 48	28 30	34 12	39 54	45 36	51 19
36	46 49	26 56	32 40	38 24	44 08	49 52	55 36
37	50 52	31 11	36 57	42 43	48 28	54 04	15 0 00
38	55 01	35 32	41 21	47 08	52 43	58 43	4 31
39	59 17	40 01	45 51	51 40	57 36	15 3 19	9 09
40	14 3 39	44 37	50 28	56 15	15 2 11	8 02	13 54
41	8 08	49 19	55 13	15 1 15	6 59	12 53	18 46
42	12 44	54 09	15 0 05	6 00	11 55	17 51	23 46
43	17 27	59 06	5 04	11 01	16 29	22 56	28 53
44	22 17	15 4 12	10 11	16 10	22 10	28 10	34 09
45	27 14	9 24	15 26	21 27	27 29	33 31	39 33
46	32 20	14 44	20 49	26 53	32 57	39 01	45 05
47	37 32	20 14	26 20	32 26	38 32	44 39	50 45
48	42 53	25 51	32 00	38 07	44 00	50 45	56 34
49	48 22	31 37	37 47	43 58	50 10	56 34	2 32
50	54 00	37 31	43 45	49 58	56 12	16 2 32	8 39
51	59 46	43 35	49 51	56 07	16 2 23	8 39	14 55
52	15 5 40	49 48	56 06	16 2 25	8 44	14 55	21 21
53	11 44	56 10	16 2 40	8 53	15 14	21 21	27 57
54	17 47	16 2 48	9 06	15 31	21 54	27 57	34 43
55	24 21	9 25	15 52	22 18	28 46	34 43	41 39
56	30 52	16 18	22 47	29 17	35 47	41 39	48 47
57	37 35	23 21	29 53	36 26	42 59	48 47	56 05
58	44 14	30 35	37 11	43 47	50 23	56 05	3 34
59	51 32	38 00	44 39	51 18	57 57	17 3 34	11 16
60	58 46	45 35	52 20	59 02	17 5 24	11 16	19 09

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.		γ Cephi.							
P.D.		12° 05'	12° 40'	12° 45'	12° 50'	12° 55'	13° 00'	13° 05'	
Lat.		° / '	° / '	° / '	° / '	° / '	° / '	° / '	
41	16 6 12	16 53 27	17 0 12	17 6 57	17 13 43	17 20 29	17 27 14		
1/2	13 50	17 1 28	8 17	15 06	22 23	28 44	35 33		
42	21 39	9 42	16 34	23 27	30 19	37 12	44 05		
1/2	29 41	18 09	25 05	32 01	38 57	45 53	52 50		
43	37 56	26 50	33 49	40 49	47 49	54 49	18 1 49		
1/2	46 24	35 44	42 48	49 51	56 55	18 3 59	11 03		
44	55 06	44 53	52 01	59 14	18 6 16	13 23	20 31		
1/2	17 4 02	54 17	18 1 29	18 8 40	15 52	23 03	30 15		
45	13 13	18 3 56	11 12	18 28	25 43	33 00	40 16		
1/2	22 38	13 52	21 11	28 31	35 51	43 11	50 32		
46	32 19	24 31	31 28	38 52	46 16	53 41	19 1 06		
1/2	42 16	34 33	42 00	49 13	56 59	19 4 28	11 57		
47	52 50	45 19	52 52	19 0 25	19 7 59	15 33	23 07		
1/2	18 3 01	56 23	19 4 01	11 39	19 18	26 55	34 36		
48	13 51	19 7 47	15 30	23 13	30 57	38 40	46 25		
1/2	24 59	19 30	27 18	34 50	42 55	50 44	58 33		
49	36 26	31 34	39 27	47 21	55 15	20 3 09	20 11 04		
1/2	48 13	43 59	51 58	59 57	20 7 57	15 57	23 56		
50	19 0 38	56 46	20 4 57	20 12 55	21 01	29 03	37 11		
1/2	12 21	20 9 56	18 07	26 17	34 28	42 39	50 51		
51	25 44	23 30	31 47	40 03	48 20	56 37	21 4 55		
1/2	39 00	37 29	45 52	54 14	21 2 38	21 11 01	19 25		
52	52 40	51 54	21 0 22	21 8 52	17 29	25 52	34 22		
1/2	20 6 47	21 6 46	15 22	23 58	32 24	41 10	49 47		
53	21 00	22 06	30 48	39 31	48 14	56 58	22 5 42		
1/2	36 19	37 55	46 45	55 35	22 4 25	22 13 01	22 07		
54	51 48	54 16	22 3 13	22 12 10	21 08	30 06	39 04		
1/2	21 7 48	22 11 08	20 13	29 18	38 23	47 29	56 35		
55	24 18	28 35	37 47	47 00	56 13	23 5 27	23 14 41		
1/2	41 24	46 36	55 57	23 5 18	23 14 29	24 02	33 24		
56	59 03	23 5 14	23 14 43	24 13	33 17	43 04	51 45		
1/2	22 17 19	24 32	34 10	43 48	53 28	24 3 07	24 12 48		
57	36 13	44 30	54 17	24 4 05	24 13 54	23 43	33 32		
1/2	55 48	24 5 11	24 15 08	25 06	35 03	45 02	55 02		
58	23 16 04	26 37	32 05	46 51	57 00	25 7 08	25 17 18		
1/2	37 06	48 50	59 07	25 9 26	25 19 45	30 04	40 24		
59	58 54	25 11 54	25 22 22	31 43	43 21	53 52	26 4 23		
1/2	24 21 32	35 50	46 30	57 10	26 7 52	26 18 34	29 17		
60	45 02	26 0 43	26 11 34	26 22 27	33 20	44 15	55 10		
1/2	25 9 27	26 34	37 38	48 44	59 50	27 10 57	27 22 05		
61	34 52	53 30	27 4 46	27 16 14	27 27 24	38 45	50 05		
1/2	26 1 18	27 21 29	33 01	44 32	56 06	28 7 40	28 19 15		
62	28 50	50 41	28 2 26	28 14 12	28 26 01	37 50	49 46		
1/2	57 32	28 21 07	33 08	45 10	57 13	29 9 17	29 21 23		
63	27 27 29	52 54	29 5 11	29 17 29	29 29 49	42 13	54 31		
1/2	54 33	29 26 07	38 41	51 16	30 3 52	30 16 30	30 24 30		
64	28 27 09	30 0 51	30 13 43	30 26 36	39 30	51 26	31 5 24		
1/2	29 4 39	37 14	50 24	31 3 36	31 16 56	31 30 57	43 23		
65	41 29	31 15 20	31 28 51	42 24	55 59	32 9 34	32 13 13		
1/2	30 18 41	55 22	32 9 14	32 23 08	32 37 59	51 13	33 5 03		
66	58 31	32 37 25	53 28	33 5 58	33 20 17	33 34 39	49 33		
1/2	31 40 00	33 21 41	33 36 20	51 03	34 5 47	34 20 34	34 35 23		
67	32 23 40	34 8 20	34 23 26	34 38 43	53 46	35 9 00	35 24 16		
1/2	33 9 46	57 36	35 13 10	35 28 47	35 44 28	36 0 10	36 15 56		
68	58 25	35 49 42	36 5 47	36 21 55	36 38 07	54 21	37 10 38		
1/2	34 49 55	36 44 55	37 1 33	37 18 14	37 35 00	37 51 48	38 8 40		
69	35 44 30	37 43 33	38 0 50	38 18 05	38 35 27	38 52 53	39 10 23		
1/2	36 42 31	38 45 57	39 3 51	39 29 49	39 39 52	39 57 59	40 16 10		
70	37 44 17	39 59 11	40 11 11	40 29 53	40 48 40	41 7 23	41 24 29		

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

		β (Kochab) Ursa Minoris.						
Star.		1875	1895	1915	1935	1955	1975	1995
P.D.		15° 20'	15° 25'	15° 30'	15° 35'	15° 40'	15° 45'	15° 50'
Lat.		° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
1		15 20 09	15 25 19	15 30 09	15 35 10	15 40 09	15 45 23	15 50 09
2		20 35	25 35	30 35	35 36	40 36	45 36	50 36
3		21 18	26 18	31 19	36 20	41 20	46 20	51 19
4		22 18	27 19	32 20	37 20	42 21	47 22	52 23
5		23 36	28 38	33 29	38 40	43 41	48 43	53 44
6		25 12	30 14	35 15	40 17	45 19	50 21	55 23
7		27 05	32 07	37 10	42 12	47 15	52 17	57 19
8		29 16	34 19	39 22	44 26	49 29	54 32	59 35
9		31 45	36 49	41 53	46 57	52 01	57 06	16 2 10
10		34 33	39 38	44 43	49 48	54 53	59 58	5 03
11		37 39	42 45	47 52	52 57	58 04	16 3 10	8 16
12		41 05	46 12	51 19	56 26	16 1 34	6 41	11 48
13		44 49	49 58	55 06	16 0 15	5 23	10 32	15 40
14		48 54	54 04	59 13	4 23	9 33	14 43	19 53
15		53 18	58 29	16 3 41	8 52	14 04	19 15	24 27
16		58 03	16 3 16	8 29	13 42	18 55	24 08	29 21
17	16	3 09	8 24	13 38	18 53	24 07	29 23	34 38
18		8 37	13 53	19 10	24 27	29 43	35 00	40 17
19		14 26	19 45	25 13	30 22	35 41	41 00	46 19
20		20 39	26 00	31 21	36 41	42 17	47 23	52 44
21		27 15	32 38	38 01	43 24	48 47	54 11	59 34
$\frac{1}{2}$		30 42	36 06	41 31	46 55	52 20	57 47	17 3 08
22		34 15	39 41	45 06	50 36	55 58	17 1 23	6 49
$\frac{1}{2}$		37 55	43 22	48 47	54 15	59 48	5 09	10 36
23		41 41	47 09	52 37	58 08	17 3 34	9 02	14 03
$\frac{1}{4}$		45 33	51 03	56 32	17 2 02	7 31	13 01	18 30
24		49 32	55 03	17 0 34	6 05	11 36	17 07	22 38
$\frac{1}{2}$		53 38	59 10	4 43	10 15	15 47	21 20	26 23
25		57 50	17 3 24	8 58	14 32	20 06	25 40	31 14
$\frac{1}{2}$	17	2 10	7 29	13 21	18 57	24 32	30 07	35 42
26		6 37	12 14	17 50	23 27	29 04	34 41	40 17
$\frac{1}{2}$		11 11	16 49	22 27	28 06	33 44	39 23	45 02
27		15 52	21 32	27 12	32 52	38 32	44 12	49 53
$\frac{1}{2}$		20 41	26 22	32 04	37 46	43 28	49 10	54 52
28		25 37	31 21	37 04	42 48	48 31	54 15	59 58
$\frac{1}{2}$		30 42	36 27	42 12	47 57	53 43	59 28	18 5 13
29		35 54	41 41	47 28	53 15	59 03	18 4 50	10 37
$\frac{1}{2}$		41 15	47 04	52 52	58 41	18 4 31	10 20	16 09
30		46 44	52 34	58 25	18 4 16	10 07	15 58	21 49
$\frac{1}{2}$		52 21	58 14	18 4 07	10 00	15 52	21 46	27 39
31		58 07	18 4 02	9 57	15 54	21 47	27 42	33 37
$\frac{1}{2}$	18	4 03	10 00	16 02	22 12	27 51	33 09	39 46
32		10 07	16 07	22 05	28 05	34 04	40 03	46 03
$\frac{1}{2}$		16 21	22 22	28 24	34 25	40 27	46 28	52 30
33		22 45	28 48	34 52	40 55	47 00	53 03	59 07
$\frac{1}{2}$		29 18	35 24	41 30	47 36	53 42	59 48	19 5 55
34		36 01	42 10	48 18	54 26	19 0 35	19 6 59	12 52
$\frac{1}{2}$		42 55	49 06	55 17	19 1 29	7 39	13 50	20 00
35		49 18	56 13	19 2 26	8 41	14 53	21 07	27 20
$\frac{1}{2}$		57 15	19 3 31	9 47	16 03	22 19	28 30	34 51
36	19	4 42	11 00	17 19	23 37	29 56	36 15	42 34
$\frac{1}{2}$		12 20	18 41	25 02	31 24	37 45	44 07	50 28
37		20 10	26 34	32 58	39 22	45 46	52 11	58 35
$\frac{1}{2}$		28 12	34 39	41 06	47 33	54 00	59 52	20 6 34
38		36 26	42 56	49 20	55 56	20 2 26	20 8 57	15 27
$\frac{1}{2}$		44 54	51 27	• 58 00	20 4 34	11 06	17 41	24 12
39		53 35	20 0 11	20 6 46	13 17	19 59	26 36	33 12
$\frac{1}{2}$	20	2 29	9 08	15 47	22 27	29 06	35 46	42 26
40		11 37	18 20	25 03	31 45	38 28	45 11	51 24

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.	(Kochab) β Ursa Minoris.																				
Year. P.D.	1875			1895			1915			1935			1955			1975			1995		
	15° 20'			15° 25'			15° 30'			15° 35''			15° 40'			15° 45'			5° 5'		
Lat.	°	'	''	°	'	''	°	'	''	°	'	''	°	'	''	°	'	''	°	'	''
41'	20	30	38	20	37	27	20	46	17	20	51	6	20	54	54	21	4	46	21	14	36
$\frac{1}{2}$	20	41		47	24		54	47		21	1	10	21	8	4	14	57		21	51	
42	50	4		57	36		21	4	33	11	30		18	29		25	25		32	22	
$\frac{1}{2}$	21	1	5	21	8	6	15	6		22	8		29	8		36	9		43	10	
43	11	48		18	52		25	57		33	1		40	1		47	11		54	10	
$\frac{1}{2}$	22	49		29	56		37	4		44	15		51	22		58	31		22	24	
44	34	6		41	18		48	31		55	44		22	2	57	22	10		19	24	
$\frac{1}{2}$	45	42		54	59		22	16		22	7	38	14	50		22	8		29	26	
45	57	38		22	5	6	21	21		19	42		27	4		34	26		41	48	
$\frac{1}{2}$	22	9	54	17	19		24	45		31	14		39	38		47	5		54	32	
46	22	30		30	2		37	31		45	3		52	34		23	0	5	23	7	37
$\frac{1}{2}$	35	29		43	4		50	39		58	15		23	5	2	10	4		21	4	
47	48	49		56	3		23	4	10	23	11	51	19	32		22	14		34	55	
$\frac{1}{2}$	23	2	23	23	10	18	18	4		25	50		30	37		41	23		48	10	
48	16	40		24	31		32	22		40	14		48	5		55	57		24	3	50
$\frac{1}{2}$	31	13		39	10		47	6		55	3		24	3	00	24	10	58	18	58	
49	46	12		54	14		24	2	16	24	10	19	18	22		26	23		34	29	
$\frac{1}{2}$	24	1	38	24	9	46	17	54		26	2		34	11		42	20		50	30	
50	17	31		25	45		34			42	14		50	29		58	45		25	7	0
$\frac{1}{2}$	33	54		42	14		50	35		58	56		25	7	17	25	15	39	24	1	
51	50	48		59	15		25	7	42	25	15	9	24	37		33	6		25	41	34
$\frac{1}{2}$	25	8	12	25	16	47	25	21		33	56		42	30		42	30		59	40	
52	26	3		34	53		43	34		51	15		26	0	46	26	9	40	25	18	22
$\frac{1}{2}$	44	46		53	33		26	2	22	26	11	10	20	00		28	51		37	41	
53	26	3	55	26	12	50	21	47		30	43		39	40		48	43		57	35	
$\frac{1}{2}$	23	44		32	46		41	50		50	54		27	0	2	27	9	4	27	18	10
54	44	10		53	22		27	2	34	27	11	45	21	0		30	14		39	36	
$\frac{1}{2}$	27	5	21	27	14	40	24			33	16		42	43		52	4	28	1	37	
55	27	13		36	41		46	16		55	40		28	5	10	14	42		24	13	
$\frac{1}{2}$	49	52		59	20		28	9	8	28	18	50	28	27		38	7		47	48	
56	28	13	18	28	23	6	32	52		42	43		52	48		29	2	23	29	12	13
$\frac{1}{2}$	87	36		47	39		57	28		29	7	31	29	17	31	27	32		37	32	
57	29	2	48	29	12	56	29	23	5	33	4		43	14		53	36		30	3	48
$\frac{1}{2}$	28	56		39	13		49	35		59	55		30	10	23	30	20	51	31	2	
58	56	4	30	6	34		30	17	57	30	27	38	38	11		48	40		59	48	
$\frac{1}{2}$	30	24	15	34	58		45	41		56	25	31	7	10		31	17	57	31	28	44
59	58	17		31	4	28	31	15	21	31	26	21	37	21		48	22		59	18	
$\frac{1}{2}$	31	24	1	35	10		46	19		57	30		32	8	43	32	15	8	32	31	4
60	55	45		32	7	7	32	18	31	32	29	55	41	23		52	49	33	4	13	
$\frac{1}{2}$	32	28	43	40	25		52	4		33	3	43	33	15	23	33	27	9	38	48	
61	33	3	17	33	15	10	33	24	4	39			50	55		34	2	54	34	14	53
$\frac{1}{2}$	39	17		51	2	34	3	37		34	15	48	34	28	2	40	17		52	33	
62	34	16	53	34	29	20	41	49		54	19		35	6	50	35	19	22	35	31	56
$\frac{1}{2}$	56	14	35	9	35	21	46	35	34	35			47	25	36	0	17		36	13	28
63	35	37	27	50	38		36	3	39	36	16	47	36	30	2	43	10		56	26	
$\frac{1}{2}$	36	20	40	36	34	6	47	34		37	1	4	37	14	37	37	28	11	37	41	48
64	37	6	4	37	19	52	37	33	42	47	38	38	1	30	38	15	29	38	29	29	
$\frac{1}{2}$	53	47	38	8	38	22	15	38	36	32	38	50	52	39	5	15	39	19	40		
65	38	44	2	38	58	40	39	13	23	39	28	12	39	42	54	39	57	44	40	12	48
$\frac{1}{2}$	39	43	38	40	7	21	40	7	21	40	22	33	40	37	51	40	53	10	41	8	33
66	40	34	29	40	48	45	41	4	26	41	20	10	41	36	0	41	51	50	42	7	46
$\frac{1}{2}$	41	32	29	41	48	40	42	4	55	42	21	14	42	37	36	42	54	2	43	10	32
67	42	35	30	42	52	18	43	9	9	43	26	4	43	43	8	44	0	11	44	17	21
$\frac{1}{2}$	43	42	34	44	0	3	44	17	35	44	35	13	44	53	5	45	10	43	45	28	36
68	44	54	8	45	12	19	45	38	43	45	49	3	46	7	33	46	33	40	46	44	50
$\frac{1}{2}$	46	10	46	46	29	34	46	40	34	47	8	13	47	27	37	47	47	7	48	6	42
69	47	33	5	47	53	5	48	13	13	48	23	28	48	53	50	49	14	20	49	35	7
$\frac{1}{2}$	49	1	57	49	23		49	14	14	50	5	34	50	27	7	50	48	47	50	13	35
70	50	38	17	51	23	35	51	23	5	51	45	43	52	8	41	52	31	37	52	54	50

TABLE XXIV.

Showing the Azimuths of Polaris when on the same vertical plane with γ (Gamma) in Cassiopea at its under transit.

All the Azimuths or bearings are North-west.

The column headings are the years or dates.

	1870	1880	1890	1900	1910	1920	1930	1940
	' "	' "	' "	' "	' "	' "	' "	' "
2	8 6	8 46	9 24	10 02	10 38	11 12	11 41	12 10
4	7	47	26	04	40	14	43	12
6	9	49	28	06	42	16	45	15
8	12	52	41	09	45	19	49	18
10	15	55	34	13	49	23	53	23
12	19	59	38	18	54	28	58	28
14	23	9 04	44	23	11 00	35	12 05	35
16	28	10	50	30	07	42	12	42
18	34	16	57	37	14	50	20	51
20	8 41	23	10 04	45	23	59	30	13 01
22	48	32	13	54	33	12 9	41	12
24	57	41	23	11 05	44	21	53	25
26	9 06	51	34	16	56	33	13 05	38
28	17	10 02	46	29	12 09	48	21	54
30	28	15	59	43	25	13 04	37	14 14
32	41	28	11 14	59	41	21	55	29
34	55	44	30	12 16	59	40	14 15	51
36	10 11	11 11	48	35	13 19	14 01	37	13
38	28	18	12 8	56	42	24	15 02	15 38
40	47	39	30	13 19	14 06	50	28	16 06
41	57	49	41	22	20	15 04	43	21
42	11 7	12 01	54	45	33	18	58	37
43	19	14	13 07	59	48	34	16 14	54
44	32	27	20	14 14	15 03	50	31	17 12
45	43	40	35	29	20	16 08	49	30
46	56	54	50	45	37	26	17 08	49
47	12 11	13 09	14 06	15 03	55	44	28	18 10
48	25	25	23	21	16 14	17 05	48	32
49	41	42	41	40	35	26	18 10	55
50	57	14 0	15 00	16 00	56	48	34	19 19
51	13 15	19	21	22	17 19	18 12	59	45
52	33	39	42	44	43	37	19 25	20 12
53	53	15 0	16 05	17 09	18 08	19 04	53	41
54	14 14	23	29	34	36	33	20 22	21 12
55	36	47	54	18 01	19 05	20 03	54	45
56	15 0	16 12	17 22	31	35	35	21 28	22 19
57	25	40	51	19 02	20 08	21 10	22 05	57
58	53	17 09	18 22	35	43	46	42	23 36
59	16 22	40	56	20 11	21 21	22 26	23 23	24 19
60	53	18 14	19 32	49	22 01	23 08	24 07	25 05
61	17 26	50	20 11	21 20	45	23 54	54	25 54
62	18 03	19 29	53	22 14	23 32	24 43	25 45	26 47
63	42	20 11	21 38	23 03	24 22	25 36	26 41	27 44
64	19 24	58	22 27	55	25 17	26 34	27 40	28 46
65	20 11	21 47	23 20	24 52	26 17	27 36	28 45	29 54
66	21 02	22 42	24 18	25 53	27 22	28 45	29 56	31 07
67	57	23 41	25 21	27 01	28 33	29 56	31 14	32 28
68	22 57	24 47	26 32	28 15	29 52	31 22	32 39	33 50
69	24 05	25 59	27 49	29 47	31 18	32 52	34 13	35 33
70	25 20	27 19	29 15	31 09	32 54	34 23	35 32	37 22

TABLE XXV.

*Showing the Azimuths of Polaris, when vertical with Alioth in Ursa
Majoris at its under transit.*

All the Azimuths or bearings are North-east.

(The top column is years, beginning Jan. 1.)

North Lat.	1870	1880	1890	1900	1910	1920	1930	1940
2	8 01	8 41	9 30	10 03	10 30	11 15	11 47	12 18
4	02	43	32	04	35	17	49	19
6	03	45	34	05	40	19	52	20
8	05	47	37	07	45	22	55	24
10	07	50	39	11	50	26	58	28
12	11	54	42	15	54	31	12 03	33
14	15	58	48	20	59	36	09	40
16	19	9 04	53	26	11 06	43	16	46
18	25	10	58	34	13	49	23	54
20	31	16	10 06	41	21	57	32	13 03
22	38	23	14	49	30	12 07	42	14
24	45	31	23	59	40	19	54	26
26	53	40	34	11 09	52	31	13 06	39
28	9 02	50	45	20	12 05	45	21	54
30	13	10 02	59	33	18	13 00	36	14 11
32	24	16	11 12	50	34	13 16	54	28
34	38	30	28	12 06	51	35	14 12	49
36	53	45	44	24	13 11	55	34	15 10
38	10 08	11 03	12 01	44	32	14 18	57	34
40	25	22	23	13 06	55	41	15 22	16 01
41	34	32	34	18	14 07	56	36	16
42	44	42	46	30	21	15 10	51	31
43	53	53	58	43	35	23	16 07	55
44	11 04	12 06	13 11	56	50	39	22	17 04
45	15	19	24	14 11	15 05	56	40	21
46	29	33	38	26	21	16 11	57	40
47	42	51	52	42	37	31	17 16	18 00
48	55	13 08	14 07	59	55	50	36	20
49	12 09	16	25	15 17	16 15	17 12	58	42
50	24	32	43	35	35	38	18 20	19 05
51	40	49	15 04	56	57	18 01	43	30
52	56	14 11	25	16 16	17 19	26	19 08	56
53	13 14	29	48	40	43	50	34	20 24
54	34	48	16 11	17 04	18 08	19 10	20 02	53
55	52	15 10	34	28	35	40	33	21 23
56	14 14	33	58	56	19 04	20 11	21 04	54
57	38	58	17 26	18 25	37	45	38	22 28
58	15 01	16 25	54	56	20 07	21 17	22 14	32
59	27	54	18 26	19 28	42	52	35	23 10
60	55	17 24	58	20 03	21 19	22 33	23 21	50
61	16 24	57	19 57	41	59	23 16	24 10	24 33
62	56	18 32	20 16	21 21	22 42	59	25 03	25 18
63	17 30	19 10	58	22 05	23 28	24 51	25 56	26 02
64	18 06	51	21 41	22 52	24 19	25 46	26 55	27 00
65	18 47	20 35	22 33	23 43	25 13	26 48	27 51	28 02
66	19 30	21 23	23 27	24 39	26 13	27 46	28 51	30 10
67	20 17	22 15	24 20	25 39	27 17	28 58	30 15	31 21
68	21 10	23 13	25 14	26 45	28 27	30 19	31 28	32 46
69	22 04	24 16	26 31	27 58	29 44	31 35	32 46	34 14
70	23 05	25 26	27 49	29 18	31 09	33 00	34 22	35 55

TABLE XXVI.

Mean places of Gamma, (Cassiopee) and Epsilon (Alioth), Ursa Majoris, at Greenwich.

Mean noon for the first day of January of each year, from 1870 to 1950.

	Gamma in Cassiopee.			Alioth in Ursa Majoris.		
	Right Ascen'n.		N. Polar Dist.	Right Ascen'n.		N. Polar Dist.
	°	'	"	°	'	"
1870	0 48	52.6	29 59 15.6	12 48	18.2	33 20 3.8
1		56.1	58 56.0		20.9	20 23.5
2		59.7	58 36.4		23.5	20 43.2
3	49	3.3	58 16.8		26.2	21 2.9
4	0 49	6.8	58 57.2		28.9	21 22.5
5		10.4	57 37.9		31.5	21 42.2
6		14.0	57 18.0		34.2	22 1.8
7		17.5	57 58.3		36.8	22 21.6
8		21.1	56 38.7		39.5	22 41.2
9		24.7	56 4.1		42.2	23 0.9
1880		28.2	55 59.5	12 48	44.8	33 23 20.5
1	0 49	31.8	55 39.9		47.5	23 40.2
2		35.4	55 20.3		50.1	23 59.9
3		38.9	55 0.7		52.8	24 19.5
4		42.5	54 41.1		55.5	24 39.2
5		46.1	54 21.4	48 58.12		24 58.9
6		49.7	54 1.9	49 0.8		25 18.5
7		53.2	53 42.3		3.4	25 38.2
8	0 49	56.7	53 22.7		6.1	25 57.8
9	0 50	0.3	53 3.9		8.7	29 17.5
1890		4.0	52 43.5	12 49	11.4	33 26 37.2
1		7.5	52 23.9		14.1	29 56.8
2		11.1	52 4.3		16.7	27 16.5
3		14.7	51 44.7		19.4	27 36.2
4		18.3	51 25.1		22.0	27 55.8
5		21.8	51 5.5		24.7	28 15.4
6		25.4	50 45.9		27.3	28 25.1
7		29.0	50 26.3		30.0	28 54.8
8		32.6	58 6.7		32.6	29 14.4
9		36.2	49 47.1		35.3	26 34.1
1900	0 58	39.7	49 27.6	12 49	37.9	29 53.7
1		43.3	49 8.0		40.6	30 13.5
2		46.9	48 48.4		43.2	30 33.0
3		50.5	48 28.8		45.9	30 52.7
4		54.1	48 9.2		48.5	31 12.3
5		57.7	47 49.6		51.2	30 32.0
6	0 51	01.2	47 30.0		53.9	31 51.6
7		04.8	47 10.4		56.5	32 11.3
8		08.4	46 50.8		59.2	32 30.9
9		12.0	46 31.7		01.9	32 50.6
1910	0 51	15.6	29 43 11.6	12 50	04.5	33 33 10.2
1911	0 51	19.2	45 12.4	12 50	07.2	29.8
12	51	22.8	44 52.8		9.8	49.5
13		26.4	44 33.2		12.5	34 09.1
14		30.0	29 44 53.2		15.12	28.7
15		33.6	44 33.6		17.8	48.0
16		37.2	44 14.0		20.5	35 08.0
17		40.8	43 54.4		23.12	27.9
18		44.4	43 34.8		25.8	47.3
19		48.0	43 24.2		28.1	36 06.0
1920	0 51	51.5	42 55.8	12 50	30.7	36 26.6
30		52 27.6	39 40.2		57.4	39 42.9
40		53 03.7	36 24.7	51	23.8	42 59.1
50		53 32.8	29 23 9.6		50.2	34 43 15.6

NOTE.— γ in Cassiopee is No. 253, and ϵ (Alioth) in Ursa Majoris is 4335, in the British Association's Catalogue of Stars, published in 1850. These stars, so essential to the Surveyor, have not been given in the British or American Nautical Almanacs. We have sent a proof of this sheet, with seven other pages, to Mr. Coffin, Superintendent of the American Ephemerides, on March 26, 1875, requesting of him to give them a place in his future years.

TABLE XXVII.

Showing the Azimuth or bearing of Alpha in the foot of the Southern Cross (Crucis), when on the same vertical plane with Beta in Hydri, or in the tail of the Serpent.

Bearings are all South-east when Alpha Crucis is at its under transit, and for the 1st of January of the years given at top.

Lat.	1850	1900	1950	2000	2050	2100	2150
	' "	' "	' "	' "	2 15	' "	' "
12	1 12	1 14	1 19	1 43	15	2 58	3 53
13	12	14	19	43	16	58	54
14	12	14	20	43	16	58	56
15	12	15	21	44	17	59	57
16	13	15	22	44	17	3 01	58
17	13	16	23	45	17	03	59
18	14	16	24	46	18	3 06	4 00
19	14	17	24	47	19	07	01
20	14	17	25	48	20	3 09	02
21	1 15	18	1 25	1 49	2 21	3 10	4 05
22	15	18	26	1 50	22	10	06
23	16	19	27	50	23	11	07
24	16	19	28	51	24	12	09
25	17	20	28	52	25	13	11
26	17	20	29	52	26	14	13
27	18	21	29	53	27	15	15
28	19	22	30	1 54	29	17	18
29	20	22	31	55	30	19	20
30	21	23	32	56	32	3 21	23
31	1 22	1 24	1 34	1 57	2 34	3 23	4 26
32	22	25	35	58	36	3 25	29
33	23	26	36	59	37	27	32
34	24	27	37	2 01	39	30	35
35	25	28	38	02	41	32	38
36	26	29	39	03	43	35	42
37	28	30	40	05	45	38	45
38	29	31	42	07	48	41	49
39	30	32	43	09	50	45	53
40	31	34	44	11	53	48	58
41	1 32	1 35	1 46	2 13	2 55	3 51	5 02
42	34	37	48	15	57	58	07
43	36	39	50	17	59	59	11
44	37	41	51	20	3 03	4 03	16
45	39	43	53	22	06	07	22
46	41	44	55	25	10	12	28
47	43	46	57	28	13	17	34
48	45	48	2 00	2 31	17	22	40
49	47	50	02	33	21	28	47
50	49	52	05	36	26	34	54
51	1 51	1 54	2 07	38	3 30	4 37	6 02
52	54	57	10	40	34	43	12
53	57	2 00	13	45	39	49	17
54	59	02	16	51	44	55	27
55	2 02	06	19	53	50	5 02	39
56	05	09	23	3 01	56	10	51
57	08	13	27	05	4 01	18	7 01
58	12	16	32	09	09	27	11
59	16	19	36	14	16	36	23
60	20	24	40	20	24	44	36
61	24	29	45	26	32	6 04	7 50
62	29	34	51	33	41	6 12	8 01
63	34	38	56	40	50	20	23
64	40	43	3 02	48	5 00	27	41
65	2 45	2 47	09	3 56	5 10	6 35	9 01

TABLE XXVIII.
ALTITUDES AND GREATEST AZIMUTHS FOR 1ST JAN., 1867, (SIDERIAL TIME.)
FOR CHICAGO—LAT. 41° 50' 30", LONG. 87° 34' 7" W.; AND BUENOS AYRES—LAT. 34° 36' 40" S., LONG. 58° 24' 3" W.

British Astronomical Catalogue of Stars.	NAME OF STAR AND CONSTELLATION.	Mag.	Pol. dist.	R. A.	Up. Meridian Passage.	Time to G. Az.	Time of G. E. Az.	Time of G. W. Az.	Greatest Azimuth.	Alt. at G. Az.
360	α (Polaris) Ursa Minoris.....	2	1 24	1 10 17	6 26 38	5 55 0	0 31 38	12 21 38	1 52 46	11 51 25
6281	α Ursa Minoris.....	4.5	3 23 44	18 15 15	23 31 36	5 47 51	17 43 45	5 19 20	4 33 36	44 55 34
5780	ϵ ".....	4.5	7 44 55	16 59 42	22 16 03	5 32 9	16 43 54	3 48 12	10 24 9	42 18 58
5285	ζ ".....	4.5	11 47 52	15 48 52	21 5 13	5 16 59	15 48 14	2 22 12	13 55 44	42 57 31
4936	β " (Kochab).....	2	15 18 5	14 51 7	20 7 28	5 3 21	15 4 7	1 10 49	20 44 46	43 45 22
8238	γ Cephei.....	3.4	6 35	23 33 55	4 50 16	5 12 5	23 38 11	10 02 21	17 43 32	43 13 47
169	α Cassiopeiæ.....	var.	11 33	0 32 59	5 49 20	3 30 56	2 18 24	9 20 16	48 58 0	53 45 9
3242	φ Ursa Majoris.....	3	27 43 7	9 23 56	14 40 7	3 5 13	10 34 54	17 15 20	55 14 9	57 29 27
3777	α " (Dubhe).....	2	37 31 55	10 53 30	16 11 51	4 9 17	12 2 34	20 21 08	38 21 11	48 47 14
4017	γ ".....	2.3	35 33 57	11 46 49	17 3 10	3 21 12	13 41 58	20 24 22	51 19 39	55 5 27
5937	β Draconis.....	3.2	37 35 57	17 27 26	22 43 47	3 6 8	19 37 39	1 49 55	54 58 59	57 20 48
6091	γ ".....	2.3	38 29 40	17 53 31	23 09 52	2 58 57	20 10 55	2 8 49	56 28 2	58 27 48
7416	α Cephei.....	3.2	27 58 39	21 15 24	2 31 45	4 6 41	22 25 04	6 38 56	39 1 42	49 3 22
5959	δ Octantis.....	6	43 17	18 0 55	23 17 51	5 58 0	17 19 51	5 17 51	0 53 48	34 36 51
88	β Hydri.....	3	11 59 45	0 18 43	5 35 39	5 26 21	0 9 18	11 02 00	14 39 21	35 23 58
4131	β Chamaeleonis.....	5	11 25 34	12 10 36	17 27 32	5 27 55	11 59 37	22 55 27	13 55 20	35 24 52
5578	α Trianguli Australis.....	2	21 13 18	16 34 37	21 51 33	4 57 47	16 53 46	2 49 20	26 4 50	37 32 27
507	α Eridani (Achenar).....	1	32 5 13	1 32 45	6 49 41	4 17 29	2 32 12	11 07 10	40 10 43	42 5 56
2096	α Argus (Canopus).....	1	37 22 33	6 21 0	11 37 56	3 53 4	7 44 52	15 31 0	47 29 59	45 37 27
3186	γ ".....	1	31 16 58	9 13 32	14 30 28	4 20 50	10 9 38	18 51 17	39 5 57	41 39 14
4187	α Crucis.....	1	27 38 22	12 19 13	17 36 9	4 35 29	13 0 41	22 11 37	34 17 37	39 52 45
4669	β Centauri.....	1	30 16 13	13 54 28	19 11 24	4 24 58	14 46 25	0 33 23	57 45 2	41 7 21
7004	α Pavonis.....	2	32 50 11	20 14 57	1 31 53	4 14 12	21 17 41	5 46 05	41 11 29	42 31 59

Northern Hemisphere.

Southern Hemisphere.

TABLE XXVIIIa.
TABLE OF EQUAL ALTITUDES.

Int'val h. m.	Log. A.	Log. B.	Int'val h. m.	Log. A.	Log. B.	Int'val h. m.	Log. A.	Log. B.
2. 0	7.7297	7.7146	4. 2	7.7451	7.6815	6. 0	7.7703	7.6198
2	98	43	4	54	07	2	08	84
4	7300	39	6	58	800	4	13	70
6	02	36	8	61	792	6	19	56
8	04	32	10	64	84	8	24	42
10	05	28	12	68	76	10	29	27
12	07	25	14	72	68	12	35	13
14	09	21	16	75	59	14	40	.6098
16	11	17	18	79	51	16	45	82
18	13	13				18	51	68
20	15	09	20	82	43	20	56	53
22	17	05	22	86	34	22	62	38
24	19	7.7101	24	90	26	24	67	23
26	21	7.7097	26	94	17	26	73	07
28	23	92	28	97	08	28	79	.5991
30	25	88	30	.7501	.6700	30	84	75
32	27	83	32	05	.6691	32	90	59
34	29	79	34	09	82	34	96	43
36	31	75	36	13	73	36	.7801	27
38	33	70	38	7.7517	7.6663	38	07	10
40	36	65	40	7.7521	7.6654	40	13	.5894
42	38	61	42	25	45	42	19	77
44	40	56	44	29	35	44	25	60
46	42	51	46	33	26	46	31	43
48	45	46	48	37	16	48	36	25
50	47	41	50	41	.6606	50	42	08
52	49	36	52	45	.6597	52	48	.5790
54	52	31	54	49	87	54	54	72
56	54	26	56	53	77	56	60	54
58	57	21	58	57	.6567	58	67	36
3.00	59	15	5.00	62	56	7.00	73	17
2	62	10	2	66	46	2	79	.5699
4	64	7.7005	4	70	36	4	85	80
6	67	7.6999	6	75	25	6	91	61
8	69	93	8	79	14	8	98	41
10	72	88	10	83	.6504	10	.7904	22
12	74	82	12	88	.6493	12	10	02
14	77	76	14	92	82	14	16	.5582
16	80	70	16	97	71	16	23	62
18	7.7383	7.6964	18	.7601	60	18	29	42
20	7.7386	7.6958	20	06	48	20	36	22
22	88	52	22	10	37	22	42	01
24	91	46	24	15	25	24	49	.5480
26	94	40	26	20	14	26	55	59
28	97	34	28	24	.6402	28	62	37
30	.7400	27	30	29	.6390	30	69	16
32	03	21	32	34	78	32	75	.5394
34	06	14	34	38	66	34	82	72
36	09	08	36	43	54	36	89	50
38	12	.6901	38	48	42	38	95	27
40	15	.6894	40	53	30	40	.8002	04
42	18	88	42	58	17	42	09	.5281
44	21	81	44	63	.6304	44	16	58
46	24	74	46	68	.6291	46	23	34
48	28	67	48	73	78	48	30	11
50	31	59	50	78	65	50	37	.5186
52	34	52	52	83	52	52	44	62
54	37	45	54	88	39	54	51	37
56	41	38	56	93	25	56	58	12
58	44	30	58	7.7698	.6212	58	65	.5087
4.00	47	23	6.00	7.7703	.6199	8.00	7.8072	72

TABLE XXVIIIb.				TABLE XXVIIIc.			
To Convert Metres into Stat. Miles.		Lat.	Statute Miles.	<i>Showing the length of a Degree of Lat. and Long. in Metres and Miles.</i>			
Metres.	Miles.			In Lat.	Length of a Degree of Lat. in Metres.	Length of a Degree of Lon. in Metres.	Length of a D. of Lon. in Stat. Miles.
		0	69.07				
		1	69.06				
		2	69.08				
10	.006	3	68.07	17	11.658.4	106.473.4	66.157
20	.012	4	68.90	18	11.669.5	105.892.6	66.796
30	.019	5	68.81	19	11.681.1	106.279.7	65.415
40	.025	6	68.62	20	693.3	104.634.8	65.015
50	.031	7	68.48	21	706.0	103.958.7	64.594
60	.037	8	68.31	22	719.2	103.250.0	64.154
70	.044	9	69.15	23	732.9	102.510.0	63.695
80	.050	10	67.95	24	747.1	101.739.7	63.216
90	.056	11	67.73	25	761.7	100.938.2	62.718
100	.062	12	67.48	26	776.7	100.105.9	62.200
200	.124	13	67.21	27	792.2	99.243.2	61.664
300	.186	14	66.95	28	808.3	98.350.2	61.109
400	.249	15	66.65	29	824.4	97.427.4	60.536
500	.311	16	69.31	30	841.9	96.474.8	59.944
600	.373			31	858.0	95.492.9	59.334
700	.435			32	875.2	94.481.9	58.706
800	.497			33	110.892.8	93.442.1	58.060
900	.559	51	43.43	34	910.7	92.373.8	57.396
1000	.621	52	42.48	35	928.8	91.277.3	56.715
2000	1.243	53	41.53	36	947.2	90.152.9	56.016
3000	1.864	54	40.56	37	995.8	89.001.0	55.300
4000	2.485	55	39.58	38	984.6	87.821.6	54.568
5000	3.107	56	38.58	39	111.003.5	86.616.0	53.819
6000	3.728	57	37.58	40	022.6	85.383.9	53.053
7000	4.319	58	36.57	41	041.8	84.125.1	52.271
8000	4.971	59	35.54	42	061.1	82.840.8	51.473
9000	5.592	60	34.50	43	080.5	81.531.1	50.659
10000	6.213	61	33.45	44	100.0	80.196.5	49.830
12000	12.427	62	32.40	45	119.4	78.837.3	48.986
12000	18.640	63	31.33	46	138.9	77.453.9	48.126
13000	24.854	64	30.24	47	158.4	79.046.8	47.251
14000	31.067	65	29.15	48	177.8	74.612.3	46.362
15000	37.281	66	28.06	49	197.2	73.162.9	45.460
16000	43.494	67	26.96	50	216.4	71.687.0	44.543

That part of Table C, from Lat. 17° to 50°, is calculated according to Bessel's formula, as given in the United States Coast Survey of 1853, page 100.

Tables C and D are from the same volume, pp. 103 and 106, excepting that showing the length of a degree of Longitude, from Latitudes 0 to 17, and 50 to 90 degrees, which is taken from Keith on the Globe, p. 193.

Those having occasion to project a map on an extensive scale, will find, in the above volume, tables from pp. 107 to 163 which have been calculated for the United States Coast Survey, under the superintendence of the late A. D. BACHE.

To find the length of a degree of Longitude in any degree of Lat. :
Rad. is to the length of a degree on the Equator—as the Cosine of the given Latitude is to the length of a degree of Longitude in that Lat.
Length of a degree on the Equator is 365144 feet. Radius of the Equator = 20921180. Polar Semiaxis = 20853180.

NOTE.—There has been nothing printed to fill from page 200 to 249.

TABLE XXIX.—To Reduce French Litres
to Cubic Feet and Imperial Gallons.

1 Litre = 0.0353166 Cubic Feet, or 0.2200967
Imperial Gallons.

Litre.	Cubic Feet	English or Imper Gallons.	Litre.	Cubic Feet.	English or Imperial Gallons.
1	0.0354	0.2201	60	.1190	13.2058
2	.0706	.4401	61	.1543	.4258
3	.1059	.6603	62	.1896	.6160
4	.1413	.8804	63	2.2249	13.8661
5	.1766	1.1005	64	.2603	14.0862
6	.2119	.3206	65	.2956	.3063
7	.2472	.5407	66	.3309	.5264
8	.2825	.7608	67	.3662	.7465
9	.3178	.9809	68	.4015	.9666
10	.3532	2.2010	69	.4368	15.1867
11	.3885	.4211	70	.4722	.4068
12	.4238	.6412	71	.5075	.6268
13	.4591	.8613	72	.5428	.8470
14	.4944	3.0814	73	.5781	16.0671
15	.5297	.3014	74	.6134	.2872
16	.5651	.5215	75	.6487	.5073
17	.6004	.7416	76	.6841	.7273
18	.6357	.9617	77	.7194	.9474
19	.6710	4.1818	78	.7547	17.1675
20	.7063	.4019	79	.7900	.3876
21	.7416	.6220	80	.8253	.6077
22	.7770	.8421	81	.8606	.8278
23	.8123	5.0622	82	.8960	18.0479
24	.8476	.2825	83	.9313	.2680
25	.8829	.5024	84	.9666	.4881
26	.9182	.7225	85	3.0019	.7082
27	.9535	.9426	86	.0372	.9283
28	.9889	6.1627	87	.0725	19.1484
29	1.0242	.3828	88	.1079	.3685
30	.0595	.6029	89	.1932	.5886
31	.0948	.8230	90	.1785	.8087
32	.1301	7.0431	91	.2138	20.0288
33	.1654	.2631	92	.2491	.2489
34	.2008	.4832	93	.1844	.4690
35	.2361	.7034	94	.3198	.6891
36	.2714	.9235	95	.3551	.9092
37	.3067	8.1436	96	.3904	21.1293
38	.3420	.3637	97	.4257	.3494
39	.3773	.5838	98	.4610	.5695
40	.4127	.8039	99	.4963	.7896
41	.4480	9.0240	100	3.5317	22.0097
42	.4833	.2441	200	7.0633	44.0193
43	.5186	.4642	300	10.5950	166.0290
44	.5539	.6843	400	14.1266	188.0387
45	.5892	.9044	500	17.6583	110.0484
46	.6246	10.1244	600	21.1900	132.0580
47	.6599	.3445	700	24.7216	154.0677
48	.6952	.5646	800	28.2533	176.0774
49	.7305	.7848	900	31.7849	198.0871
50	.7658	11.0048	1000	35.3166	220.0967
51	.8011	.2249	2000	70.6332	440.019
52	.8365	.4450	3000	105.950	660.029
53	.8718	.6651	4000	141.266	880.039
54	.9071	.8852	5000	176.583	1100.48
55	.9424	12.1053	6000	211.901	1320.58
56	.9777	.3254	7000	247.216	1540.68
57	2.0130	.5455	8000	282.533	1760.77
58	.0484	.7656	9000	317.849	1980.87
59	.0837	.9857	10000	353.166	2200.97

TABLE XXX.—Foreign
Weights and Measures.

French, new system.	English inches.
Millimetre equals	0.039371
Centimetre "	0.393708
Decimetre "	3.937079
Metre "	39.37079
Decametre "	393.7079
Hectometre "	3937.079
Kilometre "	39370.79
Myriametre "	393707.9
Foot (Pied de Roi) =	12.7925
Spanish foot =	11.034 inches
French " =	12.7325 "
Swedish " =	11.690 "
Austrian " =	12.448 "
Lisbon " =	12.96 "
Toise, or 6 ft. Fr. =	76.735 in.
Sq. metre =	1550.85 sq. in.
Sq. metre =	10.7698 sq. feet

Measure.	Sq. yds.
England Acre	4840
Amsterdam Moyen	9722
Hamburg Moyon	11545
Ireland Acre	7840
Naples Moggia	3998
Portugal Geira	6870
Prussia Morgen	3052
Rome Pizza	3158
Russia Dessitina	19066.6
Spain Fanegade	5500
Sweden Tunneland	5900
Scotland Acre	6150

SURFACE.

French, old system.	English.
Square inch =	1.1364 inches
Arpent (Paris) =	900 sq. toises
" (woodland) =	100 sq. royal perches

New system.

Are =	100 sq. metres
Are =	1076.98 sq. feet
Centare =	1 sq. metre
Decare =	10 ares
Hecatare =	100 ares

CAPACITY.

Litre taken as a standard.

Myrialitre =	10000 litres
Kilolitre =	1000 "
Hectolitre =	100 "
Decalitre =	10 "
Litre =	1 "
Decilitre =	0.1 "
Centilitre =	0.01 "
Millilitre =	0.001 "
Litre =	cubic centimetre

Troy grains.

Milligramme =	.0154
Centigramme =	.1544
Decigramme =	1.5444
Gramme =	15.4440
Decagramme =	154.4402
Hectogramme =	1544.4023
Kilogramme =	15444.0234
Milligramme =	154440.2344

For a valuable collection of
tables of weights and meas-
ures, see Oliver Byrne's Dic-
tionary of Mechanics and En-
gineering.

TABLE XXXI.—*Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		10 Centimetres.		12 Centimetres.		14 Centimetres.		16 Centimetres.	
Diam. Metre	Area of section.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Discharge in litres.
0.01	0.0001	0.3602	0.008	0.5187	0.009	0.7060	0.011	0.9221	0.013
2	03	1154	031	1662	038	2262	44	2954	050
3	07	626	071	0901	085	1226	99	1601	113
4	13	415	126	598	151	0811	176	1063	201
5	20	306	196	441	236	600	275	0784	314
6	28	241	283	347	339	472	396	617	452
7	38	198	385	285	462	387	539	506	616
8	50	168	503	241	603	328	704	428	804
9	64	145	636	208	763	283	891	370	1.018
10	79	127	785	183	943	249	1.100	326	257
11	95	114	950	164	1.140	223	330	291	521
12	113	102	1.131	148	357	201	583	262	810
13	133	093	327	134	593	183	858	239	2.124
14	154	86	539	123	847	168	2.155	219	463
15	177	79	767	114	2.121	155	474	203	827
16	201	73	2.011	106	413	144	815	188	3.217
17	227	69	270	99	724	134	3.178	176	632
18	254	64	545	93	3.054	126	563	165	4.072
19	284	61	835	87	402	119	969	155	536
20	314	57	3.142	82	770	112	4.398	146	5.027
21	346	54	464	78	4.156	106	849	139	542
22	380	51	801	74	562	101	5.322	132	6.082
23	415	49	4.155	71	980	096	817	125	648
24	452	47	524	67	5.429	92	6.333	120	7.238
25	491	45	909	64	6.091	88	872	114	854
26	531	43	5.309	62	371	84	7.433	110	8.495
27	573	41	726	59	871	81	8.016	105	9.161
28	616	40	6.158	57	7.389	77	621	101	852
29	661	38	605	55	926	75	9.247	097	10.568
30	.0707	.1037	7.069	.0153	8.482	.0072	9.896	.0094	11.310
31	755	35	548	51	9.057	69	10.567	91	12.076
32	804	34	8.043	49	651	67	11.259	88	868
33	855	33	553	48	10.264	65	974	85	13.685
34	908	32	9.079	46	895	63	12.711	82	14.527
35	962	31	621	45	11.545	61	13.470	80	15.394
36	1018	30	10.179	43	12.215	59	14.250	77	16.286
37	1075	29	752	42	903	57	15.053	75	17.203
38	1134	28	11.341	41	13.609	56	078	73	18.146
39	1195	28	946	40	14.335	54	16.724	71	19.113
40	1257	27	12.566	39	15.080	53	17.593	69	20.106
41	1320	26	13.203	38	843	51	18.484	67	21.124
42	1385	26	855	37	16.625	50	19.396	66	22.167
43	1452	25	14.522	36	17.427	49	20.331	64	23.235
44	1521	24	15.205	35	18.246	48	21.287	62	24.328
45	1590	24	904	34	19.085	47	22.266	60	25.446
46	1662	23	16.619	34	943	46	23.267	60	26.591
47	1735	23	17.350	33	20.819	45	24.289	58	27.759
48	1810	22	18.096	32	21.715	44	25.334	57	28.953
49	1886	22	858	31	22.629	43	26.401	56	30.172
50	1964	21	19.635	31	23.562	42	27.489	55	31.416
55	2376	19	23.758	28	28.510	38	33.262	49	38.013
60	2827	18	28.274	25	33.929	35	39.584	45	45.239
65	3318	16	33.183	23	39.820	32	46.456	42	53.093
70	3848	15	38.485	22	46.182	29	53.878	38	61.575
75	4418	14	44.179	20	53.015	27	61.850	36	70.686
80	5027	13	50.266	19	60.319	26	70.372	33	80.425
85	5675	12	56.745	18	68.994	24	79.443	31	90.792
90	6362	12	63.617	17	76.341	23	89.064	30	101.788
95	7088	11	70.882	16	85.059	21	99.235	28	113.412
1.00	.7854	.1010	78.540	.0115	94.248	.0020	109.956	.0016	125.664

TABLE XXXI. — *Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables of 1857.

18 Centimetres.		20 Centimetres.		22 Centimetres.		24 Centimetres.		26 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres
1.1670	0.014	1.4408	0.016	1.7434	0.017	2.0748	0.019	2.4350	0.020
9.3739	057	4616	063	0.5585	069	0.6647	75	0.7801	082
2027	127	2502	141	3028	155	3603	170	4229	184
1345	226	2661	251	2010	277	2392	302	2807	327
992	353	1225	393	1483	432	1764	471	2071	511
780	509	0.0964	566	1166	622	1388	679	1628	735
640	693	791	770	957	847	1139	924	1336	1.001
542	905	669	1.005	809	1.106	963	1.206	1130	307
469	1.145	579	272	700	400	833	527	0978	654
412	414	509	571	616	728	733	885	860	2.042
368	711	454	901	550	091	654	281	768	471
332	2.036	410	2.262	496	488	590	714	693	941
302	389	373	655	452	920	537	3.186	631	3.451
277	771	343	3.079	414	387	493	695	579	4.002
256	3.181	316	534	383	888	456	4.241	535	595
238	619	294	4.021	356	4.423	423	820	497	5.228
222	4.086	274	540	332	999	395	5.448	464	901
208	580	257	5.089	311	5.588	370	6.107	435	6.616
196	5.104	242	671	293	6.238	349	805	409	7.372
185	655	229	6.283	277	912	329	7.540	386	8.168
175	6.235	216	927	262	7.620	312	8.313	366	9.005
167	842	206	7.603	249	8.363	1296	9.123	348	883
159	7.479	196	8.310	237	9.140	82	971	331	10.802
151	8.143	187	9.048	226	953	69	10.857	316	11.762
145	836	179	818	216	10.799	57	11.781	302	12.763
139	9.557	171	10.619	207	11.680	47	12.742	290	13.804
133	10.306	164	11.451	199	12.596	37	13.741	278	14.886
128	11.084	158	12.315	191	13.547	28	14.778	267	16.010
123	889	152	13.210	184	14.531	19	15.853	257	17.174
.0119	12.723	.0147	14.137	.0178	15.551	.1211	16.965	.0248	18.378
115	13.586	142	15.095	171	16.605	04	18.114	239	19.624
111	14.476	137	16.085	166	17.693	197	19.302	231	20.910
107	15.395	132	17.106	160	18.817	91	20.527	224	22.238
104	16.343	128	18.158	155	19.974	85	21.790	217	23.606
101	17.318	124	19.242	150	21.167	79	23.091	210	25.015
98	18.322	121	20.358	146	22.393	74	24.429	204	26.465
95	19.354	117	21.504	142	23.655	69	25.805	198	27.956
92	20.414	114	22.682	138	24.951	64	27.219	93	29.487
90	21.503	111	23.892	134	26.281	60	28.670	87	31.059
87	22.620	108	25.133	131	27.646	55	30.159	82	32.673
85	23.765	105	26.495	127	29.046	51	31.686	78	34.327
83	24.938	102	27.709	124	30.480	48	33.251	73	36.022
81	26.140	100	29.044	121	31.949	44	34.853	69	37.753
79	27.370	098	30.411	118	33.452	40	36.493	65	39.534
77	28.628	95	31.809	115	34.989	137	38.170	161	41.351
75	29.914	93	33.258	113	36.562	34	39.886	57	43.210
74	31.229	91	34.699	110	38.169	31	41.639	54	45.109
72	32.572	89	36.191	108	39.810	28	43.429	50	47.049
71	33.944	87	37.715	105	41.487	25	45.258	47	49.030
69	35.343	85	39.270	103	43.197	23	47.124	44	51.051
63	42.765	77	47.517	093	52.268	11	57.020	20	61.772
57	50.894	70	56.549	85	62.204	101	67.859	19	73.513
53	59.730	65	66.366	78	73.003	093	79.646	10	86.276
49	69.272	60	76.969	73	84.666	86	92.363	102	100.06
45	79.522	56	88.358	68	97.193	81	106.03	095	114.87
42	90.478	52	100.531	63	110.58	75	120.64	88	130.69
40	102.141	49	113.490	59	124.84	71	136.19	83	147.54
38	114.511	46	127.235	56	139.96	67	152.68	78	165.41
36	127.588	44	141.765	53	155.94	63	170.12	74	184.30
34	141.372	.0042	157.080	.0050	172.79	.0060	188.50	.0070	204.20

TABLE XXXI—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Veloc'y per sec.		30 Centimetres.		34 Centimetres.		38 Centimetres.		42 Centimetres.	
Diam. Metre	Area of sect'n	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	3.2418	0.024	4.1639	0.027	5.2013	0.030	6.3539	0.033
2	03	1.0386	094	1.3340	107	1.6664	1.119	2.0357	132
3	07	0.5630	212	0.7231	240	0.9033	269	1.1035	297
4	13	3737	377	4800	427	5996	478	0.7325	528
5	20	2757	589	3541	668	4423	746	5404	825
6	28	2168	848	2785	961	3479	1.074	4249	188
7	38	1779	1.155	2285	1.308	2854	462	3487	1.616
8	50	1505	508	1933	709	2414	910	2949	2.111
9	64	1302	909	1672	2.163	2088	2.417	2551	672
10	79	1146	2.356	1471	670	1838	985	2245	3.299
11	95	1022	851	1313	231	1640	3.611	2003	991
12	113	0922	3.393	1185	845	1480	4.298	1808	4.750
13	133	840	982	1079	4.513	1348	5.044	1646	5.575
14	154	771	618	0980	5.234	1237	850	1511	6.465
15	177	712	5.301	914	6.008	1142	6.715	1395	7.422
16	201	661	6.032	849	836	1061	7.640	1296	8.445
17	227	617	809	793	7.717	0991	8.625	1210	9.533
18	254	579	7.634	744	8.652	929	9.670	1135	10.688
19	284	45	8.506	700	9.640	874	10.774	1068	11.908
20	314	15	9.425	661	10.681	826	11.938	1009	13.195
21	346	487	10.391	626	11.776	782	13.162	0955	14.547
22	380	63	11.404	595	12.925	43	14.445	907	15.966
23	415	41	12.464	66	14.126	07	15.788	864	17.450
24	452	21	13.572	40	15.381	675	17.191	825	19.000
25	491	02	14.726	17	16.690	45	18.653	788	20.617
26	531	385	15.928	495	18.052	18	20.175	755	22.299
27	573	70	17.177	75	19.467	594	21.757	725	24.047
28	616	56	18.473	57	20.936	71	23.393	697	25.862
29	661	42	19.816	40	22.458	49	25.100	71	27.742
30	.0707	.0330	21.206	.0624	24.033	.0530	26.861	.0647	29.688
31	755	19	22.643	09	25.662	511	28.681	25	31.700
32	804	08	24.127	396	27.344	494	30.561	04	33.778
33	855	298	25.659	83	29.080	78	32.501	84	35.923
34	908	89	27.238	71	30.869	63	34.501	66	38.133
35	962	80	28.863	59	32.712	49	36.560	48	40.409
36	1018	71	30.536	49	34.608	36	38.679	32	42.751
37	1075	64	32.256	39	36.557	23	40.858	17	45.159
38	1134	56	34.024	29	38.560	11	43.096	02	47.633
39	1195	49	35.838	20	40.616	00	45.395	489	50.173
40	1257	43	37.699	12	42.726	389	47.752	76	52.779
41	1320	36	39.608	04	44.889	79	50.170	63	55.451
42	1385	30	41.564	296	47.105	70	52.647	52	58.189
43	1452	25	43.566	89	49.375	61	55.184	41	60.993
44	1521	19	45.616	82	51.698	52	57.780	30	63.862
45	1590	14	47.713	75	54.075	44	60.436	20	66.798
46	1662	09	49.857	69	56.505	36	63.153	10	69.800
47	1735	05	52.049	63	58.988	28	65.928	01	72.868
48	1810	00	54.286	57	61.525	21	68.763	392	76.002
49	1886	.0196	56.573	52	64.106	14	71.659	84	79.002
50	1964	92	68.905	46	66.759	08	74.613	76	82.467
55	2376	74	71.275	23	80.778	279	90.282	40	99.785
60	2827	59	84.823	04	96.133	54	107.44	11	118.75
65	3318	46	99.550	187	112.82	34	126.10	286	139.37
70	3848	35	115.45	74	130.85	17	146.24	65	161.64
75	4418	26	132.54	62	150.21	02	167.88	47	185.55
80	5027	18	150.80	51	170.90	189	191.01	31	211.12
85	5675	11	170.24	42	192.93	77	215.63	17	238.33
90	6362	04	190.85	34	216.80	67	241.75	04	267.19
95	7088	99	212.65	27	241.00	58	269.35	193	297.71
100	.7854	.0094	235.62	.0120	267.04	.0150	298.45	.0183	329.87

TABLE XXXI. — *Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables of 1857.

46 Centimetres.		50 Centimetres.		54 Centimetres.		58 Centimetres.		62 Centimetres.	
Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres
7.6218	0.036	9.0050	0.039	10.5030	0.042	12.117	0.046	13.846	0.049
2.4419	145	2.8850	157	3.3651	170	3.8820	182	4.4360	195
1.8237	325	1.5639	353	1.8241	382	2.1044	410	2.4046	438
0.8787	578	0.831	628	2109	679	1.3969	729	1.5962	779
6482	908	0.7658	982	0.8932	1.060	0.805	1.139	1715	1.217
5097	301	6022	414	7024	527	0.8104	640	0.9260	753
4183	770	4942	924	5764	2.078	6650	2.232	7599	2.386
3538	2.312	4180	2.513	4875	714	5623	915	6427	3.116
3060	926	3616	3.181	4217	3.435	4865	3.689	5529	944
2693	613	3182	927	3712	4.241	4282	4.555	4893	4.869
2403	372	2839	4.752	3312	5.131	3821	5.511	4366	5.892
2168	5.202	562	5.655	2988	6.107	3448	6.559	3939	7.012
1970	6.106	333	6.637	2721	7.167	3139	7.698	3587	8.229
1812	7.081	141	7.697	2497	8.313	2881	8.928	3292	9.544
1674	8.129	1978	8.836	2307	9.543	2661	10.249	3041	10.956
1555	9.249	837	10.053	2143	10.857	2472	11.661	2825	12.465
1452	10.441	715	11.349	2000	12.257	2308	13.165	2637	14.073
1361	11.686	608	12.723	1876	13.741	164	14.759	2472	15.777
1281	13.042	514	14.176	765	15.310	037	16.445	2327	17.579
1210	14.451	429	15.708	667	16.965	1923	18.221	2198	19.477
1146	15.933	354	17.318	579	18.703	822	20.089	2082	21.474
1088	17.486	286	19.007	500	20.527	730	22.047	1978	23.568
1036	19.112	225	20.774	428	22.435	648	24.097	883	25.759
0999	20.810	169	22.620	365	24.429	572	26.239	797	28.048
946	22.580	118	24.544	304	26.507	503	28.471	718	30.434
906	24.423	071	26.547	249	28.670	441	30.793	646	32.917
870	26.338	028	28.628	199	30.918	383	33.208	580	35.498
836	28.325	0988	30.788	152	33.251	329	35.713	519	38.177
805	30.384	951	33.026	109	35.668	280	38.310	462	40.952
.0776	32.516	.0917	35.343	.1070	38.170	.1284	40.997	.1410	43.825
749	34.719	885	37.738	32	40.757	191	43.777	361	46.795
724	36.995	855	40.212	998	43.429	151	46.646	315	49.863
700	39.344	28	42.765	65	46.186	114	49.607	273	53.029
678	41.764	02	45.396	35	49.027	079	52.659	233	56.291
58	44.257	777	48.106	06	51.954	46	55.803	195	59.651
38	46.822	54	50.894	0880	54.965	15	59.037	160	63.108
20	49.460	32	53.761	54	58.061	0986	62.362	126	66.663
03	52.169	12	56.706	30	61.242	58	65.779	1095	70.315
586	54.951	693	59.730	08	64.508	32	69.286	65	74.065
71	57.805	74	62.832	786	67.858	07	72.885	37	77.911
56	60.732	57	66.013	66	71.294	884	76.575	10	81.856
42	63.731	40	69.273	47	74.814	62	80.356	0984	85.897
29	66.802	25	72.611	28	78.419	40	84.228	960	90.037
16	69.944	10	76.027	11	82.109	20	88.191	937	94.273
04	73.160	595	79.522	694	85.883	801	92.245	915	98.607
492	70.648	82	83.096	78	89.743	783	96.391	894	103.04
81	79.808	69	86.748	63	93.687	65	100.63	874	107.57
71	83.240	56	90.478	49	97.716	48	104.95	855	112.19
61	86.745	44	94.288	35	101.83	32	109.37	837	116.92
51	90.321	33	98.175	22	106.03	17	113.88	819	121.74
08	109.29	482	118.79	563	128.30	649	137.80	742	147.30
373	130.06	41	141.37	14	152.68	93	163.99	677	176.30
43	152.64	05	165.92	473	179.19	545	192.46	623	205.74
18	177.03	375	192.42	38	207.82	05	223.21	577	238.60
296	203.22	50	220.89	08	238.57	470	256.24	537	273.91
77	231.22	27	251.33	381	271.43	40	291.54	503	311.65
60	261.03	07	283.73	58	306.42	13	329.12	472	351.82
45	292.64	290	318.09	38	343.53	90	368.98	445	394.43
32	326.06	274	354.41	320	382.76	69	411.12	421	439.47
.0220	361.28	.0260	382.70	.0303	424.12	.0350	455.53	.0400	486.95

TABLE XXXI.—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		66 Centimetres.		70 Centimetres.		74 Centimetres.		78 Centimetres.	
Diam.	Area of section	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
Metre									
0.01	0.0001	15.690	0.052	17.650	0.055	19.725	0.059	21.915	0.061
2	03	5.0268	207	5.6547	220	6.3193	232	7.0209	245
3	07	2.7249	467	3.0652	495	3.4255	523	3.8050	551
4	13	1.8088	829	2.0347	880	2.2739	930	2.5264	980
5	20	1.3343	1.296	1.5010	1.374	1.6774	1.453	1.8637	1.532
6	28	1.0493	1.866	1.1804	.979	1.3191	2.092	4.656	2.205
7	38	0.8611	2.540	0.9686	2.694	0.825	848	2027	3.002
8	50	.7283	3.317	.8192	3.519	0.9155	3.719	0172	921
9	64	.6300	4.199	.7086	4.453	.7919	4.707	0.8799	4.962
10	79	.5544	5.183	.6237	5.498	.6970	5.811	.7744	6.126
11	95	.4947	6.272	.5565	6.652	.6219	7.032	.6910	7.412
12	113	.4464	7.464	.5021	7.917	.5611	8.369	.6234	8.821
13	133	.4065	8.760	.4572	9.291	.5110	9.822	.5677	10.353
14	154	.3730	10.159	.4196	10.776	.4689	11.391	.5210	12.007
15	177	.3446	11.663	.3876	12.370	.4332	13.077	.4813	13.783
16	201	.3201	13.270	.3601	14.074	.4024	14.079	.471	15.683
17	227	.2988	14.981	.362	15.889	.3757	16.796	.471	17.704
18	254	.2802	16.795	.352	17.813	.3522	18.831	.3913	19.849
19	284	.2637	18.713	.2966	19.847	.315	20.981	.683	22.115
20	314	.2490	20.734	.801	21.991	.131	23.247	.478	24.504
21	346	.2359	22.859	.654	24.245	.2966	25.631	.295	27.016
22	380	.2241	25.089	.521	26.609	.817	28.129	.130	29.650
23	415	.134	27.421	.400	29.083	.682	30.745	.2980	32.407
24	452	.036	29.857	.290	31.667	.560	33.477	.844	35.286
25	491	.1947	32.397	.190	34.361	.448	36.325	.720	38.288
26	531	.866	35.041	.098	37.165	.345	39.289	.606	41.412
27	573	.791	37.789	.014	40.079	.251	42.369	.501	44.659
28	616	.721	40.639	.1936	43.103	.164	45.565	.404	48.029
29	661	.657	43.594	.864	46.236	.083	48.878	.315	51.521
30	.0707	.1598	46.653	.1797	49.480	.2008	52.307	.2231	55.135
31	.755	.542	49.859	.735	52.834	.1939	55.853	.154	58.871
32	.804	.490	53.080	.677	56.297	.874	59.514	.082	62.731
33	.855	.442	56.449	.622	59.871	.813	63.292	.014	66.713
34	.908	.397	59.923	.571	63.555	.756	67.186	.1951	70.817
35	.962	.354	63.499	.523	67.348	.702	71.196	.891	75.045
36	1018	.314	67.179	.478	71.251	.652	75.323	.835	79.394
37	1075	.276	70.964	.436	75.265	.604	79.565	.782	83.866
38	1134	.240	74.851	.395	79.388	.559	83.925	.733	88.461
39	1195	.207	78.843	.357	83.622	.517	88.399	.685	93.178
40	1257	.175	82.938	.321	87.965	.477	92.991	.641	98.017
41	1320	.144	87.137	.287	92.418	.439	97.699	.598	102.98
42	1385	.116	91.439	.255	96.982	.402	102.52	.558	108.07
43	1452	.088	95.845	.224	101.66	.368	107.46	.520	113.27
44	1521	.062	100.36	.195	106.44	.335	112.52	.483	118.60
45	1590	.037	104.97	.167	111.32	.304	117.69	.449	124.05
46	1662	.014	109.69	.140	116.33	.274	122.98	.416	129.63
47	1735	.0991	114.51	.115	121.45	.246	128.29	.384	135.32
48	1810	.969	119.43	.090	126.67	.218	133.91	.354	141.15
49	1886	.948	124.46	.067	132.00	.192	139.55	.325	107.09
50	1964	.928	129.59	.044	137.45	.167	145.30	.297	153.15
55	2376	.840	156.81	.0945	166.31	.056	175.81	.174	185.32
60	2827	.767	186.61	.863	197.92	.0965	209.23	.072	220.54
65	3318	.706	219.01	.794	232.28	.888	245.56	.0986	258.83
70	3848	.654	254.00	.736	269.39	.822	284.79	.913	300.18
75	4418	.609	291.58	.685	309.25	.766	326.92	.851	344.60
80	5027	.570	331.75	.641	351.86	.716	371.97	.796	392.07
85	5675	.535	374.52	.602	397.22	.673	419.91	.748	442.61
90	6362	.505	419.88	.568	445.22	.634	470.77	.705	496.22
95	7088	.477	467.82	.537	496.18	.600	524.53	.667	552.88
1.00	.7854	.0452	518.36	.0510	549.78	.0569	581.20	.0633	612.61

TABLE XXXI. — *Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables in 1857.

82 Centimetres.		86 Centimetres.		90 Centimetres.		94 Centimetres.		98 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
24.220	0.064	26.640	0.068	29.176	0.071	31.827	0.074	34.594	0.078
7.7594	258	8.5350	270	9.3474	283	10.197	295	11.083	307
4.2062	580	4.6266	608	5.0670	636	5.5274	664	6.0078	693
2.7921	1.030	3.0712	1.081	3.3635	1.131	3.6691	1.181	3.9881	1.232
2.0597	610	2.2655	689	2.4812	767	2.7066	846	2.9418	924
1.6197	2.319	1.7816	2.432	1.9512	2.545	2.1285	2.658	2.3135	2.771
3292	3.156	4620	3.310	6012	3.464	1.7467	3.618	1.8985	3.771
1242	4.121	2365	4.323	3542	4.524	4773	4.725	6057	4.926
0.9724	5.217	0696	5.471	1714	5.726	2778	5.980	3889	6.234
8558	6.440	0.9414	6.754	0810	7.068	1246	7.383	2224	7.697
7637	7.793	8400	8.173	0.9199	8.553	035	8.933	0907	9.313
6890	9.274	7579	9.726	8300	10.179	9054	10.631	0.9841	11.083
6274	10.884	6902	11.415	7558	11.946	8245	12.477	8962	13.007
5758	12.623	6333	13.229	6936	13.854	7567	14.470	8224	15.085
5319	14.491	5859	15.197	6407	15.904	6990	16.611	7597	17.318
4941	16.487	435	17.291	5952	18.096	493	18.895	7058	19.704
613	18.612	078	19.520	557	20.428	462	21.336	6589	22.244
325	20.866	4757	21.884	210	22.902	683	23.880	6177	24.938
071	23.249	477	24.383	4904	25.518	649	26.651	5814	27.785
3844	25.761	228	27.017	631	28.274	052	29.531	491	30.787
641	28.401	005	29.787	387	31.173	4785	32.557	201	33.943
459	31.171	3804	32.691	167	34.212	545	35.732	4940	37.253
293	34.069	623	35.731	3967	37.393	328	39.055	704	40.717
143	37.096	457	38.205	786	40.715	130	42.525	489	44.334
006	40.251	306	42.215	621	44.177	3950	46.142	293	48.105
2880	43.536	168	45.660	469	47.584	3784	49.907	113	52.031
764	46.949	040	49.239	330	51.530	632	53.820	3948	56.110
657	50.491	2923	52.955	201	55.418	492	57.881	795	60.343
558	54.163	814	56.805	082	59.447	362	62.089	654	64.731
2466	57.962	2713	60.789	2971	63.627	3241	66.445	3522	69.272
381	61.891	618	64.910	868	67.929	128	70.948	402	73.967
301	65.948	531	69.165	771	72.382	023	75.599	286	78.816
226	70.135	448	73.555	681	76.977	2925	80.398	179	83.819
156	74.449	371	78.081	597	81.713	833	85.345	079	88.976
090	78.893	299	82.741	518	86.590	747	90.439	2982	94.287
028	83.466	231	87.537	443	91.609	665	95.680	897	99.752
1970	88.167	167	92.468	373	96.769	589	101.07	814	105.37
915	92.997	106	97.534	307	102.07	516	106.61	735	111.14
863	97.957	049	102.74	244	107.51	448	112.29	661	117.07
813	103.10	1995	108.07	184	113.10	383	118.13	590	123.15
767	108.26	943	113.54	128	118.82	321	124.10	523	129.39
722	113.61	804	119.15	074	124.69	263	130.23	460	135.77
680	119.08	848	124.89	024	130.70	207	136.51	399	142.32
640	124.68	803	130.77	1975	136.85	154	142.93	342	149.01
601	130.42	761	136.78	929	143.14	104	149.50	287	155.86
564	136.38	721	142.92	885	149.57	056	156.22	235	162.87
529	142.27	682	149.21	842	156.15	010	163.09	185	170.03
496	148.38	646	155.62	802	162.86	1966	170.10	137	177.34
464	154.63	610	162.17	764	169.72	924	177.26	091	184.80
433	161.01	572	168.86	727	176.72	883	184.57	047	192.42
297	194.82	427	204.32	563	213.83	705	223.33	1853	232.83
185	231.85	203	243.16	427	254.47	557	265.78	692	277.09
090	272.10	199	285.38	313	298.65	433	311.92	557	325.20
010	315.57	110	330.97	216	346.37	327	361.76	442	377.15
0940	362.27	034	379.94	132	397.61	235	415.28	343	432.95
879	412.18	967	432.28	059	452.59	156	472.50	256	492.60
826	465.31	909	488.01	995	510.71	086	533.40	180	556.10
779	521.66	857	547.11	939	572.56	024	598.00	113	623.45
737	581.24	811	609.59	888	637.94	969	666.29	053	694.65
.0699	644.03	.0769	675.44	.0842	706.86	.0919	738.28	.0999	769.69

TABLE XXXI. — Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		102 Centimetres.		106 Centimetres.		110 Centimetres.		114 Centimetres.	
Diam. Metre	Area of section.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	37.475	0.080	40.472	0.083	43.584	0.086	46.812	0.090
2	03	12.006	320	12.966	333	13.964	346	14.997	358
3	07	6.5083	721	7.0287	749	7.5692	778	8.1297	806
4	13	4.3208	1.282	4.6658	1.332	5.0245	1.382	5.3966	1.433
5	20	3.1870	2.003	3.4418	2.081	3.7065	2.160	3.9809	2.238
6	28	2.5062	884	2.7066	997	2.9148	3.110	3.306	3.223
7	38	0566	3.925	2.2211	4.079	3919	4.233	2.5690	4.387
8	50	1.7394	5.128	1.8785	5.327	0230	5.529	1728	5.731
9	64	5046	6.489	6249	6.743	1.7499	6.998	1.8795	7.251
10	79	3242	8.011	4391	8.325	5401	8.639	6541	8.953
11	95	1816	9.693	2761	10.072	3742	10.454	4760	10.832
12	113	0661	11.536	1514	11.988	2399	12.441	3317	12.892
13	133	9708	13.538	0485	14.069	1291	14.601	2127	15.131
14	154	8909	15.702	0.9621	16.318	1362	16.933	1129	17.548
15	177	8230	18.025	8888	18.731	0.9571	19.439	0280	20.145
16	201	7645	20.508	8257	21.312	8892	22.117	9550	22.920
17	227	7137	23.152	7708	24.060	8301	24.968	8916	25.876
18	254	6692	25.956	7227	26.973	7783	27.992	8359	29.009
19	284	6298	28.920	6802	30.053	7325	31.188	7868	32.321
20	314	5948	32.044	6424	33.300	6918	34.558	7430	35.814
21	346	634	35.329	6085	36.715	553	38.100	7088	39.485
22	380	352	38.773	5780	40.294	224	41.815	6685	43.334
23	415	096	42.379	503	44.041	5927	45.702	6365	47.365
24	452	4863	46.144	252	47.953	656	49.763	6075	51.571
25	491	651	50.070	023	52.032	409	53.996	5809	55.960
26	531	456	54.155	4812	56.278	182	58.402	5566	60.525
27	573	277	58.401	619	60.691	4974	62.981	5342	65.271
28	616	111	62.807	440	65.270	781	67.733	5135	70.196
29	661	3958	67.374	275	70.015	603	72.657	4944	75.299
30	.0707	.3816	72.100	.4121	74.928	.4438	77.755	.4766	80.582
31	755	683	76.987	3978	80.206	284	83.025	4601	86.044
32	804	560	82.033	844	85.250	140	88.467	4447	91.682
33	855	444	87.241	720	90.661	006	94.083	4302	97.505
34	908	336	92.608	603	96.238	880	99.071	4167	103.50
35	962	234	98.136	493	101.98	761	105.83	4040	109.68
36	1018	138	103.82	389	107.89	650	111.97	3920	116.04
37	1075	048	109.67	292	113.97	545	118.27	3807	122.57
38	1134	2963	115.68	200	120.22	446	124.75	3701	129.29
39	1195	882	121.25	113	126.63	352	131.41	3600	136.18
40	1257	836	128.18	030	133.20	263	138.23	3505	143.26
41	1320	733	134.67	2952	139.95	179	145.23	3414	150.51
42	1385	665	141.32	878	146.86	098	152.40	3328	157.94
43	1452	596	148.13	807	153.93	023	159.74	3247	165.55
44	1521	537	155.09	740	161.18	2950	167.26	3169	173.34
45	1590	477	162.23	676	168.59	881	174.95	3095	181.31
46	1662	421	169.52	614	176.16	815	182.81	3024	189.46
47	1735	367	176.97	556	183.90	752	190.85	2956	197.78
48	1810	315	184.58	500	191.81	692	199.05	2891	206.29
49	1886	265	192.35	446	199.89	634	207.43	2830	214.98
50	1964	217	200.28	395	208.13	579	215.99	2770	223.84
55	2376	007	242.34	168	251.84	334	261.34	2507	270.84
60	2827	1833	288.40	1980	299.71	132	311.02	2289	322.33
65	3318	687	338.47	822	315.74	1962	365.01	2107	378.29
70	3848	562	392.54	687	407.94	817	423.33	1951	438.72
75	4418	455	450.62	571	468.29	692	485.97	1817	503.64
80	5027	361	513.71	470	532.82	583	552.92	1700	573.03
85	5675	278	578.80	381	601.50	487	624.20	1597	646.89
90	6362	205	648.90	302	674.34	142	699.79	1506	725.24
95	7088	140	723.00	232	751.35	326	779.71	1424	808.06
1.00	.7854	.1082	801.11	.1168	832.52	.1258	863.94	.1351	895.36

TABLE XXXI.—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

118 Centimetres.		122 Centimetres.		126 Centimetres.		130 Centimetres.		134 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
50.154	0.093	53.612	0.096	57.185	0.099	60.874	0.102	64.678	0.105
16.068	0.371	17.176	0.383	18.321	0.396	19.503	0.408	20.721	0.421
8.7102	0.834	9.3107	0.862	9.9313	0.891	10.572	0.919	11.232	0.947
5.7819	1.483	6.1806	1.533	6.5925	1.583	7.0177	1.634	7.4562	1.684
4.2652	2.317	4.5593	2.395	4.8631	2.474	5.1768	2.553	5.5003	2.631
3.3542	3.336	3.5854	3.449	3.8244	3.563	4.0710	3.676	4.3256	3.789
2.7524	4.541	2.9432	4.695	3.1383	4.849	3.3407	5.003	3.5594	5.157
2.3279	5.931	4884	6.132	2.6543	6.333	2.8255	6.535	3.0620	6.735
2.0137	7.505	2.1525	7.761	2.2960	8.015	4440	8.270	2.5968	8.525
1.7722	9.267	1.8944	9.582	2.0207	9.895	1510	10.210	2.2854	10.523
5814	11.212	6904	11.594	8031	11.974	1.9193	12.354	0393	12.734
4268	13.344	5252	13.797	6268	14.250	7318	14.703	1.8400	15.154
2993	15.661	3889	16.192	4815	16.723	5770	17.255	6755	17.785
1924	18.164	2746	18.781	3595	19.396	4472	20.012	5376	20.626
1.1014	20.851	1.1774	21.558	1.2558	22.265	3368	22.973	1.4204	23.679
0232	23.724	0938	24.529	1666	25.332	2419	26.138	3195	26.942
0.9552	26.784	0211	27.691	0891	28.600	1594	29.507	2318	30.416
8956	30.027	0.9574	31.045	0212	32.063	0870	33.081	1549	34.099
8429	33.457	9011	34.590	0.9611	35.725	0231	36.859	0870	37.993
7960	37.070	8509	38.327	9076	39.582	0.9662	40.841	0265	42.096
7540	40.871	8060	42.256	8598	43.641	9152	45.027	0.9724	46.411
7162	44.854	7656	46.376	8166	47.896	8693	49.417	9236	50.938
6820	49.025	7290	50.688	7776	52.349	8278	54.012	8795	55.673
6509	53.383	6957	55.192	7421	57.001	7899	58.811	8393	60.619
6224	57.924	6653	59.887	7097	61.851	7554	63.814	8027	65.776
5963	62.648	6375	64.773	6799	66.896	7238	69.021	7690	71.144
5724	67.561	6118	69.852	6526	72.141	6947	74.432	7381	76.723
5502	72.658	5881	75.122	6273	77.586	6678	80.048	7095	82.510
5297	77.941	5662	80.583	6040	83.225	6429	85.868	6831	88.509
0.5107	83.408	0.5459	86.237	0.5823	89.064	0.6198	91.892	6585	94.720
4930	89.064	5269	92.082	5621	95.100	5983	98.120	6357	101.14
4764	94.900	5093	98.117	5432	101.33	5782	104.55	6144	107.77
4609	100.93	4927	104.35	5256	107.77	5595	111.19	5944	114.61
4464	107.13	4772	110.77	5090	114.40	5419	118.03	5757	121.66
4328	113.53	4627	117.38	4935	121.23	5253	125.08	5581	128.92
4200	120.11	4490	124.18	4789	128.25	5098	132.32	5416	136.39
4079	126.87	4360	131.18	4651	135.48	4951	139.78	5260	144.08
3965	133.83	4238	138.36	4521	142.90	4813	147.44	5113	151.97
3857	140.96	4123	145.74	4398	150.52	4682	155.30	4974	160.08
3755	148.28	4014	153.31	4281	158.34	4558	163.36	4842	168.39
3658	155.79	3910	161.07	4171	166.35	4440	171.63	4717	176.92
3566	163.48	3812	169.03	4066	174.57	4328	180.11	4599	185.65
3478	171.36	3718	177.17	3966	182.98	4222	188.79	4486	194.59
3395	179.42	3629	185.50	3871	191.59	4121	197.67	4378	203.75
0.3316	187.67	0.3544	194.03	0.3780	200.40	4024	206.76	0.4276	213.12
3240	196.11	3463	202.75	3694	209.40	3932	216.05	4177	222.70
3167	204.72	3386	211.66	3611	218.60	3844	225.54	4084	232.48
3098	213.53	3311	220.77	3532	228.01	3760	235.24	3995	242.48
3032	222.52	3241	230.06	3457	237.60	3679	245.14	3900	252.69
2968	231.69	3173	239.55	3384	247.40	3602	255.26	3827	263.11
2686	280.35	2871	289.85	3063	299.35	3260	308.86	3464	318.36
2453	333.64	2622	344.95	2797	356.26	2978	367.57	3164	378.88
2288	391.56	2413	404.84	2574	418.11	2740	431.38	2911	444.66
2091	454.12	2235	469.51	2384	484.91	2537	500.30	2696	515.69
1947	521.31	2081	538.98	2210	556.65	2363	574.32	2510	591.99
1821	593.13	1947	613.24	2077	633.35	2210	653.45	2349	673.56
1711	669.59	1829	692.29	1951	714.99	2077	737.69	2206	760.38
1613	750.69	1725	776.13	1839	801.58	1958	827.03	2080	852.47
1526	836.41	1631	864.76	1740	893.12	1852	921.47	1968	949.81
0.1447	926.77	0.1548	958.19	0.1651	989.60	0.1757	1021.00	0.1867	1052.44

TABLE XXXI. — Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Veloc'y per sec		138 Centimetres.		142 Centimetres.		146 Centimetres.		150 Centimetres.	
Diam. Metre	Area of section.	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'ge in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	68.596	0.108	72.631	0.112	76.780	0.115	81.045	0.118
2	03	21.977	0.434	23.269	446	24.599	0.459	25.965	0.471
3	07	11.913	0.975	12.613	1.004	13.334	1.032	14.075	1.060
4	13	7.9080	1.734	8.3731	1.784	8.8515	1.835	9.3431	1.885
5	20	5.8336	2.710	6.1766	2.788	6.5295	2.867	6.8922	2.945
6	28	4.5873	3.902	4.8573	4.015	5.1348	4.128	5.4200	4.241
7	38	3.7645	5.311	3.9859	5.405	4.2136	5.619	4.4477	5.773
8	50	3.1839	6.937	3.3712	7.138	3.5638	7.339	3.7617	7.540
9	64	2.7541	8.777	2.9161	9.033	3.0827	9.287	3.2539	9.543
10	79	2.4239	10.839	2.5665	11.153	2.7131	11.467	2.8638	11.781
11	95	2.1628	13.114	2.2900	13.495	2.4809	13.874	2.5553	14.255
12	113	1.9519	15.608	2.0662	16.606	2.1843	16.512	3056	16.965
13	133	7771	18.315	1.8816	18.849	1.9891	19.379	0996	19.910
14	154	6308	21.242	7267	21.859	1.8254	22.474	1.9266	23.091
15	177	5064	24.387	5950	25.093	1.6862	25.799	1.7798	26.507
16	201	3995	27.746	4818	28.551	5664	29.354	6534	30.159
17	227	3065	31.324	3833	32.231	4623	33.140	5436	34.047
18	254	2249	35.115	2976	36.135	3711	37.151	4472	38.170
19	284	1529	39.125	2207	40.261	2904	41.395	3621	42.529
20	314	0887	43.354	1528	44.610	2186	45.866	2863	47.124
21	346	0313	47.797	0920	49.184	1544	50.569	2185	51.954
22	380	0.9796	52.458	0372	53.979	0965	55.498	1574	57.020
23	415	9328	57.335	0.9876	58.998	1.0440	60.659	1020	62.322
24	452	8902	62.429	9425	64.240	0.9964	66.049	0517	67.859
25	491	8513	67.740	9014	69.705	9528	71.668	1.0058	73.631
26	531	8156	73.268	8636	75.392	9129	77.516	0.9636	79.640
27	573	7828	79.011	8289	81.303	8762	83.593	9249	85.884
28	616	7525	87.974	7968	87.488	8423	89.900	8891	92.363
29	661	7245	91.151	7671	93.794	8109	96.435	8560	99.078
30	.0707	6985	97.546	0.7395	100.37	0.7818	103.20	0.8252	106.03
31	755	6742	104.16	7139	107.18	7546	110.20	7966	113.22
32	804	6516	110.99	6899	114.20	7293	117.42	7698	120.64
33	855	6304	118.03	6675	121.45	7056	124.87	7448	128.30
34	908	6106	125.29	6465	128.93	6834	132.56	7214	136.19
35	962	5920	132.77	6268	136.62	6626	140.47	6994	144.32
36	1018	5744	140.47	6082	144.54	6430	148.61	6787	152.69
37	1075	5579	148.38	5907	152.68	6245	156.98	6592	161.28
38	1134	5423	156.21	5742	161.04	6070	165.58	6407	170.12
39	1195	5276	164.85	5586	169.63	5905	174.41	6233	179.19
40	1257	5136	173.42	5438	178.44	5748	183.47	6068	188.50
41	1320	5003	182.20	5297	187.48	5600	192.76	5911	198.04
42	1385	4877	191.19	5164	196.73	5459	202.28	5762	207.82
43	1452	4757	200.40	5037	206.21	5325	212.02	5621	217.83
44	1521	4643	209.83	4916	215.92	5197	222.00	5486	228.08
45	1590	4535	219.48	0.4801	225.84	0.5076	232.20	0.5358	238.57
46	1662	4431	229.34	4692	235.99	4959	242.64	5235	249.29
47	1735	4332	239.42	4587	246.36	4849	253.30	5118	260.24
48	1810	4237	249.72	4486	256.96	4743	264.20	5006	271.43
49	1886	4146	260.23	4390	267.78	4641	275.32	4899	282.86
50	1964	4059	270.96	4298	278.82	4544	286.67	4796	294.53
55	2376	3674	327.86	3890	337.37	4112	346.87	4341	356.38
60	2827	3355	390.19	3553	401.50	3756	412.81	3964	424.12
65	3318	3088	457.93	3269	471.20	3456	484.48	3648	497.75
70	3848	2859	531.09	3027	546.48	3200	561.88	3378	577.27
75	4418	2662	609.67	2819	627.34	2980	645.01	3146	662.68
80	5027	2491	693.67	2637	713.77	2788	733.88	2943	753.98
85	5675	2340	783.08	2478	805.78	2619	828.48	2765	851.18
90	6362	2207	877.92	2336	903.87	2470	928.81	2607	954.26
95	7088	2087	978.17	2210	1006.53	2336	1034.9	2466	1063.24
1.00	0.7864	0.1980	1083.9	0.2097	1115.27	0.2217	1146.7	0.2340	1178.12

TABLE XXXI. — Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables in 1857.

154 Centimetres.		158 Centimetres.		162 Centimetres.		166 Centimetres.		170 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
85.425	0.121	89.920	0.124	94.531	0.127	99.257	0.130	104.10	0.134
27.368	0.484	28.808	0.496	30.286	0.509	31.800	0.522	33.351	0.534
14.836	1.089	15.616	1.117	16.417	1.145	17.238	1.173	18.078	1.202
9.8441	1.935	10.366	1.985	10.898	2.035	11.443	2.086	12.001	2.136
7.2647	3.024	7.6470	3.102	8.0391	3.181	8.441	3.259	8.8527	3.338
5.7129	4.354	6.0136	4.467	6.3219	4.580	6.6380	4.694	6.9617	4.807
4.6880	5.927	4.9347	6.081	5.1878	6.235	5.4471	6.388	5.7128	6.542
3.9650	7.741	4.1737	7.943	4.3877	8.144	4.6070	8.343	4.8317	8.545
3.4298	9.797	3.6102	10.051	3.7953	10.306	3.9851	10.561	4.1795	10.815
3.0186	12.095	3.1774	12.409	3.3393	12.723	3.5073	13.037	3.6784	13.352
2.6934	14.034	2.8352	15.014	2.9806	15.395	1296	15.776	3.2822	16.156
4302	17.416	5581	17.868	6883	18.322	2.8237	18.774	2.9614	19.227
2130	20.439	3295	20.971	4489	21.502	5714	22.033	6968	22.665
0309	23.706	1377	24.322	2473	24.938	3597	25.554	4748	26.170
1.8760	27.213	1.9747	27.919	2.0760	28.627	2.1798	29.335	2.2861	30.042
7428	30.962	1.8345	31.768	1.9285	32.572	2.0250	33.376	1237	34.181
6270	34.954	7126	35.862	8004	36.771	1.8904	37.678	1.9826	38.587
5254	39.187	6057	40.207	6880	41.224	7724	42.241	8589	43.260
4357	43.663	5113	44.797	5888	45.932	6682	47.065	7495	48.200
3558	48.280	4272	49.636	5004	50.894	5754	52.150	6522	53.407
2843	53.339	3519	54.725	4212	56.111	4923	57.495	5651	58.881
2199	58.540	2841	60.060	3489	61.581	4174	63.102	4866	64.623
1616	63.983	2227	65.645	2854	67.307	3497	68.969	4155	70.631
1086	69.667	1669	71.477	2267	73.287	2881	75.097	3509	76.906
0601	75.594	1159	77.558	1731	79.522	2318	81.484	2919	83.449
0157	81.764	0692	83.886	1240	86.011	1802	88.134	2377	90.258
0.9749	88.173	1.0262	90.463	0788	92.754	1327	95.043	1886	97.335
9371	94.826	9865	97.290	0370	99.753	0889	102.21	1420	104.68
9022	101.72	9497	104.36	9984	107.00	0483	109.65	0994	112.29
8698	108.86	0.9156	111.68	0.9625	114.51	1.0106	117.34	1.0599	120.17
8396	116.23	8838	119.25	9291	122.27	0.9756	125.29	0231	128.31
8114	123.85	8541	127.07	8979	130.29	9428	133.50	9888	136.72
7851	131.72	8264	135.14	8688	138.56	9122	141.98	9567	145.40
7604	139.82	8004	143.45	8414	147.08	8835	150.71	9266	154.35
7373	148.17	7760	152.01	8158	155.86	8566	159.71	8983	163.56
7154	156.76	7530	160.82	7916	164.90	8312	168.97	8717	173.04
6948	165.58	7313	169.88	7688	174.18	8073	178.48	8466	182.79
6753	174.65	7109	179.19	7473	183.73	7847	188.26	8230	192.80
6570	183.97	6915	188.75	7270	193.52	7633	198.30	8006	203.08
6396	193.52	6732	198.55	7077	203.58	7431	208.60	7794	213.63
6231	203.32	6558	208.60	6895	213.88	7239	219.16	7592	224.44
6074	213.36	6393	218.90	6721	224.44	8057	229.99	7401	235.53
5925	223.64	6236	229.45	6556	235.26	6884	241.07	7220	246.88
5783	234.16	6087	240.24	6399	246.32	6719	252.41	7047	258.49
0.5647	244.93	0.5944	251.29	0.6249	257.65	0.6562	264.01	0.6882	270.37
5518	255.93	5808	262.58	6106	269.23	6411	275.88	6724	282.58
5394	267.18	5678	274.12	5969	281.06	6268	289.00	6574	294.94
5276	278.67	5554	286.00	5839	293.15	6131	300.39	6430	307.68
5163	290.41	5435	297.95	5714	305.49	5999	313.03	6292	320.58
5055	302.38	5321	310.23	5594	318.09	5874	325.94	6160	333.80
4575	365.88	4816	375.38	5063	384.89	5316	394.39	5575	403.89
4179	435.43	4398	446.74	4624	458.05	4855	469.36	5092	480.67
3845	511.02	4047	524.30	4255	537.57	4468	550.84	4685	564.11
3561	592.66	3748	608.06	3940	623.45	4137	638.84	4390	654.24
3316	680.25	3490	698.03	3669	715.70	3852	733.37	4040	751.04
3102	774.09	3265	794.20	3433	814.30	3604	834.41	3780	854.52
2914	873.88	3068	896.57	3225	919.27	3386	941.97	3551	964.67
2748	979.71	2892	1005.0	3041	1031.0	3193	1056.0	3348	1081.0
2599	1091.6	2736	1120.0	2877	1148.0	3020	1177.0	3168	1205.0
0.2466	1209.5	0.2596	1241.0	0.2729	1272.0	0.2866	1304.0	0.3005	1335.0

TABLE XXXI.—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Veloc'y per sec. Diam. Metre	Area of sect'n.	174 Centimetres.		178 Centimetres.		182 Centimetres.		186 Centimetres.	
		Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	109.05	0.187	114.13	0.140	119.31	0.143	124.62	0.146
2	03	34.939	0.547	36.563	0.159	38.225	0.572	39.924	0.584
3	07	18.939	1.230	19.820	1.258	20.721	1.287	21.646	1.315
4	13	12.572	2.187	13.157	2.237	13.755	2.287	14.367	2.337
5	20	9.2741	3.416	9.7054	3.495	10.147	3.574	10.597	3.652
6	28	7.2932	4.920	7.6324	5.033	7.9792	5.146	8.3338	5.259
7	38	5.9848	6.696	6.2631	6.850	6.5478	7.004	6.8387	7.158
8	50	5.0618	8.747	5.2972	8.947	5.5379	9.149	5.7840	9.349
9	64	4.3784	11.069	4.5821	11.323	4.7903	11.578	5.0032	11.833
10	79	3.8535	13.665	4.0327	13.979	4.2160	14.294	4.4034	14.607
11	95	3.4385	16.536	3.5984	16.916	3.7619	17.296	3.9291	17.676
12	113	3.1024	19.678	3.2467	20.130	3.3943	20.584	3.5451	21.035
13	133	2.8252	23.095	2.9566	23.625	3.0909	24.157	3.2283	24.687
14	154	5926	26.786	2.7132	27.400	2.8365	28.017	2.9626	28.632
15	177	2.3949	30.747	2.5063	31.455	2.6201	32.162	2.7366	32.867
16	201	2248	34.984	3283	35.788	4841	36.591	5423	37.396
17	227	2.0770	39.494	1736	40.402	2724	41.311	3734	42.218
18	254	1.9474	44.277	2.0379	45.215	1306	46.314	2253	47.251
19	284	8328	49.333	1.9181	50.467	0053	51.602	0944	52.737
20	314	7309	54.662	8114	55.920	1.8937	57.177	1.9779	58.484
21	346	6396	60.267	7158	61.651	7938	63.038	8735	64.423
22	380	5573	66.142	6298	67.662	7038	69.184	7796	70.704
23	415	4829	72.293	5519	73.953	6224	75.617	6645	77.277
24	452	4152	78.715	4810	80.525	5483	82.355	6171	84.143
25	491	3534	85.412	4163	87.376	4807	89.340	5465	91.802
26	531	2967	92.382	3570	94.504	4186	96.630	4817	98.752
27	573	2445	99.623	3024	101.92	3616	104.21	4221	106.50
28	616	1964	107.14	2520	109.60	3089	112.07	3671	114.53
29	661	1518	114.93	2053	117.57	2601	120.22	3161	122.86
30	.0707	1.1104	122.99	1.1620	125.82	1.2148	128.65	1.2688	131.48
31	755	0718	131.33	1217	134.35	1727	137.37	2248	140.39
32	804	0359	139.94	0841	143.15	1333	146.37	1837	149.59
33	855	0023	149.82	0489	152.24	0965	155.67	1452	159.09
34	908	0.9707	157.98	1.0159	161.61	0620	165.24	1092	168.87
35	962	9411	167.41	0.9849	171.26	1.0296	175.11	0754	178.95
36	1018	9132	177.11	9557	181.18	0.9991	185.25	0435	189.33
37	1075	8870	187.09	9282	191.39	9704	195.69	1.0135	199.99
38	1134	8622	197.34	9022	201.87	9432	206.41	0.9852	210.95
39	1195	8387	207.86	8777	212.64	9176	217.42	9584	222.19
40	1257	8165	218.66	8544	223.68	8933	228.71	9330	233.74
41	1320	7954	229.73	8324	235.01	8702	240.29	9089	245.57
42	1385	7754	241.07	8114	246.61	8483	252.15	8860	257.69
43	1452	7563	252.68	7915	258.49	8275	264.30	8642	270.11
44	1521	7382	264.57	7725	270.66	8076	276.74	8435	282.82
45	1590	0.7209	276.74	0.7544	283.10	0.7887	289.46	0.8238	295.82
46	1662	7044	289.17	7372	295.82	7707	302.47	8940	309.11
47	1735	6887	301.88	7207	308.82	7534	315.76	7869	322.70
48	1810	6736	314.86	7049	322.10	7370	329.34	7697	336.58
49	1886	6592	328.12	6898	335.66	7212	343.21	7532	350.75
50	1964	6453	341.65	6754	349.50	7060	357.36	7374	365.21
55	2376	5841	413.39	6113	422.90	6390	432.40	6674	441.90
60	2827	5334	491.97	5582	503.29	5836	514.59	6096	525.90
65	3318	4909	577.39	5137	590.66	5370	603.94	5609	617.21
70	3848	4546	669.63	4757	685.03	4973	700.42	5194	715.81
75	4418	4233	768.71	4429	786.38	4631	804.05	4837	821.73
80	5027	3960	874.62	4144	894.73	4333	914.83	4525	934.94
85	5675	3720	987.36	3893	1010.0	4070	1033.0	4251	1055.0
90	6362	3508	1107.0	3671	1132.0	3838	1158.0	4008	1203.0
95	7088	3318	1233.0	3472	1262.0	3631	1290.0	3792	1318.0
100	0.7854	0.3148	1367.0	0.3295	1398.0	0.3445	1429.0	0.3598	1461.0

TABLE XXXI. -- *Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables of 1857.

190 Centimetres.		194 Centimetres.		200 Centimetres.		210 Centimetres.		220 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'r. in litres.	Hgt. in 100 met.	Disch'r. in litres.	Hgt. in 100 met.	Disch'ge in litres.
130.03	0.149	135.56	0.152	144.08	0.157	158.85	0.165	174.34	0.173
41.659	0.597	43.432	0.609	46.160	0.628	50.891	0.660	55.854	0.691
22.582	1.343	23.543	1.372	25.022	1.414	27.587	1.484	30.277	1.555
34.981	2.388	15.628	2.438	16.610	2.513	18.313	2.639	20.098	2.765
11.058	8.731	11.529	8.809	12.253	8.927	13.509	4.123	14.826	4.320
8.696	5.372	9.066	5.485	9.636	5.655	10.623	5.938	11.659	6.220
7.136	7.312	7.440	7.466	7.907	7.697	8.717	8.082	9.567	8.446
6.036	9.551	6.292	9.751	6.687	10.053	7.373	10.556	8.092	11.058
6.221	12.087	5.443	12.341	5.785	12.723	6.373	13.361	6.999	13.996
4.695	14.923	4.790	15.235	5.091	15.708	5.613	16.493	6.160	17.279
4.100	18.056	4.274	18.436	543	19.007	5.009	19.957	5.497	20.907
3.699	21.489	3.857	21.940	099	22.620	4.519	23.751	4.960	24.881
369	25.219	512	25.749	3.733	26.547	4.115	27.874	516	29.201
091	29.248	229	29.864	425	30.788	3.776	32.327	145	33.866
2.856	33.576	2.9771	34.283	3.164	35.343	488	37.110	3.829	38.877
653	38.208	766	39.004	2.939	40.212	241	42.223	557	44.234
477	43.126	582	44.034	744	45.396	025	47.666	320	49.936
322	48.349	421	49.367	573	50.894	2.837	53.430	113	55.883
185	53.871	278	55.005	422	56.706	670	59.541	2.930	62.376
064	59.690	151	60.946	287	62.832	521	65.974	767	69.115
1.955	65.809	038	67.195	166	69.272	388	72.736	621	76.200
857	72.225	1.936	73.746	058	76.027	268	79.828	490	83.629
768	78.941	843	80.601	1.959	83.095	160	87.250	371	91.405
687	85.954	759	87.763	870	90.478	061	95.002	262	99.526
614	93.266	682	95.230	788	98.175	1.971	103.08	164	107.99
546	100.88	612	103.00	713	106.16	889	111.50	073	116.81
484	108.79	547	111.08	644	114.51	813	120.24	1.990	125.96
427	116.99	487	119.46	581	123.15	743	129.31	913	135.47
373	125.50	432	128.14	522	132.10	678	138.71	841	145.32
1.324	134.30	1.3803	137.13	1.467	141.37	1.617	148.44	1.775	155.51
278	143.41	332	146.42	416	150.95	561	158.50	714	166.05
235	152.81	288	156.02	361	160.85	509	168.89	656	176.94
195	162.51	246	165.93	324	171.06	460	179.61	602	188.17
158	172.51	207	176.14	282	181.68	414	190.66	552	199.74
122	182.80	170	186.65	243	192.42	371	202.04	505	211.67
089	193.40	135	197.47	207	203.58	330	213.75	460	223.93
058	204.29	103	208.59	172	215.04	292	225.80	418	236.55
028	215.48	072	220.02	139	226.82	256	238.17	378	249.51
000	226.97	043	231.75	108	238.92	222	250.87	341	262.81
0.974	238.76	015	243.79	079	281.33	189	263.89	305	276.46
948	250.85	0.989	256.13	051	264.05	159	277.26	272	290.46
925	263.24	964	268.78	024	277.09	129	290.95	240	304.80
902	275.92	940	281.73	0.999	290.44	102	304.96	209	319.49
880	288.90	918	294.98	975	304.11	075	319.31	180	334.52
0.860	302.18	0.896	308.54	0.953	318.09	1.050	333.99	1.153	349.89
940	315.76	876	322.41	921	332.38	026	349.00	126	365.62
821	329.64	856	336.58	910	346.99	003	364.34	101	381.69
803	343.82	837	351.05	890	361.91	0.981	380.01	077	398.10
786	358.29	819	365.84	871	377.15	961	396.01	054	414.87
769	373.07	802	380.92	853	392.70	940	412.34	032	431.97
696	451.41	726	460.91	772	475.17	851	498.92	0.934	522.68
636	537.21	663	548.52	705	565.29	777	593.76	853	622.04
585	630.48	610	643.76	749	663.66	715	696.85	785	730.03
542	731.21	565	746.60	607	759.69	662	808.18	727	846.66
505	839.40	526	857.07	559	883.58	617	927.76	677	971.93
472	955.05	492	975.15	523	1005.0	577	1056.0	633	1106.0
444	1078.0	462	1101.0	492	1155.0	542	1192.0	595	1248.0
418	1209.0	436	1234.0	463	1272.0	511	1336.0	561	1400.0
396	1347.0	412	1375.0	438	1478.0	483	1489.0	531	1559.0
0.375	1492.0	0.3914	1524.0	0.416	1571.0	0.459	1649.0	0.503	1728.0

